# Higgs Bosons in Superconductors

Based on
Littlewood and CMV (1981)
CMV (2000)
Barlas and CMV (2012)

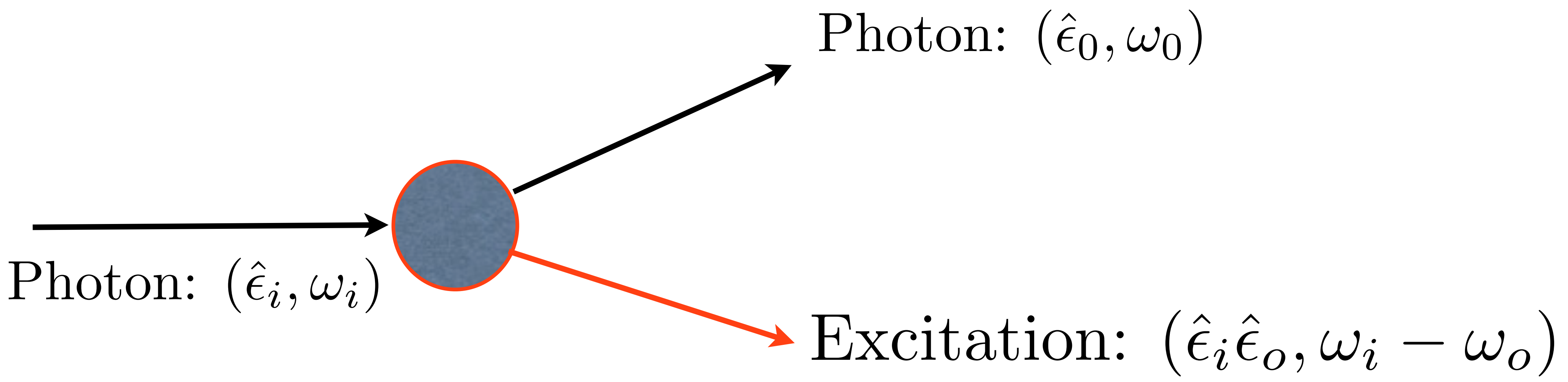
Spurred by Raman Scattering expts.in $NbSe_2$
at Urbana ($1980's$) by Klein and Sooryakumar,
Klein and Dierker

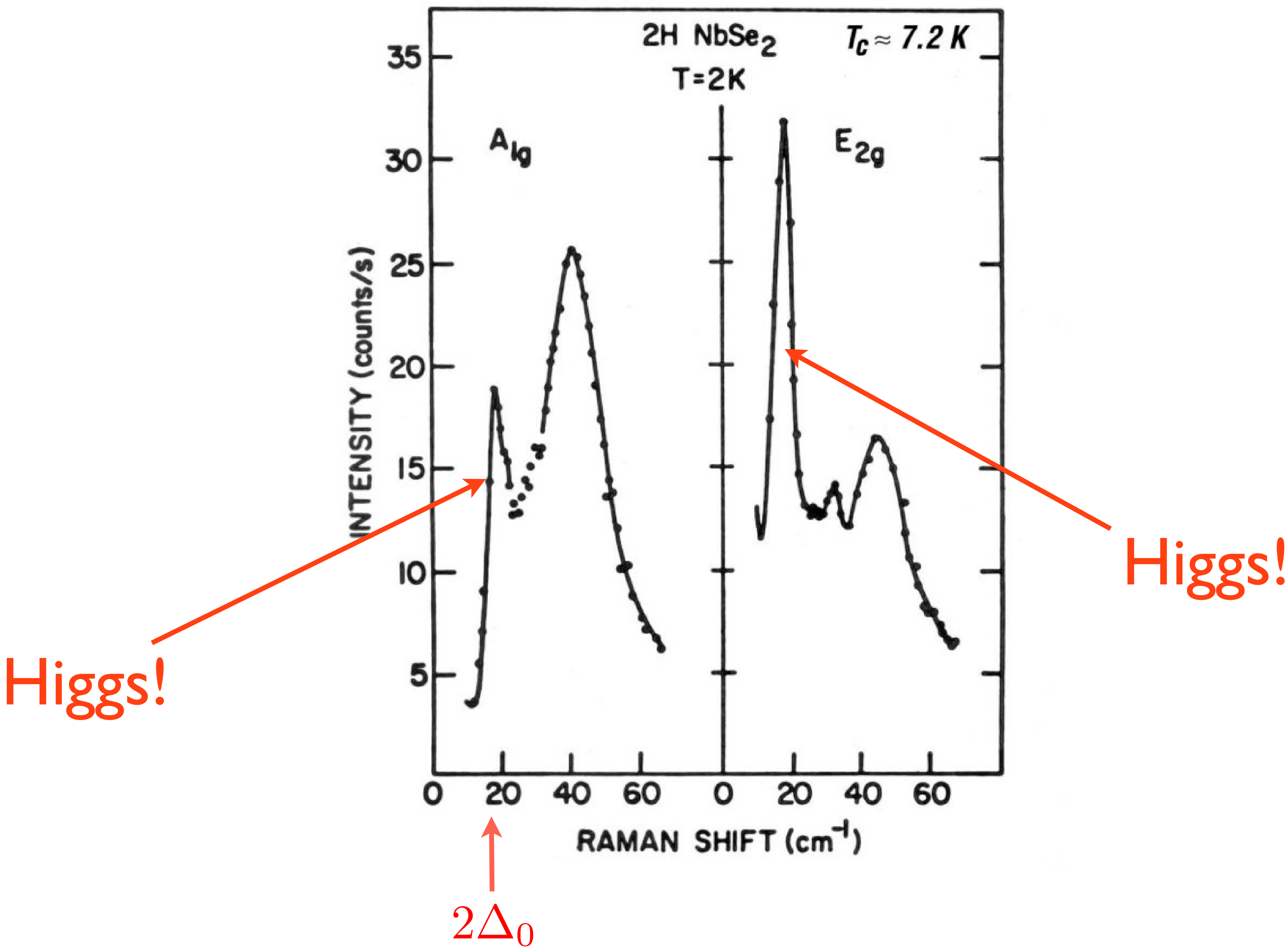
Recent Expts.- M-A M$\acute{e}$asson, A. Sacuto, et al. (Paris)
M. Le Tacon, Y. Li, Keimer, et al., (stuttgart)

Many Discussions with EPT's especially
Andy Cohen and David Gross

## Urbana Raman Scattering group (1979-80)

Unusual Modes detected in NbSe(2) in Raman Scattering on entering the superconducting phase.




2H $NbSe_2$
T=2K
$T_c \approx 7.2$ K
$A_{1g}$
$E_{2g}$
INTENSITY (counts/s)
RAMAN SHIFT ($cm^{-1}$)
Higgs!
Higgs!
$2\Delta_0$

## Higgs Bosons in Superconductors

1. Basics of Superconductivity:
Ginzburg-Landau
Gauge Invariance in BCS and Phase Modes (Plasmons)
Amplitude Modes or Higgs
Relation to the Standard model of particle physics.
2. How does one couple to the Higgs?

3. Emergent particle-hole symmetry of BCS.

4. More Experiments in $NbSe_2$
Higgs in d-wave superconductors
Experiments on Cuprates

5. Questions from a CM Theorist to EP Theorists

Phenomenological understanding of Superconductivity:

London(s) (1935), Ginzburg-Landau (1950).

Following the Meissner-Ochsenfeld Effect.

Superconductivity is
a MACROSCOPIC COHERENT QUANTUM STATE in
which the metallic electrons develop a STIFFNESS.

B=0 inside a Supercond., different from a perfect cond.

Ginzburg and Landau (1950)
Describe a superconductor by a complex wave-function

$$\Psi(r) = |\Psi(r)| e^{i\phi(r)}$$

with a Hamiltonian

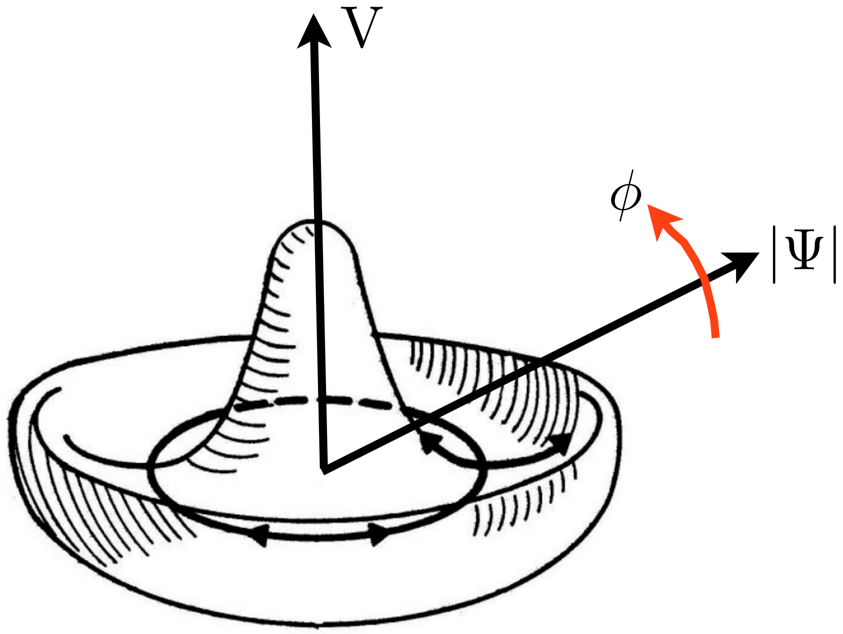
$$H = \int d^d(r) \left( \frac{1}{2m} |(\nabla - e^*/c\mathbf{A})\Psi|^2 + r|\Psi|^2 + u|\Psi|^4 \right)$$

Then current:

$$\mathbf{j} = -\left( \frac{2e^2}{mc}\mathbf{A} + \frac{e\hbar}{m}\nabla\phi \right) \| Psi|^2.$$

London's Eqn.,
$\nabla \times \nabla \times \mathbf{B} = \lambda^{-2}\mathbf{B}$; $\lambda^{-2}$: (Mass of W)
i.e the eqn. giving stiffness
against magnetic field penetration follows
from stiffness of $\Psi$.

$$H = \int d^d(r)\left(\frac{1}{2m}|(\nabla - e^*/c\mathbf{A})\Psi|^2 + r|\Psi|^2 + u|\Psi|^4\right)$$



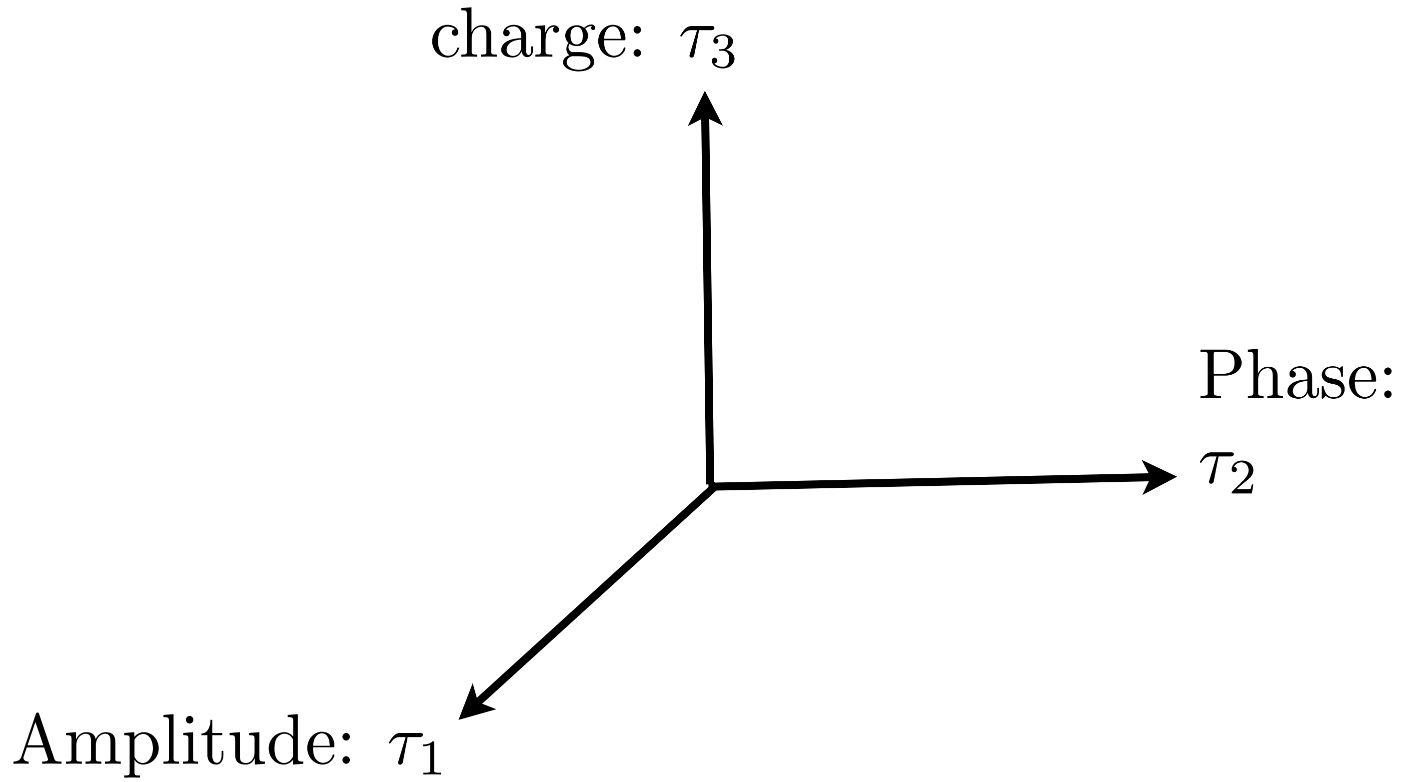

The Ginzburg Landau Model was used to calculate various properties of superconductors.

It did not give Higgs particle to condensed matter theorists.

But it did to Higgs (1964).
And
There is also no Higgs in Superfluid Helium(4).

BCS:

$$\Psi_{\mathbf{k}} = \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} \qquad \Psi^\dagger_{\mathbf{k}} = \begin{pmatrix} c^\dagger_{\mathbf{k}\uparrow} & c_{-\mathbf{k}\downarrow} \end{pmatrix}$$




$$\mathcal{H} = \sum_{\mathbf{k}} \Psi^\dagger_{\mathbf{k}} \tau_3 \epsilon_{\mathbf{k}} \Psi_{\mathbf{k}} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V(\mathbf{k},\mathbf{k}',\mathbf{q}) \Psi^\dagger_{\mathbf{k}+\mathbf{q}} \tau_3 \Psi_{\mathbf{k}} \Psi^\dagger_{\mathbf{k}'-\mathbf{q}} \tau_3 \Psi_{\mathbf{k}'}$$

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_{BCS} + \mathcal{H}_1, \\ \mathcal{H}_{BCS} &= \sum_{\mathbf{k}} \Psi^\dagger_{\mathbf{k}} \left( \epsilon_{\mathbf{k}} \tau_3 + \Delta_{\mathbf{k}} \tau_1 \right) \Psi_{\mathbf{k}}. \end{aligned}$$

Local Gauge Invariance:

$$\Psi \to \exp(i\alpha(\mathbf{r},t)\tau_3)\psi$$

Continuity Equation

$$\frac{\partial}{\partial t}(\psi^\dagger \tau_3 \psi) + \nabla \cdot \Psi^\dagger \frac{\mathbf{P}}{m} \Psi = 0.$$

BCS Theory does not satisfy this, yet they got right answers for things they calculated !

Need to do one loop calculations (RPA) using

$\mathcal{H} - \mathcal{H}_{BCS}$

Gorkov (1959): From BCS theory to Ginzburg-Landau

Eternal Quest for Knowledge - Lev Gorkov (1961)

The artist and the discoverer of the relationship of microscopic theory to the phenomenology.

Lev Gor'kov (2010)

Anderson (1959), and some others did such a calculation: Obtained the "Goldstone mode" for oscillations of phase:

$$\tau_2 \text{ oscillation} : \omega \propto k$$

But this couples to Longitudinal fluctuations of EM field: Showed that the oscillation is at the plasmon frequency as in the normal metallic state:

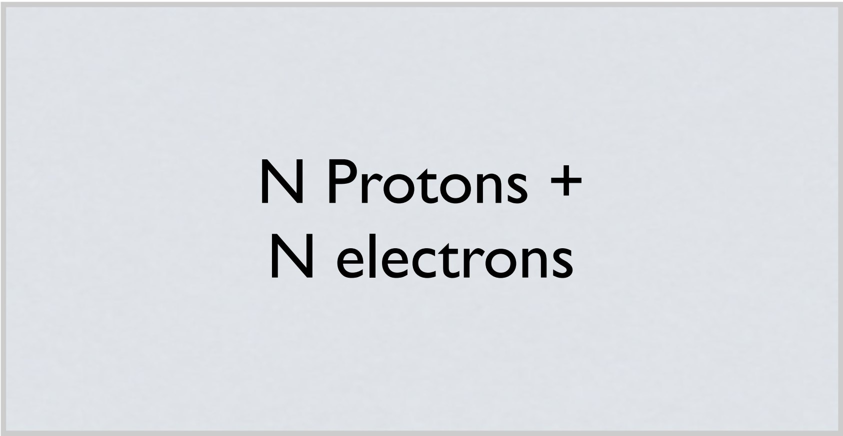
$$\omega = \omega_P = \sqrt{\frac{4\pi n e^2}{m}} - O(\Delta^2/\omega_P)$$

"Anderson mechanism".

$\omega_P^{-1}$ sets also the scale for the London penetration depth.

Or of the mass of W or the range of weak interactions.

# Plasmons:



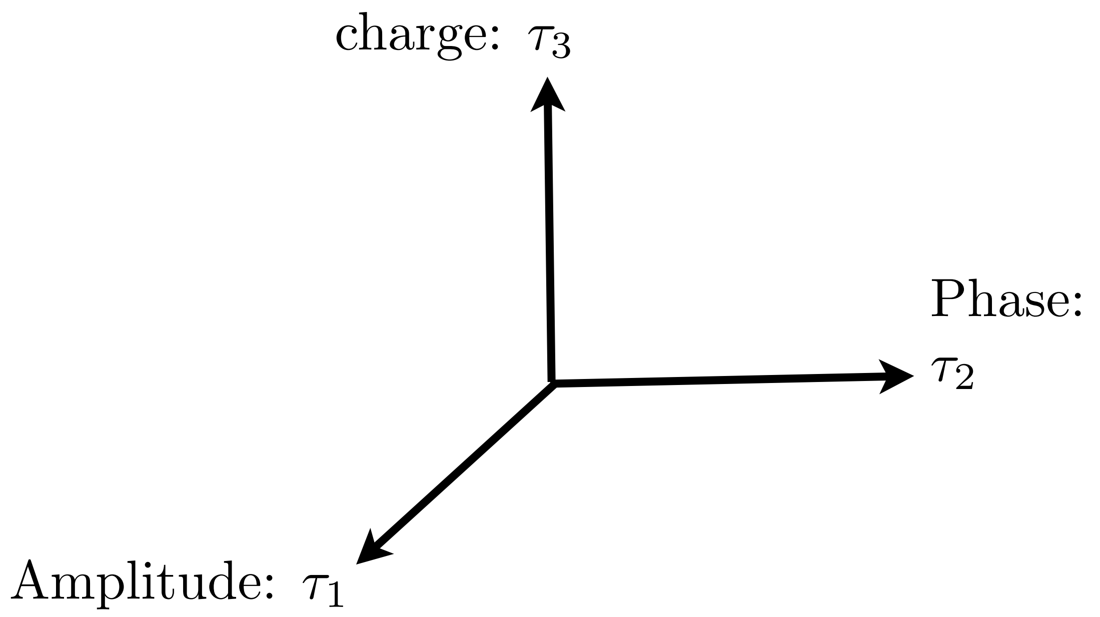

Now move the electrons to one side by $\delta x$


The electrons will move back and forth at a frequency $\omega_p = \sqrt{\frac{4\pi e^2 (N/V)}{m}}$

## Amplitude Mode

$\mathcal{H}$, unlike $\mathcal{H}_{BCS}$ is also invariant to

$$\begin{aligned} \Psi &\rightarrow \exp(\alpha(\mathbf{r},t)\tau_1)\Psi \\ \nabla &\rightarrow \nabla + \alpha(\mathbf{r},t)\tau_1, \end{aligned}$$




leading to a "continuity" equation:

$$i\Psi^\dagger\tau_1\left(\frac{\overleftarrow{\partial}}{\partial t} - \frac{\vec{\partial}}{\partial t}\right)\Psi + \nabla\cdot\Psi^\dagger\tau_2\left(\frac{\overleftarrow{P}}{m} + \frac{\vec{P}}{m}\right)\Psi = 0.$$

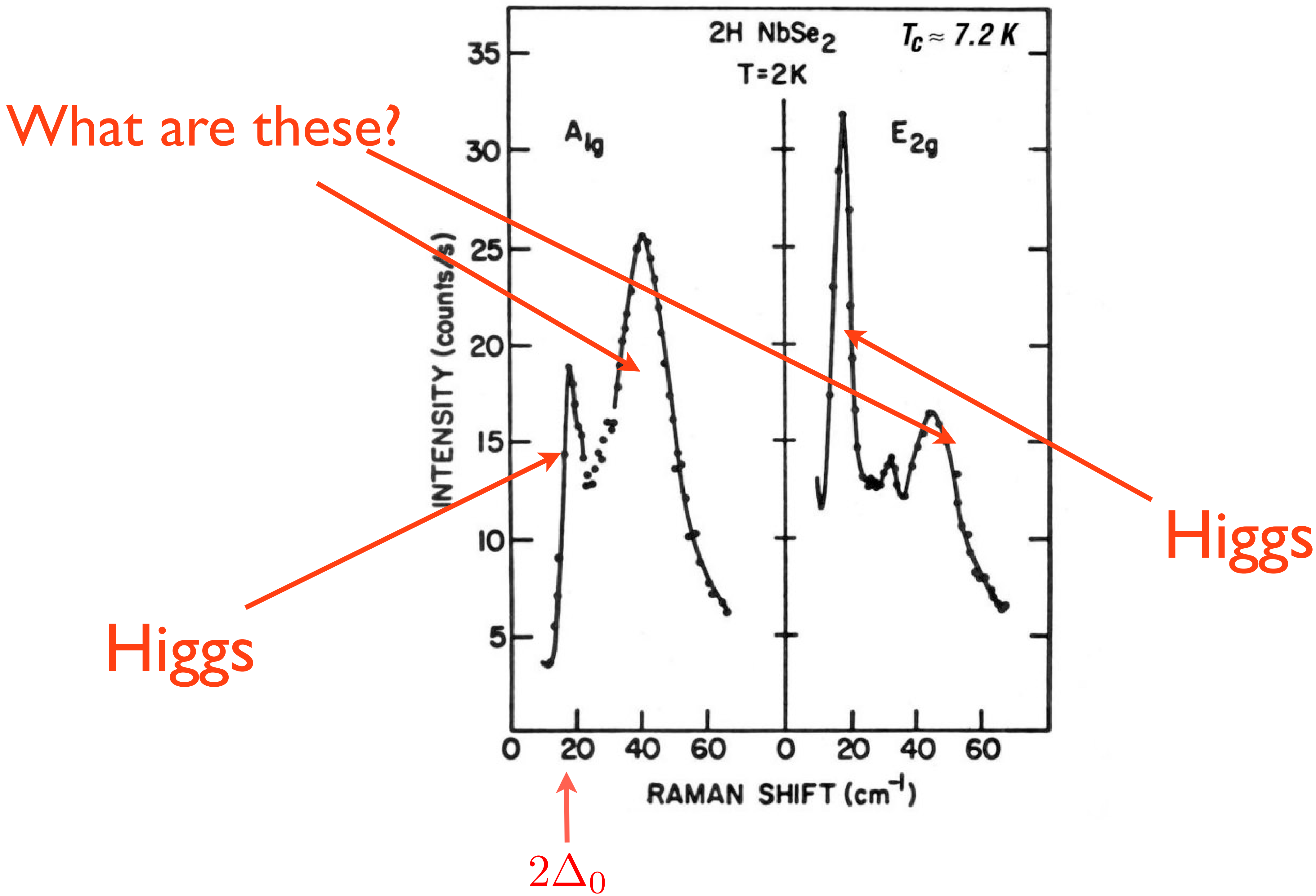
Littlewood, CMV (1981): Calculation consistent with this invariance yields an excitations in the

$\tau_1$ or amplitude sector with
$\nu_q^2 \approx 4\Delta^2 + \frac{1}{3} v_F^2 q^2 + i \frac{\pi^2}{12} \Delta v_F q.$

This does not help at all with the observed sharp mode

How would one couple to it anyway?

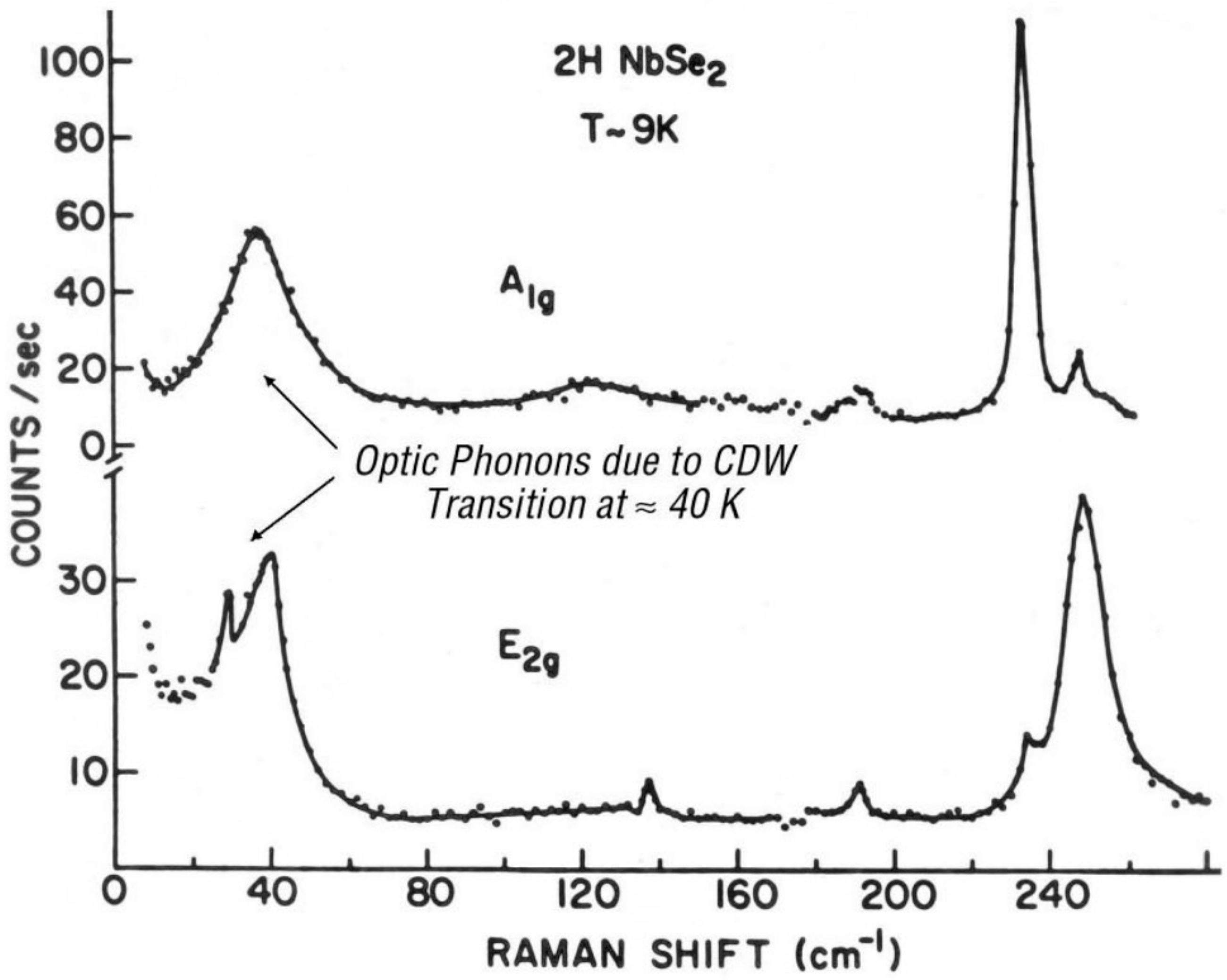
It has no charge, no dipole moment,
no magnetic moment, etc.
to which we couple excitations
with external probes.



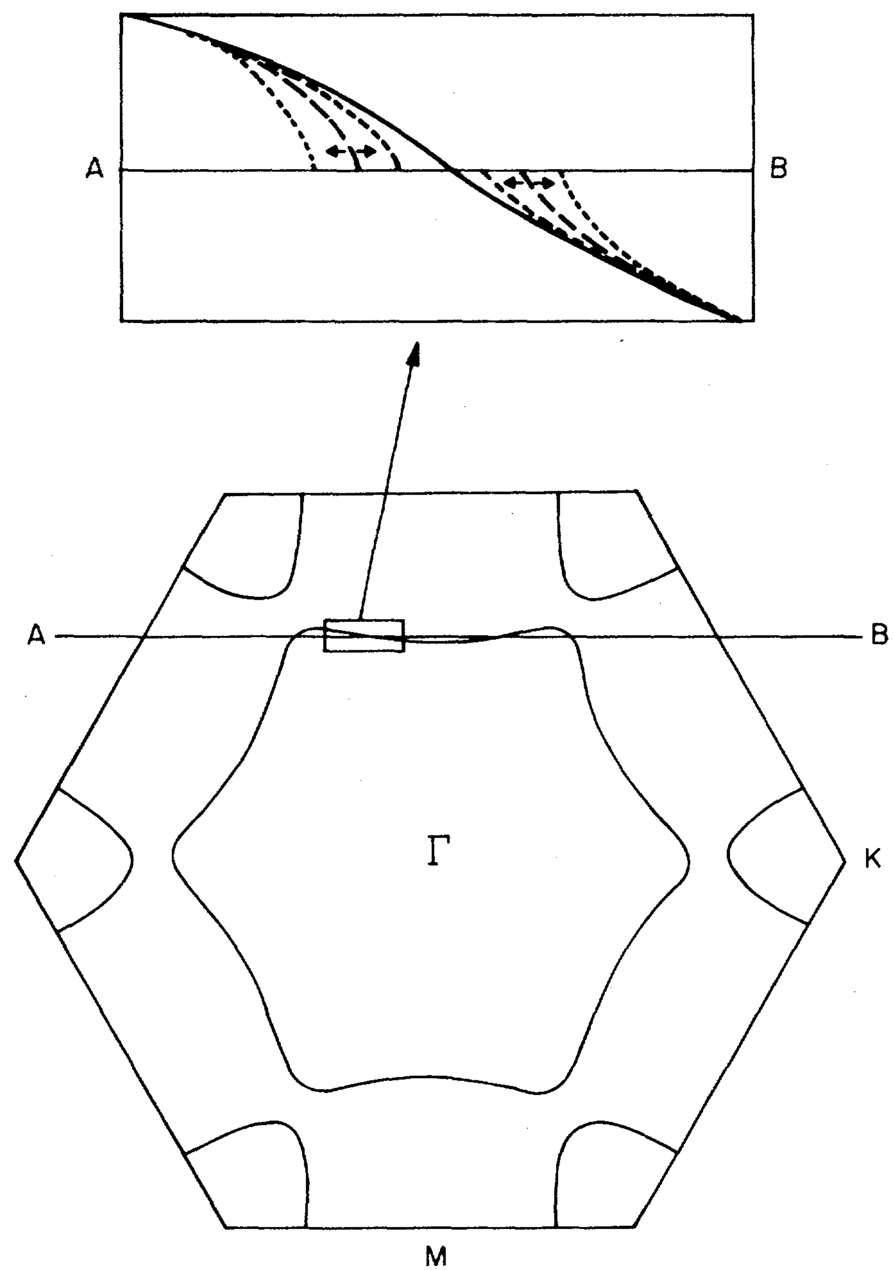
What are these?
Higgs
Higgs
$2\Delta_0$
2H NbSe2
T=2K
Tc ≈ 7.2 K
A1g
E2g
INTENSITY (counts/s)
RAMAN SHIFT (cm⁻¹)

**$NbSe_2$** has a charge density wave transition at 33 K.

This is a structural transition which gaps part of the Fermi-surface.
In the low T phase, altered periodicity gives new optical phonons



Oscillation of the CDW induced phonon oscillates the density of electrons at the FS and therefore the superconducting condensate.



## Coupling of CDW to Superconducicitivity

Under pressure the distortion of CDW, $< u_0 >$ and its $T_d$ decreases, but $T_c$ goes up.
So, we know $dT_c/dT_d$ or $\frac{dT_c}{d<u_0>}$.

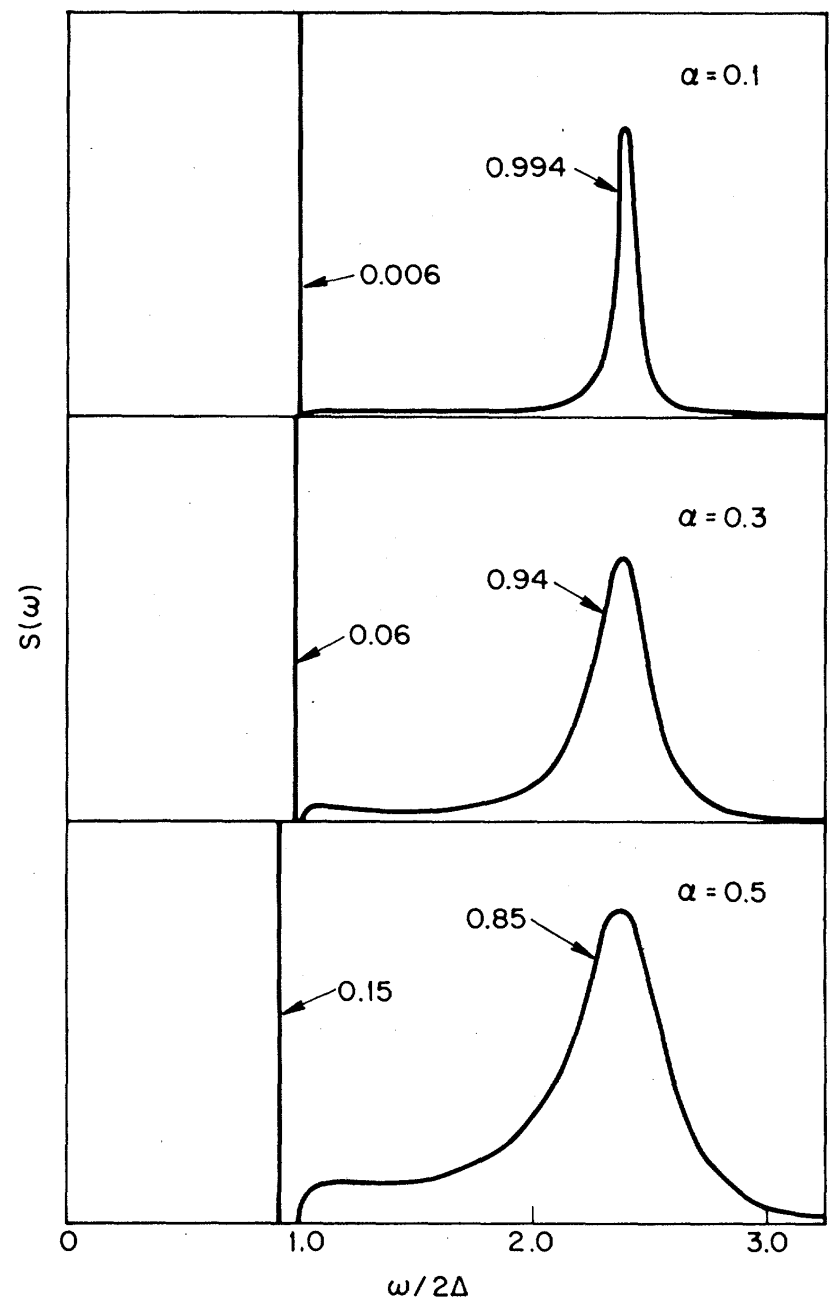
So, an oscillation $u(t)$ about $< u_0 >$ must tickle the superconducting condensate.

A coupling
$H' = g\ u(t)\Psi_{\mathbf{k}}^{+}\tau_1\Psi_{\mathbf{k}}$, must therefore exist. $g = V\partial\Delta/\partial u$

The $\tau_1$ mode or "Amplitude" mode must therefore appear as a pole in the self-energy of the CDW induced phonon

# Raman Scattering couples to such Phonons

So calculate the Self-energy of such phonons (one loop enough) and study the spectral weight.



Points of Agreements with the experiment:
(1) new mode sharper than its parent.

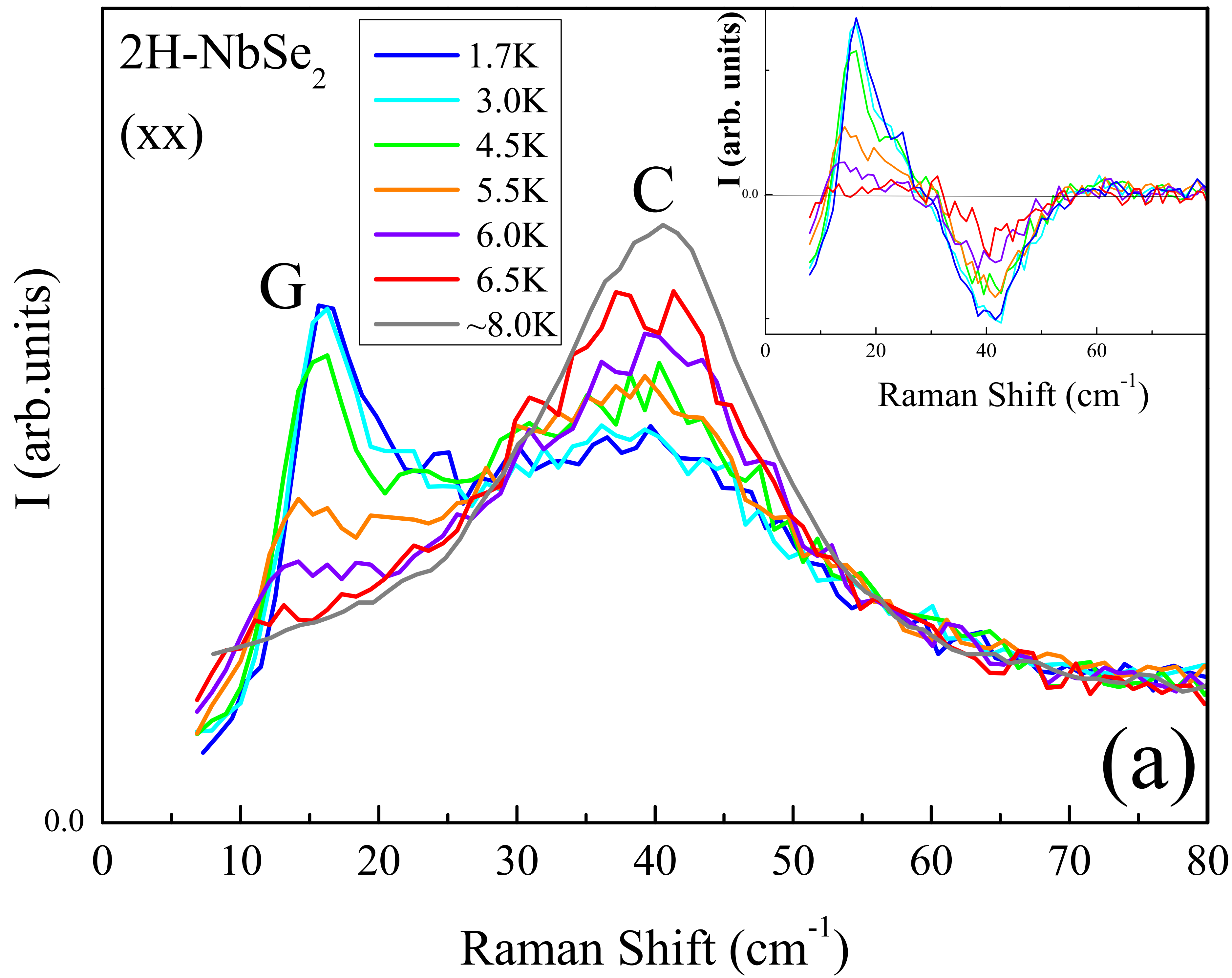
(2)parent gets broader when progeny forms.

Never fully tested:
The total spectral weight as a function of temperature must be conserved,

Although shift of intensity with magnetic field was studied by the Urbana group.

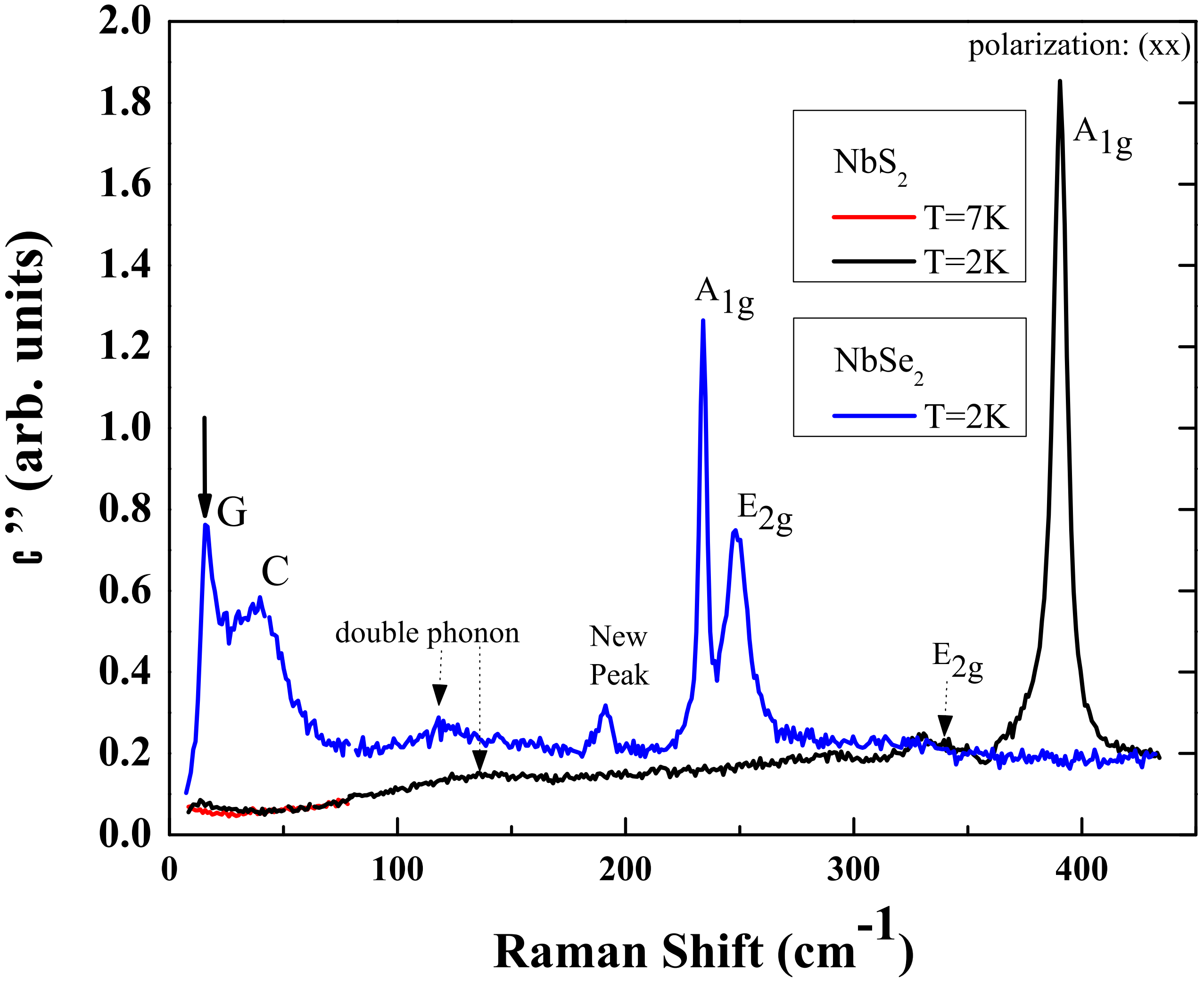
The amplitude mode can be observed only in rather special situation.

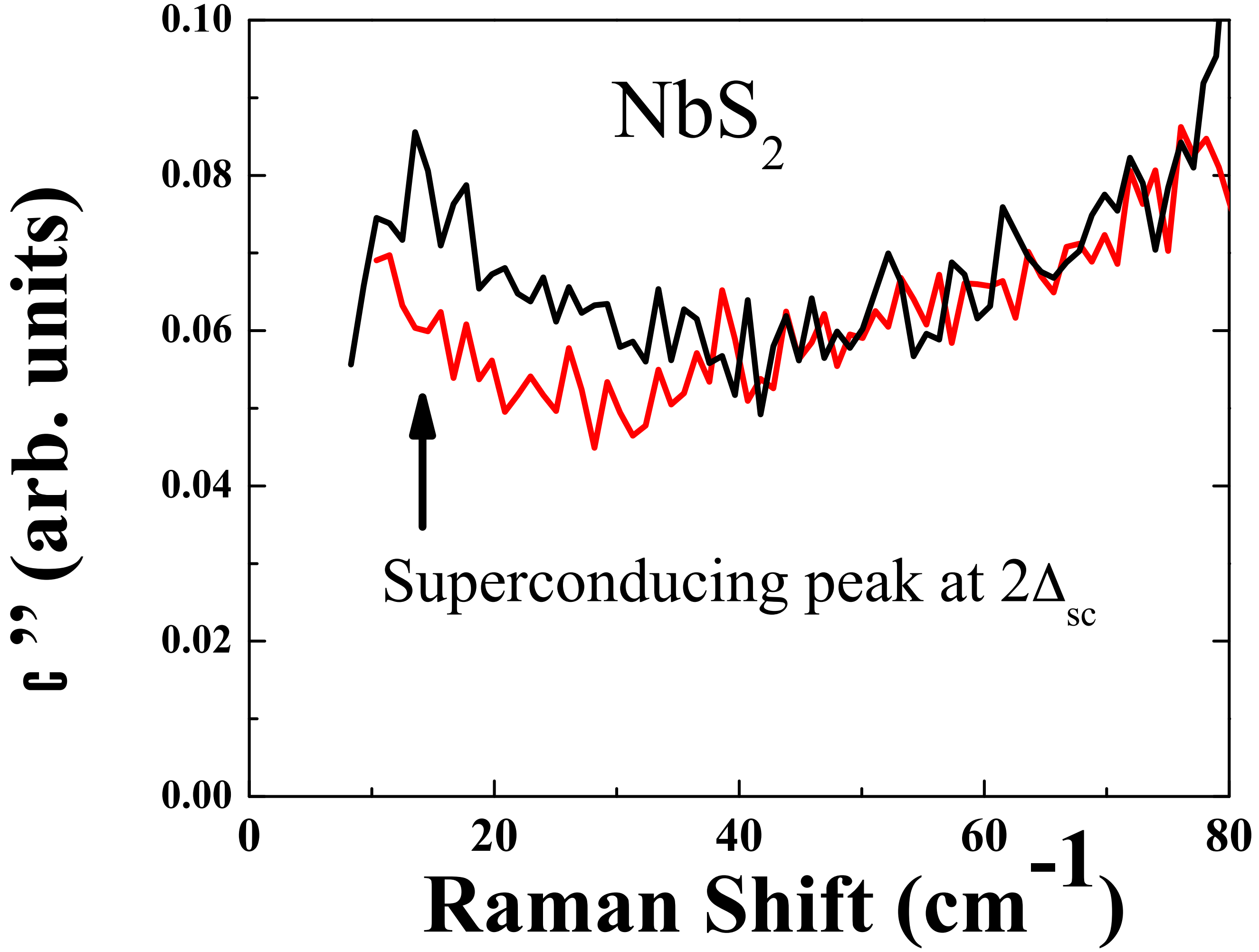
New Experiments: (M-A. Méasson, A. Sacuto, Paris)



Just as interesting

$NbS_2$ - Same structure, similar $T_c$, no CDW.



## Why is this the "Higgs"

Peter Higgs in "The Rise of the standard Model, Ed. (Hoddeson et al. 1997)

"The existence of the characteristic massive spin-zero modes had not been noticed by Anderson or by Englert and Brout. Indeed the theory of what particle physicists would call the Higgs mode in a superconductor was not published until 1981, after it had been detected in the Raman spectrum of superconducting NbSe(2)! (Ref. to expts. by Klein et al, and to theory by Littlewood and cmv)."

**Why was the theory not done earlier for a superconductor?**

**Standard Model of Particle Physics**

1. Consistency of theory requires that weak-interactions just as electromagnetic interactions be locally gauge-invariant theories. So the universe is a condensate of some field $\Psi$: $SU(2) \times U(1)$. $\Psi$ has a Landau-Ginzburg type Lagrangian.

2. Unification gets rid of the problem of the un-observed Goldstone modes - they become the plasmons - W and Z particles. It also provides the amplitude or Higgs mode which is a scalar $< \Psi >$ .

3. $\Psi$ Being a (complex) scalar terms of the form
$g_i|\Psi|\bar{\phi}_i\phi_i$ are allowed in the Lagrangian, where $\phi$ are fermions. This provides mass to the fermions $m_i = g_i < \Psi >$.

4. Strong interactions are combined in the theory with no problems.

## Why was Higgs missed in CMP over the years?

Suppose we try Ginzburg-Landau Lagrangian:

$$H = \int d\mathbf{r} \left( |\nabla\psi|^2 + r|\psi|^2 + u|\psi|^4 \right).$$

$$\langle\, \psi \,\rangle = [-r/2u]^{1/2} \equiv \rho_0^{1/2}$$

$$\psi(\mathbf{r},t) = [\rho_0 + \delta\rho(\mathbf{r},t)]^{1/2} \exp(i\phi(\mathbf{r},t))$$

Suppose now one calculates dynamics by linear in time term in the Lagragian

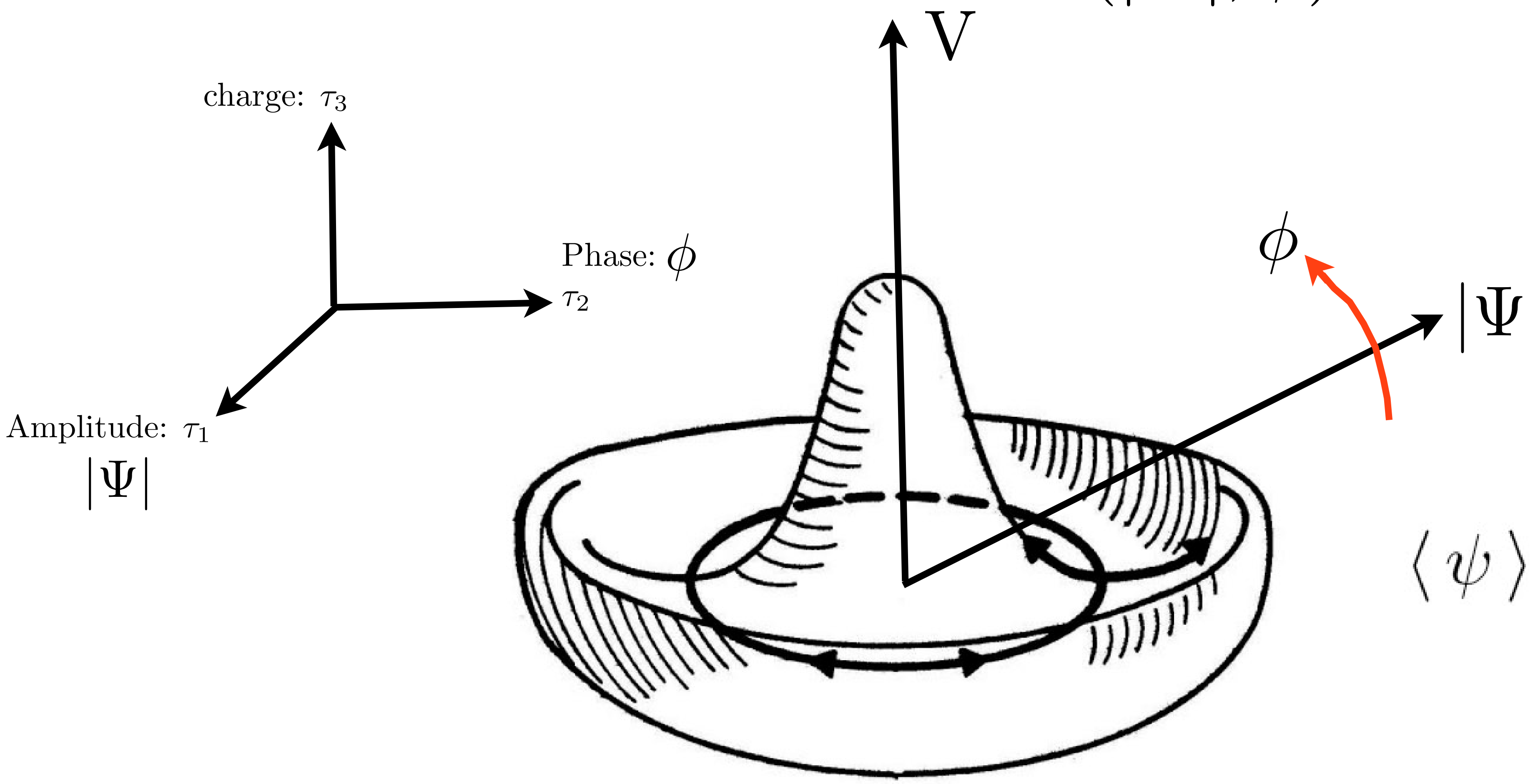
$$\begin{aligned} \partial\phi/\partial t &= (\nabla^2\delta\rho + V\delta\rho)/2\rho_0 \\ (1/2\rho_0)\,\partial\delta\rho/\partial t &= \nabla^2\phi \end{aligned}$$

$$\frac{\partial^2(\phi, \delta\rho/\rho_0)}{\partial t^2} = \nabla^2(\phi, \delta\rho/\rho_0) + \nabla^4(\phi, \rho/\rho_0),$$

$$\omega^2 = k^2 + k^4$$

No Higgs! as in superfluid $^4He$.

# Effective Potential $V(|\Psi|, \phi)$ for $r < 0$




$$\langle \psi \rangle = [-r/2u]^{1/2} \equiv \rho_0^{1/2}$$

In Equilibrium all values of $\phi$ must have the same energy

Therefore there must exist a collective mode of zero energy at long wavelengths

This is the content of the so-called Goldstone Theorem.

Try 'Lorentz-invariant' Lagrangian as done by Higgs (1964)

$$\mathcal{L} = |\partial_\mu \psi|^2 + r|\psi|^2 + u|\psi|^4 \; ; \quad \mu = (it, \mathbf{r})$$

$$\text{Higgs Immediately} : \Omega_{Higgs} = \sqrt{-2r/u}$$

**Essential Physical Point** (CMV -2001)
Weak-coupling equations for superconductivity are mathermatically identical to the Dirac equation, i.e. thery are particle-hole symmetric, although the normal metallic state (and the state just below $T_c$) is not. Then first order time-derivatives are zero. This point is being displayed in the "Higgs" observations in the cold-atom experiments (Bloch-et al. 2012).

# Higgs Bosons in Cuprate Superconductors.

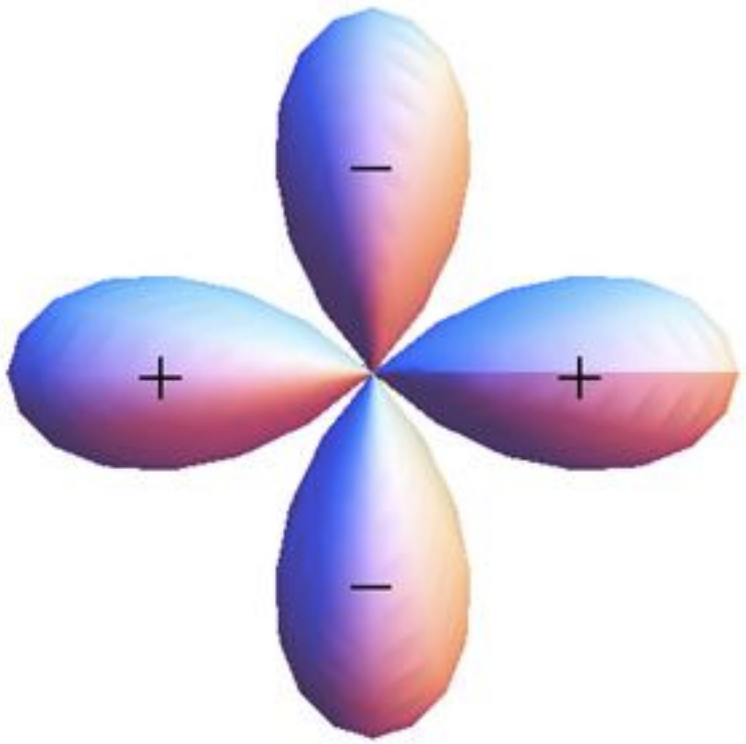
"Higgs" Modes in D-Wave Supeconductors
(with Yafis Barlas)

Superconducting Condensate:

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \Psi(\mathbf{r}, \mathbf{R})$$

$$\mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)$$

$$\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2))/2$$

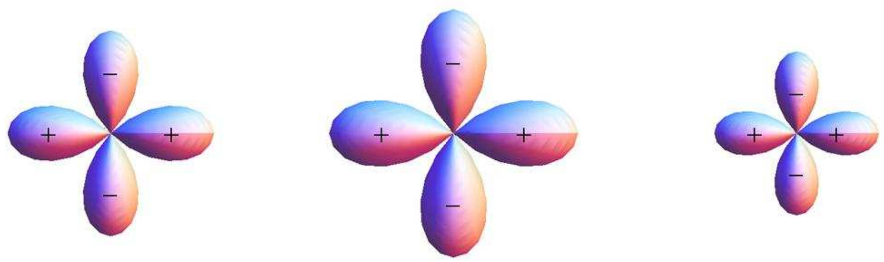



$$\Psi(\mathbf{r}, \mathbf{R}) = \Psi(\mathbf{r})e^{i\phi(\mathbf{R})}$$

$$\Psi(\mathbf{r}) = \Psi(|\mathbf{r}|)P_2(\hat{\mathbf{r}})$$

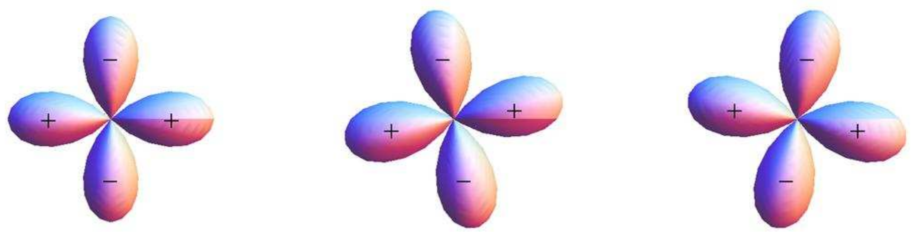
# Various Possible Oscillation Modes

Breathing

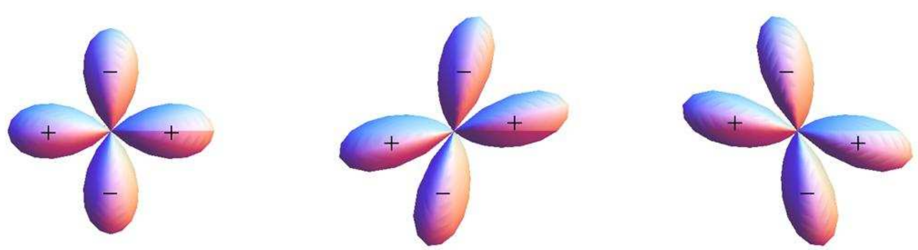


Calculate $\omega_H = \frac{1}{\sqrt{2}}(2\Delta_0)$.

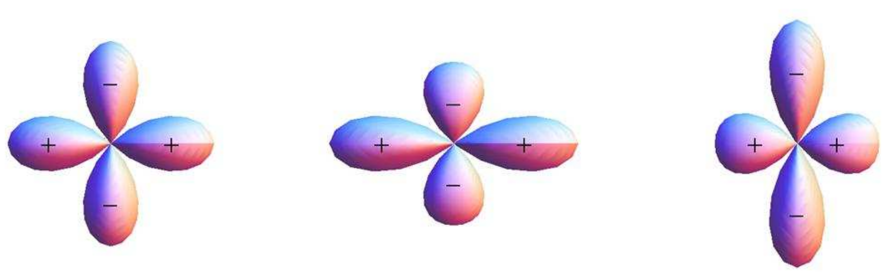
Nodding



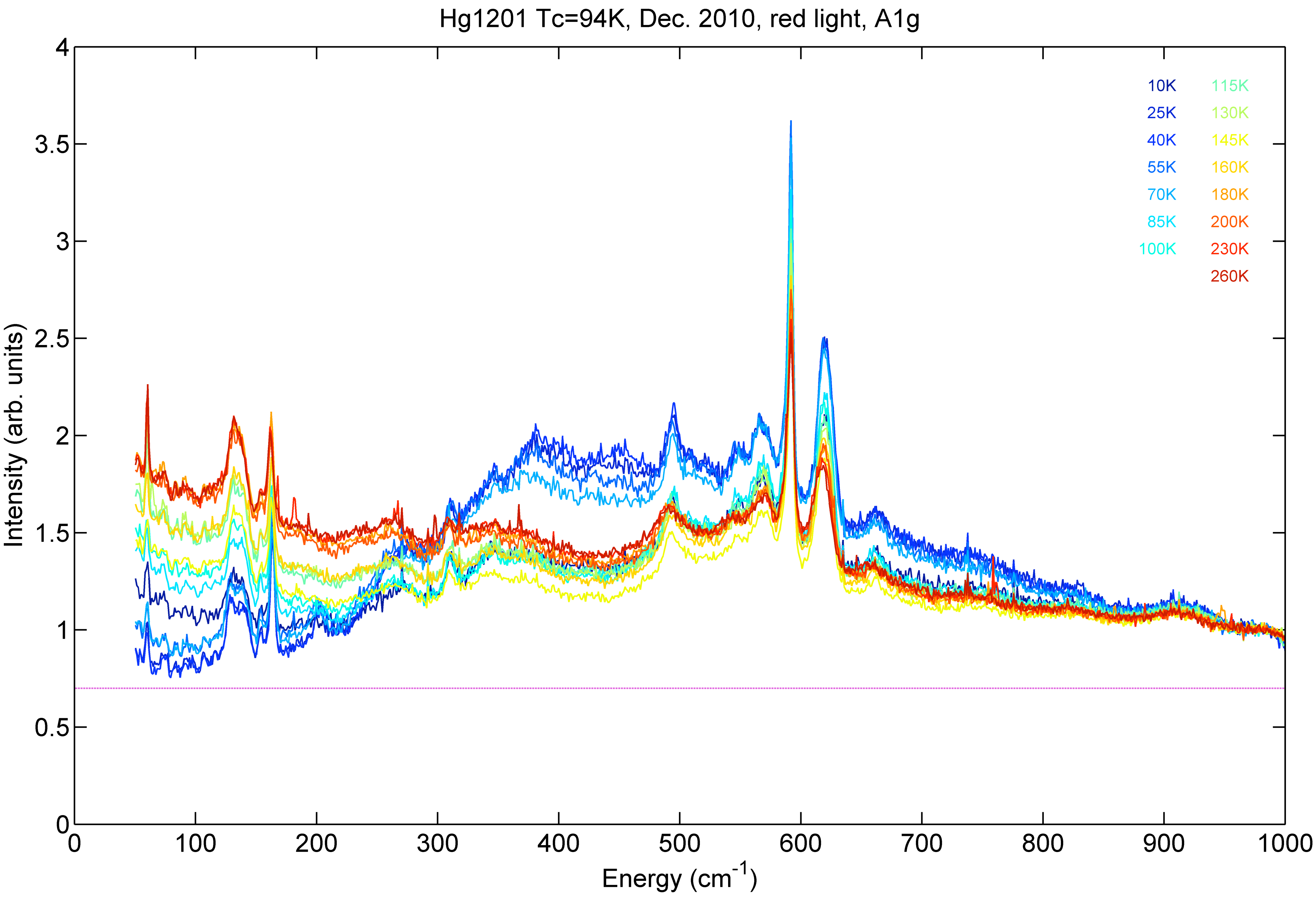
Clapping



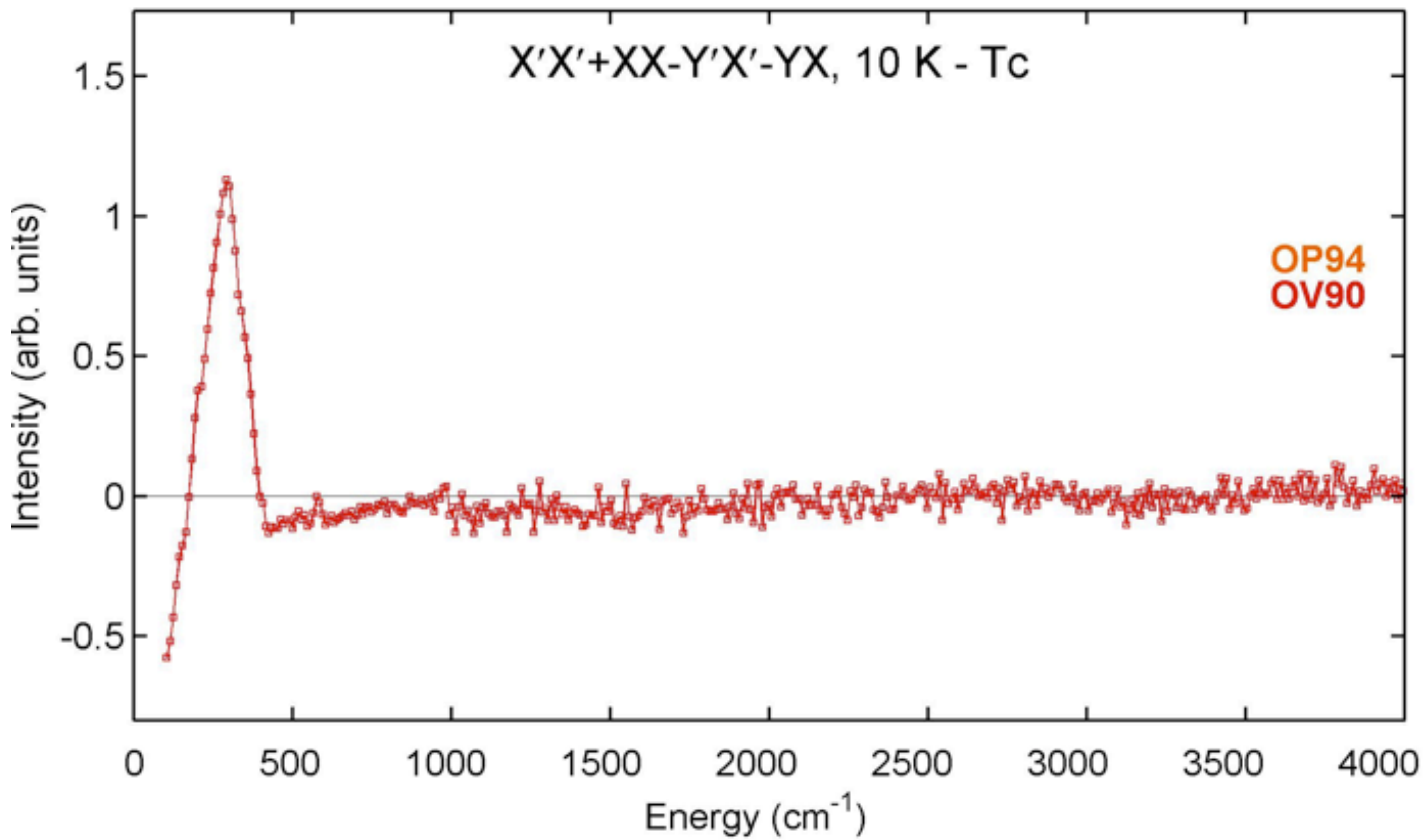
Osculating



# M. Le Tacon, Yuan Li, et al. (2012)



Exptl. Results of A1g Geometry (Le Tacon, Y. Li, et al. (2012))



The breathing mode steals weight from the broad background and appears as a sharp Resonance?

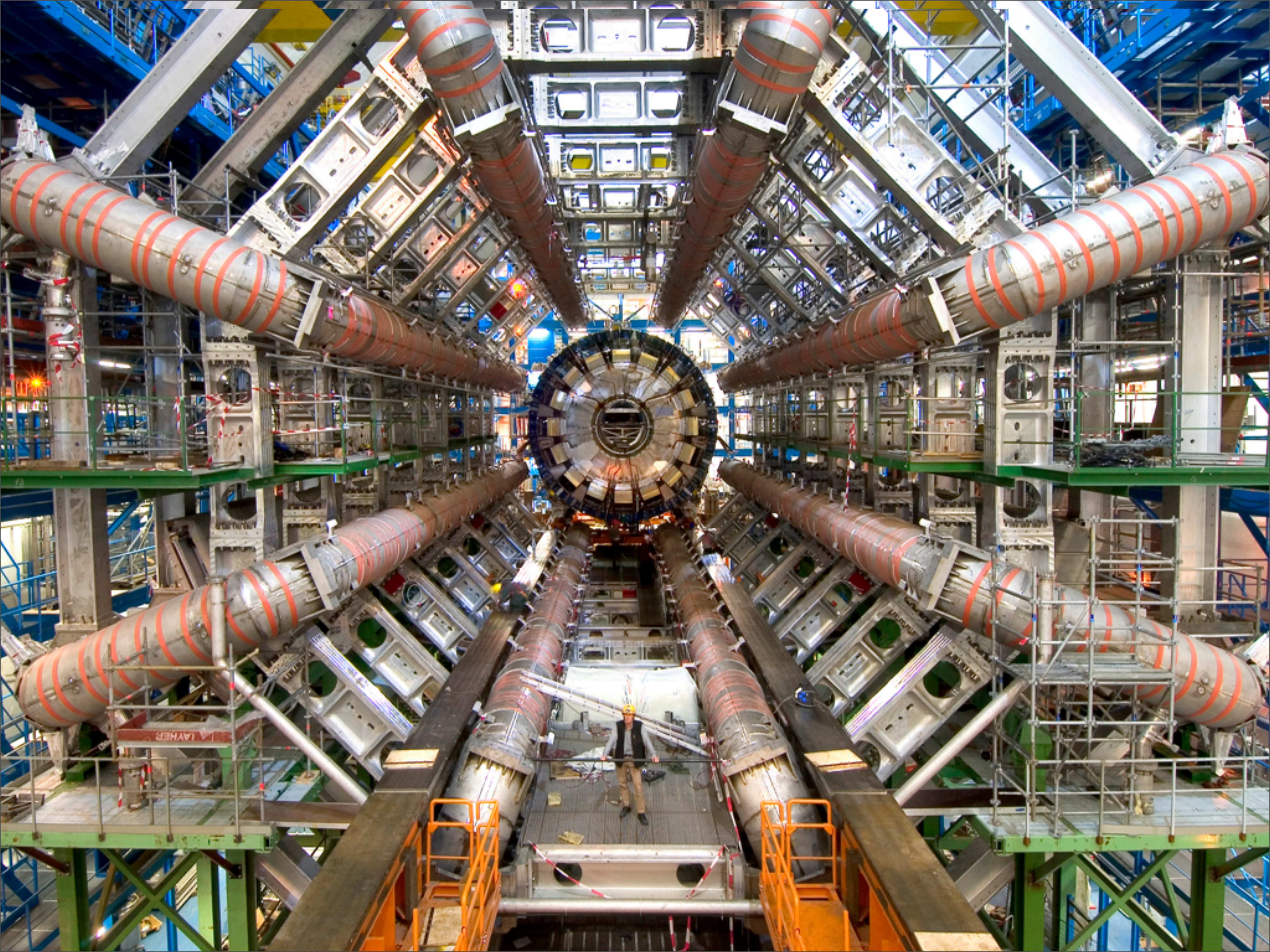
The broad background oscillates the condensate for Cuprates just as phonons do for ordinary Superconductors

The broad background is consistent with the quantum-critical fluctuations hypothesized (1989), which lead to a marginal-Fermi-Liquid and which are shown to promote superconductivity (2007) and tested as such in other experiments.

## Anything from superconductivity of interest to HEP?

Since there is a basic theory of metals and superconductors, the Higgs is not all that important in CMP.
It would have been if Ginzburg-Landau theory was all we had.

In EPP, it is of great importance. It completes a consistent Phenomenological theory built on about 50 years of expts. and probably limits the form of any theory beyond (both in terms of new particles and "microscopic" basis for the phenomenology.



## A Few Questions in a CM Theorist's mind to ask EP Theorists

1. Why is it so difficult for you to see the Higgs?
-1 Higgs per $10^9$ proton collisions?

2. Should'nt you just increase the temperature and see the electron mass disappear with a phase transition or crossover at T of order 200 GeV. Should'nt you see the electron mass flow?

3. You have a condensate $U(1) \times SU(2)$ pervading the universe. Something(s) anaologous to magnetic field(s) must be zero. What are they? What are the penetration depths? What are the coherence lengths?

Questions Contd.

4. Talking of the above, where are terms like $|\nabla\phi|^2$, with which we have the most fun? Josephson Effects, for example.

5. What are the difficulties in constructing the theory whose low energy form is the Ginzburg Landau type theory, i.e the Standard Model?

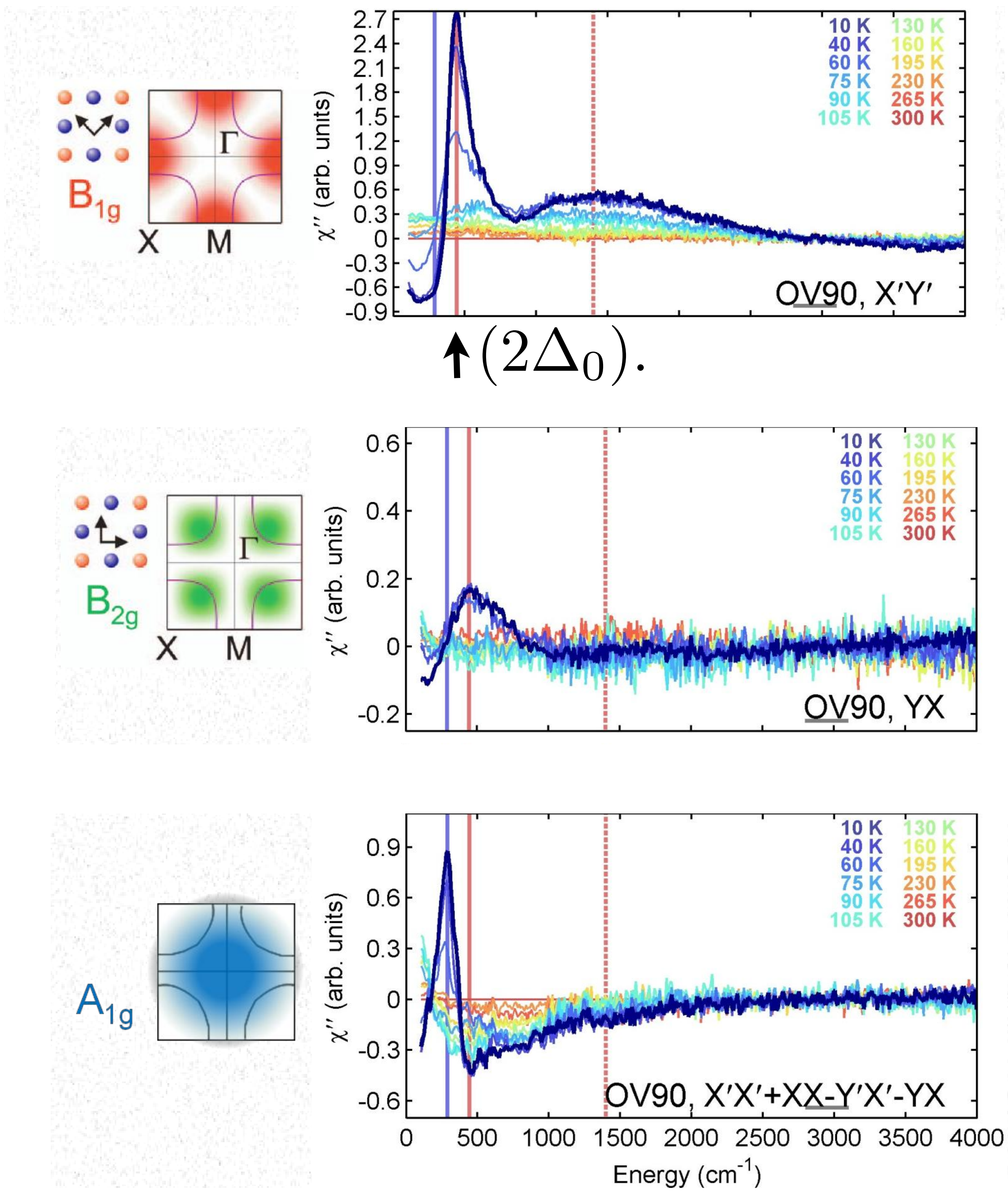
6. Is it possible that the phenomenology is the "theory"; i.e. will it become an axiom?
Is it good enough for that -
infra-red stable, ultraviolet stable, etc.?

An Epistemological Question:

The Higgs field is introduced to have a gauge invariant theory of massive fermions which is renormalizable.

Observation of Higgs seems to imply that our demand for a consistent theory of Nature determines what Nature does!

# Le Tacon, Yuan Li, Keimer, et al. (Stuttgart- 2011-2012, Unpublished)




Peak close to

$$\frac{1}{\sqrt{2}}(2\Delta_0).$$