Do Cuprate High-Temperature Superconductors harbor Extra Dimensions?

Thanks to: NSF, EFRC (DOE)

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IS HINCHLIFFE'S RULE TRUE?

Boris Peon

Abstract

Hinchliffe has asserted that whenever the title of a paper is a question with a yes/no answer, the answer is always no. This paper demonstrates that Hinchliffe's assertion is false, but only if it is true.

What would be a Signature of Extra Dimensions?

And it came to pass that...

$$\oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0$$

Gauss' law for electricity

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

Gauss' law for magnetism

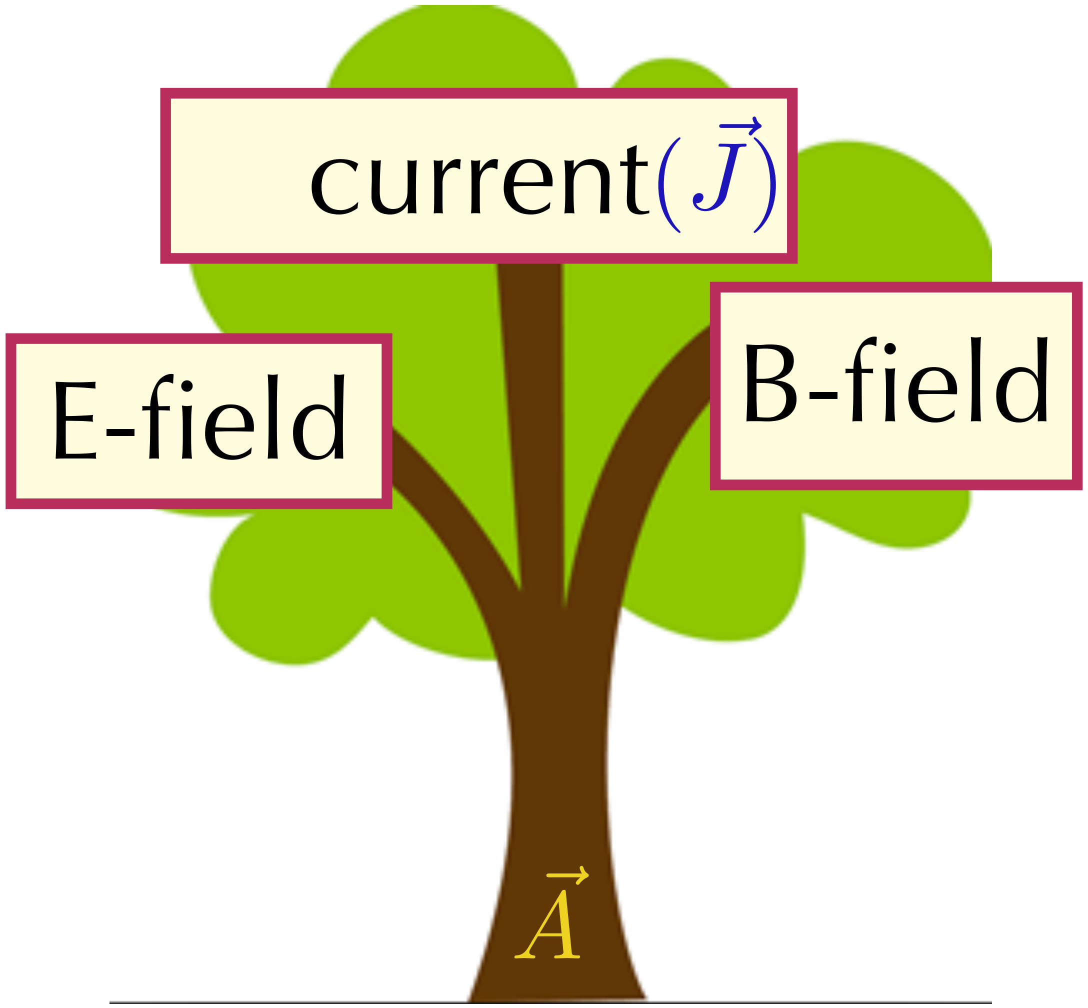
$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Faraday's law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i$$

Ampere-Maxwell law

and there was Light!


current($\vec{J}$)
E-field
B-field
$\vec{A}$

what God really said:

let there be F

$$F = dA$$

$$F_{0i} = -E_i \qquad F_{ij} = \epsilon_{ijk} B_k$$

A Maxwell equation

$$dF = J$$

redundancy

gauge invariance

$$A \to A + \partial\Lambda$$

has no units

$$[A_\mu] = \frac{1}{L} = 1$$

$$F \to F$$

gauge invariance= current conservation

$$S = \int d^dx \left(F^2 + J_\mu A^\mu + \cdots\right)$$

$$A \to A + \partial\Lambda$$

$$S \to S + \int d^dx \, J_\mu \partial\Lambda$$

integrate by parts

$$\partial_\mu J^\mu = 0$$ current conservation

gauge invariance

$[A_\mu] = 1$

current conservation

$$[d^d x J A] = 0$$

fixes dimension of current

$$[J] = d - 1$$

conserved quantities have fixed dimensions

renormalization does nothing

what
if

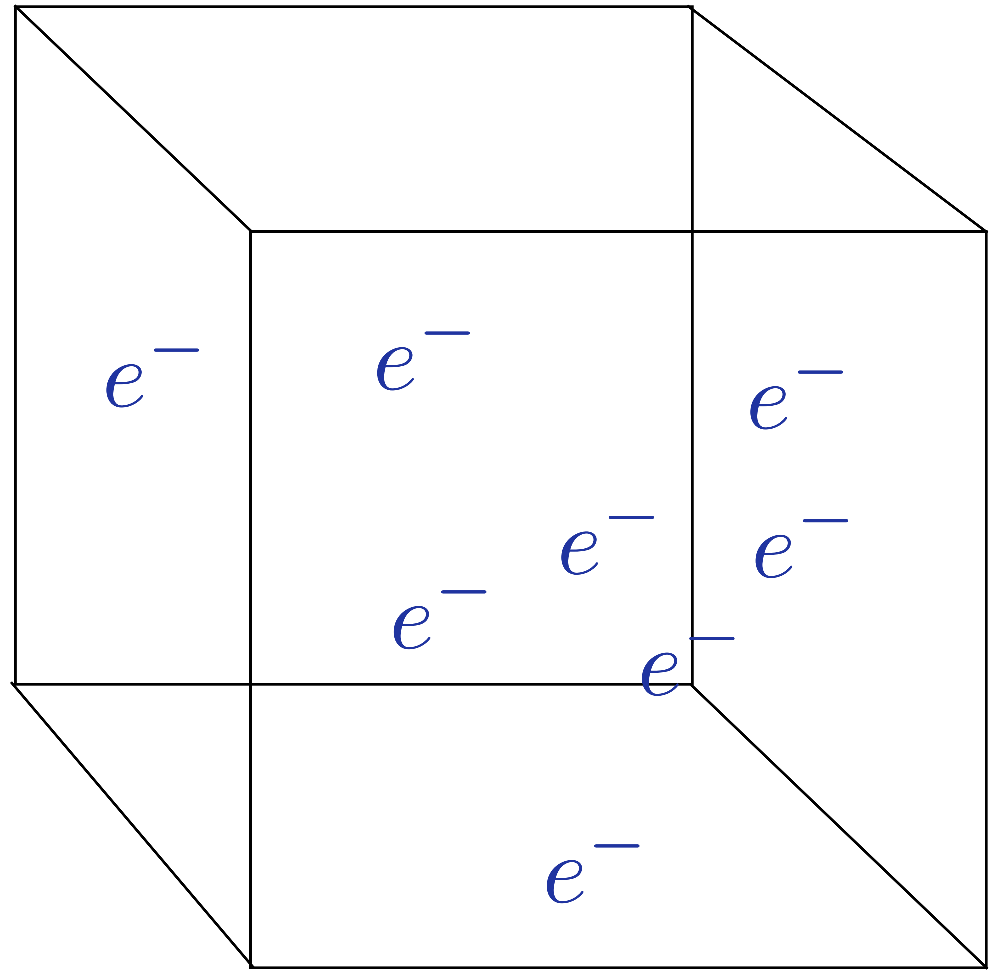
$$[A_\mu] \neq 1$$

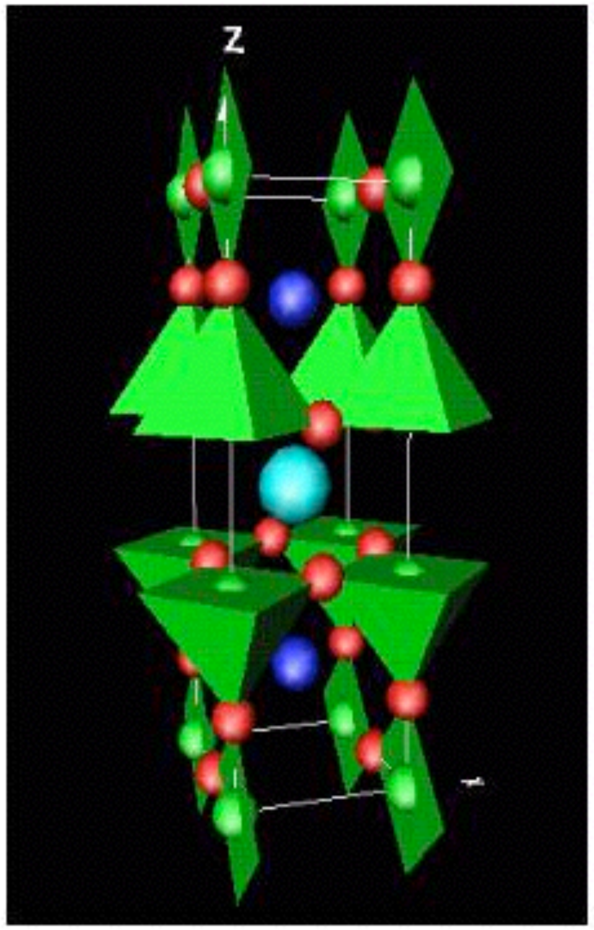
but the current is
conserved

extra dimensions

where might this be seen?

physics at strong coupling (interactions)

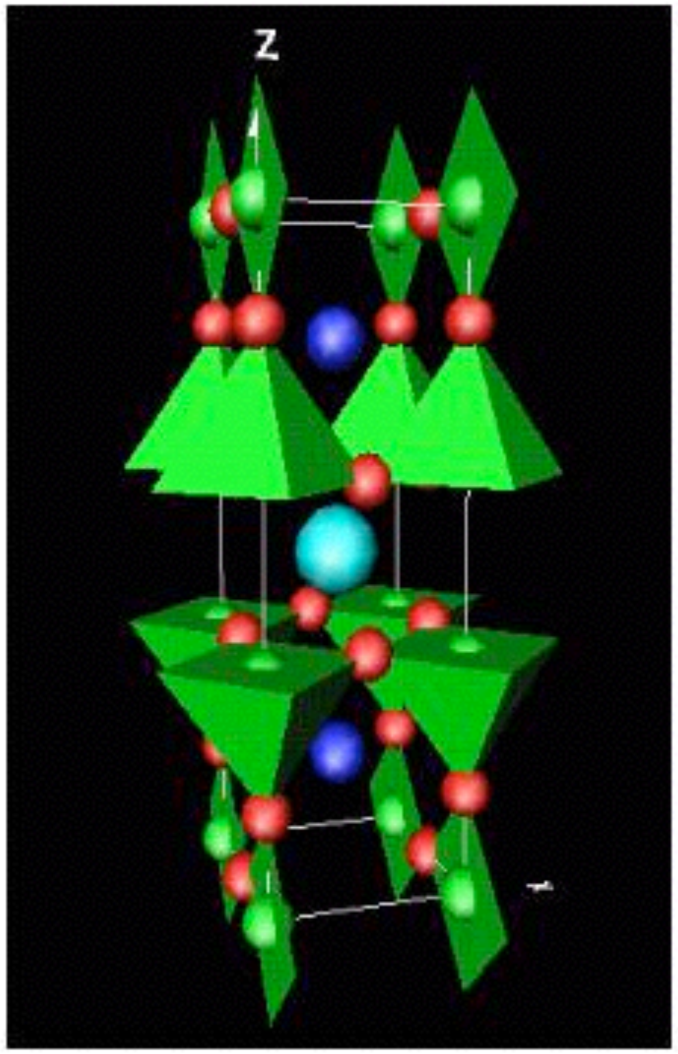




$YBa_2Cu_3O_7$
Cuprate Superconductors

not independent particles
(collective phenomena)

strong coupling
metallic state
(strange metal)



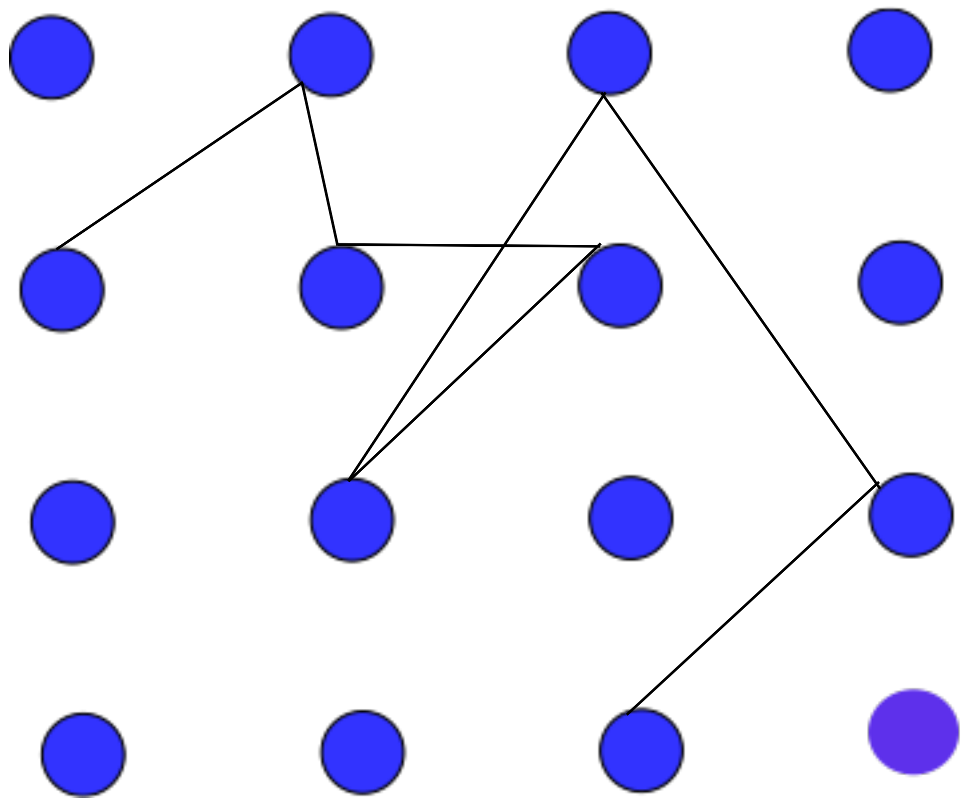
$YBa_2Cu_3O_7$
Cuprate Superconductors

?

superconductivity

what is the strange metal?

Drude metal



momentum relaxation

$$\dot{p} = e(E + \frac{p \times B}{m}) - \frac{p}{\tau}$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \Rightarrow \lim_{\tau \to 0} \Re\sigma \to \infty$$

standard metals

resistivity

$$\rho \propto T^2$$

Weidemann-Franz law

$$\frac{\kappa_{xx}}{T\sigma_{\rm xx}} = \frac{\pi^2}{3}$$

optical conductivity

$$\Re\sigma \propto 1/\omega^2$$

strange metal: experimental facts

Quantum critical behaviour in a high-T_c superconductor

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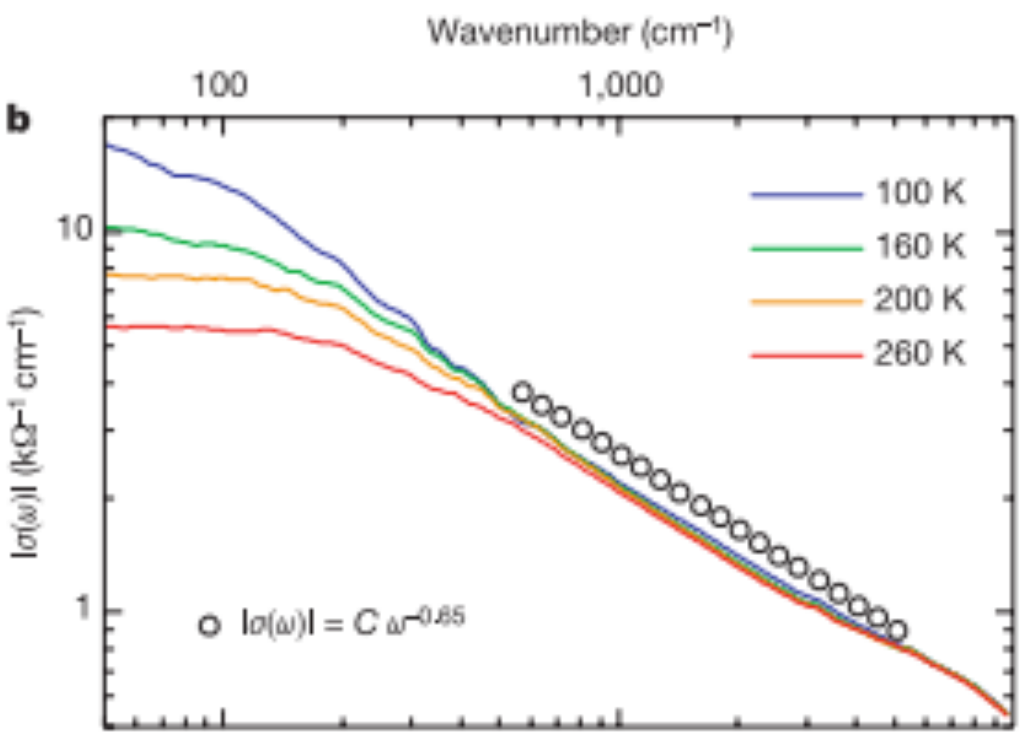
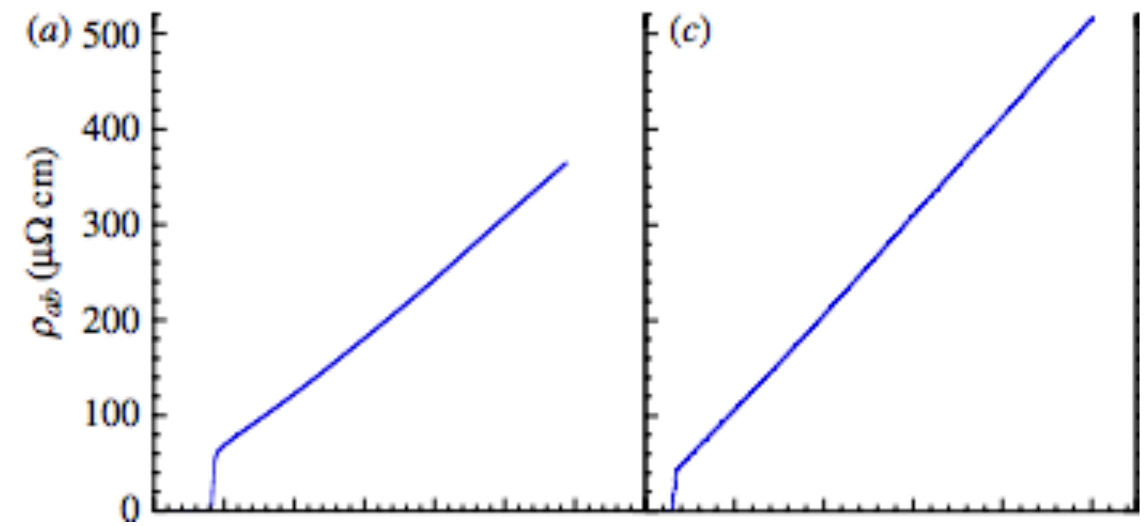


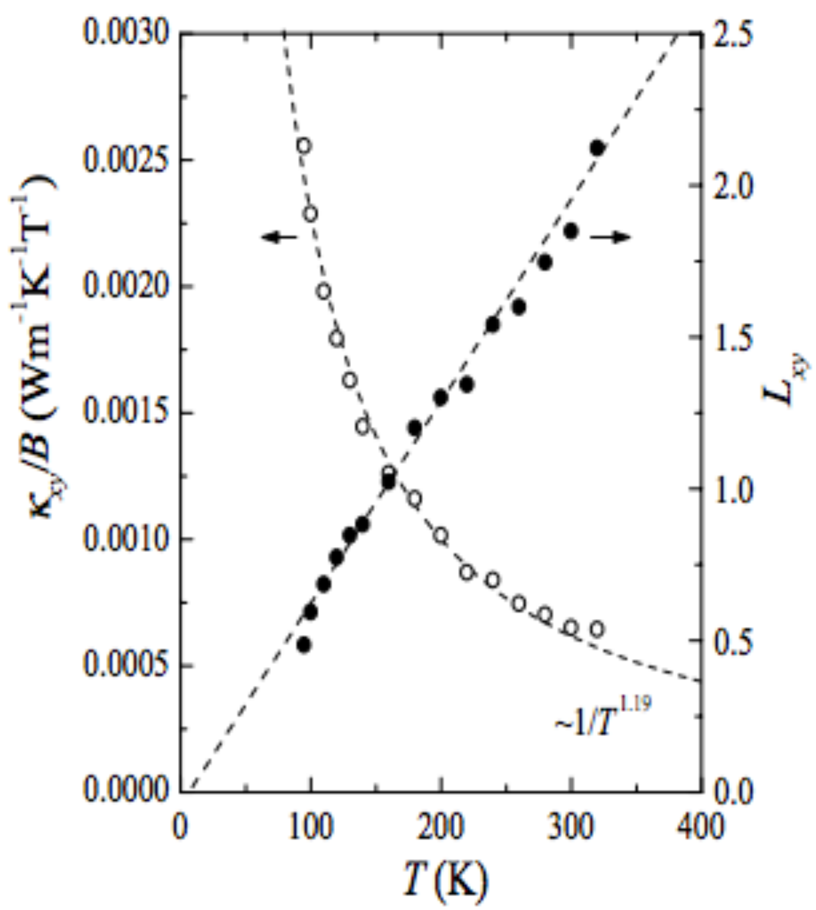
$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

T-linear resistivity



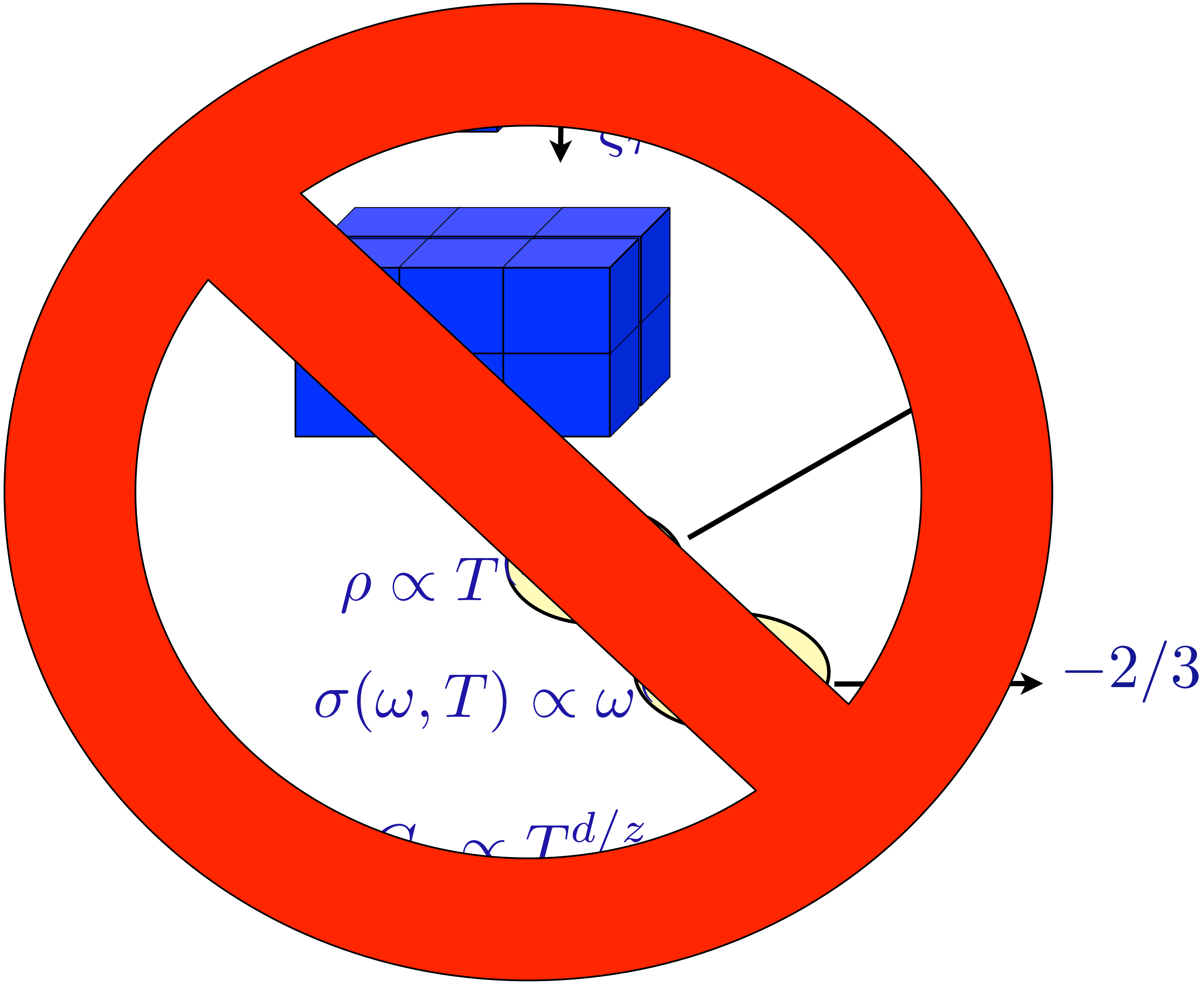
$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



$$\frac{\text{Theories of cuprates}}{\text{Theories of strange metal}} = \infty$$

why is the problem hard?

single-parameter scaling



new length scale?

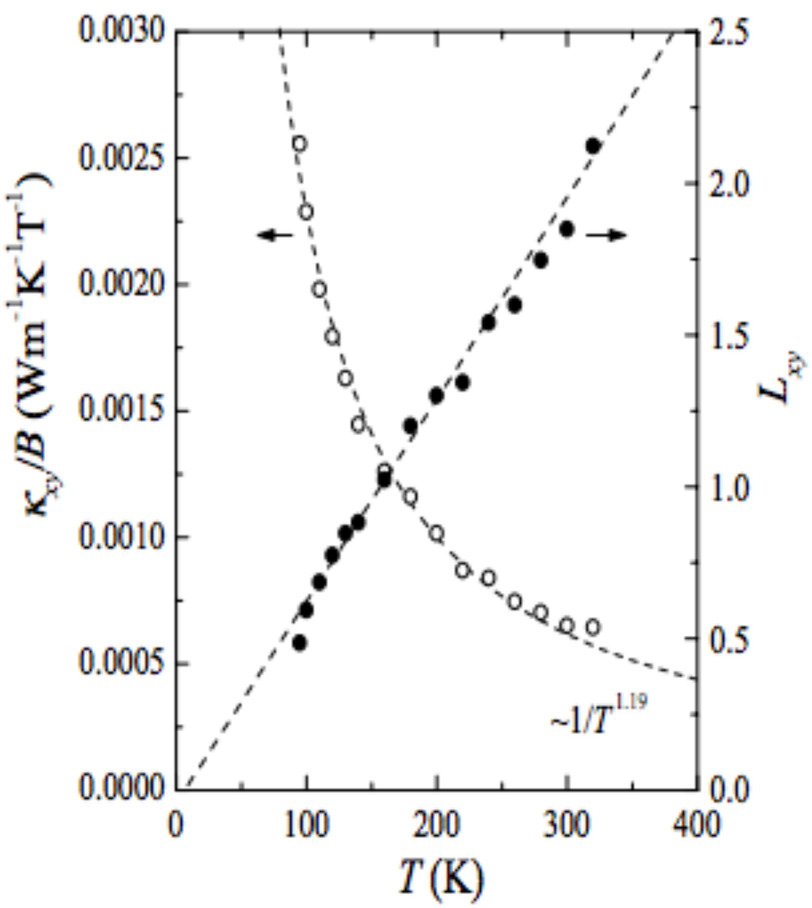
strange metal: experimental facts

Hall Angle

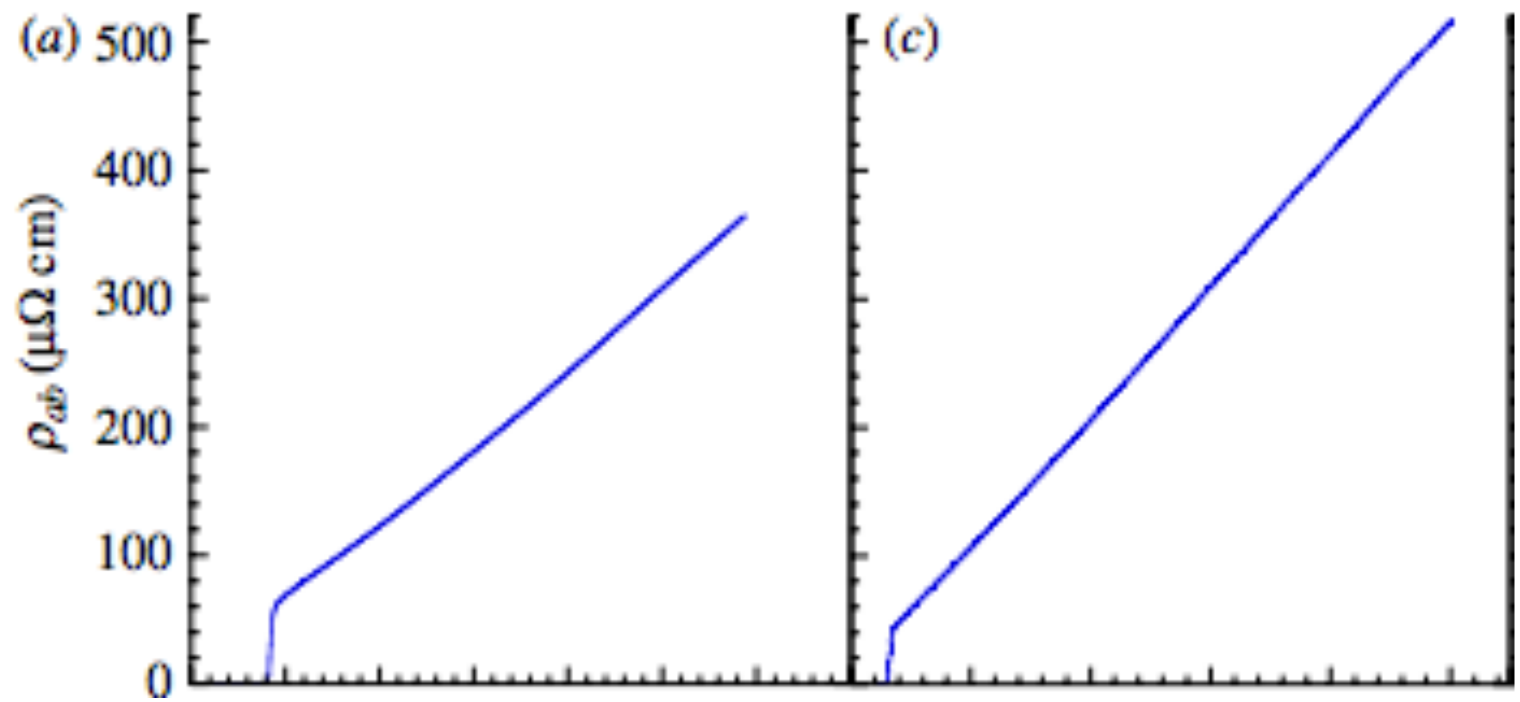
$$\cot\theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2$$

Hall Lorenz ratio

$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



T-linear resistivity



all explained if

$$[J_\mu] = d - \theta + \Phi + z - 1$$

$$[A_\mu] = 1 - \Phi$$

$$\Phi = -2/3$$

Hartnoll/Karch

Recall in
standard E&M

$$[A_\mu] = 1$$

$$[J_\mu] = d - 1$$

strange metal

$$[J_\mu] = d - \theta + \Phi + z - 1$$

$$[A_\mu] = 1 - \Phi$$

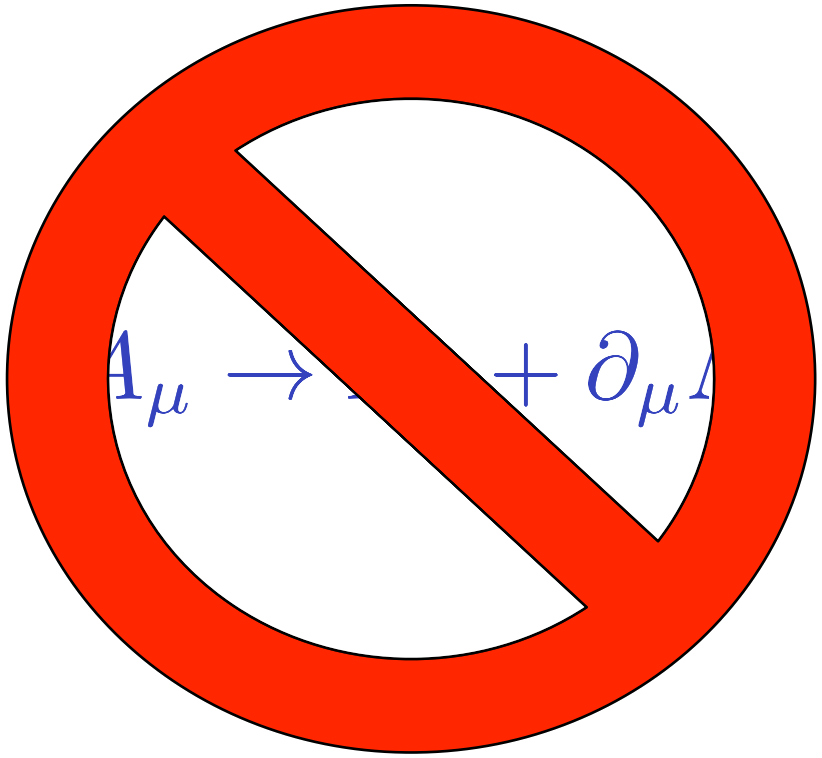
$$\Phi = -2/3$$

How is this possible?

what is the new gauge principle?

if

$$[A_\mu] \neq 1$$



hint

$$\partial_\mu J^\mu = 0$$

current conservation

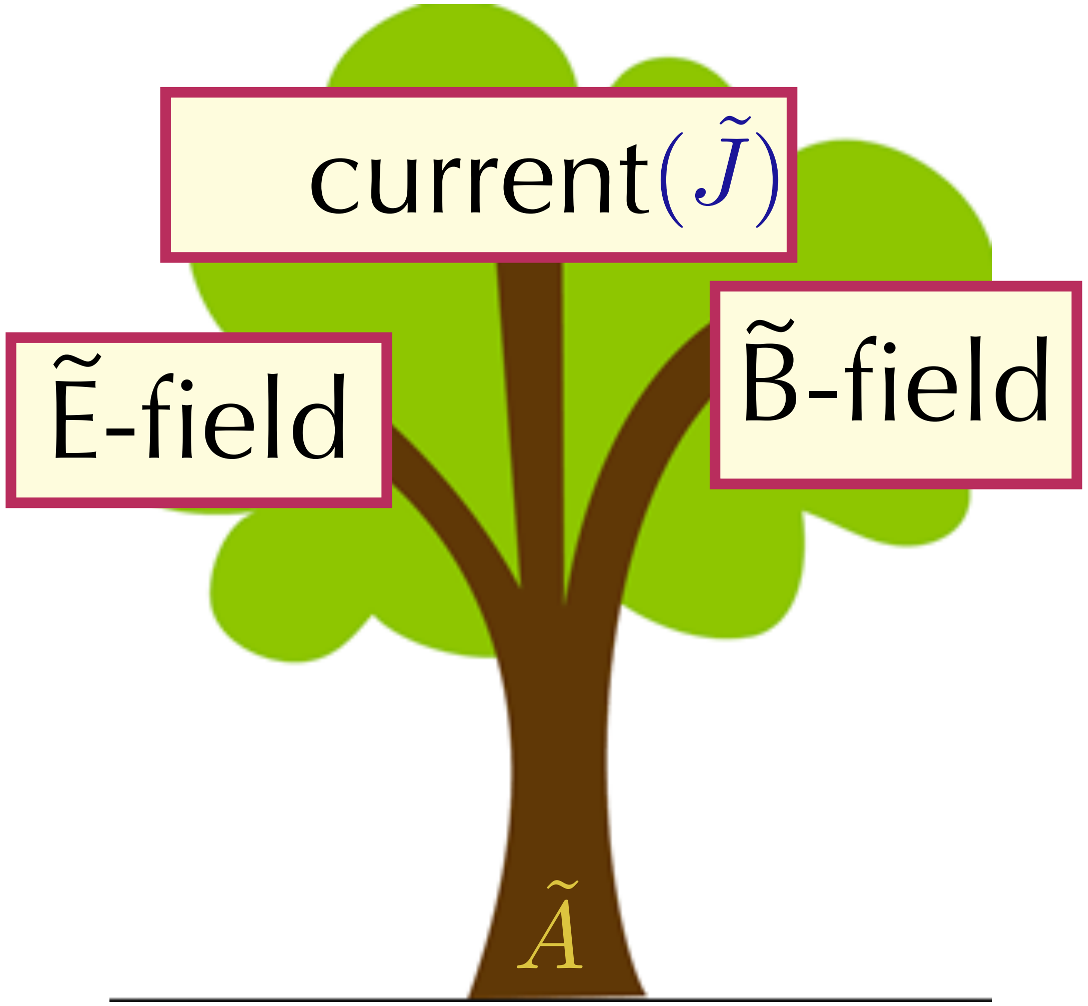
what if

$$[\partial_\mu, \hat{Y}] = 0$$

new current

$$[\tilde{J}] = d - 1 - D_Y$$

$$\partial_\mu \hat{Y} J^\mu = \partial_\mu \tilde{J}^\mu = 0$$



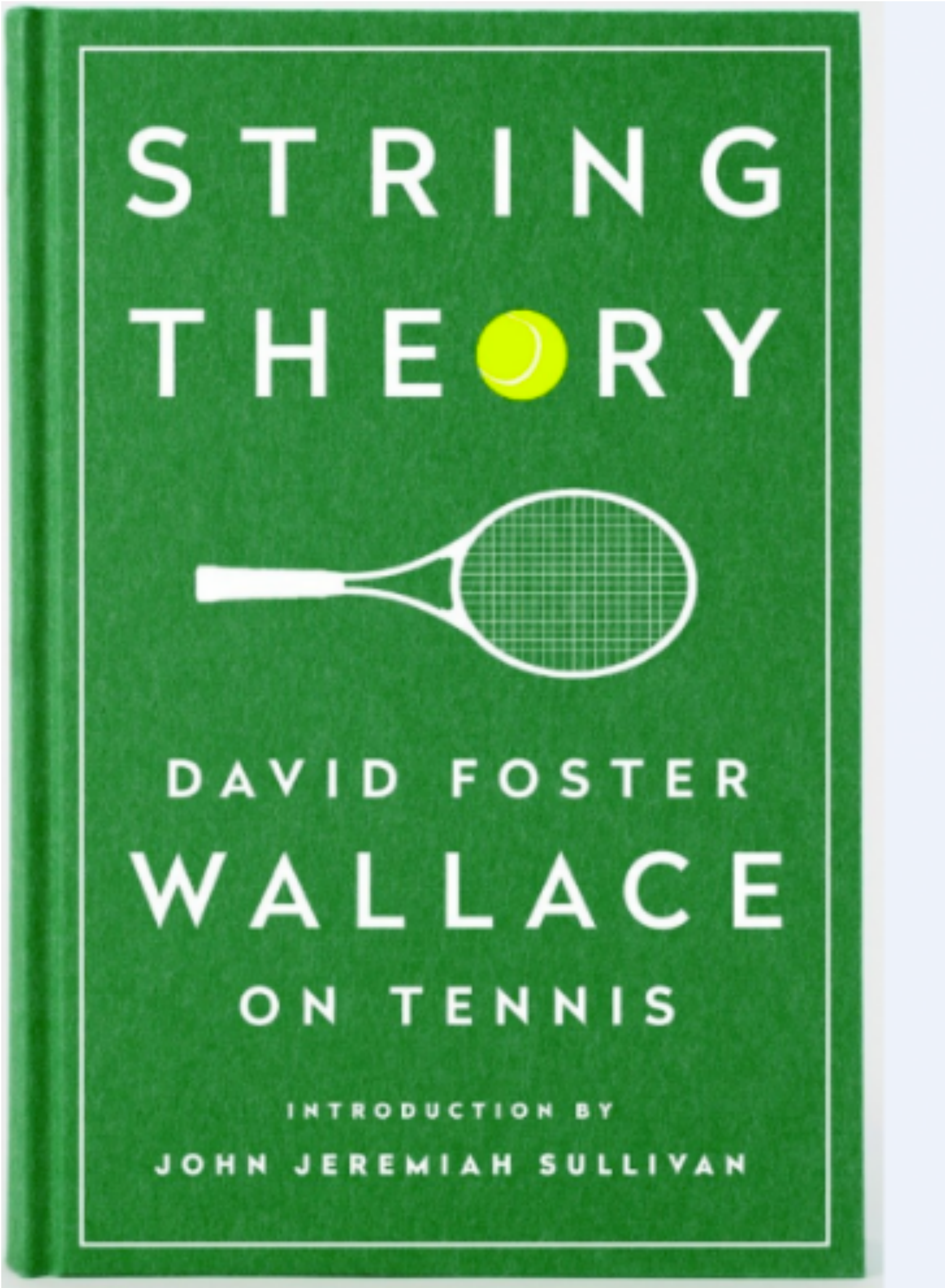
current($\tilde{J}$)
Ẽ-field
B̃-field
$\tilde{A}$

standard E&M has an added ambiguity

$$\partial_\mu J^\mu = 0$$

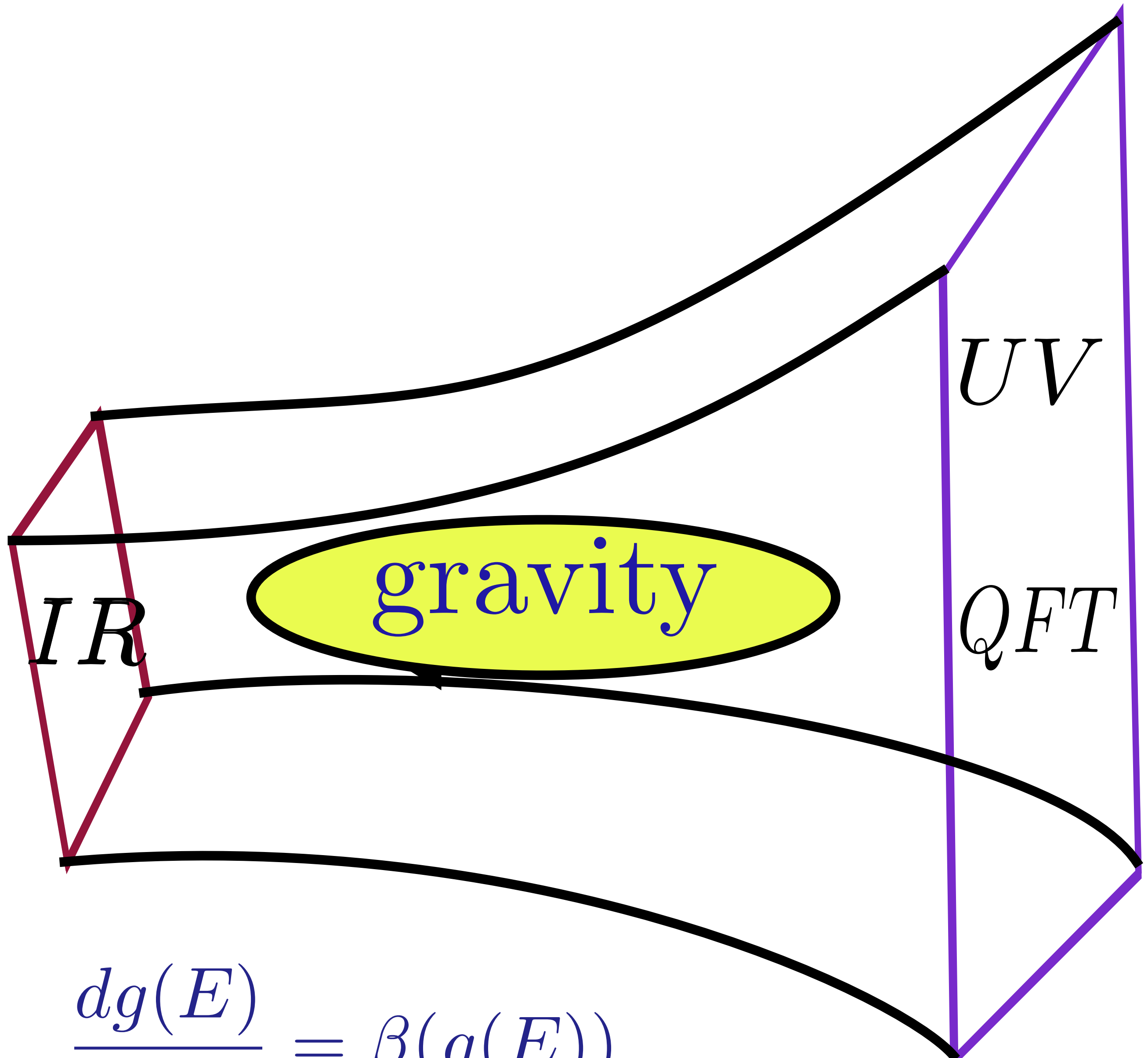
$$[\partial_\mu, \hat{Y}] = 0$$

Does a non-trivial solution exist?


STRING THEORY
DAVID FOSTER WALLACE
ON TENNIS
INTRODUCTION BY
JOHN JEREMIAH SULLIVAN

?

gauge-gravity duality
(Maldacena, 1997)



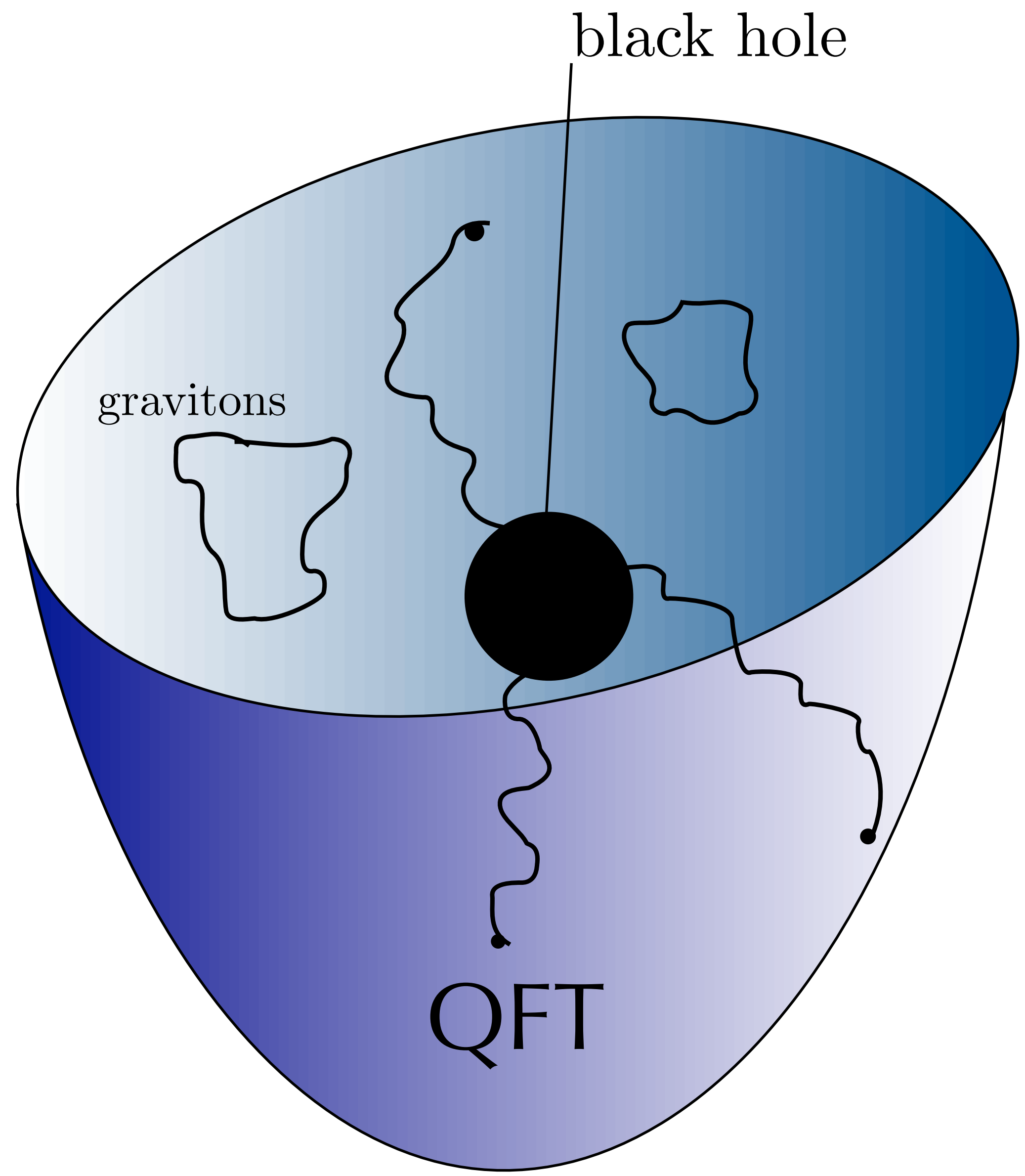
coupling constant

$g = 1/\text{ego}$

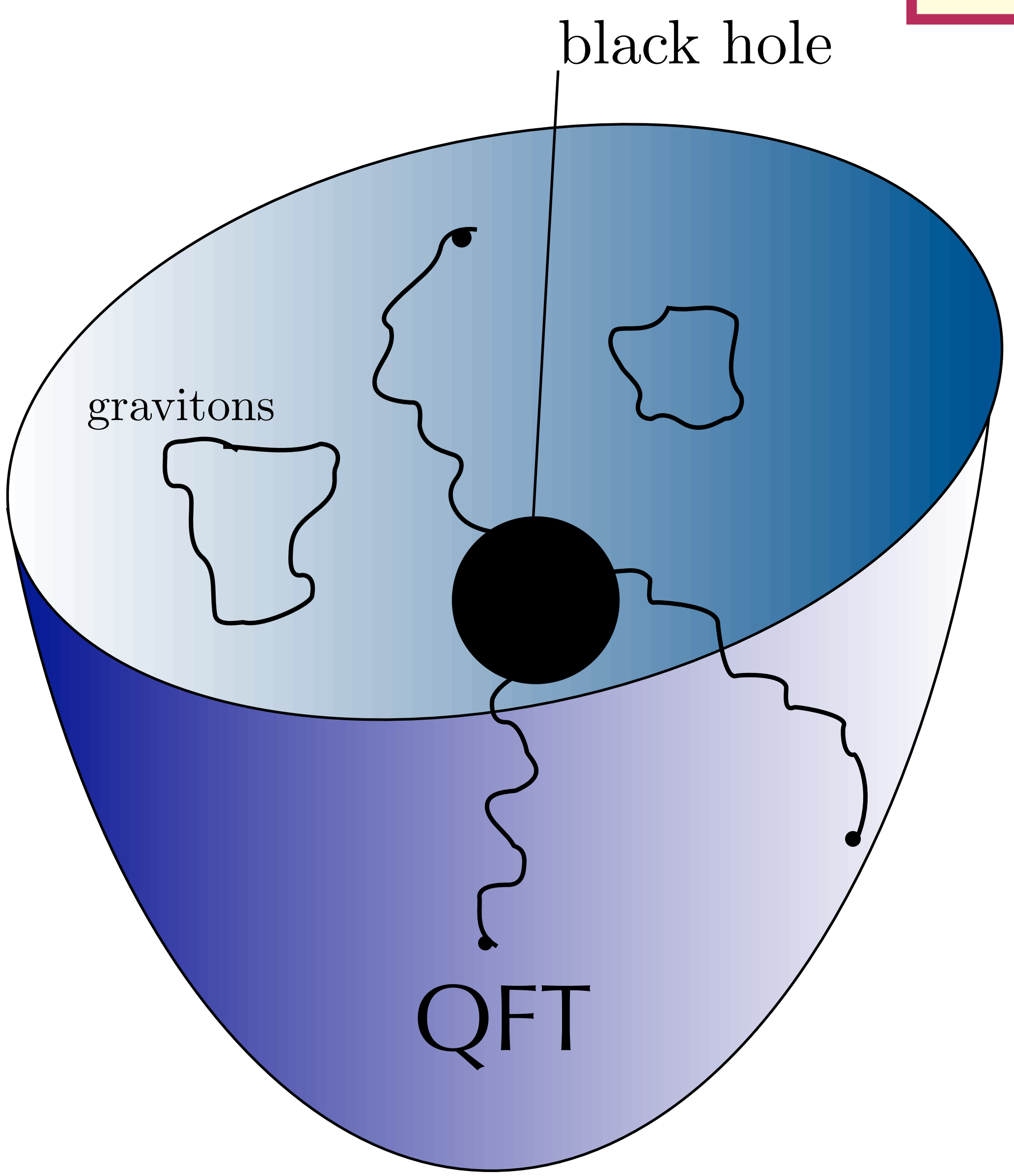
$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

certain strongly coupled theories have gravity duals (holography)



Maldacena
conjecture
(1997)



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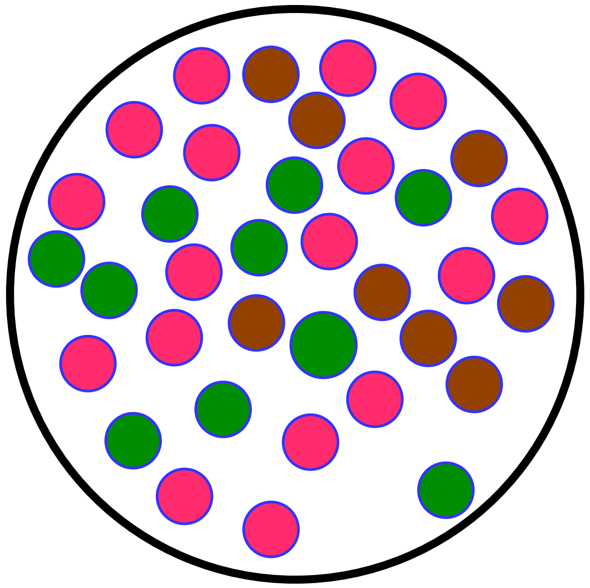
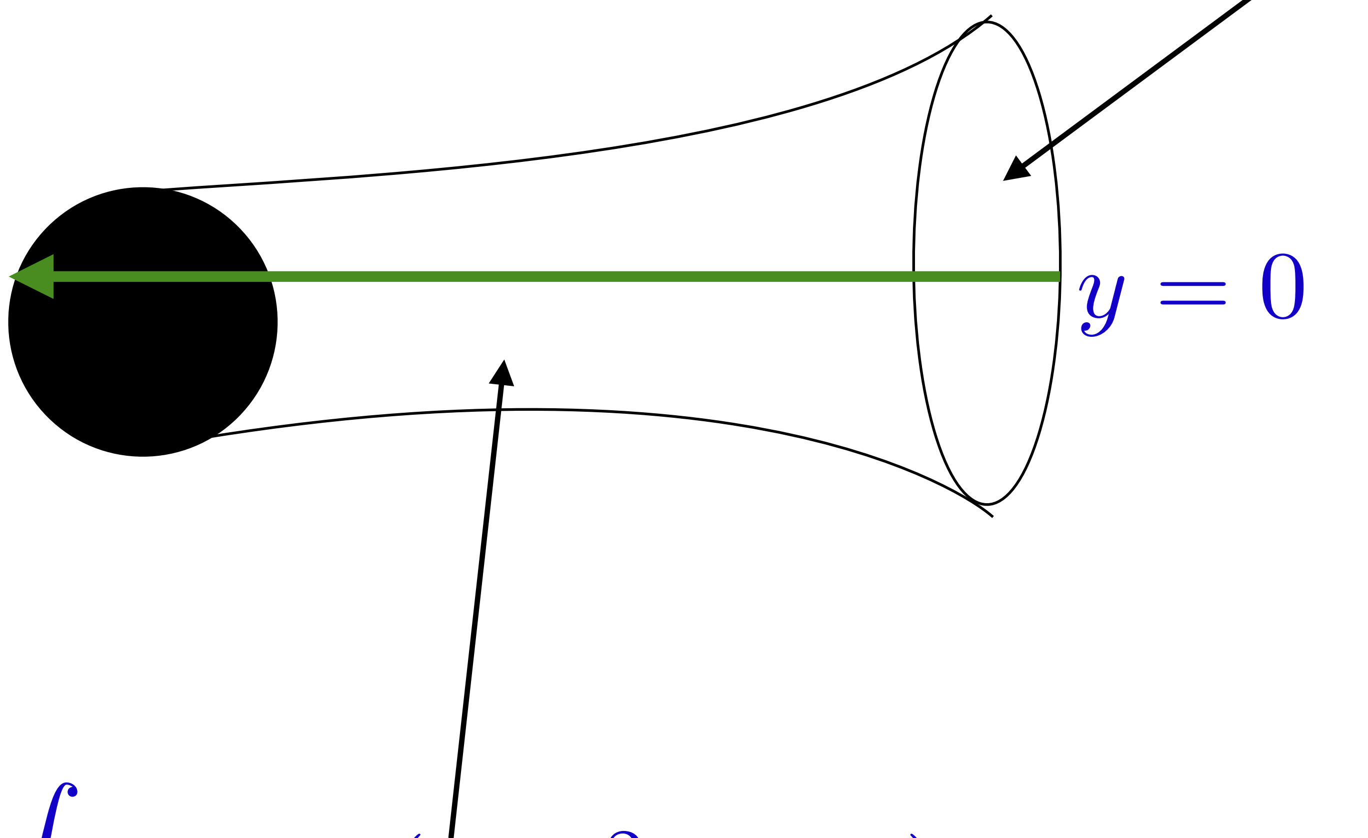


strongly coupled
quantum system
(nucleons)

claim



how?

?

$$S = \int dV_d dy \left(y^a F^2 + \cdots \right)$$

$$F = dA$$

if holography is RG then how can it lead to an anomalous dimension?

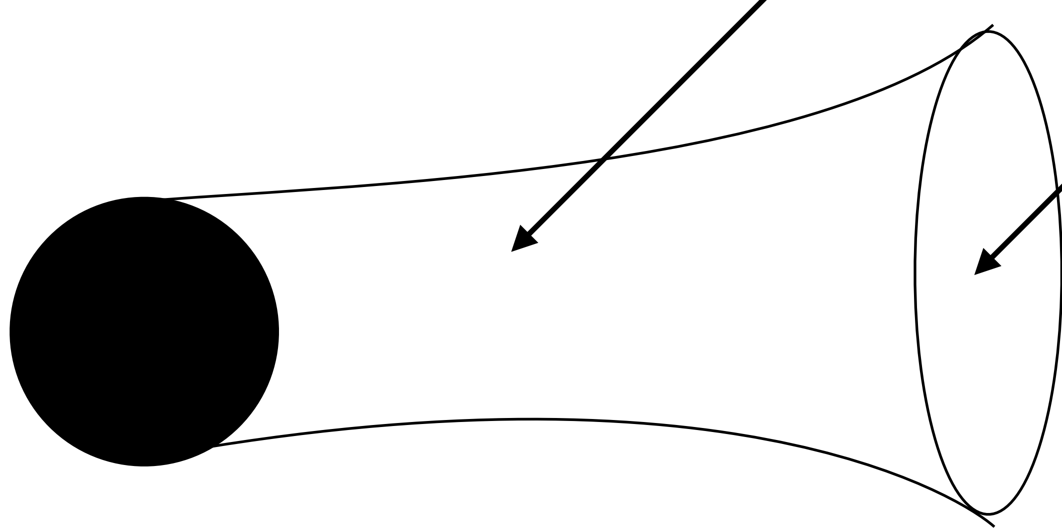
$$S = \int dV_d dy \left(y^a F^2 + \cdots \right)$$

eom

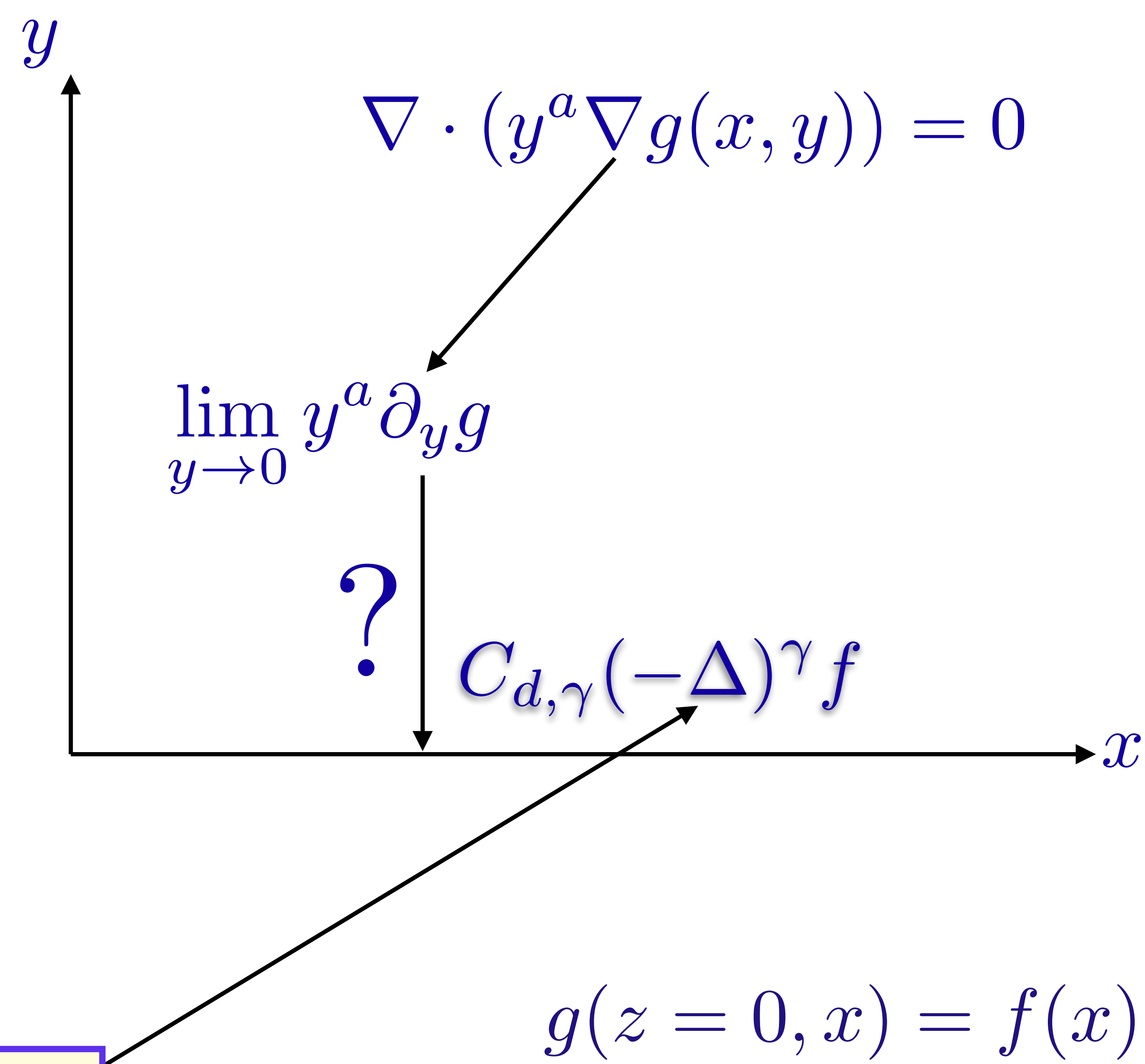
$$d(y^a \star dA) = 0$$

$$y \neq 0 \;\longrightarrow\; A \to A + \partial\Lambda$$

what about the boundary?



Caffarelli-Silvestre extension theorem (2006)



fractional Laplacian

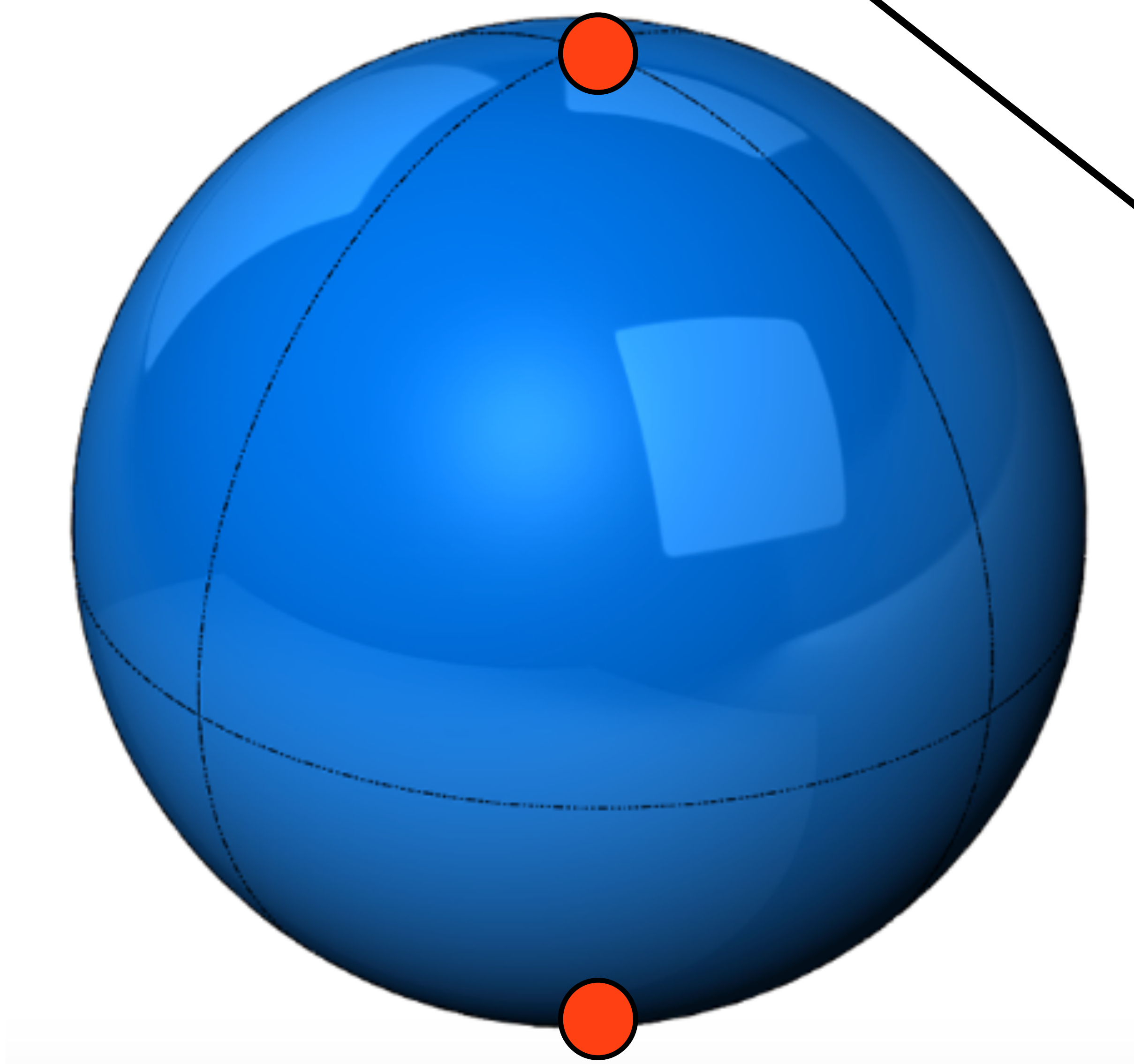
$$g(z=0,x) = f(x)$$

$$\gamma = \frac{1-a}{2}$$

fractional Laplacians

$\Delta^\gamma f(x)$

requires information everywhere: non-local



closer look

$$\nabla \cdot (y^a \nabla u) = 0$$

scalar field
(use CS theorem)

$$d(y^a \star dA) = 0$$

holography

similar equations

generalize CS
theorem to p-forms
GL,PP:1708.00863

boundary action: fractional Maxwell equations

$$\Delta^{\gamma} A_{\perp} = 0$$

boundary action has anomalous dimension (non-locality)

$$F_{ij} = \partial_i^{\gamma} A_j - \partial_j^{\gamma} A_i \equiv d_{\gamma} A = d\Delta^{\frac{\gamma-1}{2}} A,$$

new gauge transformation

$$A \to A + d_\gamma \Lambda,$$

$$d_\gamma \equiv (\Delta)^{\frac{\gamma-1}{2}} d$$

$$[A] = \gamma$$

$$\partial_\mu J^\mu = 0$$

current conservation

what if

$$[\partial_\mu, \hat{Y}] = 0$$

answer

?

$$[\partial_\mu, \hat{Y}] = 0$$

$$[d, \Delta^\alpha] = 0$$

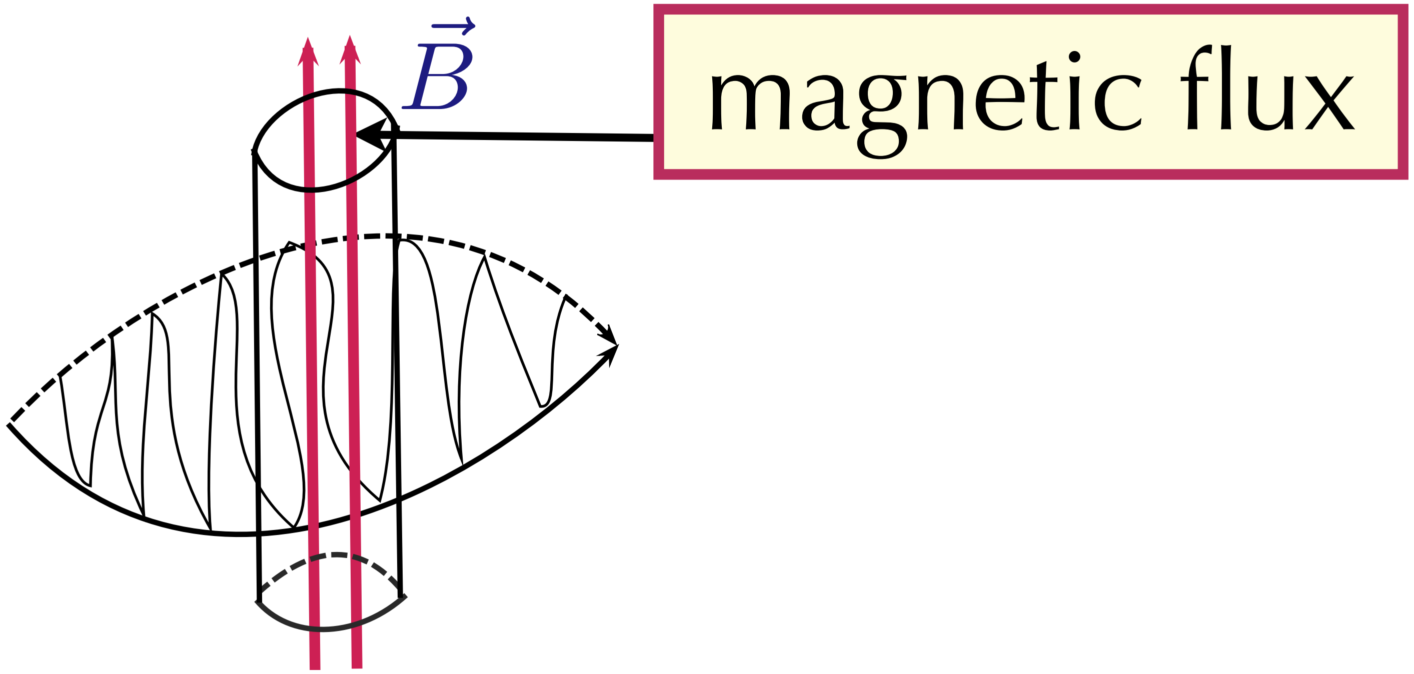
$$\hat{Y} = \Delta^\alpha$$

$$J \to \Delta^\alpha J \qquad [J] = d - 1 - \alpha$$

is there a knock-down experiment?

Aharonov-Bohm Effect

$$\oint A \cdot d\ell$$



physical consequences of anomalous dimension for $\mathcal{A}_\mu$

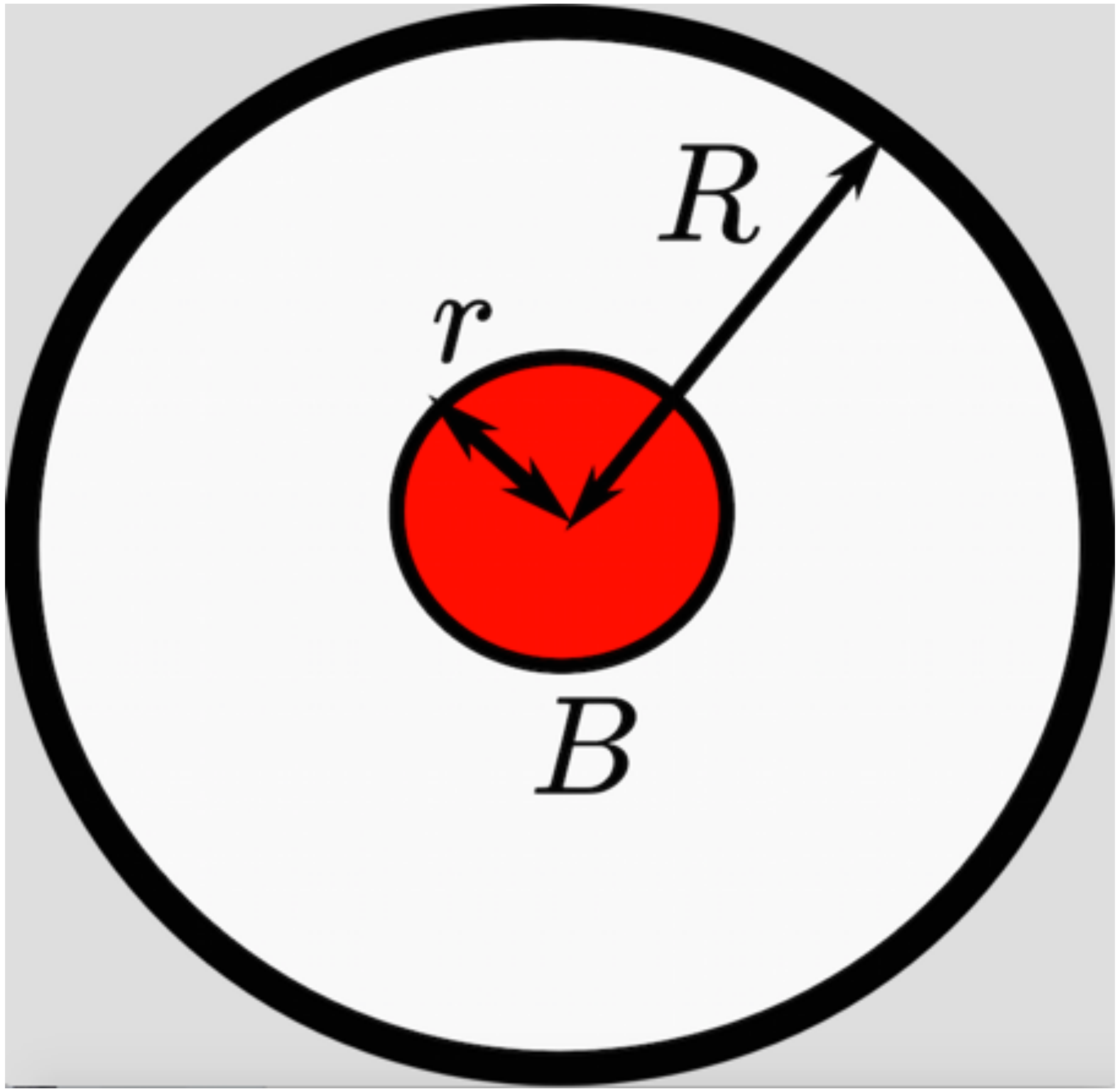
$$\mathcal{A}_\mu \rightarrow \mathcal{A}_\mu + \partial_\mu^\alpha \mathcal{G}$$

$$\vec{\nabla}^\alpha \times \vec{A} = \vec{B}$$

no Stokes' theorem

$$\oint \vec{A} \cdot d\ell \neq \int_S B \cdot d\vec{S}$$

Aharonov-Bohm Effect must change



$$\Delta\phi_D = \frac{e}{\hbar}\pi r^2 B F(R, \alpha)$$

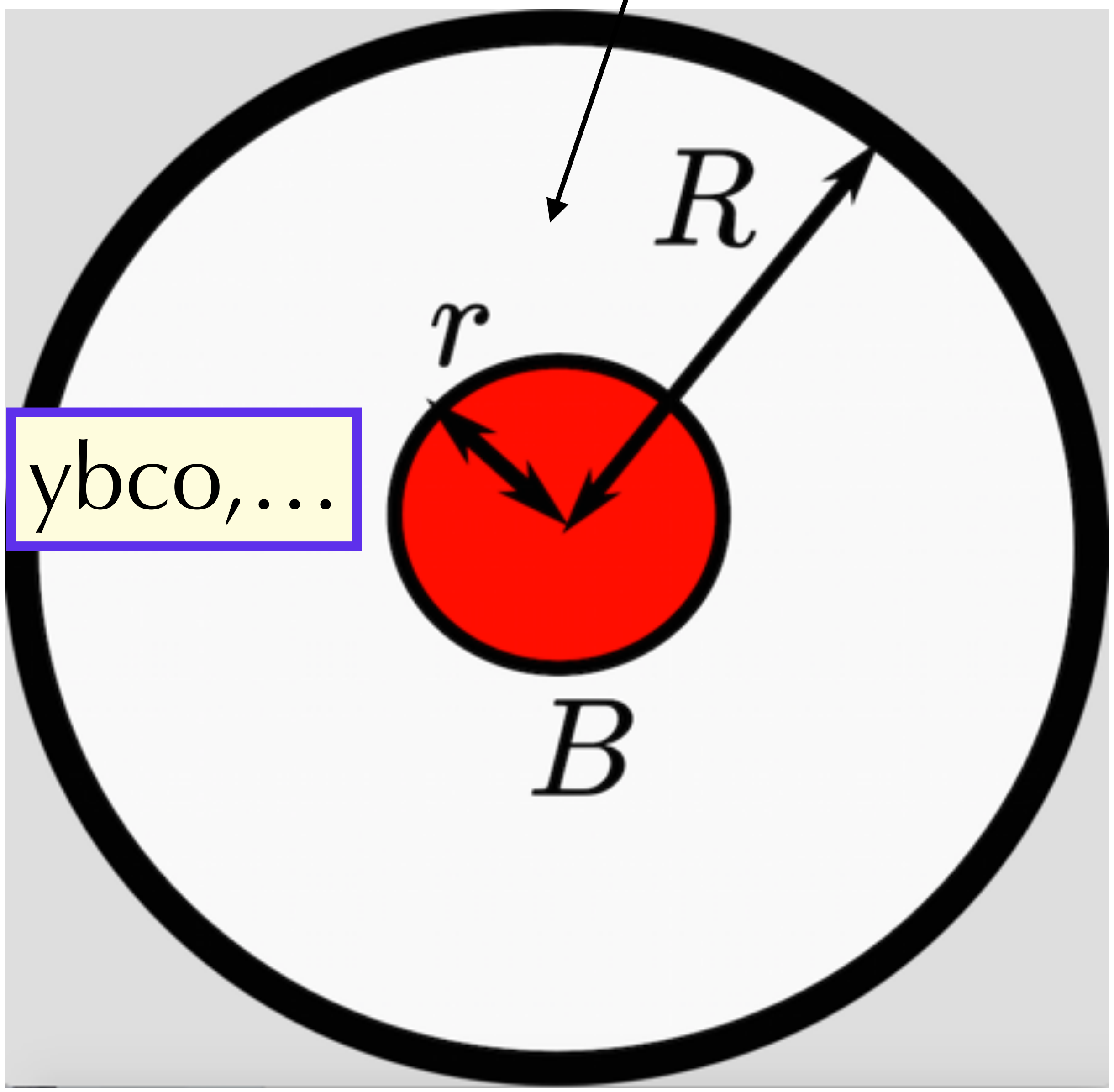
is the correction large?

$$\alpha = 1 + 2/3 = 5/3$$

$$\Delta\Phi_R = \frac{eB\ell^2}{\hbar} L^{-5/3}/(0.43)^2$$

yes!

experiment



Planckian dissipation

$$\tau = \frac{\hbar}{k_B T}$$

$$\tau \approx 10^{-14} s$$

Table 11.1

Element	77 K	273 K
Li	$7.3 \times 10^{-14} s$	$8.8 \times 10^{-15} s$
Na	$1.7 \times 10^{-13} s$	$3.2 \times 10^{-14} s$
K	$1.8 \times 10^{-13} s$	$4.1 \times 10^{-14} s$
Rb	$1.4 \times 10^{-13} s$	$2.8 \times 10^{-14} s$
Cs	$8.6 \times 10^{-14} s$	$2.1 \times 10^{-14} s$

strange metal

combine
AC+DC
transport

$[J] = \Phi$

fraction
not d

non-local
action

$[A_\mu] = d_A \neq 1$

probe by Aharonov-Bohm effect on
underdoped cuprates:
extra dimensions