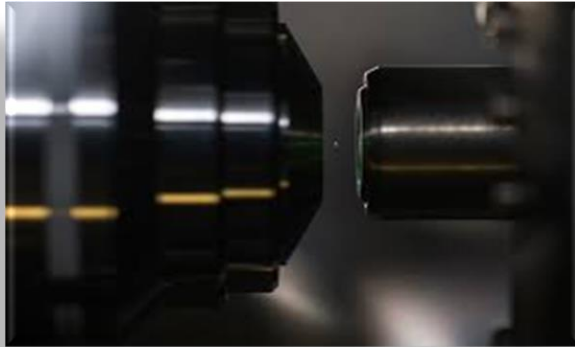


Flying lasers for sound



*Courtesy J. Adam Fenster & A. N. Vamivakas



Optics & Photonics News 27 42 (2016)

R. Pettit^{1,2}, W. Ge³, P. Kumar³, K. Xiao³, L. Neukirch^{1,2}, D. Luntz-Martin¹,
J. Schulz^{1,2}, **Mishkat Bhattacharya**^{2,3,4} and A. N. Vamivakas^{1,2}

¹Institute of Optics & ²Center for Coherence and Quantum Optics
University of Rochester

³School of Physics and Astronomy & ⁴Future Photon Initiative
Rochester Institute of Technology



AMO Theory group @ RIT

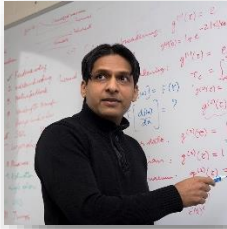


Rochester Institute of Technology:
School of Physics and Astronomy:

20,000 students + 1100 faculty
~38 fulltime faculty,
PhD Program in Astrophysics
MS in physics started in 2018

THEORY
(RIT)

mxbsps@rit.edu



MB



Brandon R.



Wenchao Ge



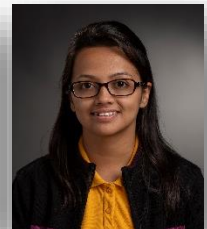
Pardeep K.



Kewen X.



Sandeep S.



Arpita P.

EXP
(UofR)



University of Rochester



Nick V.



Levi Neukirch



Robert Pettit



nick.vamivakas@rochester.edu



Talk outline

- Introduction
- Theory
- Experiment
- Conclusion



B. Rodenburg, L. P. Neukirch, A. N. Vamivakas and **M. Bhattacharya**
Quantum model of cooling and force sensing with an optically trapped
nanoparticle, *Optica* **30**, 318 (2016).

*R. M. Pettit, W. Ge, P. Kumar, D. R. Luntz-Martin, J.T. Schultz,
L. P. Neukirch, **M. Bhattacharya** and A. N. Vamivakas, *An Optical
Tweezer Phonon Laser*, *Nature Photonics* **13**, 402 (2019).



- * [News and Views](#), R. Huang & H. Jing, *Nature Photonics*, **13**, 371 (2019)
- * [Optics and Photonics News](#), May 2019, [Breakthroughs of 2019](#), Dec 2019.



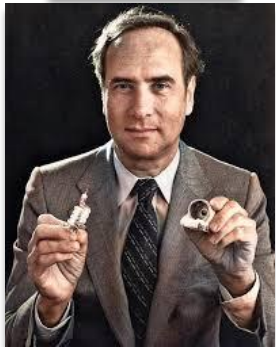
Optical laser



Laser history



1. Maser invented by Townes in 1953.
2. Laser proposed by Townes and Schawlow in 1958.



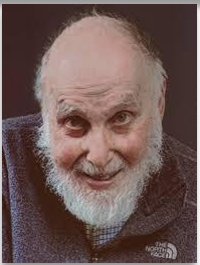
3. Laser built in 1960 by T. Maiman.
4. Quantum theory proposed in 1960s (Haken, Lamb, Lax...)

5. Multiple Nobel prizes

- 1981 Bloembergen, Schawlow
- 1997 Chu, Cohen-Tannoudji, Phillips
- 2001 Cornell, Wieman, Ketterle
- 2005 Hansch, Hall, Glauber
- 2012 Haroche, Wineland
- 2017 Weiss, Thorne, Barish
- 2018 Ashkin, Morou, Strickland

Edible laser
with jello..

\$10B industry



....

The Laser Inventor (Springer, 2018)

Cavity optomechanics: Displacement sensing

PRL 116, 061102 (2016)

PHYSICAL REVIEW LETTERS

12 FEBRUARY 2016

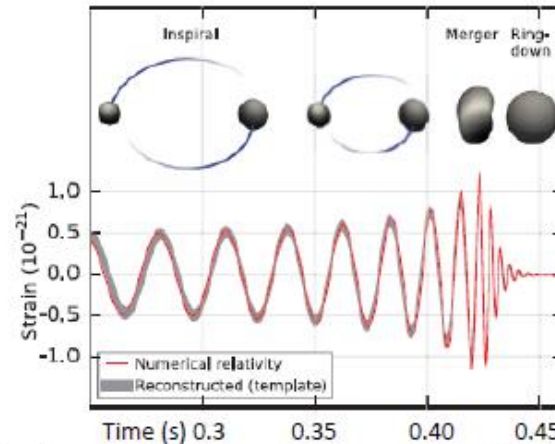
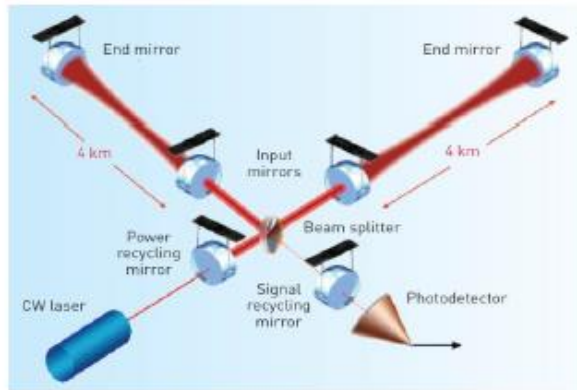
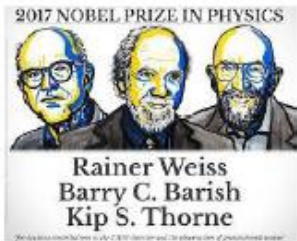


Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)



Also...

Accelerometry

A. G. Krause, M. Winger, T. D. Blasius, Q. Lin and O. Painter, *Nat. Photon.* 6 768 (2012)

Magnetometry

S. Forstner *et. al*, *PRL* 108 120801 (2012)

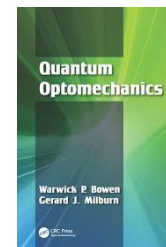
Thermometry

J. Millen *et. al*, *Nat. Nano.* 9 425(2012)

LIGO is a cavity optomechanical device !! Displacement = Strain x Length of int. arm $\approx 10^{-18}m$



Quantum-classical interface, sensing, transduction, Slow light, OMIT, memory.



Photons and phonons

Photons

~

Phonons

Both are realizations of the harmonic oscillator

(E, B)

~

(Q, P)

→ Can we make a laser for phonons?



The role of feedback



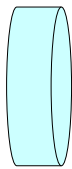
Fabry



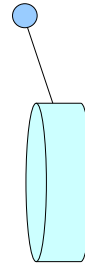
Perot

Laser

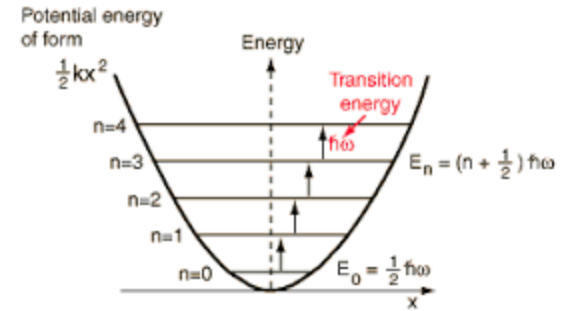
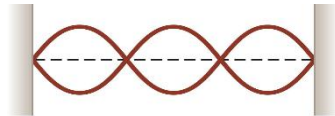
λ
→



$\hbar k_n$
→
←
 $-\hbar k_n$



—|—|—
 $-L$ 0 q



Radiation Force \propto *optical intensity*

Light-matter interaction energy

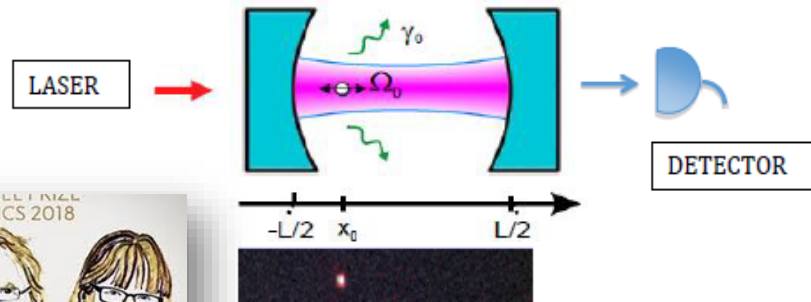
\propto *optical intensity* $\times q$



$a^\dagger a$

- C. K. Law, PRA **51**, 2537(1995).
- MB, et. al, AJP **81**, 267 (2013).

Cavity optomechanics: Levitation



Theory: D. E. Chang et. al, PNAS **107**, 1005 (2007)
O. Romero-Isart...I. Cirac, PRL **107**, 020405 (2011)

Expt: N. Keisel...M. Aspelmeyer et al., PNAS **110**, 14180 (2013): **64 K**
J. Millen...P. Barker et. al, PRL **114**, 123602 (2015): **10 K**

Limitations

1. Manipulation only possible along cavity axis
2. Physical access to particle limited
3. Scaling is complicated
4. Optical wavelengths have to be resonant with the cavity
5. Interaction is given

Question:

Can we do optomechanics without cavities?

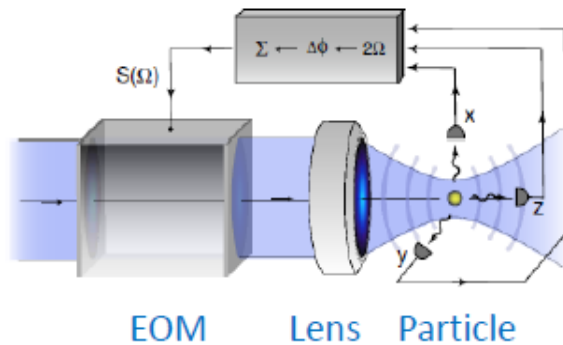
→ Ashkin, A., Dziedzic, J. M., Bjorkholm, J. E. & Chu, S. *Observation of a single-beam gradient force optical trap for dielectric particles*. Opt. Lett. **11**, 288–290 (1986)

O. Romero-Isart et al. PRL **107**, 020405 (2011)
P Asenbaum, S Kuhn, S Nimmrichter, U Sezer, M Arndt, Nat. Comm **4**, 111 (2013)



Cavityless optomechanics: Levitation

Single beam nanoparticle trapping



$$E = E_t e^{-a q^2}$$

Gaussian beam

Optical force on a Rayleigh particle ($r \ll \lambda$)

= Gradient force + Scattering force

$$V_t = -d \cdot E$$

Trapping potential

$$= -\frac{\alpha E^2}{2} \sim b I_t q^2$$

Harmonic trap along each spatial direction

Note: All manipulation of the nanoparticle will be done by modulating I_t .

J. Gieseler, B. Deutsch, R. Quidant, and L. Novotny, PRL **109** 103603 (2012)

L. Neukirch, E. von Hartmann, J. M. Rosenholm, and A. N. Vamivakas, Nat. Phot. **9**, 653–657 (2015).

J. Millen, T. Deesuwana, P. Barker, J. Anders, Nat. Nanotech. **9**, 425 (2014)

M Rashid...H. Ulbricht, PRL **117**, 273601 (2016).

R. Gambhir...A. A. Geraci, PRA **91**, 051805(R) (2015).

Thai M. Hoang...Tongcang Li. Nat. Comm. **7**, 12550 (2016)

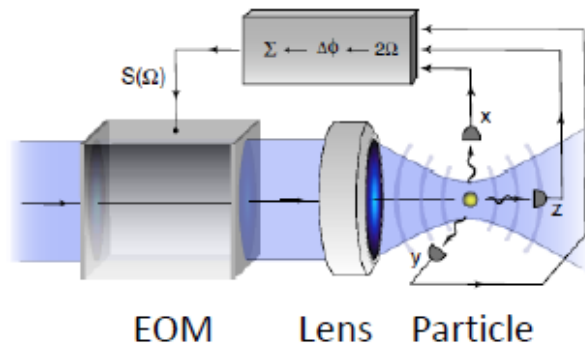
G. Conangla...N.Meyer,...R. Quidant, PRL **122**, 223602 (2019).



Cavityless optomechanics: Feedback



Cavityless optomechanics: Feedback



Linear damping: $F_l \propto -p$

- This is present due to collisions with gas particles
- It can also be engineered using **parametric feedback**

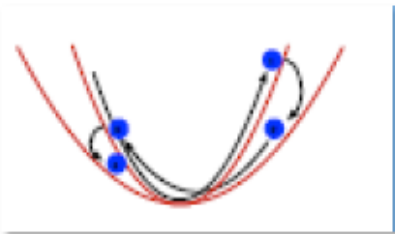
Measure q

Calculate p

Modulate the trap laser $V_t \sim (I_t + \epsilon_l \frac{p}{q})q^2$

$$F_l = -\partial V_t / \partial q \propto -\epsilon_l p$$

Nonlinear damping: $F_n \propto -q^2 p$

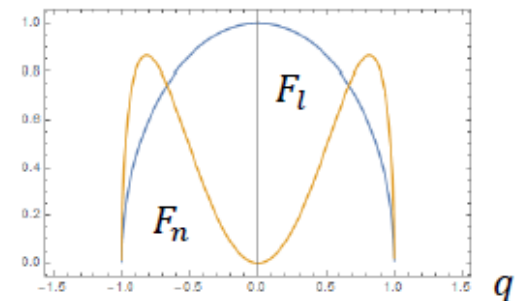


Measure q

Calculate p

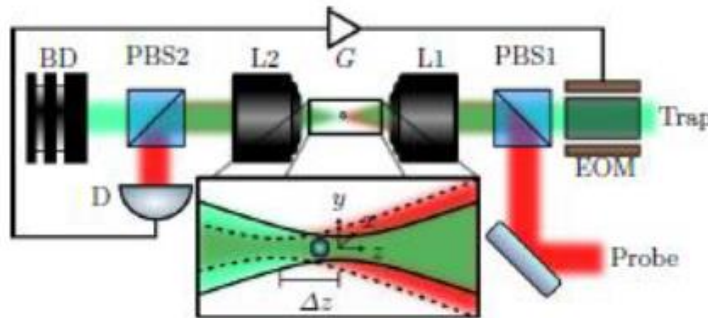
Modulate the trap laser $V_t \sim (I_t + \epsilon_n p q)q^2$

$$F_n = -\partial V_t / \partial q \propto -\epsilon_n q^2 p$$



Theoretical model: microscopic Hamiltonian

A theorist's view of the experiment



$$H = H_m + H_f + H_{int}$$

Kinetic energy

$$H_m = |\mathbf{p}|^2 / 2m$$

Field energy

$$H_f = \epsilon_0 \int |\mathbf{E}(\mathbf{r})|^2 d^3\mathbf{r}$$

Interaction energy

$$H_{int} = - \int_V \mathbf{P}(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r}) d^3\mathbf{r} / 2$$

Total field

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_t + \mathbf{E}_p + \mathbf{E}_b$$

Polarizable nanoparticle

$$\mathbf{P}(\mathbf{r}) = \alpha \mathbf{E}(\mathbf{r})$$

Procedure: Eliminate bath modes using Born-Markov approximation
 Trace over probe field and x, y motion
 Add suitable terms for gas collisions
 Add Markovian feedback and backaction



Theoretical model – Master equation

$\dot{\rho}$			
$= -i[\omega_m b^\dagger b, \rho]$	Unitary dynamics		
$-\frac{(A_t + D_p)}{2} D[q]\rho$	Position diffusion	}	L. Diosi, Quantum master equation of a particle in a gas environment <i>Europhysics Letters</i> 30 , 63 (1995)
$-\frac{D_q}{2} D[p]\rho$	Momentum diffusion		
$-i\frac{\gamma_g}{2} [q, \{p, \rho\}]$	Gas damping		
$-i\gamma_f [q^3, \{p, \rho\}]$	Nonlinear feedback cooling	}	H. M. Wiseman and G. J. Milburn, Quantum theory of optical feedback via homodyne detection <i>Physical Review Letters</i> 70 , 548 (1993)
$-\Gamma_f D[q^3]\rho$	Backaction		
$+i\gamma_l [q, \{p, \rho\}]$	Linear heating	}	
$-\Gamma_l D[q]\rho$	Backaction		



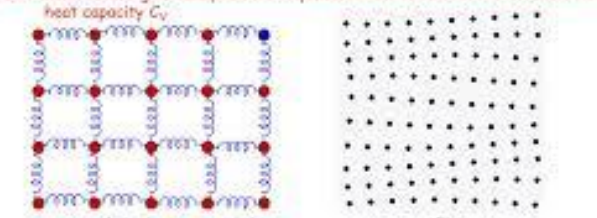
$$D[0]\rho = 0^\dagger 0\rho + \rho 0^\dagger 0 - 0\rho 0^\dagger$$



Phonons

Atomic Vibrations in Solids: phonons

Goal: understanding the temperature dependence of the lattice contribution to the heat capacity C_V

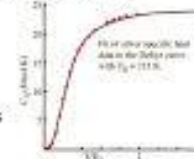


concept of the harmonic solid

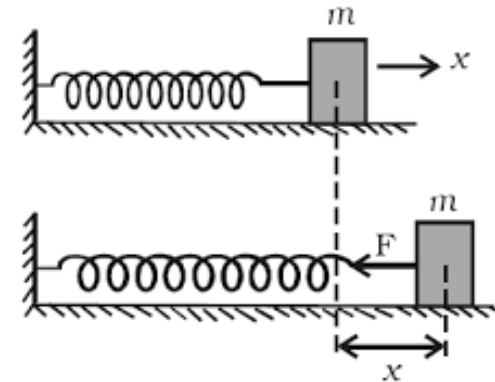
vibrational modes quantized

→

phonons with properties in close analogy to photons



Center of mass vibrations



Linear harmonic oscillator

Phonon lasers in cavity optomechanics

PRL 108, 223904 (2012)

PHYSICAL REVIEW LETTERS

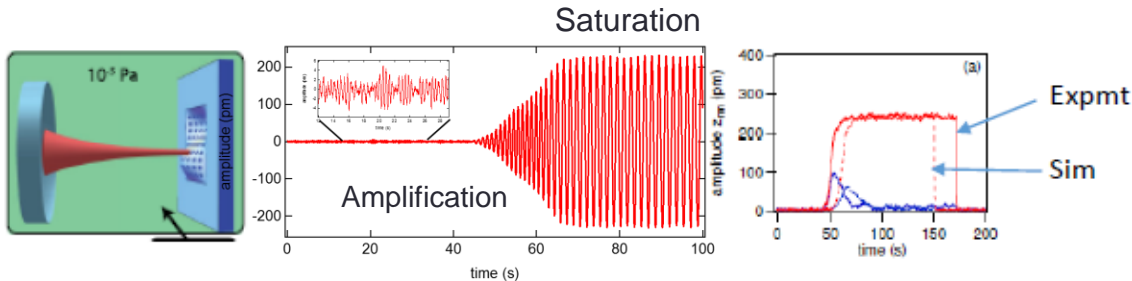
(Lots of work in
the solid state!!)

Laser-Rate-Equation Description of Optomechanical Oscillators

J. B. Khurgin,¹ M. W. Pruessner,² T. H. Stievater,² and W. S. Rabinovich²

“Optomechanical systems in which optically furnished gain enables self-sustained mechanical oscillation are properly called **phonon lasers**.”

Mode competition and Anomalous Mode cooling in a Multimode Phonon Laser
- Kemiktarak et. al, PRL **113**, 030802(2014)



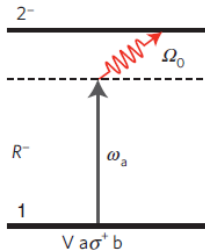
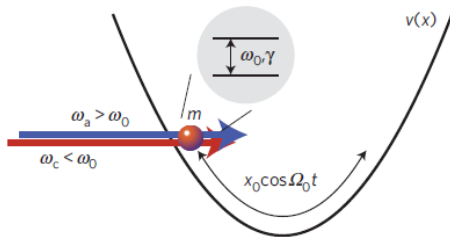
*H. Jing, et al.
PRL **113**, 053604 (2014)

*D. Navarro-Urrios et al.,
J. Opt. **18**, 094206 (2016)

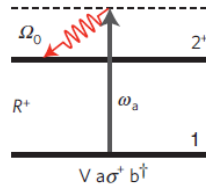
*Phonon lasers in cavity optomechanics, K. Vahala et al.,
OSA Technical Digest (2010).

Levitated optomechanics: Phonon laser with a trapped ion

Using a trapped Mg^+ ion



Cooling

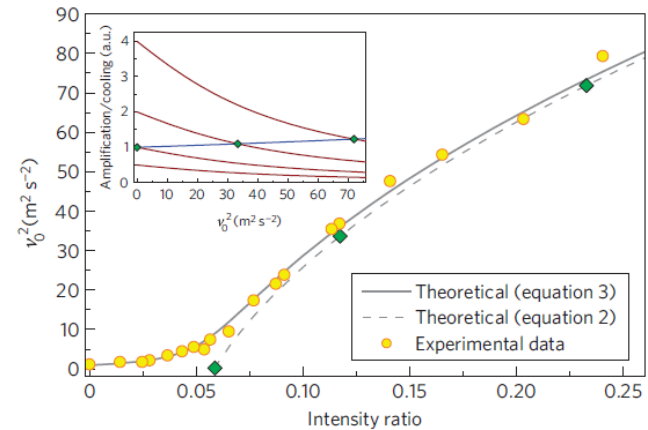
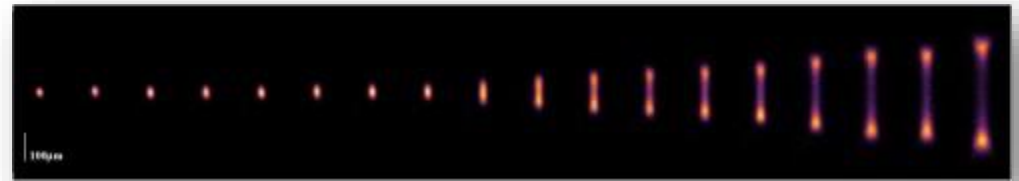


Amplification



Nobel Prize 2005

Threshold Behavior

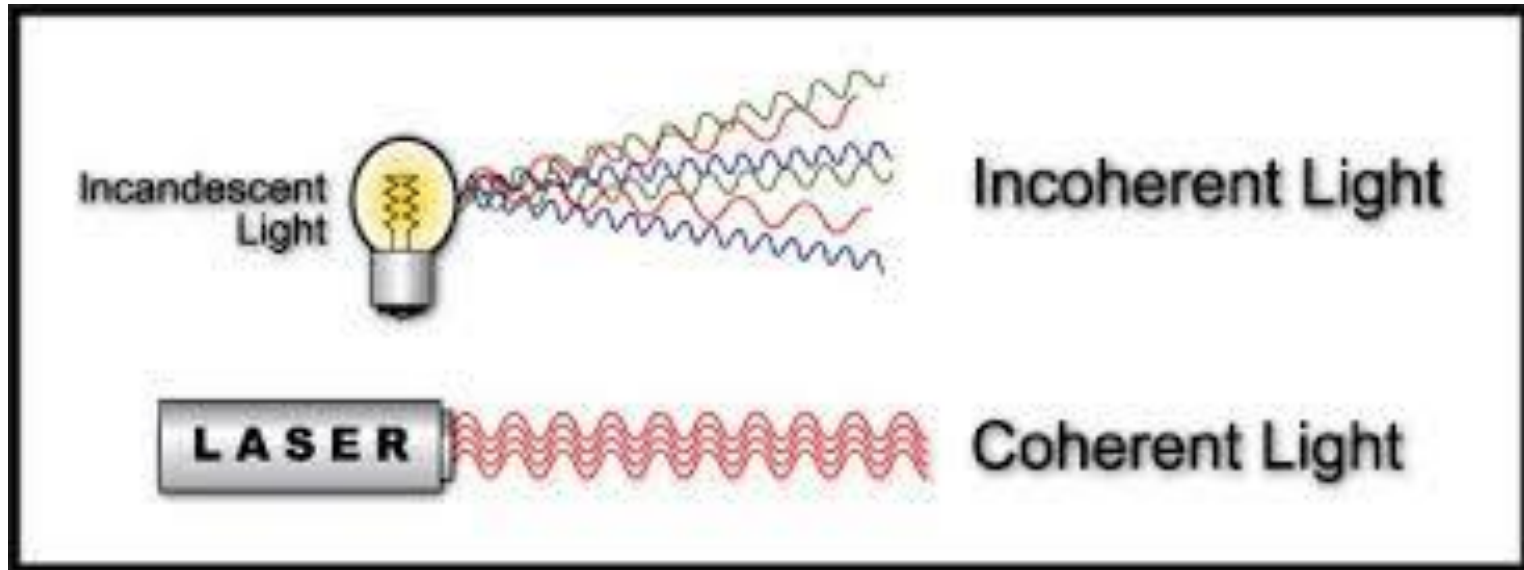


A phonon laser, K. Vahala et. al, Nature Physics **5**, 682 (2009)

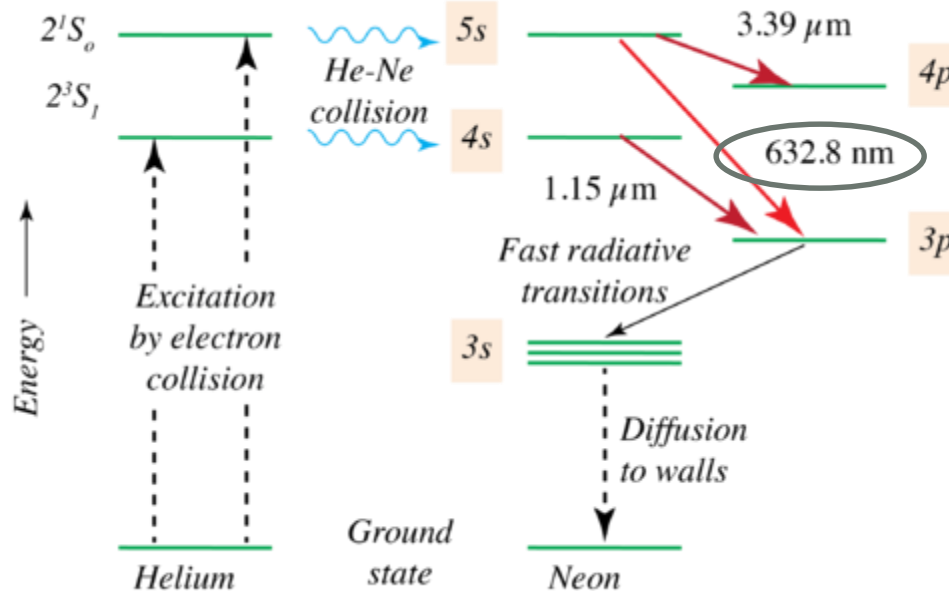
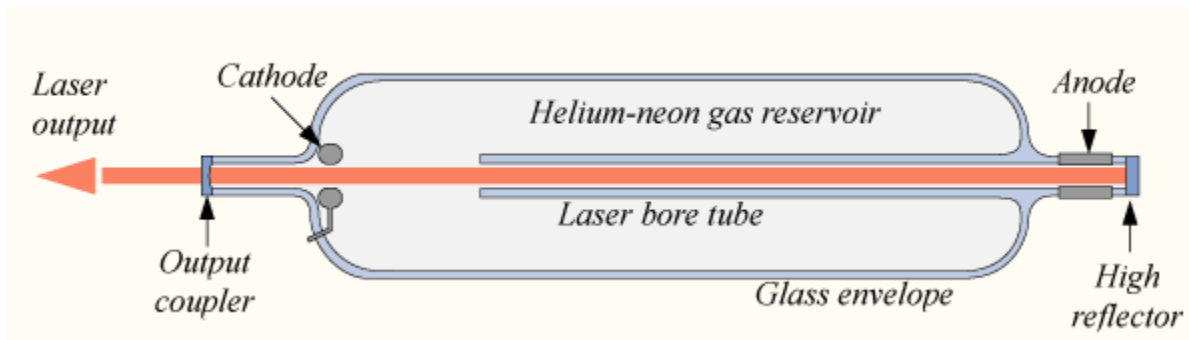
Phonon lasing from optical frequency comb... Michael Ip et al., PRL **121**, 04320 (2018)



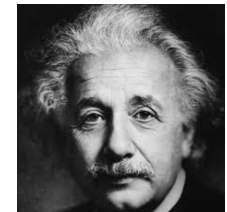
Optical laser



Optical laser



GAIN MEDIUM



What is a (phonon) laser?

Nature Photonics

Optical **L**aser Checklist
(March 2017)

- Stimulated emission
- Threshold behavior of output
- Linewidth narrowing
- Coherence
- Polarization
- Output beam above threshold

Exceptions:

1. *Light amplification without stimulated emission...*, H. Wiseman, PRA **60**, 4083 (1999).
2. *Thresholdless nanoscale coaxial lasers*, M. Khajavikhan et al. Nature **482**, 204 (2012).

.....

Note: There is no explicit mention of a quantized gain medium

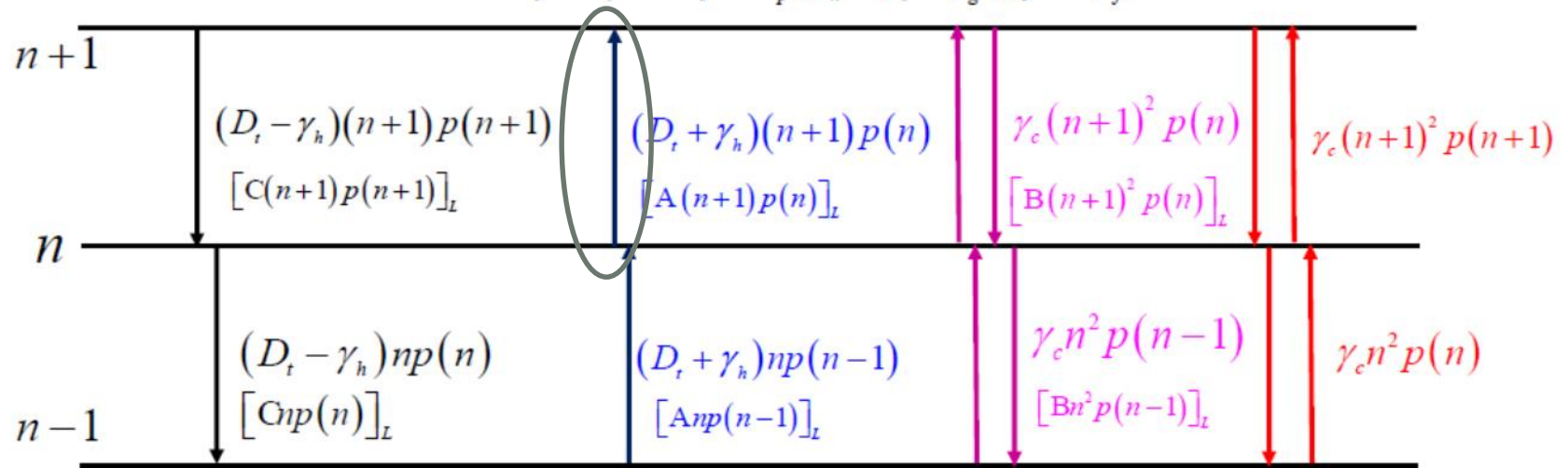
*In principle, the last two can also be demonstrated, but need more experimental work.



Stimulated emission

$$\dot{p}(n) = -(D_t + \gamma_h)(n+1)p(n) + (D_t + \gamma_h)np(n-1) + \gamma_c(n+1)^2 p(n) - \gamma_c n^2 p(n-1) \\ - (D_t - \gamma_h)np(n) + (D_t - \gamma_h)(n+1)p(n+1) + \gamma_c(n+1)^2 p(n+1) - \gamma_c n^2 p(n)$$

$$D_t = A_t + 2\Gamma_l + D_p; \gamma_h = \gamma_l - \gamma_g; \gamma_c = 6\gamma_f$$



* *Quantum Optics* by Scully and Zubairy



Phonon dynamics

$$\langle N \rangle = \frac{m\omega_m^2 \langle q_{rms}^2 \rangle}{\hbar \omega_m}$$

Phonon dynamics

nonlinear feedback cooling

linear heating

gas damping

gas scattering +
light scattering +
feedback backaction

$$\langle \dot{N} \rangle = -2J\langle N \rangle^2 + (\gamma_l - \gamma_g - J)\langle N \rangle + L$$

nonlinearity

gain

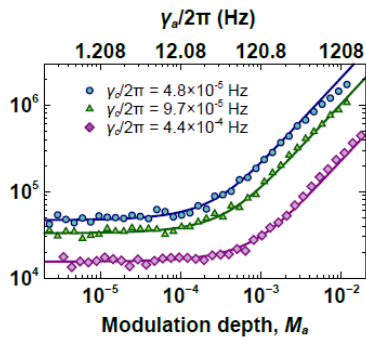
loss

fluctuations

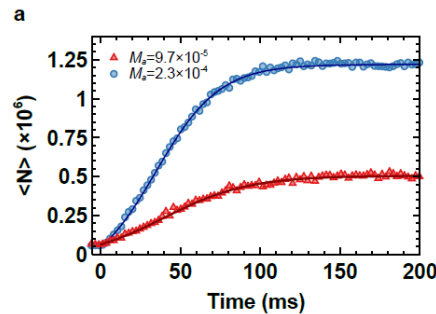
Analogous to photon number equation near optical laser threshold !!

(For class A lasers)

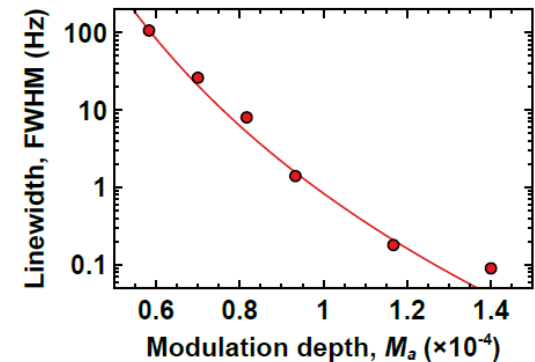
Threshold



Phonon number transient



Linewidth narrowing



$$\langle N(t) \rangle = -\frac{(24\gamma_c + 2\gamma_g - 2\gamma_a)}{48\gamma_c} + \frac{1}{24\gamma_c\tau} \tanh \left[\frac{t}{\tau} + \theta \right]$$

* Quantum Optics by Scully and Zubairy

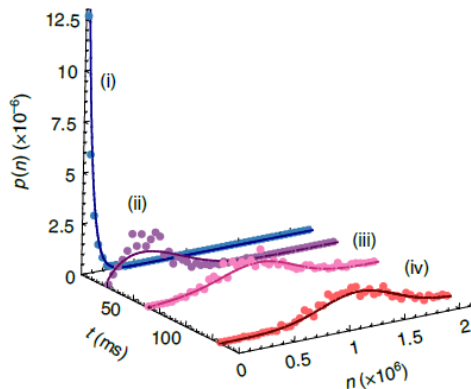


Phonon statistics

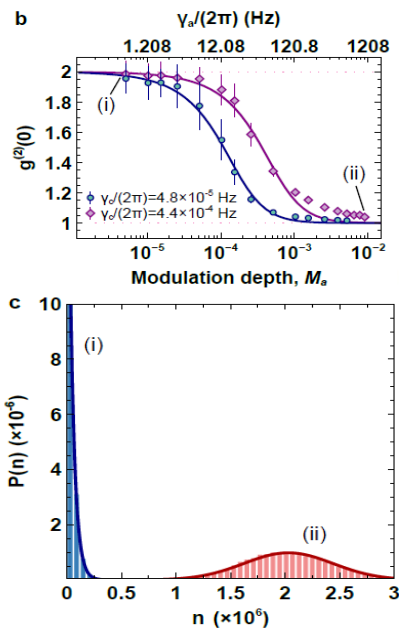
Time transient

Coherence: $g^2(0)$

Phase space portrait



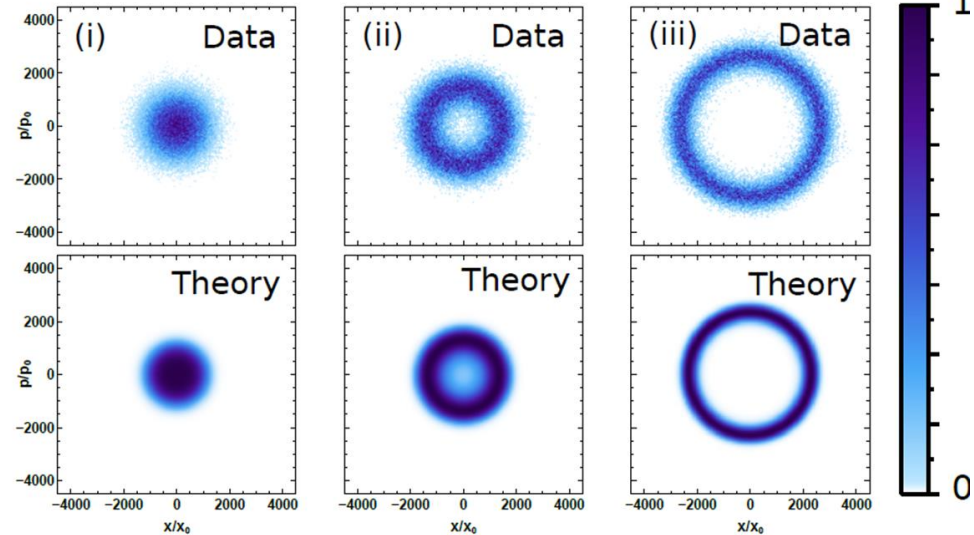
Boltzmann
distribution



Subthermal number squeezing
(Poissonian for lower pressures)

Brownian

Coherent



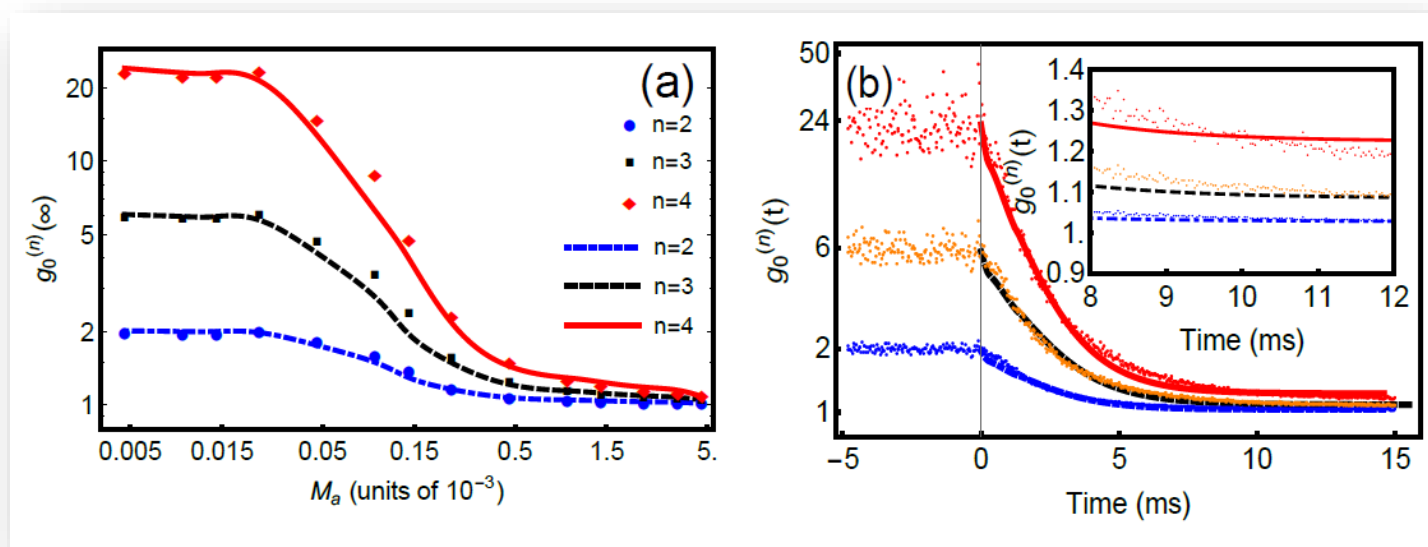
P function



More recent work: Higher order correlations

Equal time correlation functions

$$g^{(n)}(t, 0) = \frac{\langle \hat{a}^{\dagger n}(t) \hat{a}^n(t) \rangle}{\langle a^\dagger(t) a(t) \rangle^n}$$

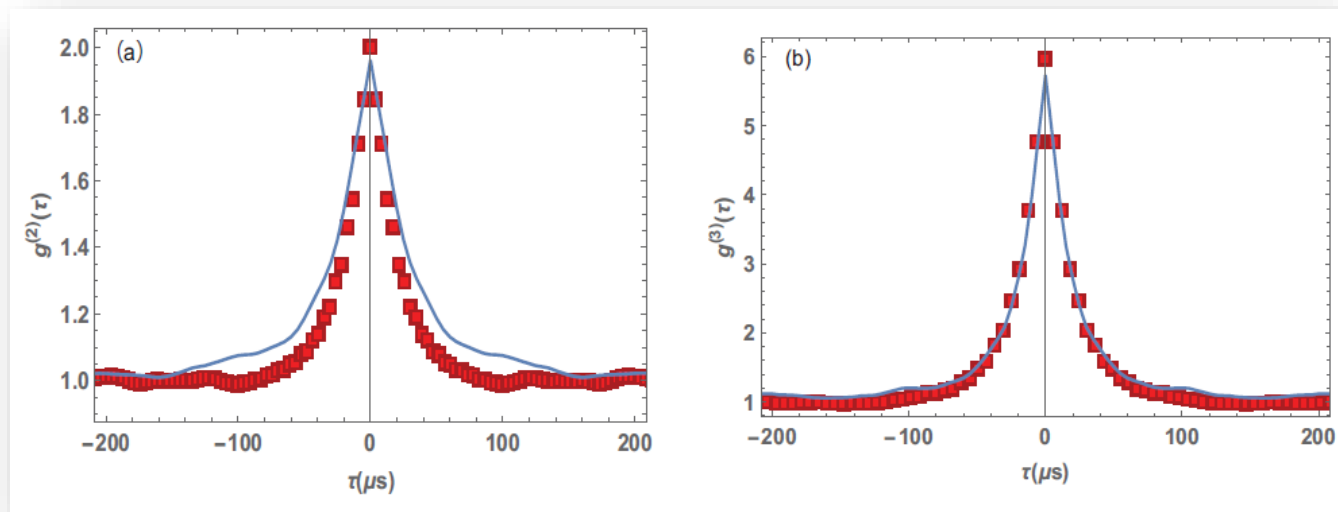


More recent work: Higher order correlations

Unequal time
correlation
functions

$$g^{(2)}(t, \tau) = \frac{\langle \hat{a}^\dagger(t) \hat{a}^\dagger(t + \tau) \hat{a}(t + \tau) \hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t) \hat{a}(t) \rangle^2}$$

$$g^{(3)}(t, \tau_1, \tau_2) = \frac{\langle \hat{a}^\dagger(t) \hat{a}^\dagger(t + \tau_1) \hat{a}^\dagger(t + \tau_2) \hat{a}(t + \tau_2) \hat{a}(t + \tau_1) \hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t) \hat{a}(t) \rangle^3}$$



$$\tau_1 = \tau_2 = \tau$$

More recent work: Higher order correlations

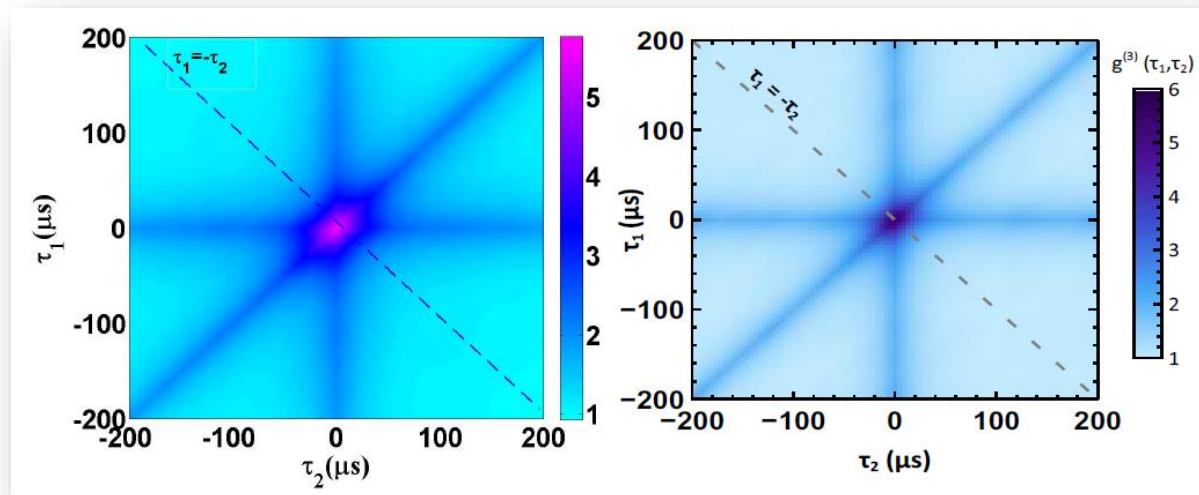
Unequal time
correlation
functions

$$g^{(3)}(t, \tau_1, \tau_2) = \frac{\langle \hat{a}^\dagger(t) \hat{a}^\dagger(t + \tau_1) \hat{a}^\dagger(t + \tau_2) \hat{a}(t + \tau_2) \hat{a}(t + \tau_1) \hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t) \hat{a}(t) \rangle^3}$$

Theory

Experiment

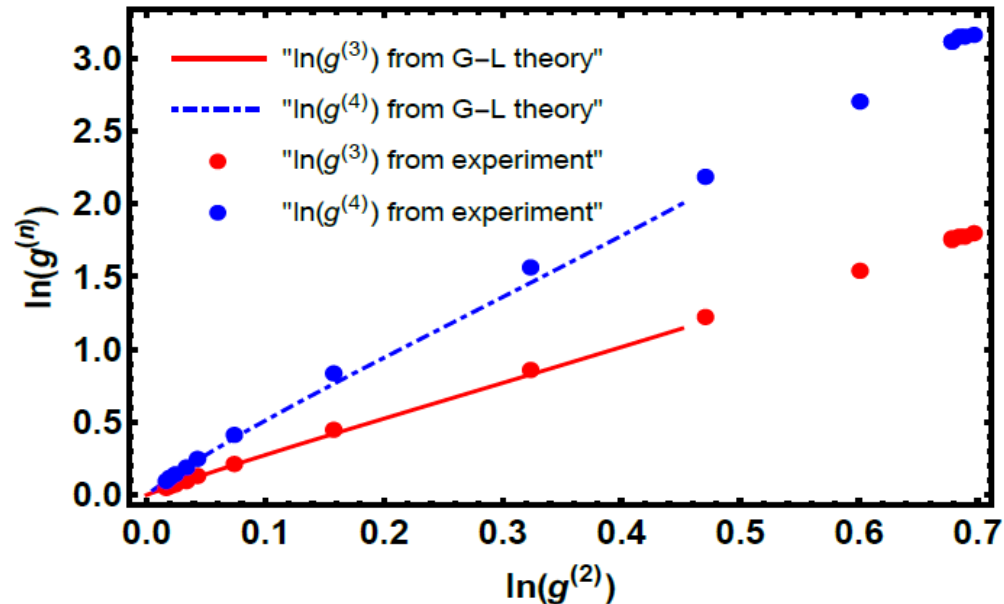
$g^{(3)}(\tau_1, \tau_2)$



$$\tau_1 \neq \tau_2$$



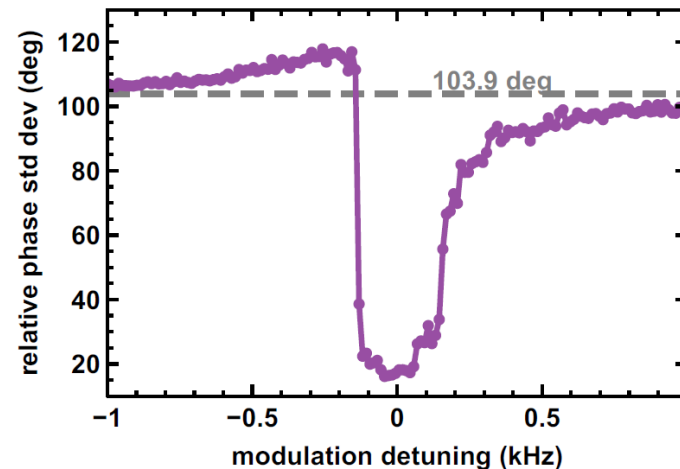
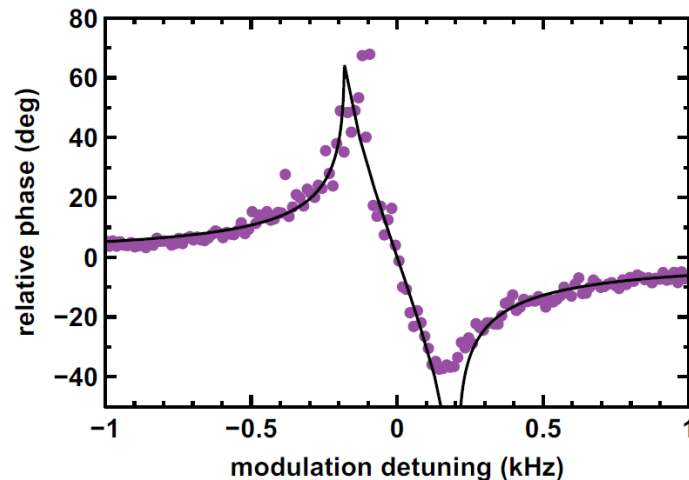
Comparison with Landau-Ginsburg theory



*Probing the Ginzburg-Landau potential for lasers using higher-order photon correlations, N. Takemura et al., arxiv:1908.08679v1 (2019)

More recent work: Injection locking

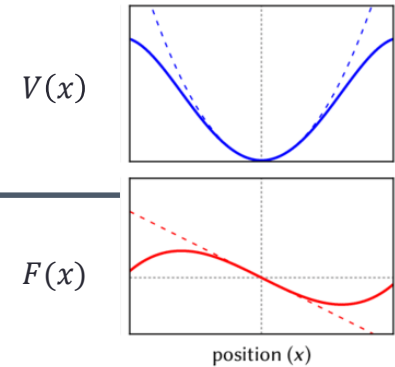
The phase of the free-running phonon laser can be locked to an external modulation



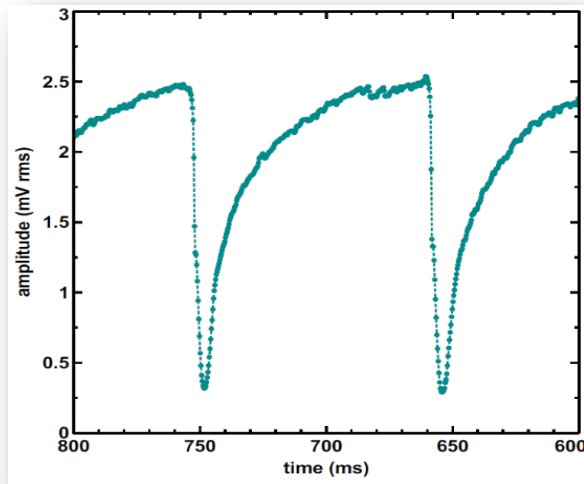
- *Injection locking of a trapped ion phonon laser*, S. Knunz et al. PRL **105**, 013004 (2010).
- PhD Thesis, Jan Gieseler.



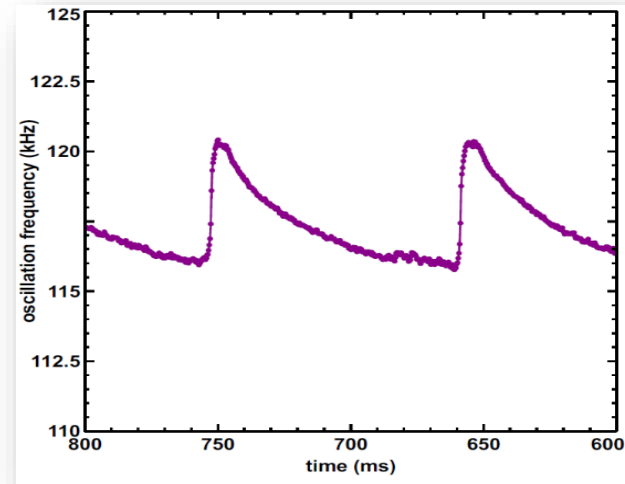
More recent work: Q-switching ??



Amplitude



Frequency



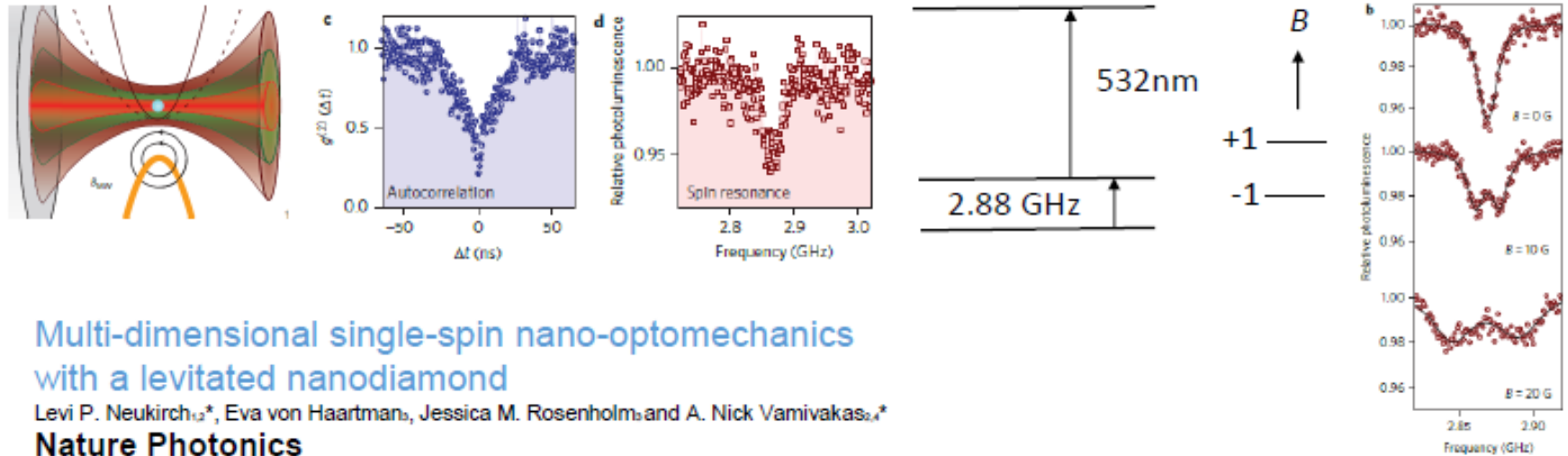
*Observed in the regime where mechanical amplitude $\sim \omega_0$
Looks like the Duffing nonlinearity could be responsible

Pulsed phonon lasing in trapped ions

Y. Xie, W. Wan, H. Y. Wu, F. Zhou, L. Chen and M. Feng
Phys. Rev. A **87**, 053402 (2013).



Levitated quantum emitters: spin+mechanics



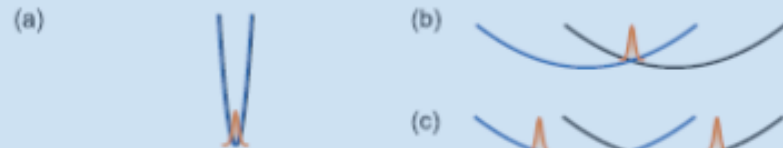
Multi-dimensional single-spin nano-optomechanics with a levitated nanodiamond

Levi P. Neukirch^{1,2*}, Eva von Haartman³, Jessica M. Rosenholm³ and A. Nick Vamivakas^{1,4*}

Nature Photonics

Large quantum superpositions of a levitated nanodiamond, Yin et al., PRA **88**, 033614 (2013)

$$H_c = -\mu \cdot B_z \sim S_z \frac{\partial B}{\partial z} Q_z$$



Single and two-mode mechanical squeezing of an optically levitated nanodiamond via dressed-state coherence, W. Ge and MB, NJP **18**, 103002 (2016)

*Generating spin squeezing states....PRB **94**, 205118 (2016), K. Xia and J. Twamley



Cavity optomechanics: charged dielectrics

PRL 114, 123602 (2015)

PHYSICAL REVIEW LETTERS

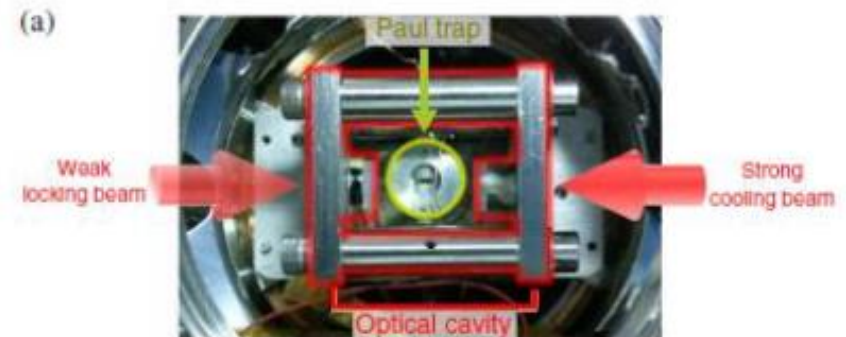
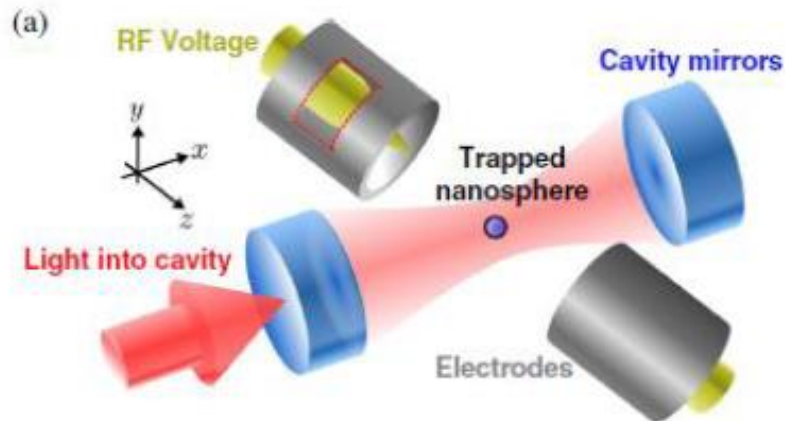
week ending
27 MARCH 2015

Cavity Cooling a Single Charged Levitated Nanosphere

J. Millen, P.Z. G. Fonseca, T. Mavrogordatos, T. S. Monteiro, and P.F. Barker*

Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom

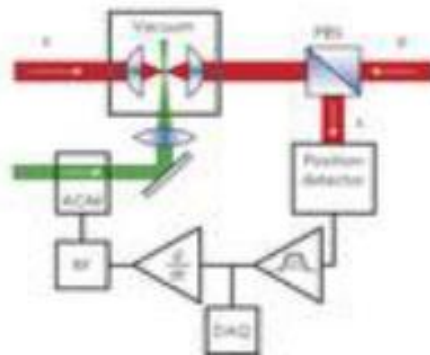
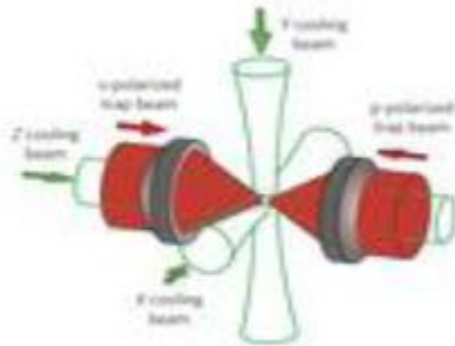
(Received 31 December 2014; published 27 March 2015)



Larger objects: Mie regime

$$r = 2\mu\text{m}$$

300K \rightarrow 1 mK



Millikelvin cooling of an optically trapped microsphere in vacuum

[T. Li](#), [S. Kheifets](#) & [M. G. Raizen](#)

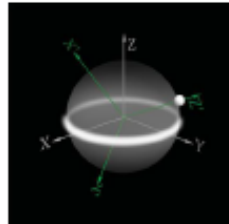
Nature **7**, 527 (2011)

Cooling the Motion of Diamond Nanocrystals in a Magneto-Gravitational Trap in High Vacuum

J. -F. Hsu et. al, Scientific Reports **6**, 30125 (2016)

Rotation

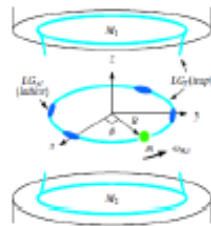
Rotational motion



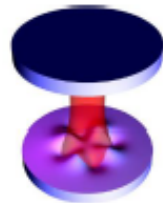
- J. T. Rubin and L. I. Deych
PRA **84**, 023844 (2011)
- L. Deych and V. Shuvayev
PRA **92**, 013842 (2015)

Focus: The Fastest Spinners

July 20, 2018•
Physics 11,73.



- MB, JOSA B **32**, B55 (2015)



- Briant et al., PRA **68**, 033823 (2003)
- H. Shi and M. Bhattacharya,
J. Phys. B, **46** 151001 (2013)

*GHz Rotation of
an Optically Trapped
Nanoparticle in
Vacuum*

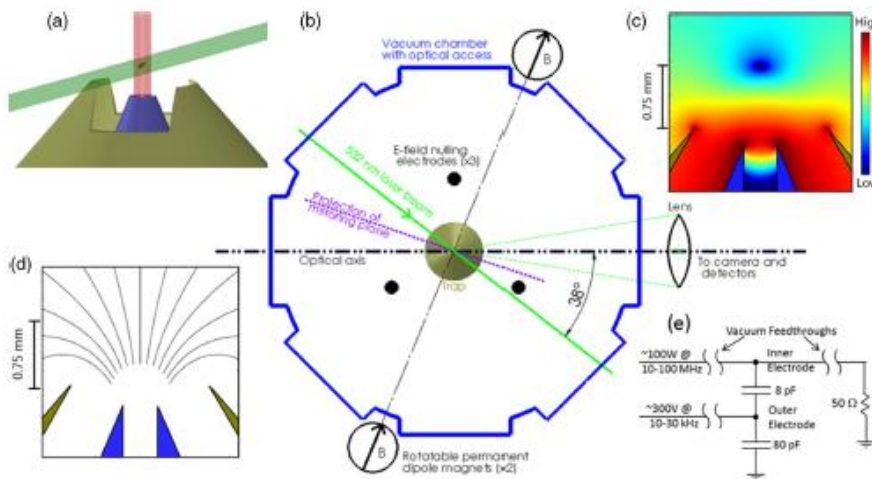
René Reimann et. al
PRL **121**, 033602 (2018)

*Optically Levitated
Nanodumbbell Torsion
Balance and GHz
Nanomechanical Rotor,*
Jonghoon Ahn et. al
PRL **121**, 033603 (2018)

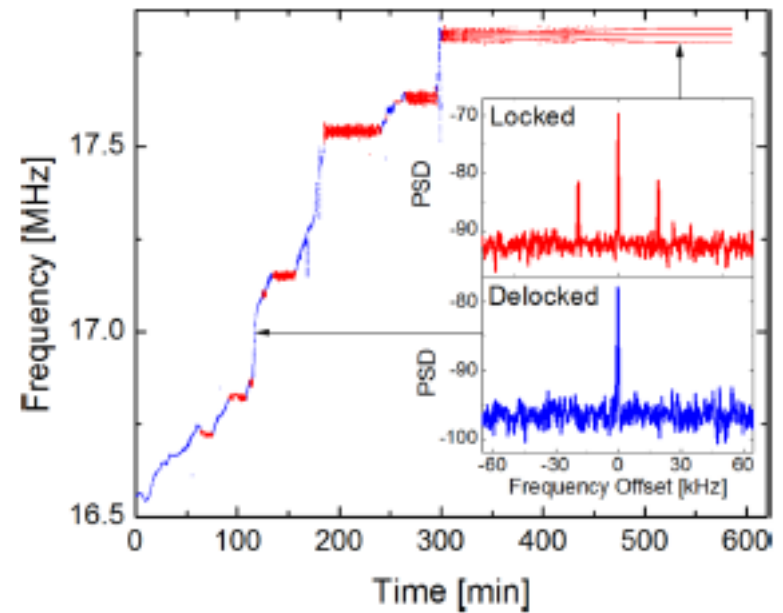


Rotation – Graphene nanoplatelets

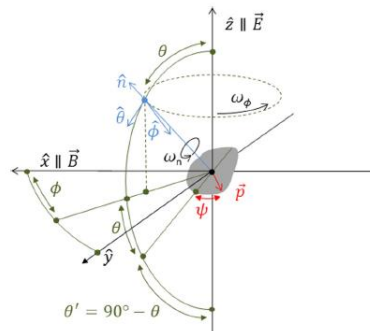
B. E. Kane et. al, Phys. Rev. B **82**, 115441 (2010), Phys. Rev. B **96**, 035402 (2017)



Levitation

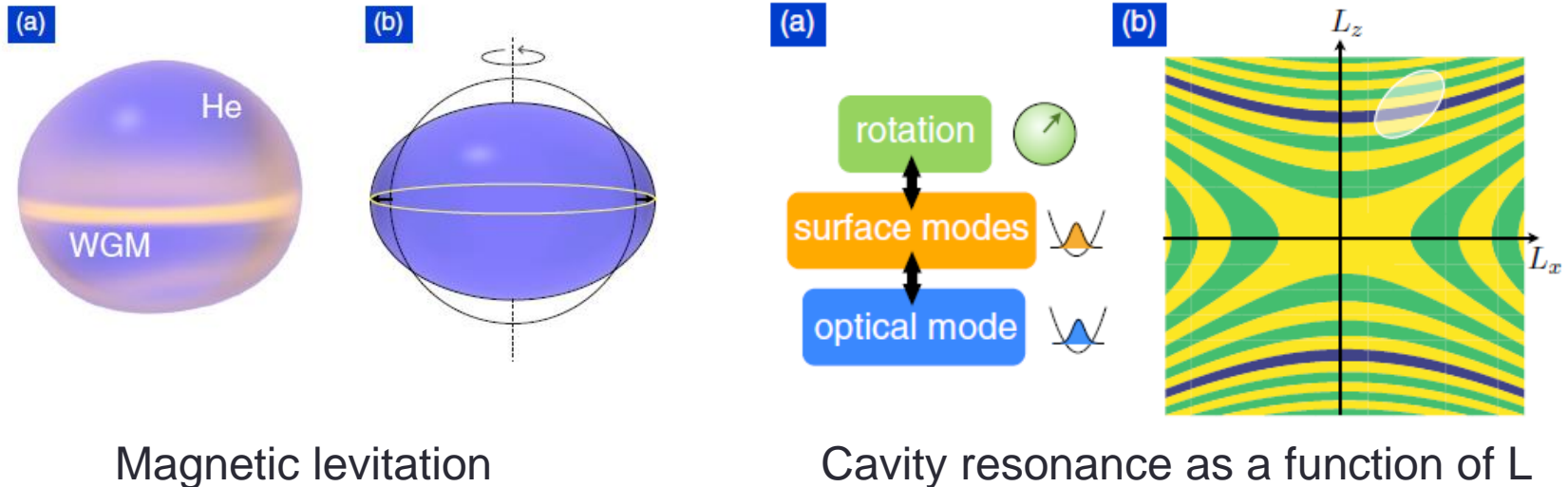


Frequency locking



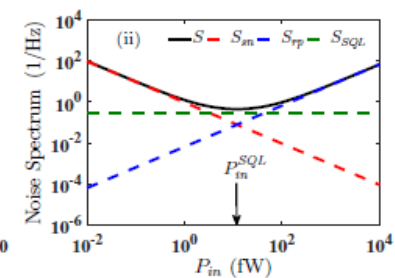
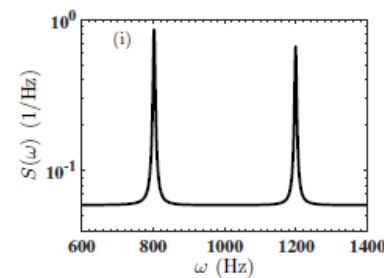
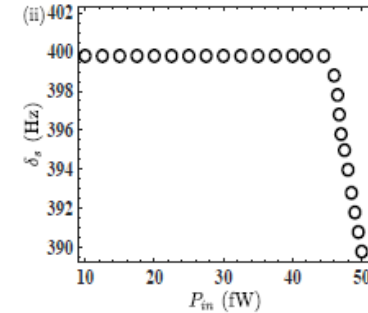
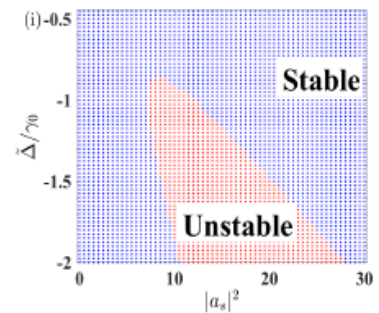
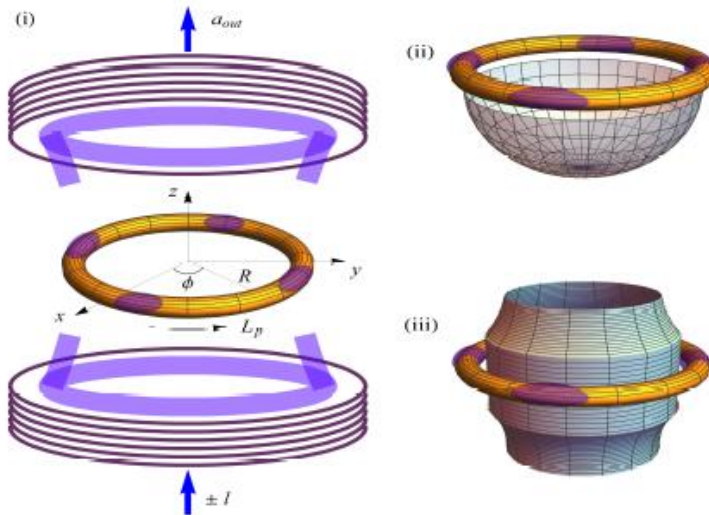
Rotation – Cavity optomechanics in a levitated helium drop

L. Childress et. al, PRA **96**, 063842 (2017)



$$\hat{H}_{\text{QND}} = \hbar g_L \left(\frac{\hat{L}_z}{\hbar} \right)^2 \hat{a}^\dagger \hat{a} \quad \text{Non-demolition measurement of angular momentum}$$

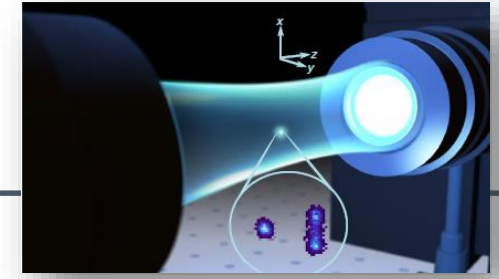
Rotation – Cavity optomechanics with a rotating BEC



*P. Kumar, T. Biswas, K. Feliz, R. Kanamoto, M. S. Chang, A. K. Jha and MB,
In review.



Conclusions



Mg^+ ion

Nanoparticle

Cavity optomechanics



$10^{-25}kg$

$10^{-18}kg$

$10^{-9}kg$

Atomic

Mesoscopic

Microscopic

1. Generate coherent states with good fidelity: Schrodinger cat state.
2. Nonlinear force measurement across the lasing transition.
3. Mie particles.
4. Our technique is very general and applies to any harmonic oscillator. All it requires is position measurement + feedback.



THANKS !!!

Optical Tweezer Phonon Laser: FAQs

Q. What constitutes the phonon in your case ?

A. The center of mass oscillations of the nanoparticle along one direction in space.



Optical Tweezer Phonon Laser: FAQs

Q. Can standard cavity optomechanics theory be used to describe your system?

A. No.

- There is no cavity.
- The light-matter interaction is single-pass.
- ...



Optical Tweezer Phonon Laser: FAQs

Q. Why do you use a quantum model? There is nothing quantum in the experiment...??

A. The theory came first, is valid in the quantum regime, and needs to be verified in that limit.

A fully quantum theory is required for predicting **ground state occupation**.

The quantum model helps us establish the presence of **stimulated emission**. In that sense something quantum *is* happening in our phonon laser.



Optical Tweezer Phonon Laser: FAQs

Q. Why do you have both heating and cooling?

A. → Heating supplies gain

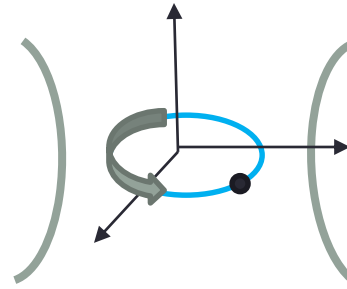
→ Cooling supplies nonlinearity



Optical Tweezer Phonon Laser: FAQs

Q. How about polarization and an output beam ?

A. Polarization



B. Output beam

We are in the zero dimensional limit of no vibronic output coupling, but...

