A Comparative Anatomy of the Atmosphere and Ocean:

Waves, Turbulence and Thermodynamics

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Outline

- What makes the ocean overturn?
- What is the role of large-scale turbulence?
- The atmosphere:
  - A heat engine, in part
  - Role of waves and turbulence
- The ocean:
  - Not a heat engine
  - Role of waves and turbulence

Along the way we'll learn some meteorology and oceanography...

Why bother? Let's just compute it! Degrees of Freedom

- Reynolds number

  $$UL/\nu = 10 \times 10^6/10^{-5} = 10^{12}$$

- Equivalent to viscous scale of about 1mm

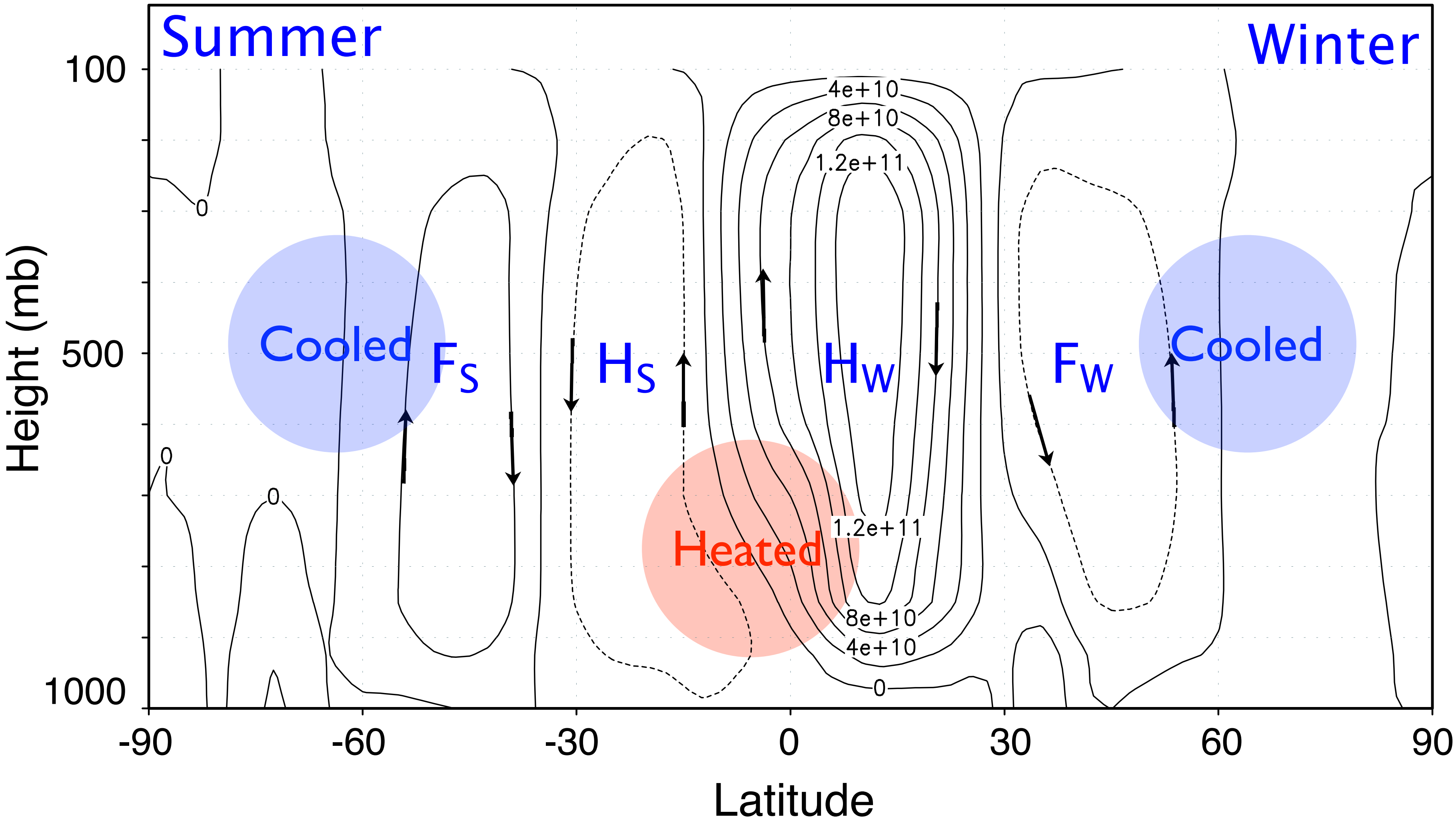
- Degrees of freedom in atmosphere

  $$N \sim \mathrm{Re}^{9/4} \approx 10^{27}$$

Degrees of Freedom (more sensibly?)

- Large-eddy scales:
  - Atmosphere: $L_e \sim 1000\,\mathrm{km}$
  - Ocean: $L_e \sim 100\,\mathrm{km}$
- To resolve ocean eddies globally need 1000x1000x100 gridpoints - marginal at best, and impossible on a routine basis. Atmospheric eddies are well resolved (so there is less excuse for our lack of understanding!).
- Emphasizes the need to understand as well as simulate....

Lower Atmosphere Overturning Circulation



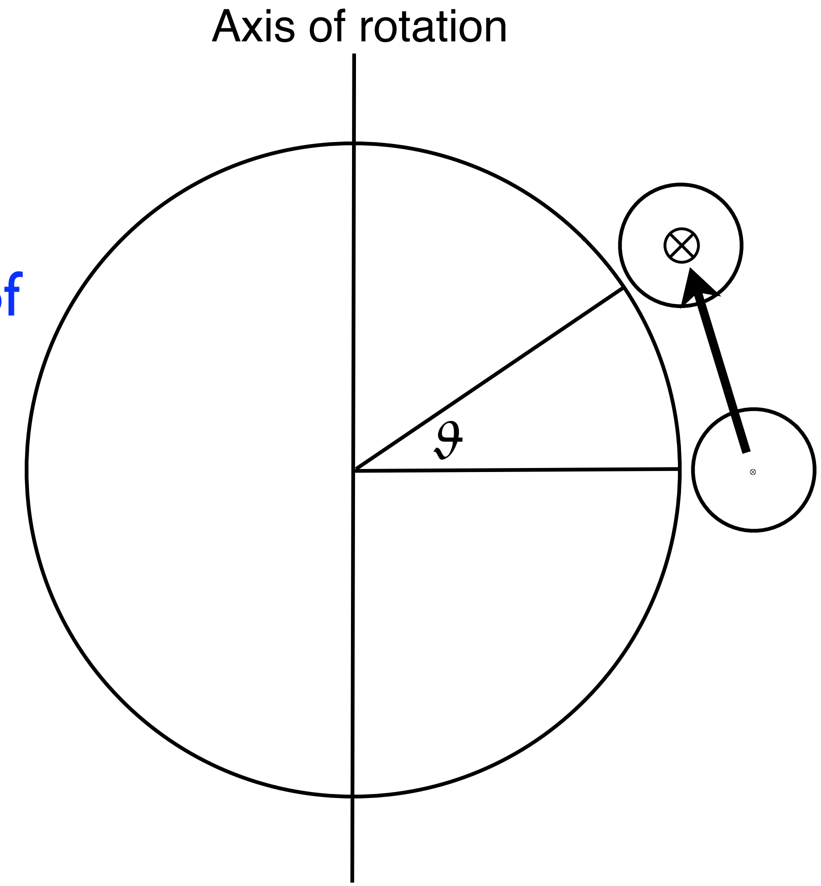
A heat engine in part - heated at equator, cooled at pole, with the circulation produced by that differential heating. Atmosphere transparent to solar radiation, opaque to IR - so heating occurs at the ground, at a *lower level* than the cooling.

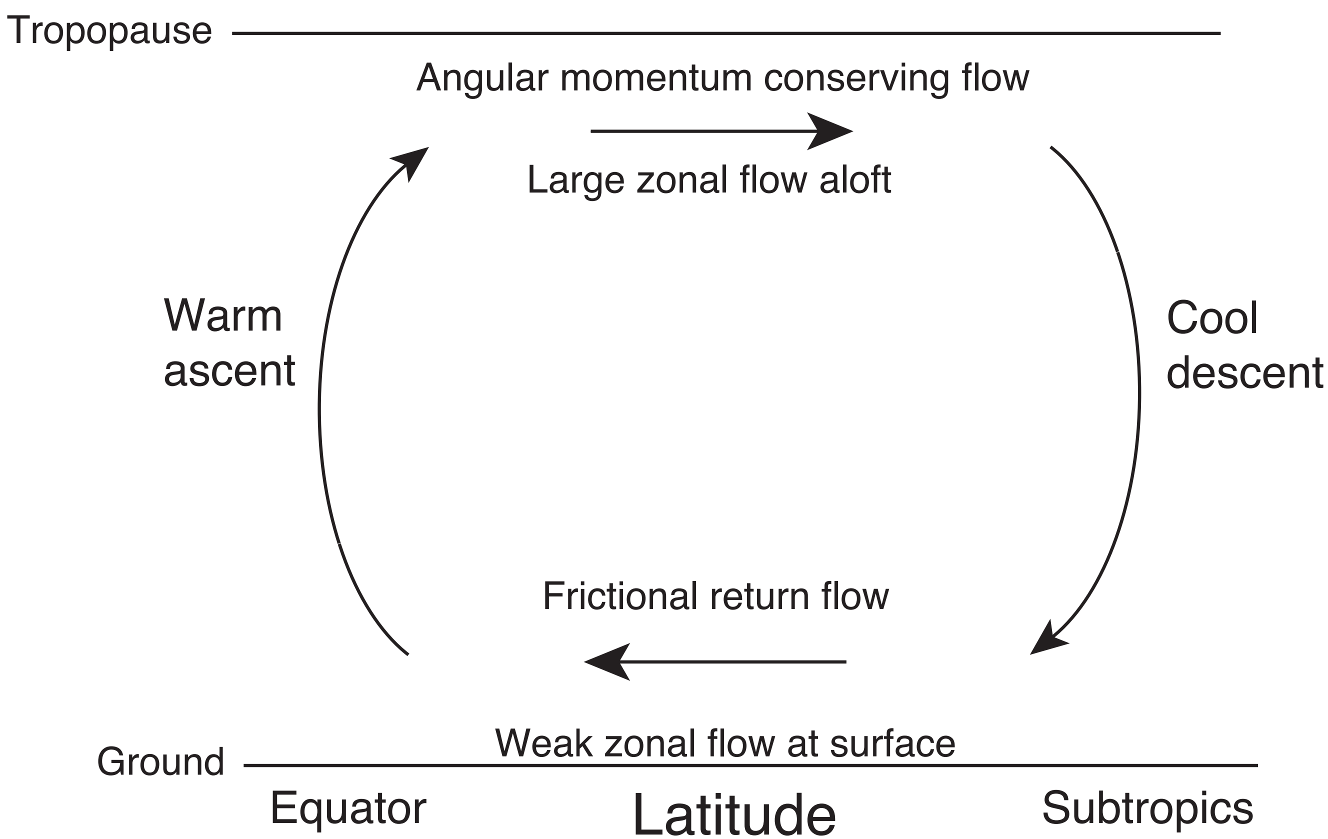
A bit like a classical thermodynamic cycle

- Atmosphere (or at least the troposphere) is heated (by solar radiation) where it is warm, and cooled (net cooling) where it is cold.
- Quasi-adiabatic transfer of heat polewards in the circulation. Hadley (1735) envisioned one big overturning cell.
- Heating creates (available) potential energy, converted to kinetic energy and ultimately `lost' to the system. KE energy dissipation is returned as heat to the system, but you can't use it to do work - could create perpetual motion and violate the 2nd law.

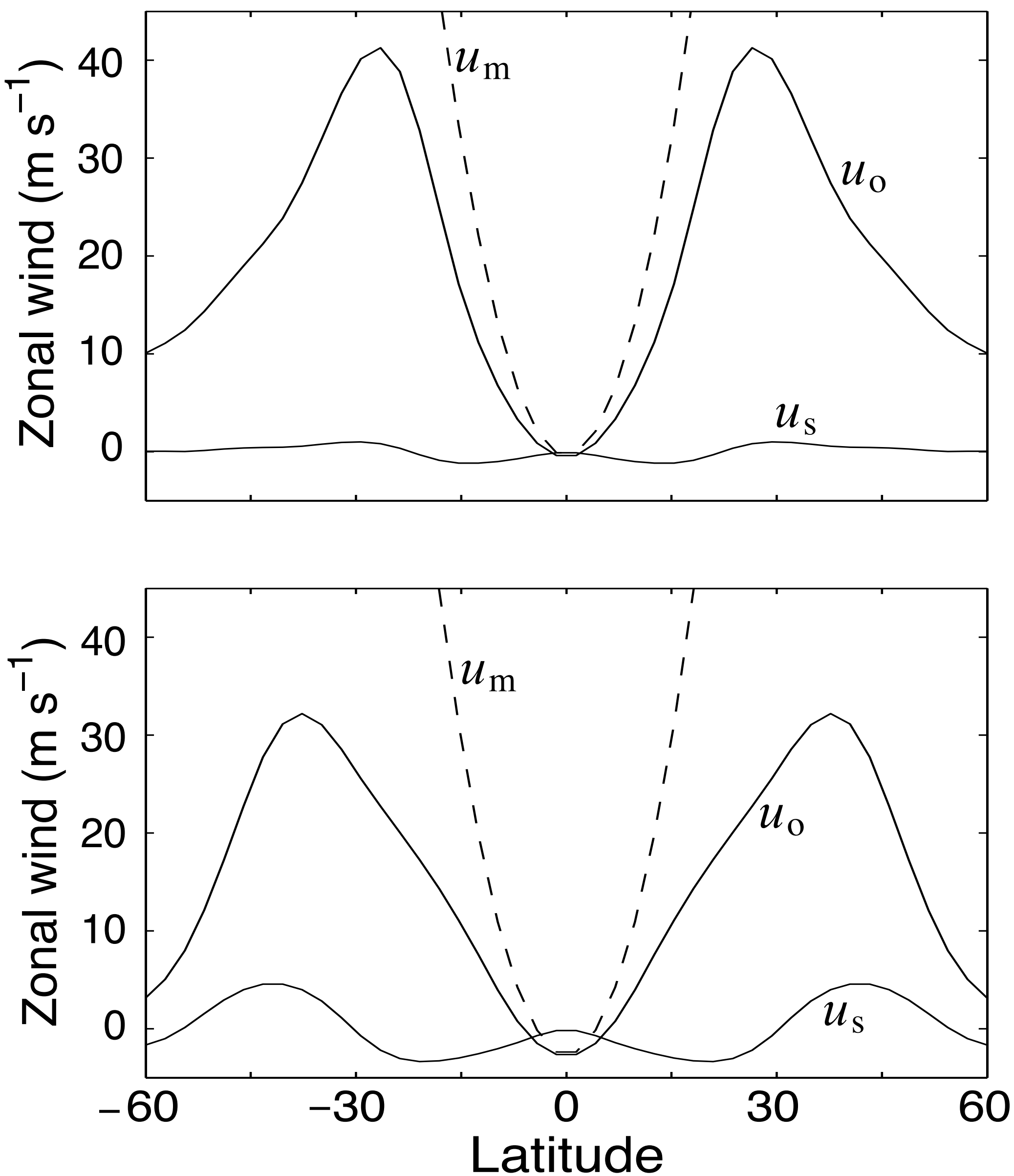
Some dynamics...
Hadley Cell

Air moving polewards moves closer to axis of rotation, and zonal flow must increase to maintain angular momentum conservation





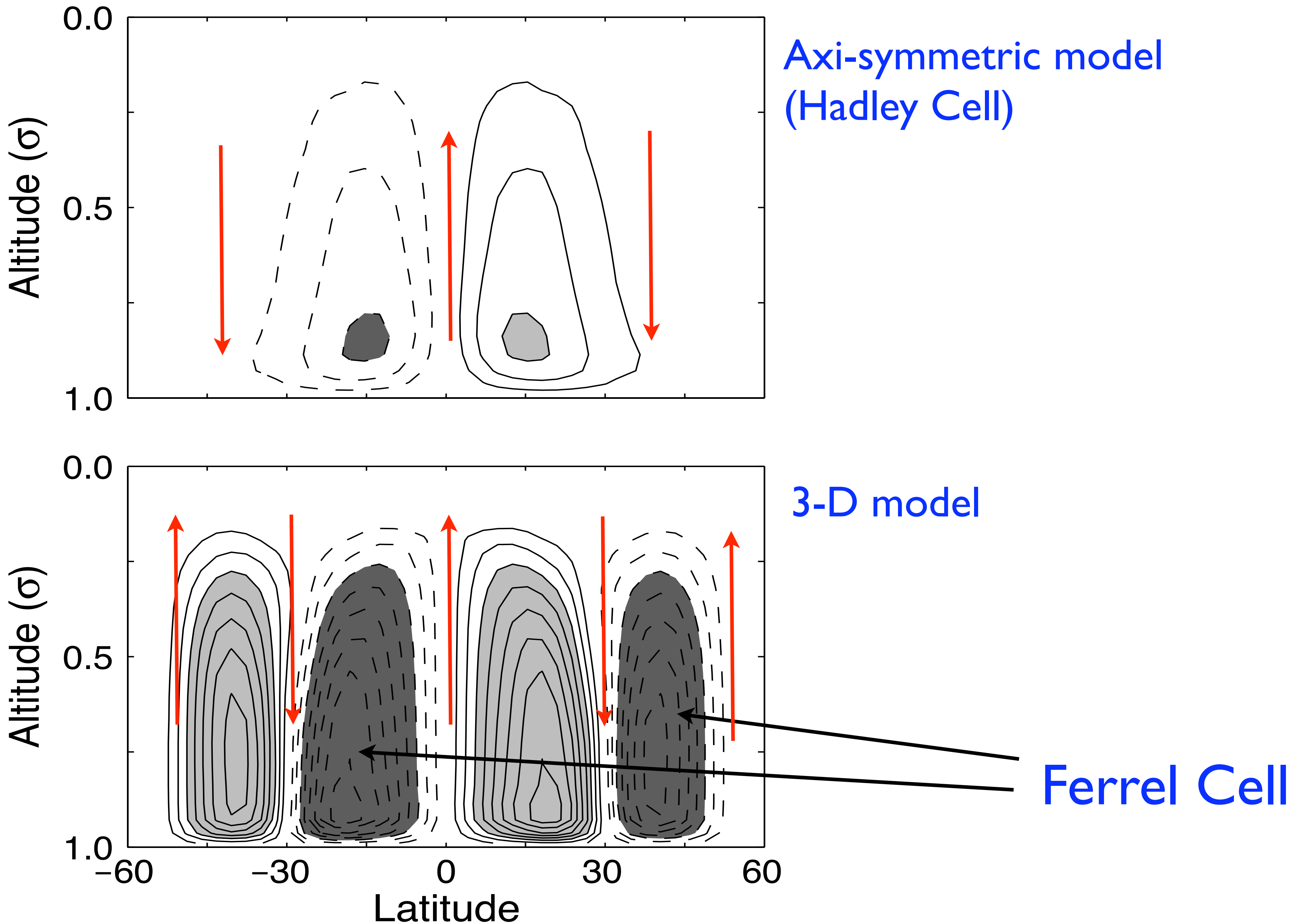
Numerical Experiments



Axi-symmetric model, nearly angular-momentum conserving

3-D model, not angular-momentum conserving, largely because of baroclinic eddies

(C. Walker)

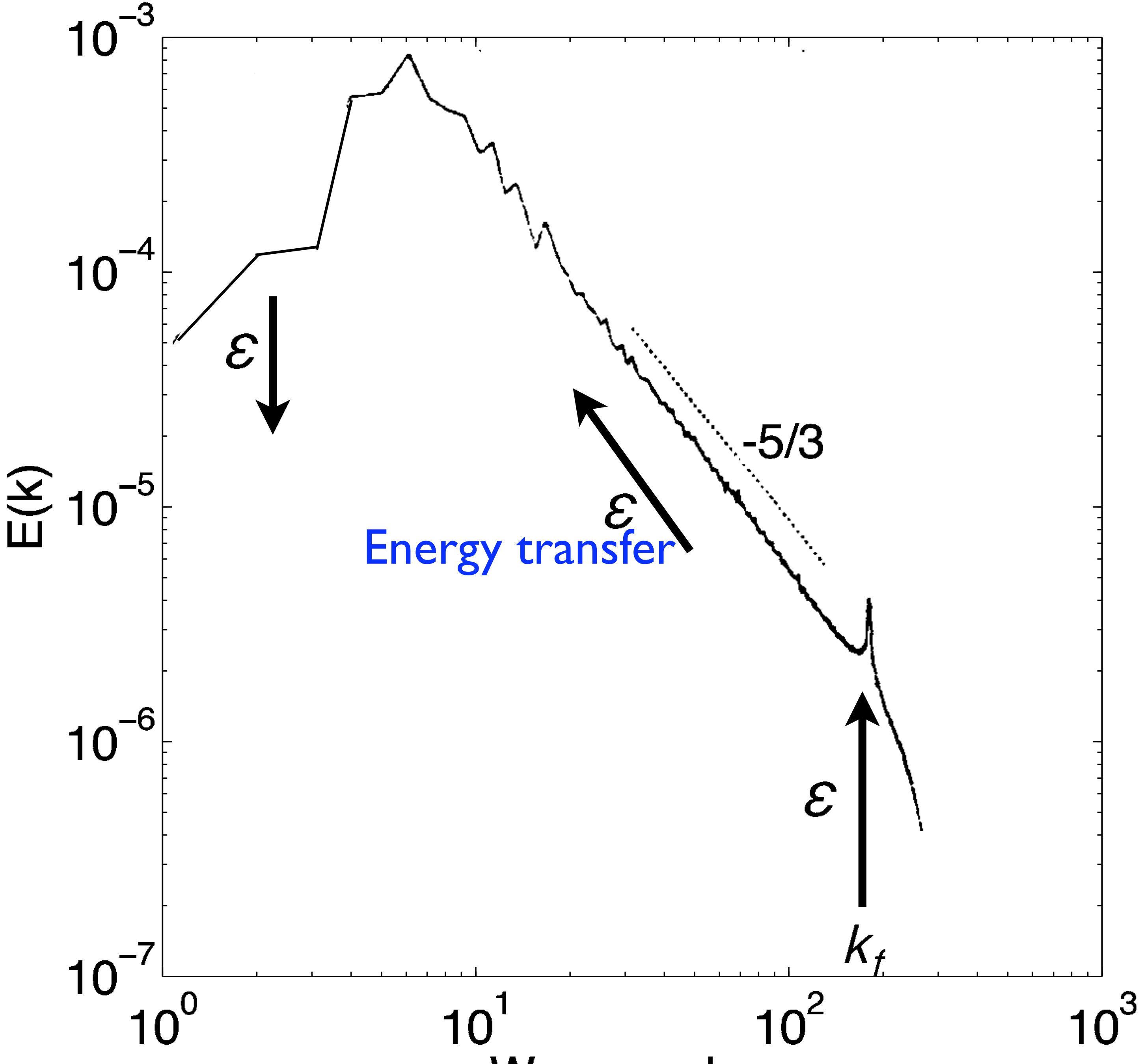

0.0
0.5
1.0
Altitude (σ)
Axi-symmetric model
(Hadley Cell)
0.0
0.5
1.0
Altitude (σ)
3-D model
Ferrel Cell
−60
−30
0
30
60
Latitude

- Why should the mid-latitude cell overturn in the wrong direction?
- Combination of waves and turbulence.

Waves and turbulence

- It is a profound consequence of Rossby waves and 2D turbulence that waves and turbulence almost inevitably co-exist in the atmosphere.
    - In 2D turbulence energy cascades to *larger* scales!
    - The larger the scale, the more dominant are Rossby waves.
    - At the largest scales, Rossby waves dominate the spectrum.

Upscale energy transfer in 2D turbulence



Maltrud & V.

Rossby wave dispersion relation

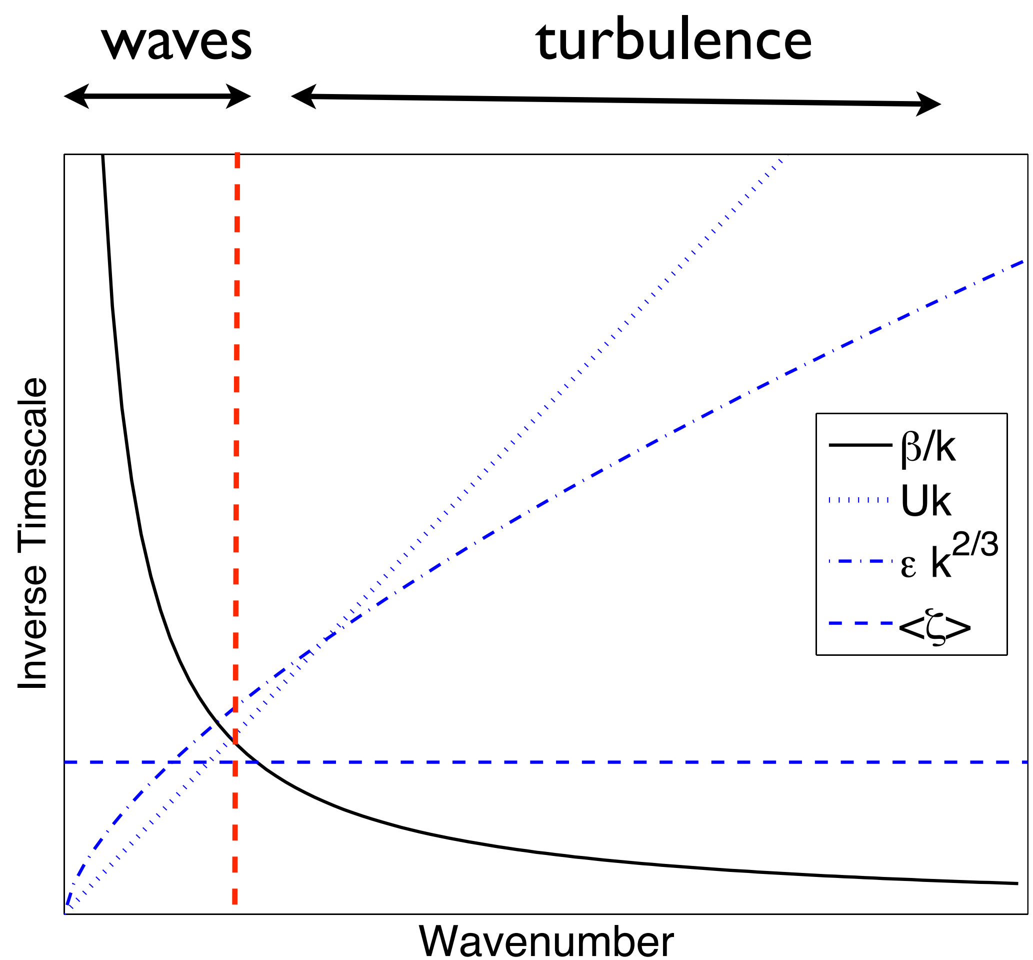
$$\omega_{Rossby} = \frac{\beta k}{(k_x^2 + k_y^2)} \sim \frac{\beta}{k}$$

Turbulence inverse timescale

$$\omega_{turb} \approx U_{rms} k$$

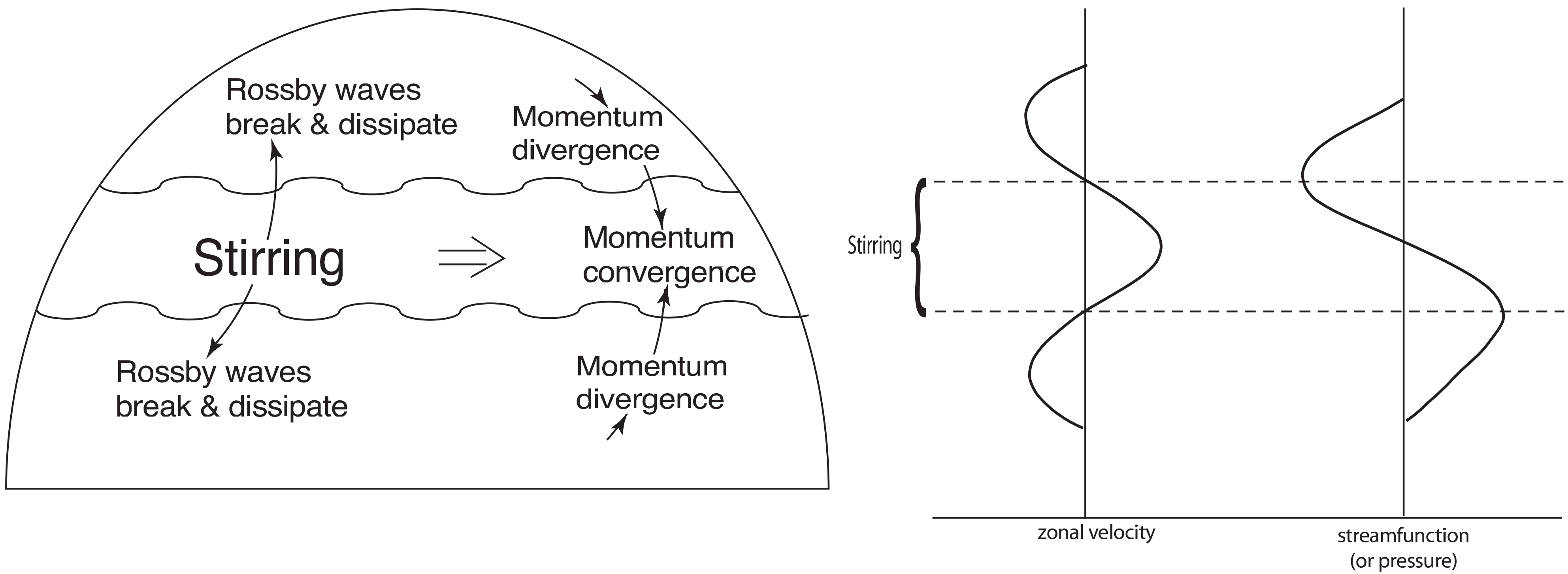
Crossover

$$\boxed{k_\beta \sim \sqrt{\frac{\beta}{U_{rms}}}}$$



Rossby waves dominate the dynamics at large scales

Schematic of generation of eddy-driven jet



Generation of Zonal Motion — Momentum Fluxes and Rossby Waves

Transport of momentum by Rossby waves,

$$\psi = C\exp[i(k_x x + k_y y - \omega t)]$$

with dispersion relation

$$\omega = ck = \overline{u}k - \frac{\beta k_x}{k_x^2 + k_y^2},$$

The meridional component of the group velocity is given by

$$c_{gy} = \frac{\partial \omega}{\partial k_y} = \frac{2\beta k_x k_y}{(k_x^2 + k_y^2)^2}.$$

Must be directed *away* from the source region (a radiation condition).
Northwards (southwards) of the source $k_x k_y$ is positive (negative).

$$u' = -\Re iCk_y e^{i(k_x x + k_y y - \omega t)}$$
$$v' = \Re iCk_x e^{i(k_x x + k_y y - \omega t)}.$$

The associated momentum flux is

$$\boxed{\overline{u'v'} = -\frac{1}{2}C^2 k_x k_y}$$

Northwards of the source Momentum flux, $\overline{u'v'} < 0$.

Southwards of the source Momentum flux, $\overline{u'v'} > 0$.

Momentum *converges* in the region of the stirring.

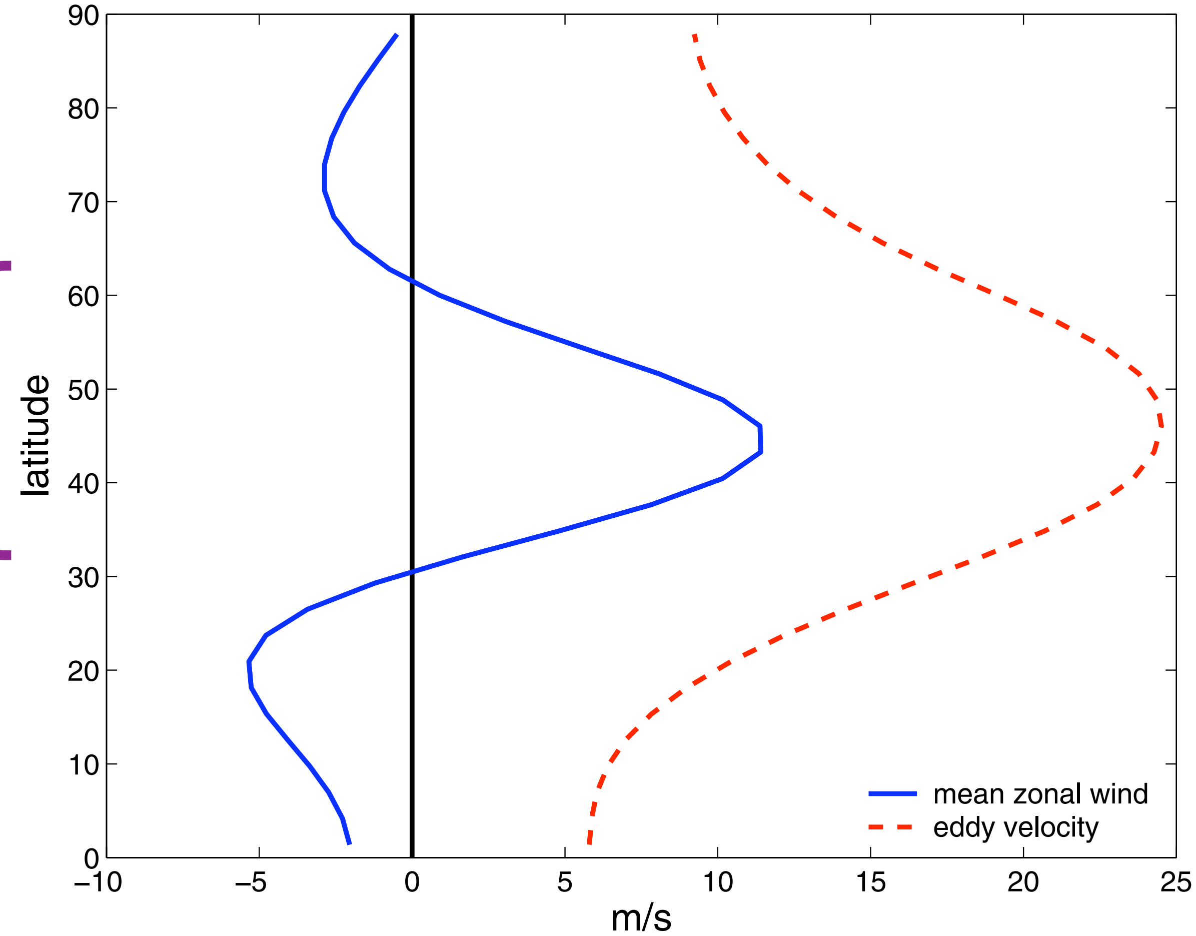
$$\boxed{\frac{\partial}{\partial y}\overline{u'v'} < 0}$$

Diffusion?

- A simple consequence is that in Rossby-wave turbulence momentum is transported upgradient!
- Turbulent diffusion (at least of momentum) simply does not happen.
- We shouldn't be surprised, because of pressure forces.
- We can diffuse other quantities (potential vorticity) but that is a story for another day.

Time & Zonally Averaged Flow in Barotropic model

$$\frac{\partial}{\partial y}\overline{u'v'} < 0 \left\{ \right.$$



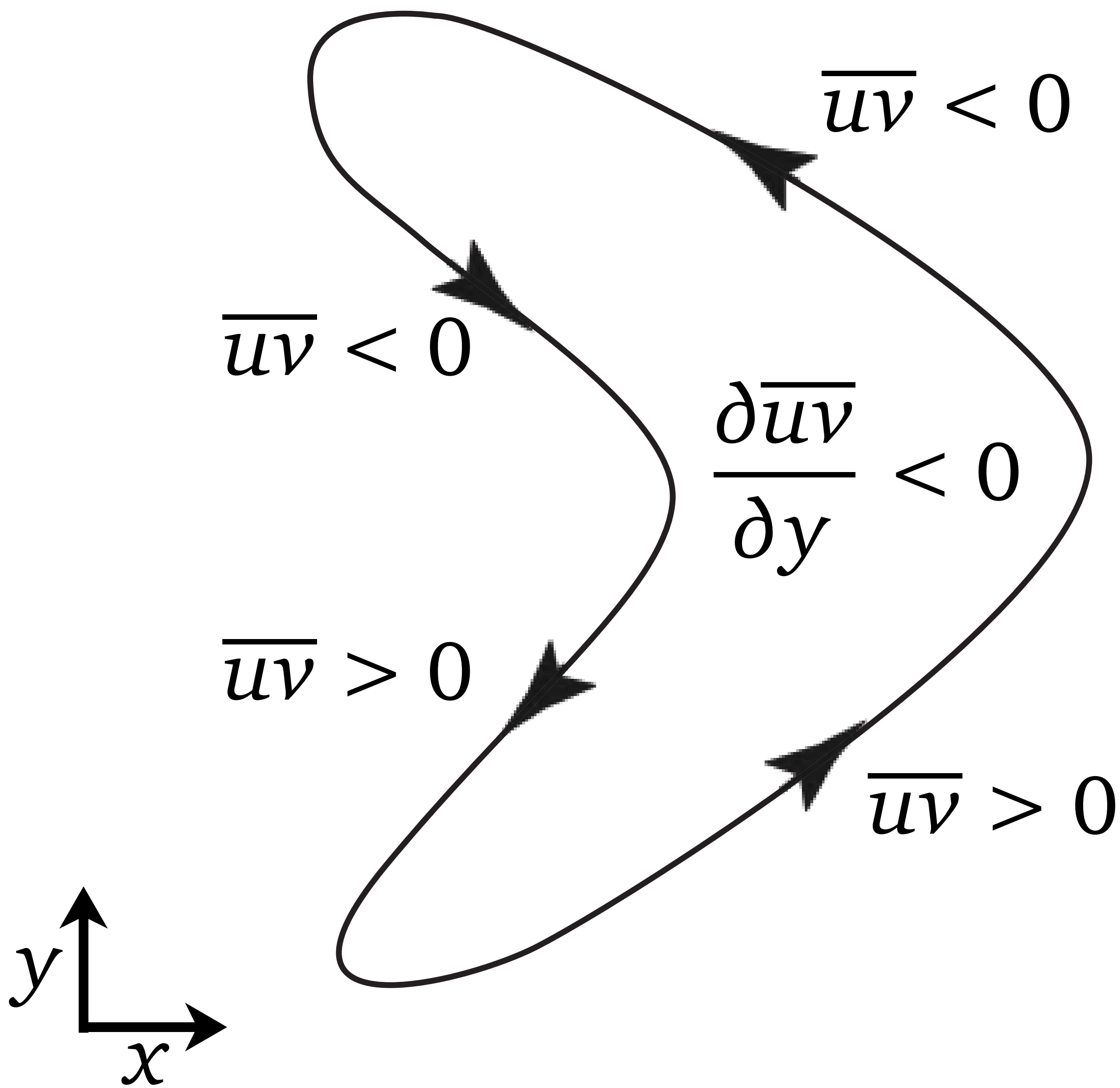
Solid line — $\overline{u}$

Dashed line $\cdots$ $u_{\rm rms}$

Equation of motion:

$$\frac{\partial \zeta}{\partial t} + J(\psi, \zeta) = S - r\zeta + \nu \nabla^4 \zeta$$

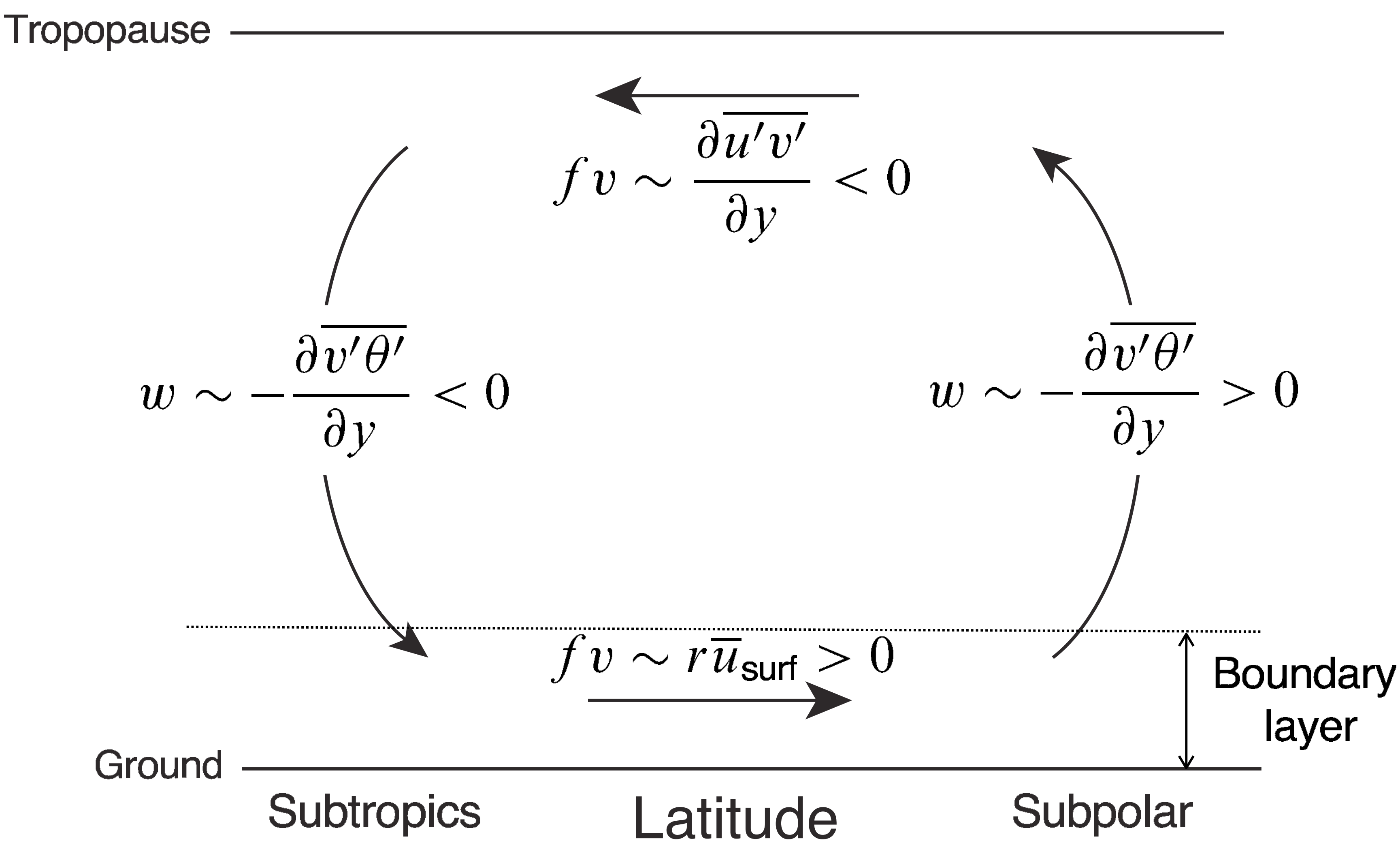
Consequence - bow-shaped eddies.



Ferrel Cell

$$\frac{\partial \overline{u}}{\partial t} - f\overline{v} \approx -\frac{\partial}{\partial y}\overline{u'v'}$$

Rossby waves give momentum convergence



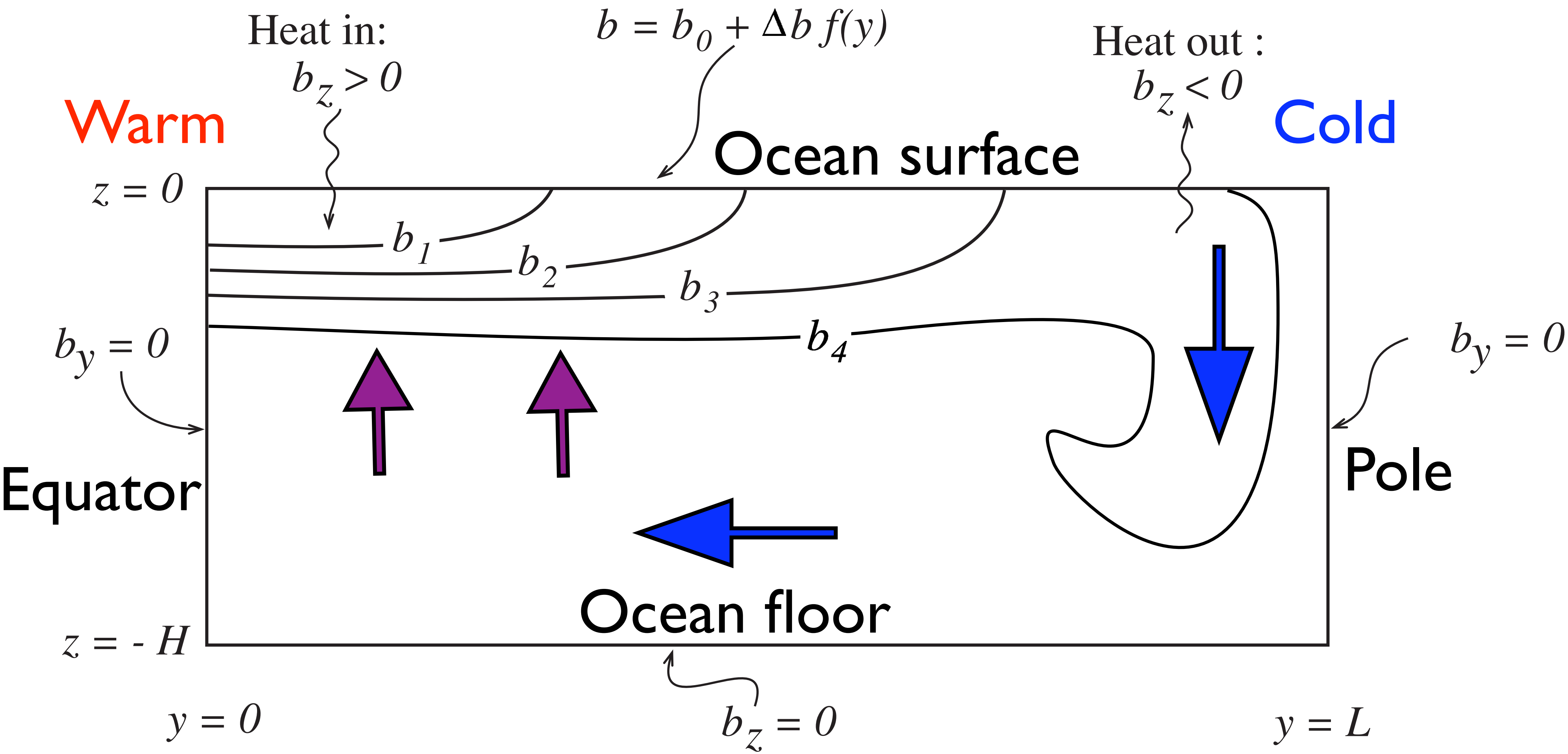
Heat Transport

- But the Ferrel Cell must still transport heat polewards...
- Express in terms of a ‘residual transport’....stay tuned....

The Ocean

- Thermodynamics: Heated at equator, cooled at pole -- heat engine?
- We expect the ocean to be baroclinically unstable (ocean has weather, too). Eddy KE approx 10 times mean KE in ocean.
- But, eddy scales are 10 times smaller.
- So are eddies important? What about Rossby waves?

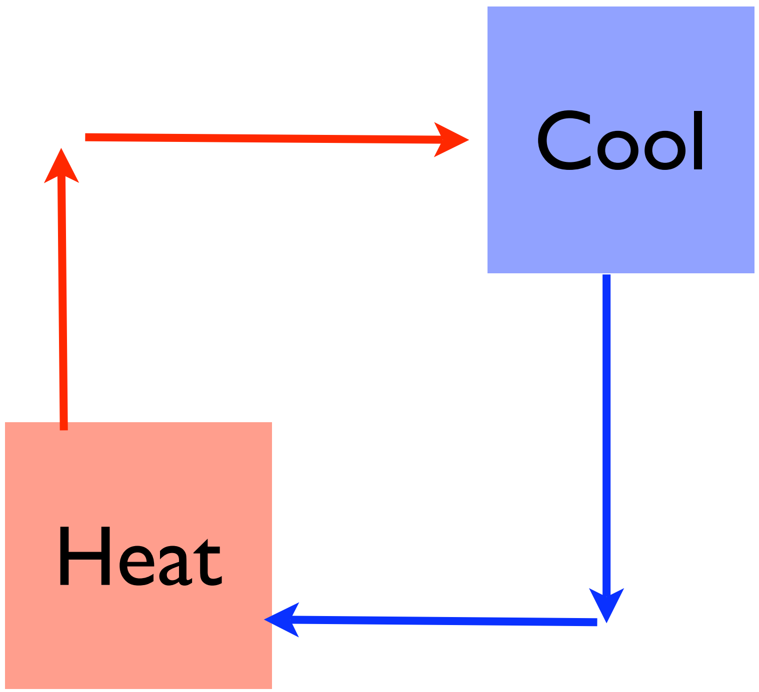
Thermal driving of the Ocean



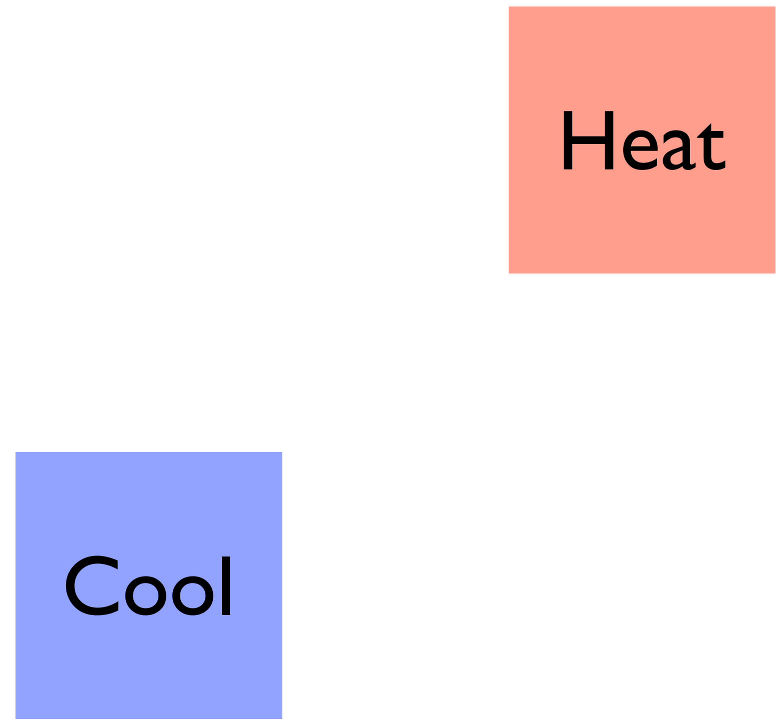
Sandstrom's so-called theorem

- In 1908 Sandstrom proposed a `theorem' that has been ignored, deified and misunderstood (and caused confusion, to boot).
- Seems to show that a deep ocean circulation can't exist under certain circumstances, but with much hand waving.
- Revisit this, with a little more rigour...

Sandstrom's 'theorem'



Realizable steady circulation



No steady circulation without mechanical forcing

Kelvins circulation

$$\frac{\mathrm{D}C}{\mathrm{D}t} = \oint p\,\mathrm{d}\alpha + \oint \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{r} = \oint T\,\mathrm{d}\eta + \oint \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{r},$$

Friction retards flow

$$\oint T\,\mathrm{d}\eta > 0 \qquad \oint p\,\mathrm{d}\alpha > 0$$

Transform

$$\oint T\,\mathrm{d}\eta = \oint c_p \frac{T}{\theta}\,\mathrm{d}\theta = \oint c_p \left(\frac{p}{p_\mathrm{R}}\right)^{\kappa} \mathrm{d}\theta$$

Heating is at higher pressure than cooling.

$$\boxed{\oint c_p \left(\frac{p}{p_\mathrm{R}}\right)^{\kappa} \mathrm{d}\theta > 0}$$

Equations of motion

(b = temperature)

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{f} + 2\boldsymbol{\omega}) \times \boldsymbol{v} = -\nabla B + b\mathbf{k} + \nu \nabla^2 \boldsymbol{v},$$

$$\frac{\mathrm{D}b}{\mathrm{D}t} = \frac{\partial b}{\partial t} + \nabla \cdot (b\boldsymbol{v}) = \dot{Q} = J + \kappa \nabla^2 b,$$

$$\nabla \cdot \boldsymbol{v} = 0,$$

($\dot{Q}$ = heating)

(ε = energy dissipation)

Kinetic energy

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \frac{1}{2}\boldsymbol{v}^2 \right\rangle = \langle wb \rangle - \varepsilon$$

Potential energy

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle bz \rangle = \left\langle z\dot{Q} \right\rangle + \langle bw \rangle$$

$$\boxed{\left\langle z\dot{Q} \right\rangle = -\varepsilon < 0}$$

Heating must occur below cooling in order to sustain a dissipative circulation!

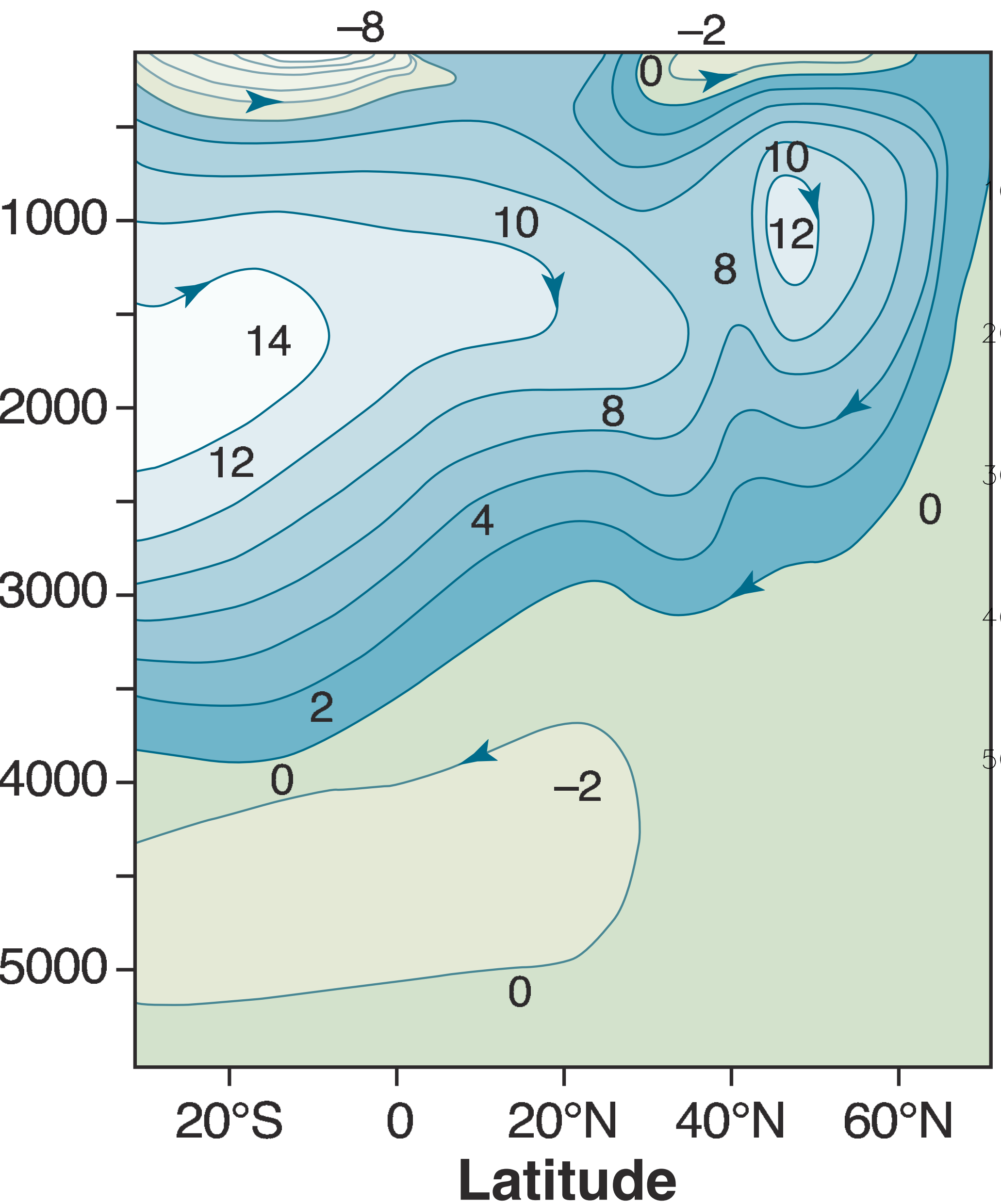
Corollary *(Paparella and Young, 2002)*

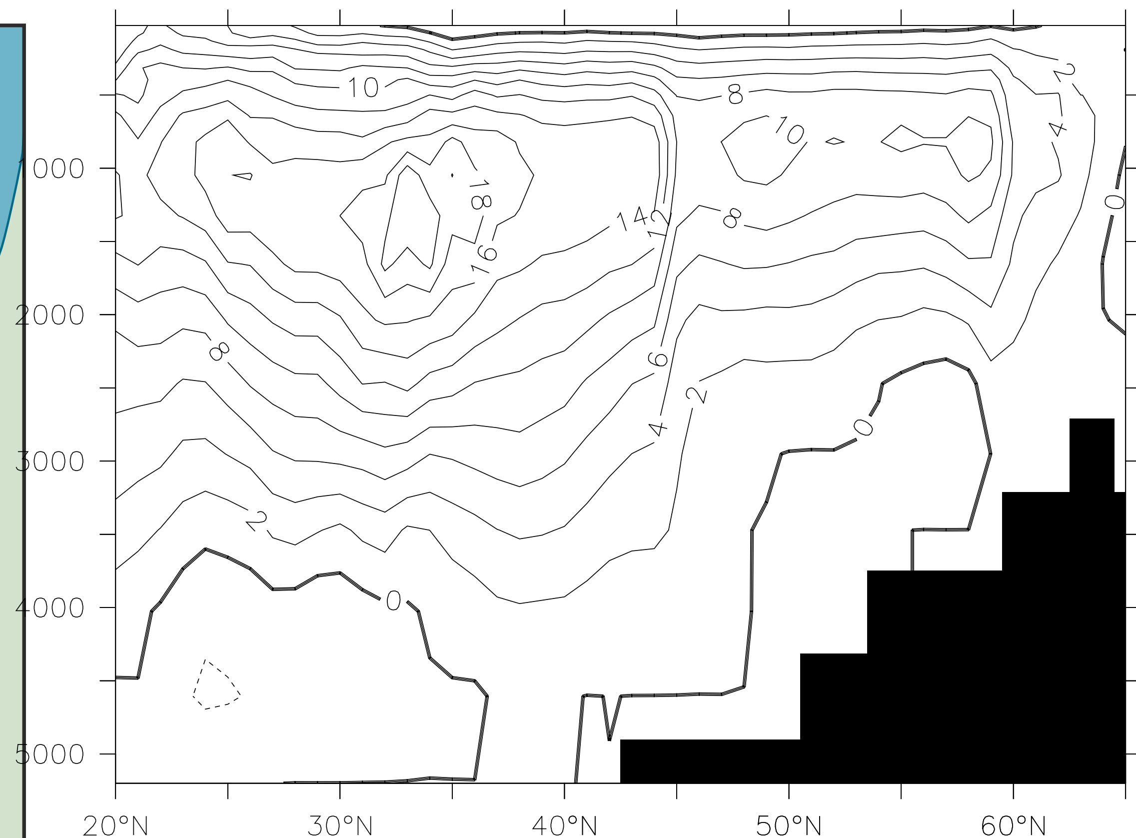
- If the heating is at the surface, and occurs via the imposition of a surface boundary condition, then
$$\varepsilon = H^{-1}\kappa\left[\overline{b}(0) - \overline{b}(-H)\right]$$

- Therefore, for small κ the energy dissipation is correspondingly small (one hairdryer of energy per cubic kilometer).

- This is *much smaller* than observed. Other factors must therefore be important.

But there is a deep circulation!





From Wunsch, and Zhang and Vallis

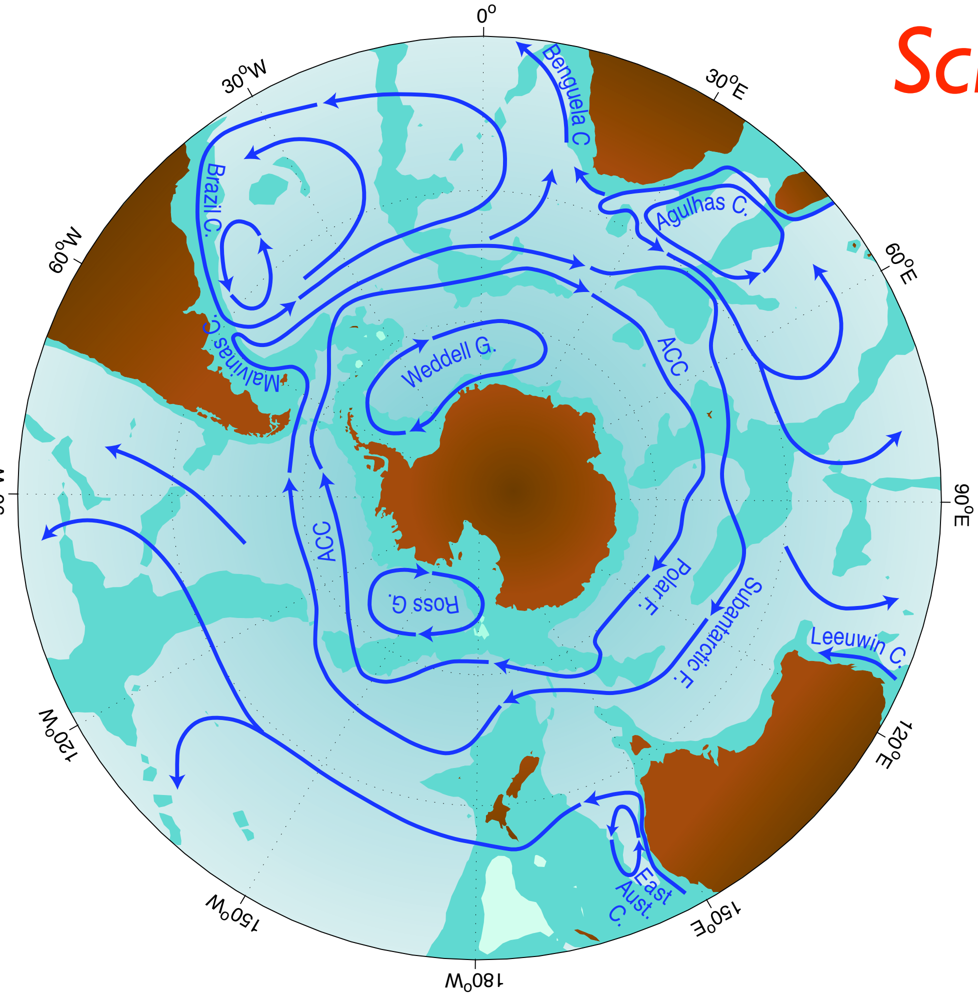
- Deep Ocean Circulation relies on mechanical forcing, and this has two effects:

  I. Directly provides a mechanism ('conveyor belt') for deep circulation.

  II. Allows for a finite 'eddy diffusivity' to exist.

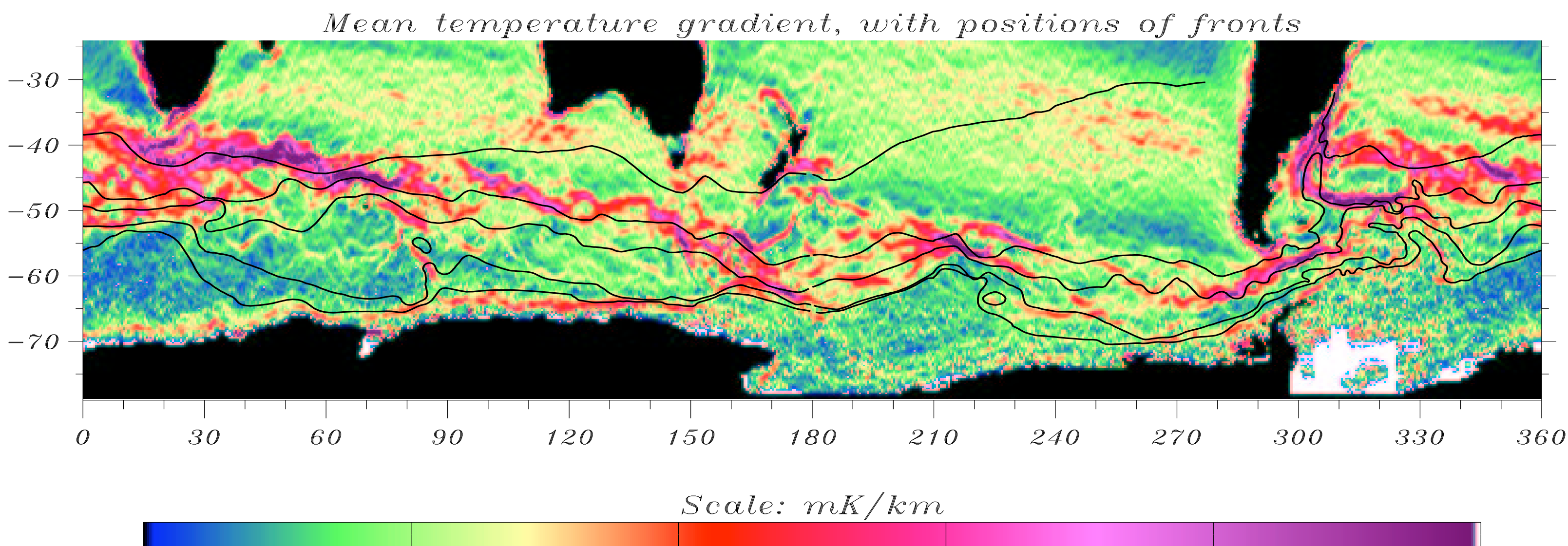
- Ocean is *not* a thermally-driven heat engine.

Schematic of the ACC



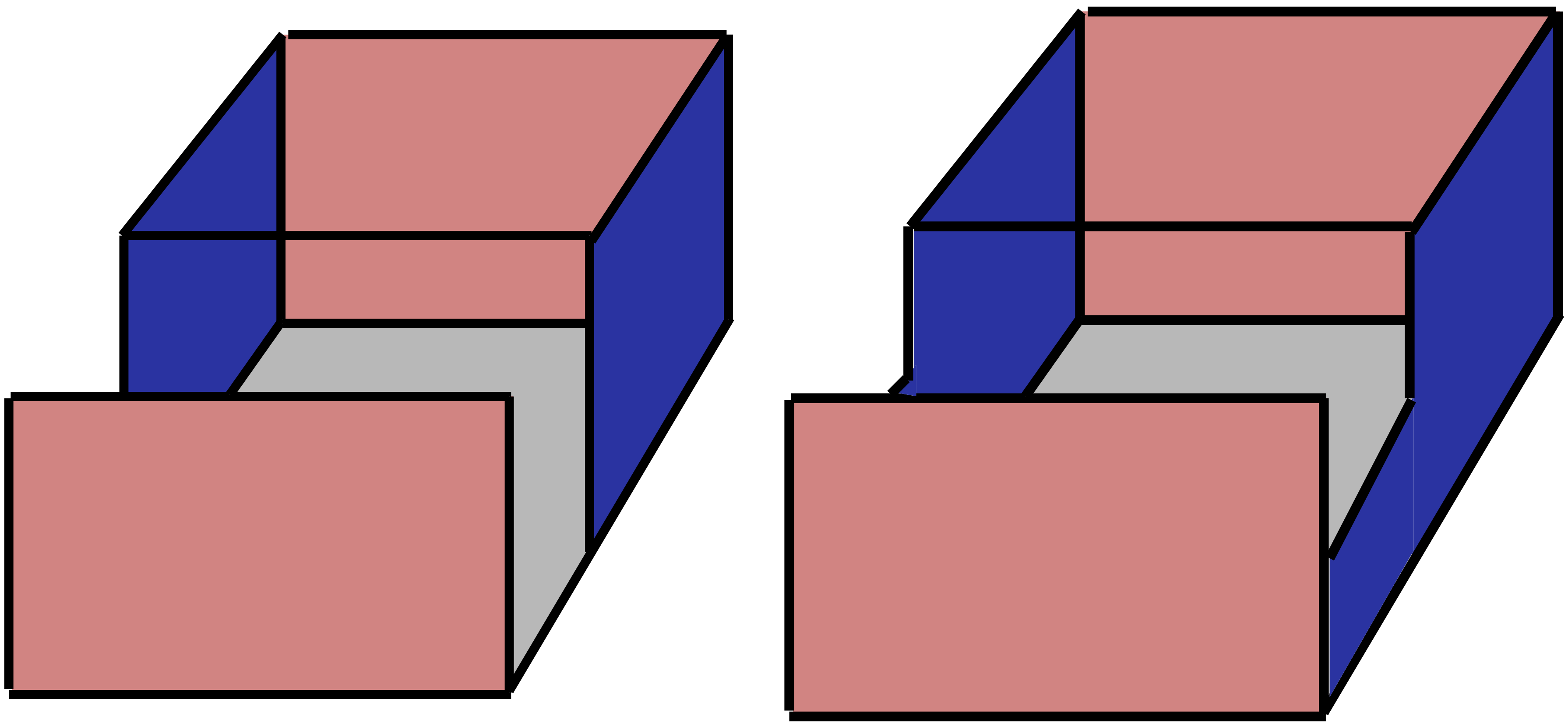
- The ocean system most like the atmosphere.
- Complicated set of currents and fronts.

Temperature gradient:



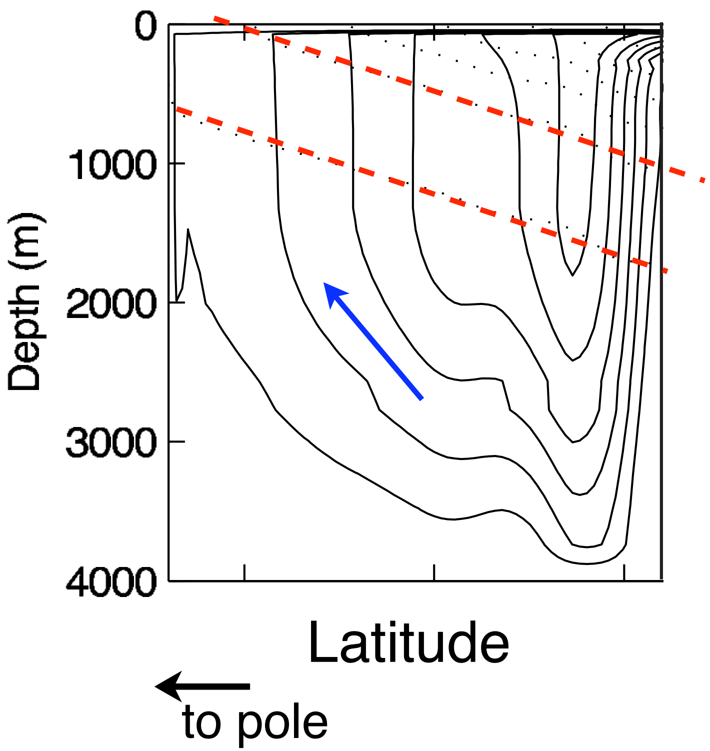
Olbers et al 2004

A Simple Model of the Antarctic Circumpolar Channel (ACC)



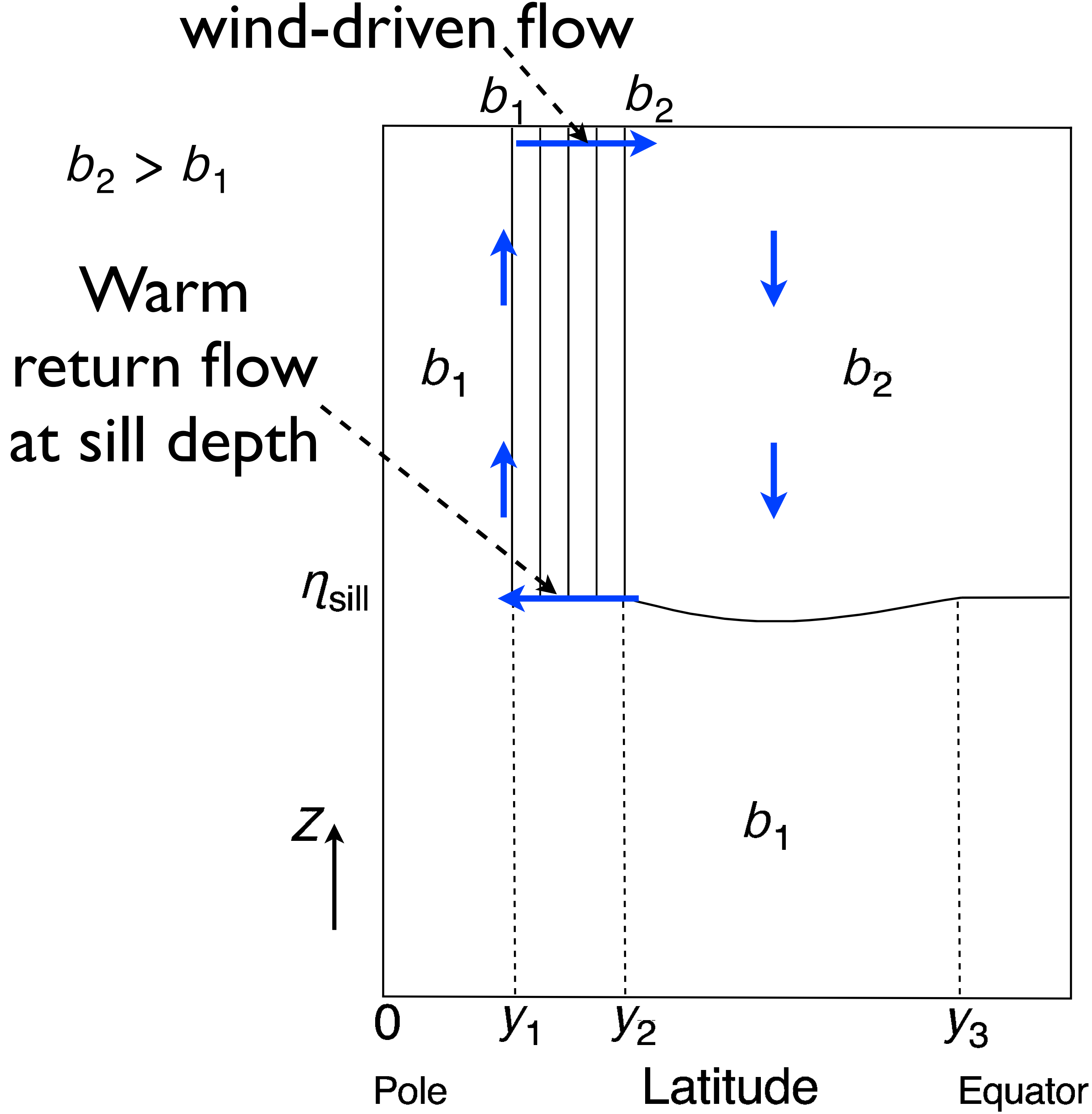
Overturning circulation

Eulerian zonally averaged circulation



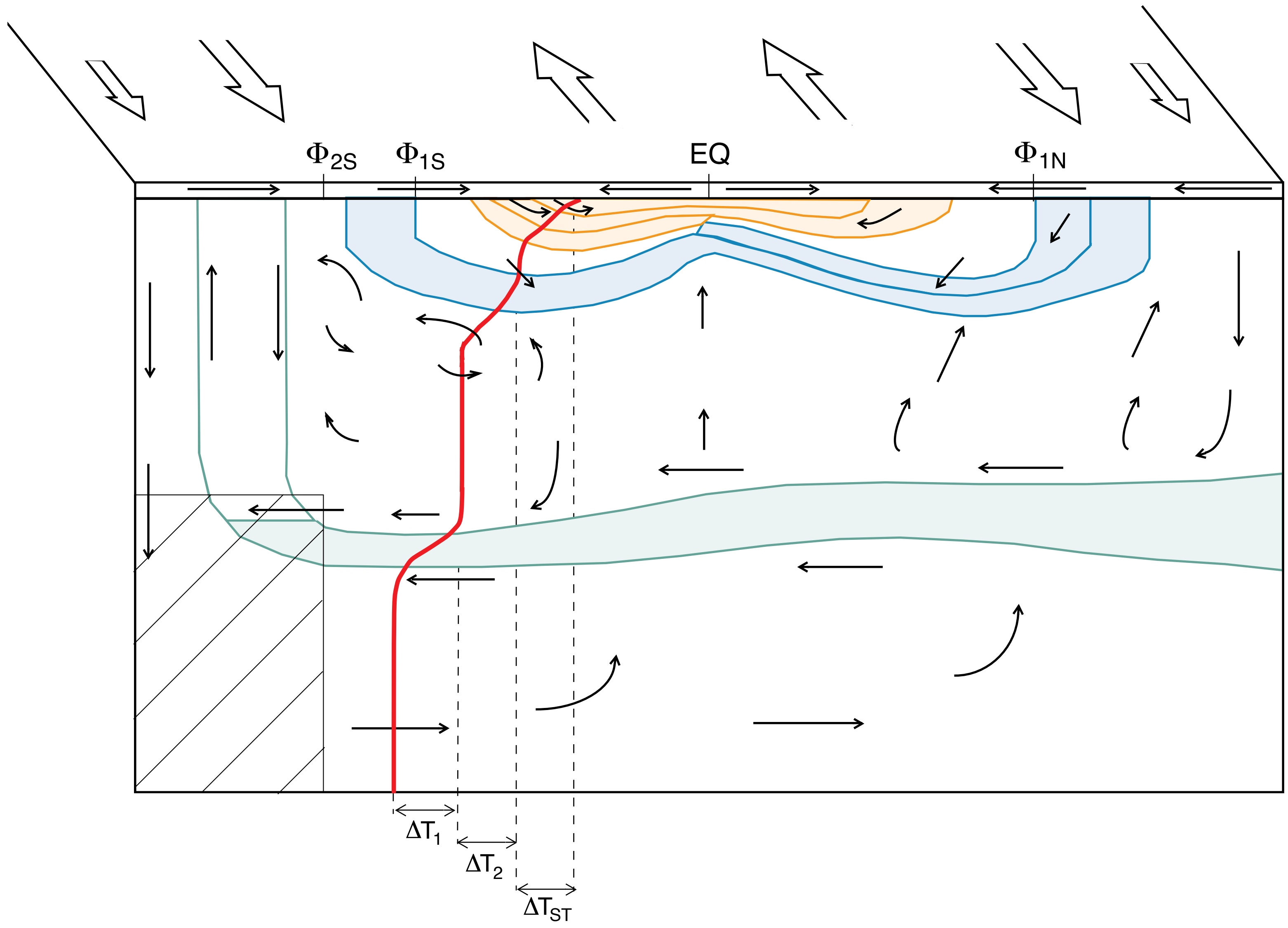
isotherms

- There *is* a deep circulation

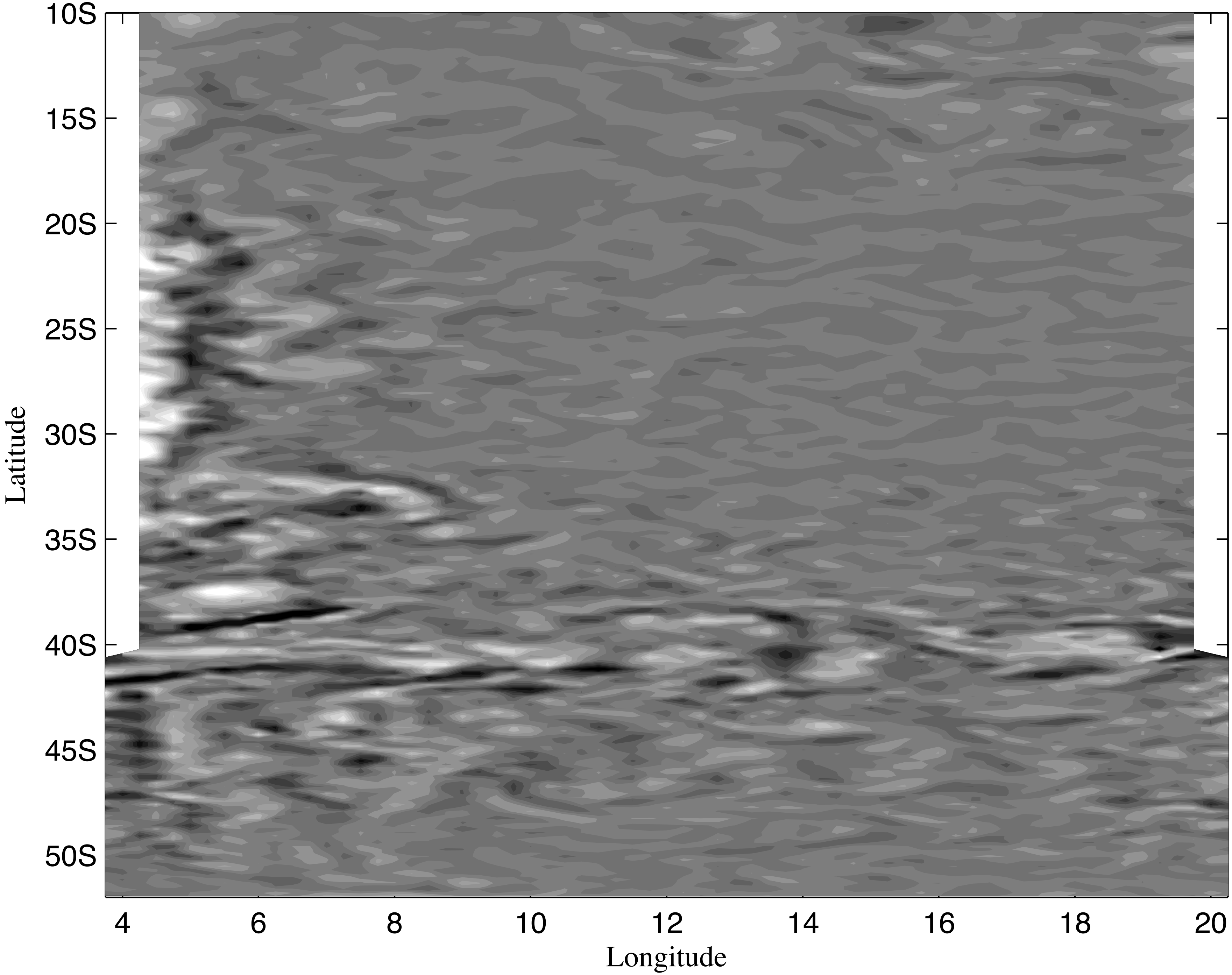


- In ACC, the non-eddying flow has vertical isopycnals, and so is extremely baroclinically unstable!

Schematic of interhemispheric overturning circulation in ocean

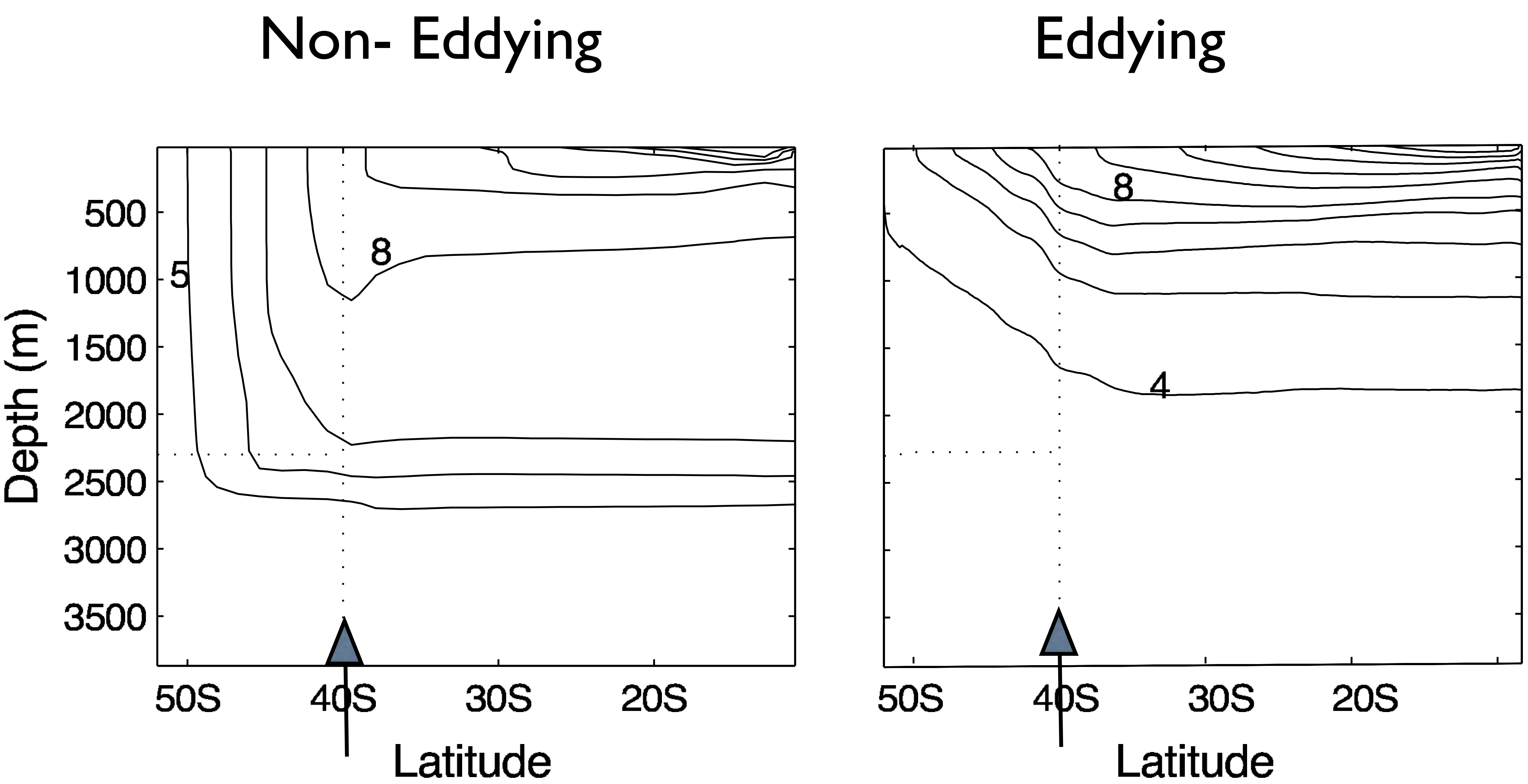


Eddy Permitting Vorticity Snapshot



Henning and Vallis

Isotherms in a channel + gyre.

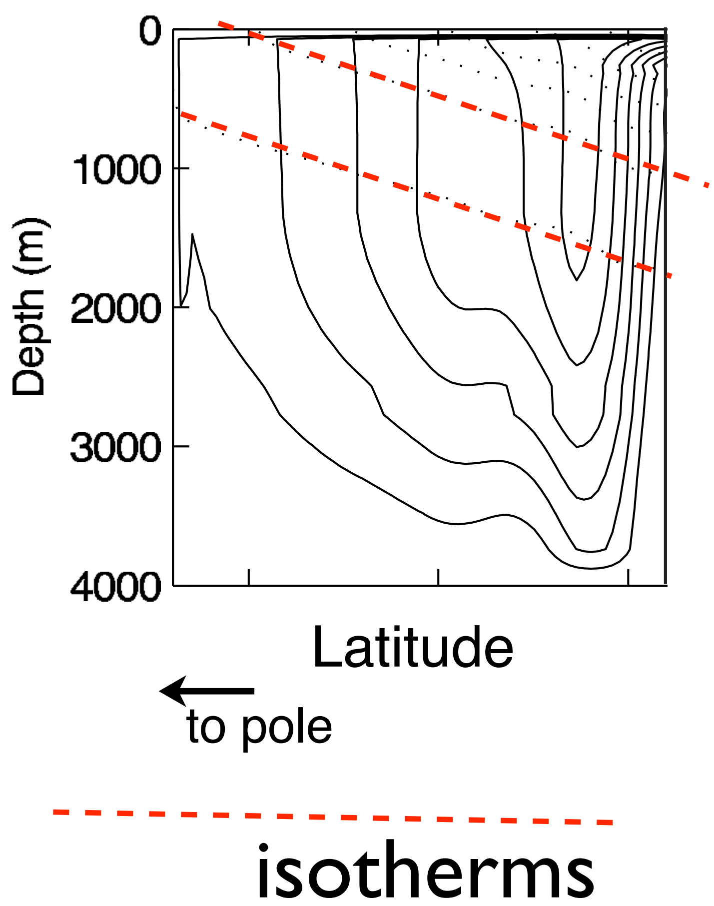


What’s going on?

- Without eddies, convection extends all the way to the bottom, producing vertical isotherms.
- This situation is baroclinically unstable.
  - The isotherms slump, producing an order unity change in the stratification.

Overturning circulation

Eulerian



- As noted, there *is* a deep circulation
- Now, it is generally believed that the flow in the ocean interior is adiabatic.
    - This means the flow cannot cross isotherms

The Transformed Eulerian Mean (TEM)

- Consider a fluid layer of variable thickness h. Total transport of thickness is $\overline{v}\overline{h} + \overline{v'h'}$
- Define $\overline{v}^* = \frac{1}{\overline{h}}\left(\overline{v}\overline{h} + \overline{v'h'}\right)$
- Transform equations to new variables, and use $\overline{v}^*$ instead of v

The Momentum Equation

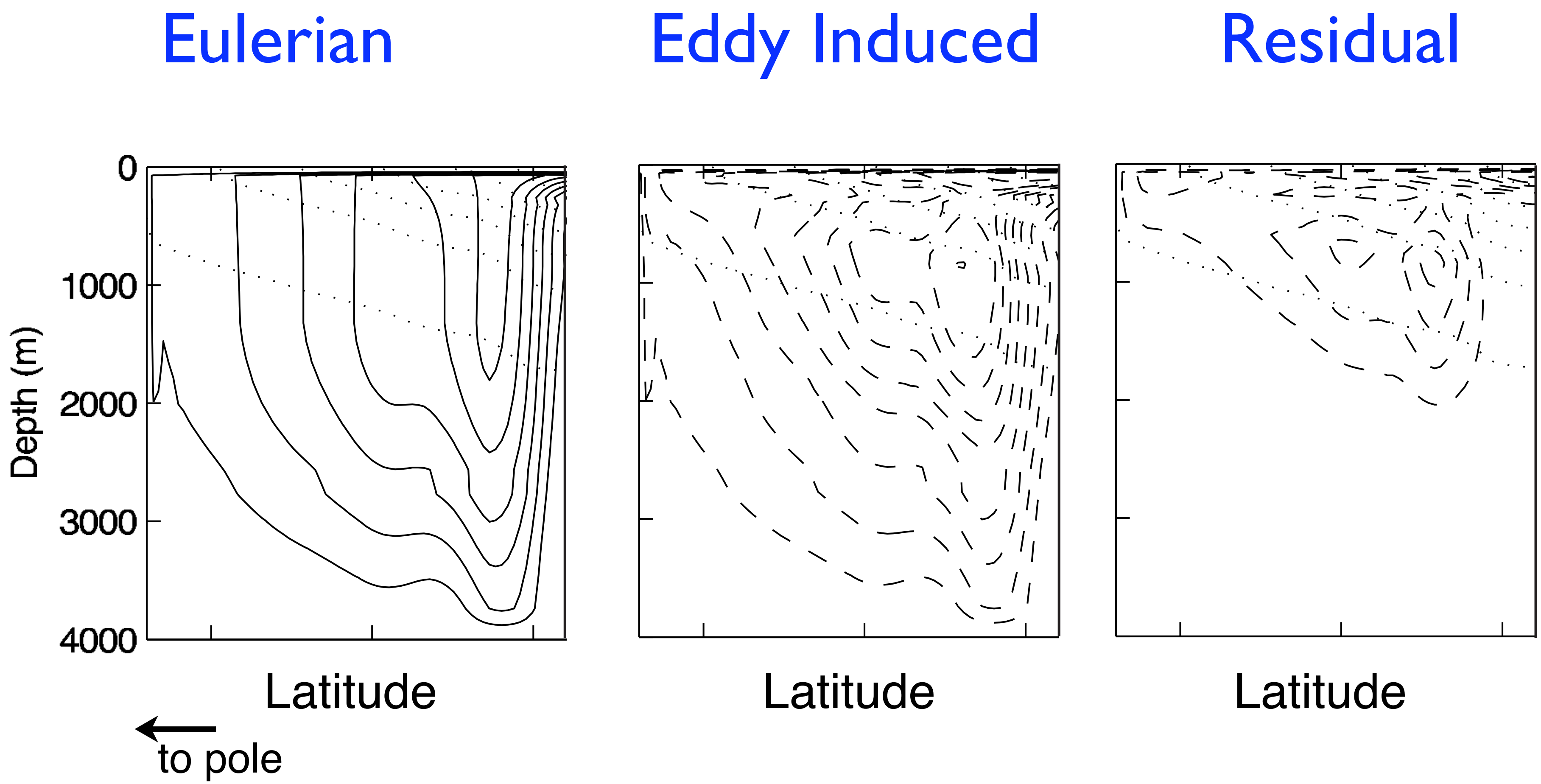
On transforming the momentum equation similarly, a miracle occurs....

Approximately: $$\frac{\partial \overline{u}}{\partial t} - f_0 \overline{v}^* = \overline{v'q'} + \frac{\partial \tau}{\partial z}$$

Right-hand side contains potential vorticity flux, $\overline{v'q'}$ not momentum flux. *Yes!*

Potential vorticity is a materially conserved variable, with all sorts of nice properties.

Overturning circulation



- Large cancellation between Eulerian and eddy-induced transport; residual flow is *weak*.
- Residual flow is approximately parallel to isopycnals.

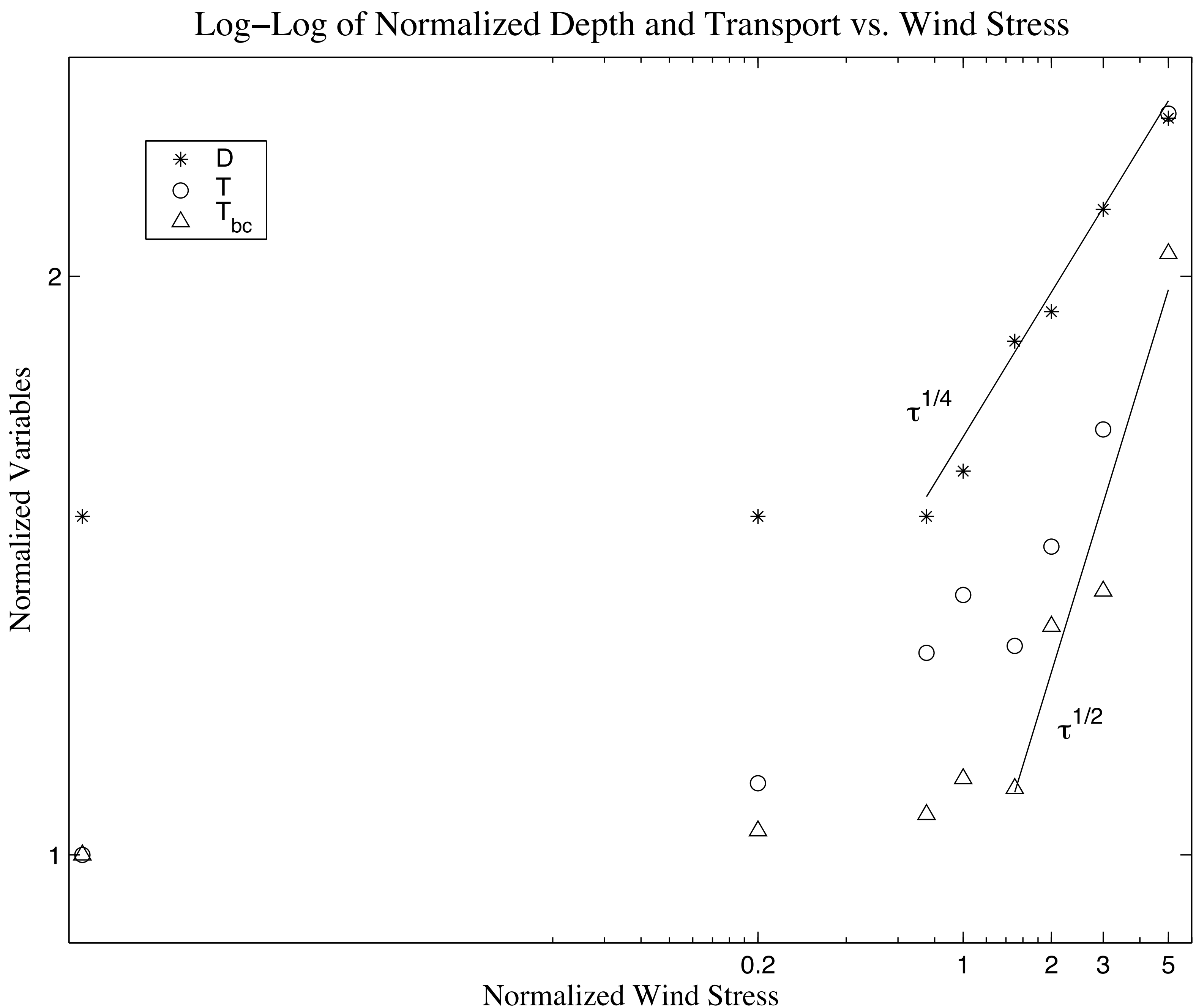
Elements of a Theory of the ACC..

- Assume that the total (or residual) streamfunction is zero. i.e. exact cancelletion between mean and eddy.
- From the momentum equation we find $$\frac{\partial \tau}{\partial z} \approx -\overline{v'q'}$$
- Make a closure, e.g., $$\overline{v'q'} = -\kappa \nabla q$$

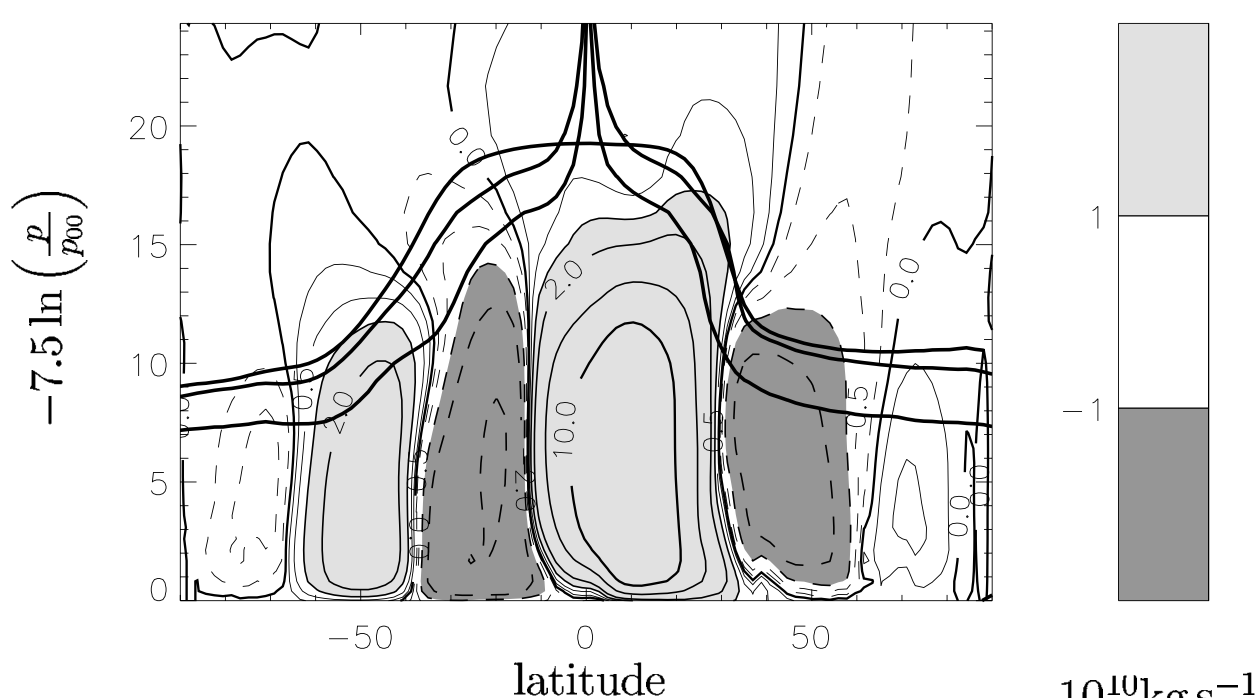
Into the breach...

- If eddy diffusivity is given by $\kappa \sim U_e L_e$
- and eddy length is the Rhines scale $L_e \sim \left(\frac{U_e}{\beta} \right)^{1/2}$
- and the velocity scale is obtained from the APE $U_e \sim (\Delta b D)^{1/2} (L_e / L)$
- Then we obtain a prediction for the depth of the stratification $D \sim \left(\frac{\tau L^2 f}{\Delta b^{3/2}} \right)^{2/5} \left(\frac{L}{a} \right)^{4/5}$

Alas the theory is not perfect....Transport and depth vs wind, gives power laws in disagreement with the theory.

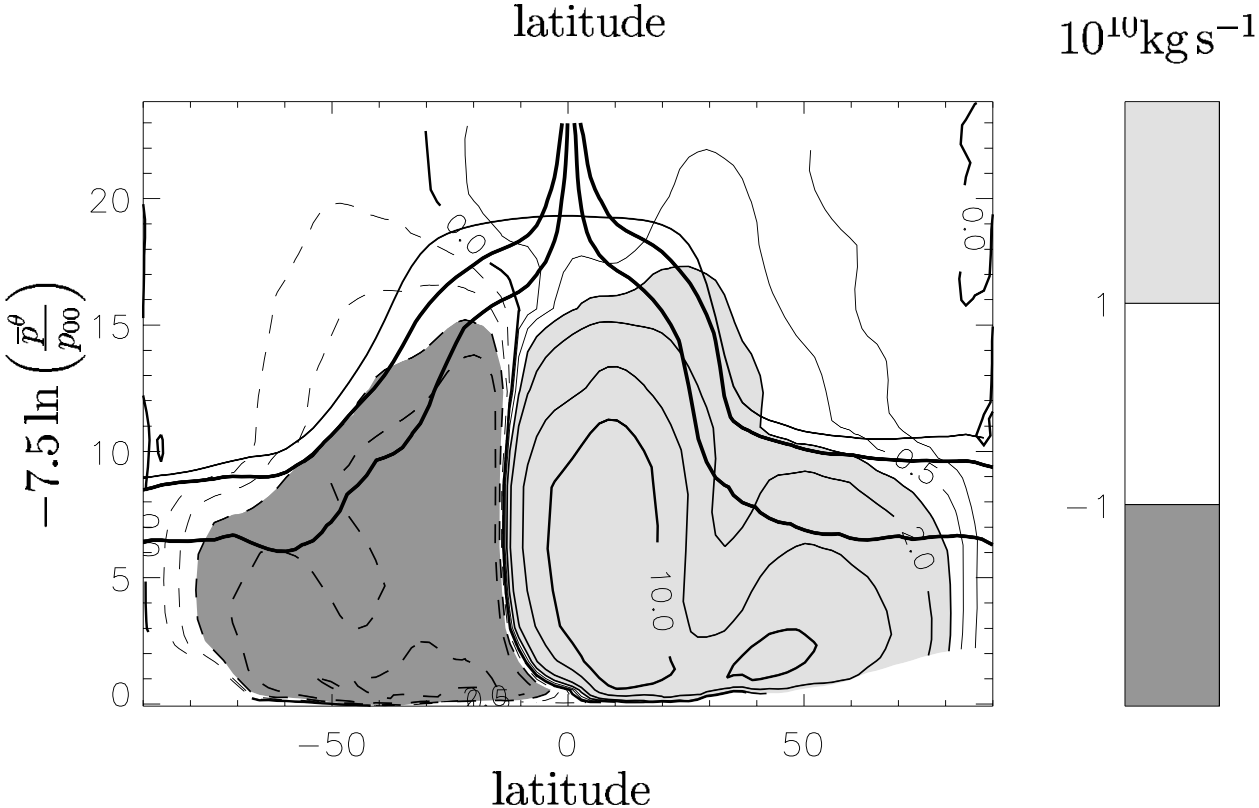


Back to the atmosphere:



- Eulerian circulation, indirect because mechanically driven.

$$\frac{\partial \overline{b}}{\partial t} + \boldsymbol{v} \cdot \nabla \overline{b} = Q - \nabla \overline{\boldsymbol{v}' b'}$$



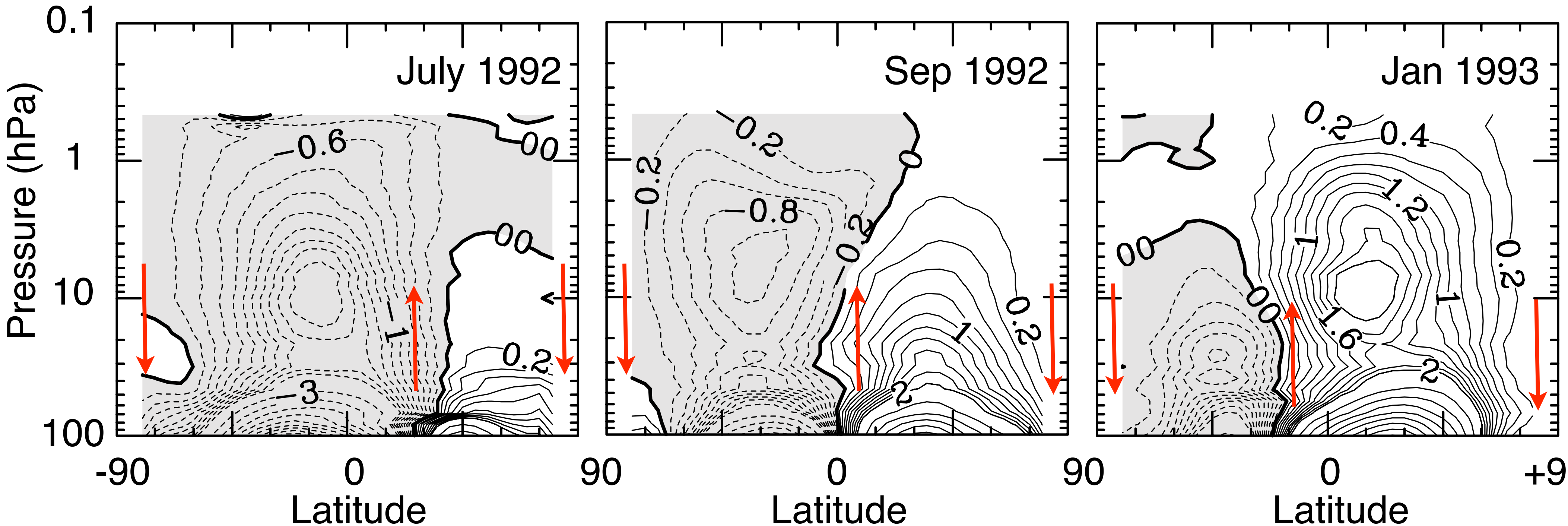
- Residual circulation is direct, because it is thermodynamic.

$$\frac{\partial \overline{b}}{\partial t} + \overline{v}^* \cdot \nabla \overline{b} = Q$$

Courtesy of M. Juckes

The Stratosphere

The stratosphere is, in part, a mechanically-driven natural refrigerator!



Observations of Brewer-Dobson circulation. Air descending over the summer pole, where it is being heated!

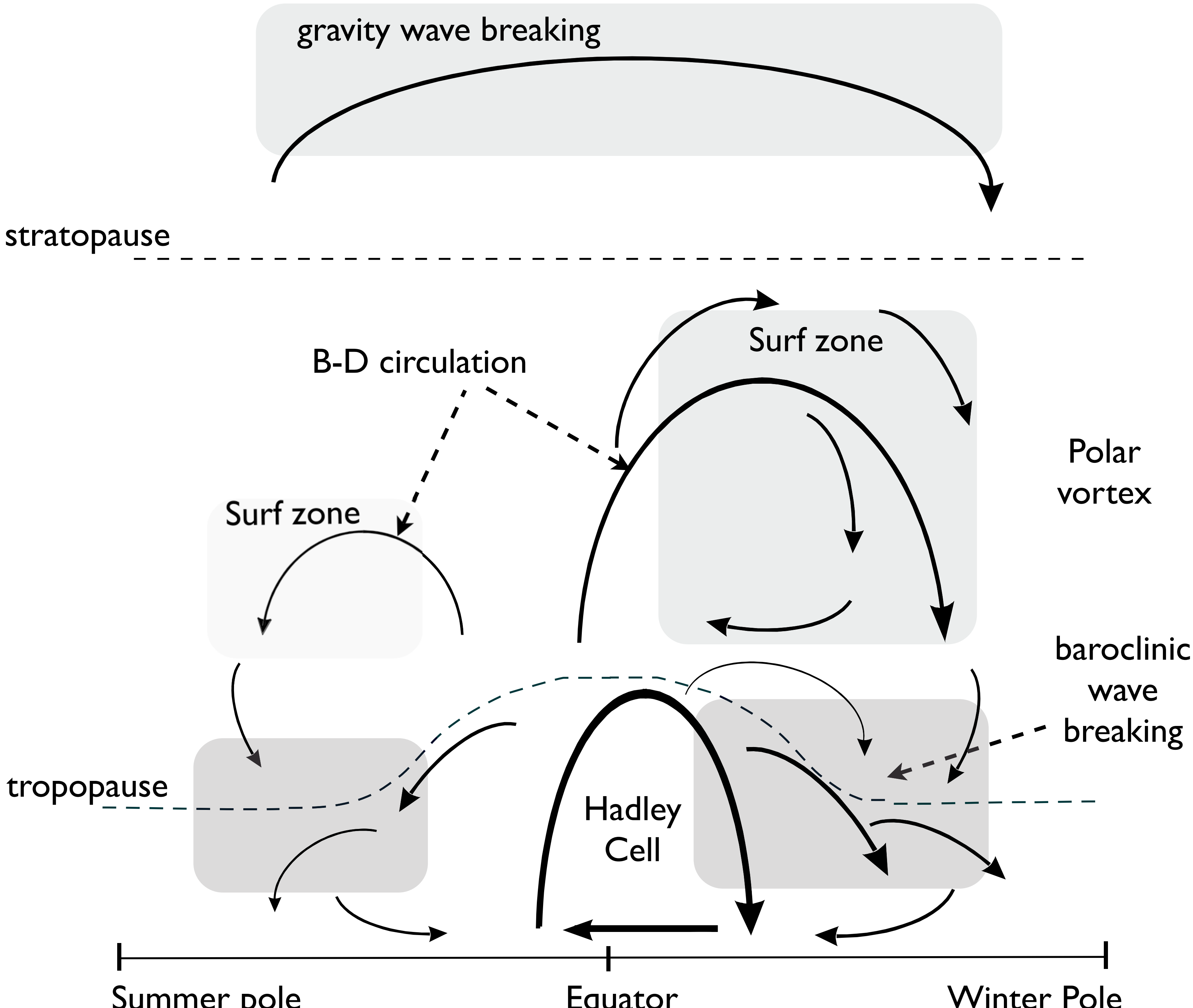
Waves in stratosphere

- Rossby waves propagate upwards into stratosphere where they break and deposit westward momentum.

$$f\overline{v}^* \approx \overline{v'q'}$$

Meridional circulation ‘Wave drag’

- This generates a ‘mechanically-driven’ overturning circulation, discovered by Brewer and Dobson in 1930s. (Brewer looking at water vapour, Dobson independently looking at ozone.)


gravity wave breaking
stratopause
B-D circulation
Surf zone
Surf zone
Polar
vortex
baroclinic
wave
breaking
tropopause
Hadley
Cell
Summer pole
Equator
Winter Pole

Concluding Remarks

- Circulation of both atmosphere and ocean are an intertwining of thermodynamics and dynamics, the latter being an intertwining of waves and turbulence.

- The atmosphere is in part a heat engine, but not the ocean. Stems from opaqueness of seawater to solar radiation, so that heating, and cooling, is nearly all at the top.

- Wind has two effects on ocean circulation:

  - Provides a direct means to drive the deep circulation

  - Provides an energy input that enables small scale turbulence to exist.

- Atmosphere is also, in part, mechanically driven by Rossby waves interacting with turbulence, especially middle and upper atmosphere.

- Not and the end of our understanding, nor even the beginning of the end. Perhaps at the end of the beginning.