

Quantum Optics with Electrical Circuits:

‘Circuit QED’

Experiment

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ARDA

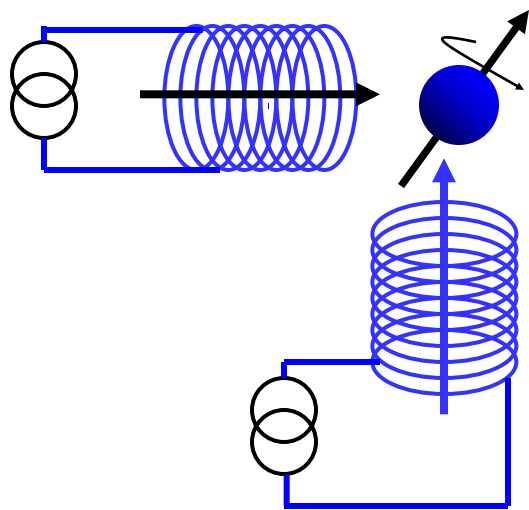
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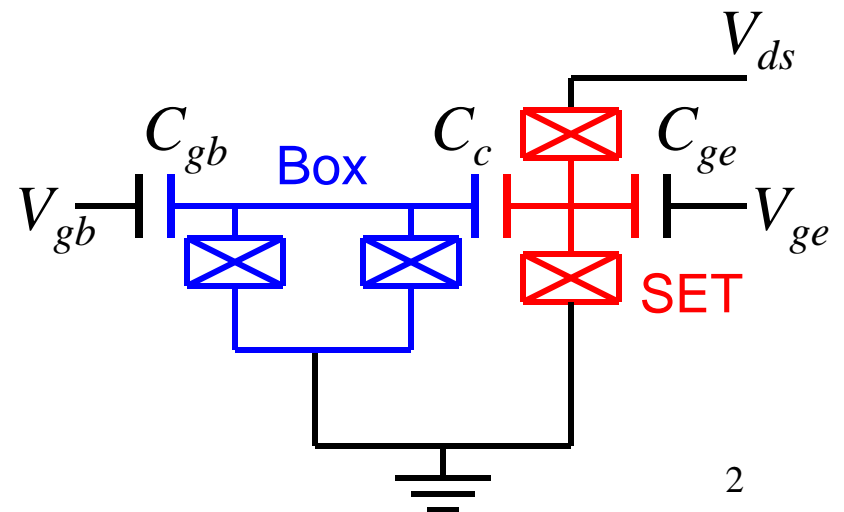
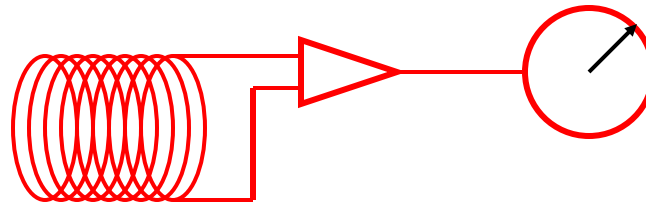


Quantum Computation and NMR of a Single „Spin

Single „Spin $\frac{1}{2}$



Quantum Measurement

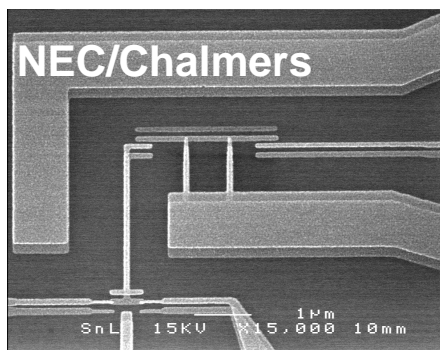


(After Konrad Lehnert)

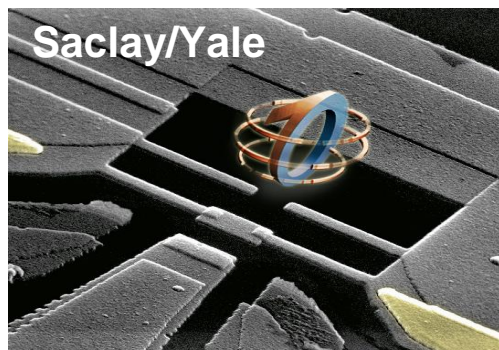
State of the Art in Superconducting Qubits

- Nonlinearity from Josephson junctions (Al/AlO_x/Al)

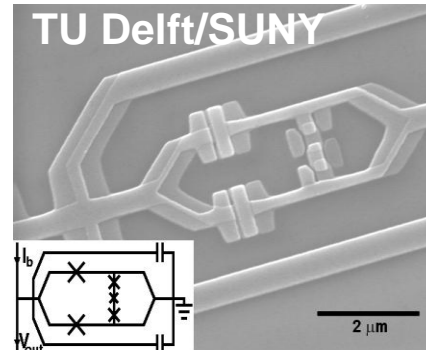
Charge



Charge/Phase



Flux



Phase



Junction size \longrightarrow $E_J = E_C$ \longleftarrow # of Cooper pairs

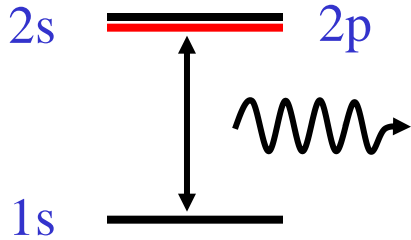
- 1st qubit demonstrated in 1998 (NEC Labs, Japan)
- “Long” coherence shown 2002 (Saclay/Yale)
- Several experiments with **two** degrees of freedom
- C-NOT gate (2003 NEC, 2006 Delft and UCSB)
- Bell inequality tests being attempted (2006, UCSB)

So far only classical E-M fields: atomic physics with circuits

Our goal: interaction w/ quantized fields \longrightarrow Quantum optics with circuits

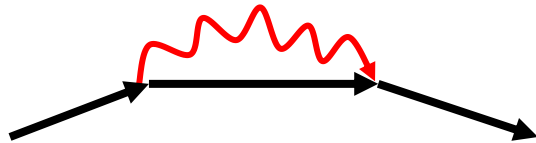
Communication between discrete photon states and qubit states

Atoms Coupled to Photons

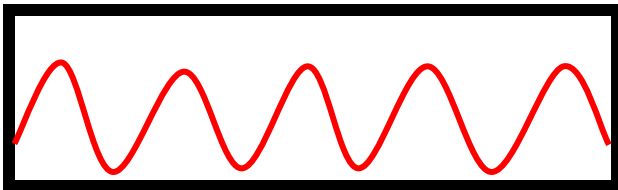


Irreversible spontaneous decay into the photon continuum:

$$2p \quad 1s \quad T_1 \sim 1 \text{ ns}$$



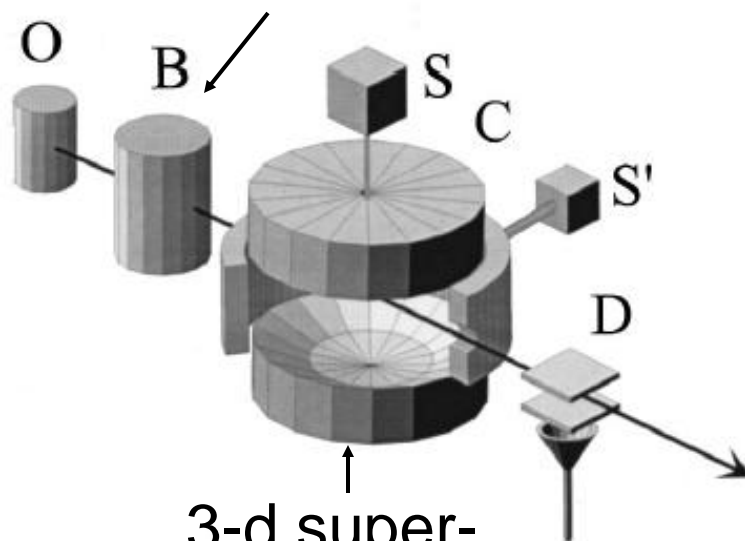
Vacuum Fluctuations:
(Virtual photon emission and reabsorption)
Lamb shift lifts 1s 2p degeneracy



Cavity QED: What happens if we trap the photons as discrete modes inside a cavity?

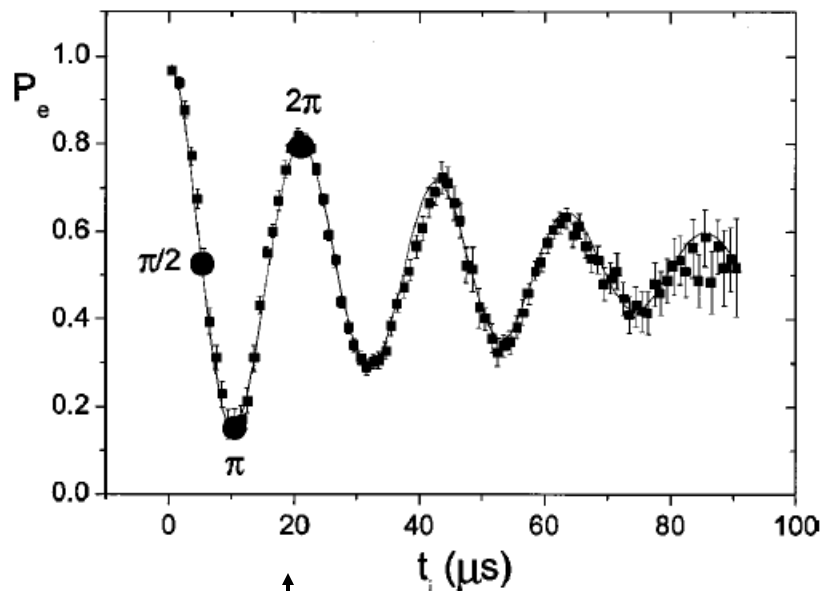
Microwave cQED with Rydberg Atoms

beam of atoms;
prepare in $|e\rangle$



3-d super-
conducting
cavity (50 GHz)

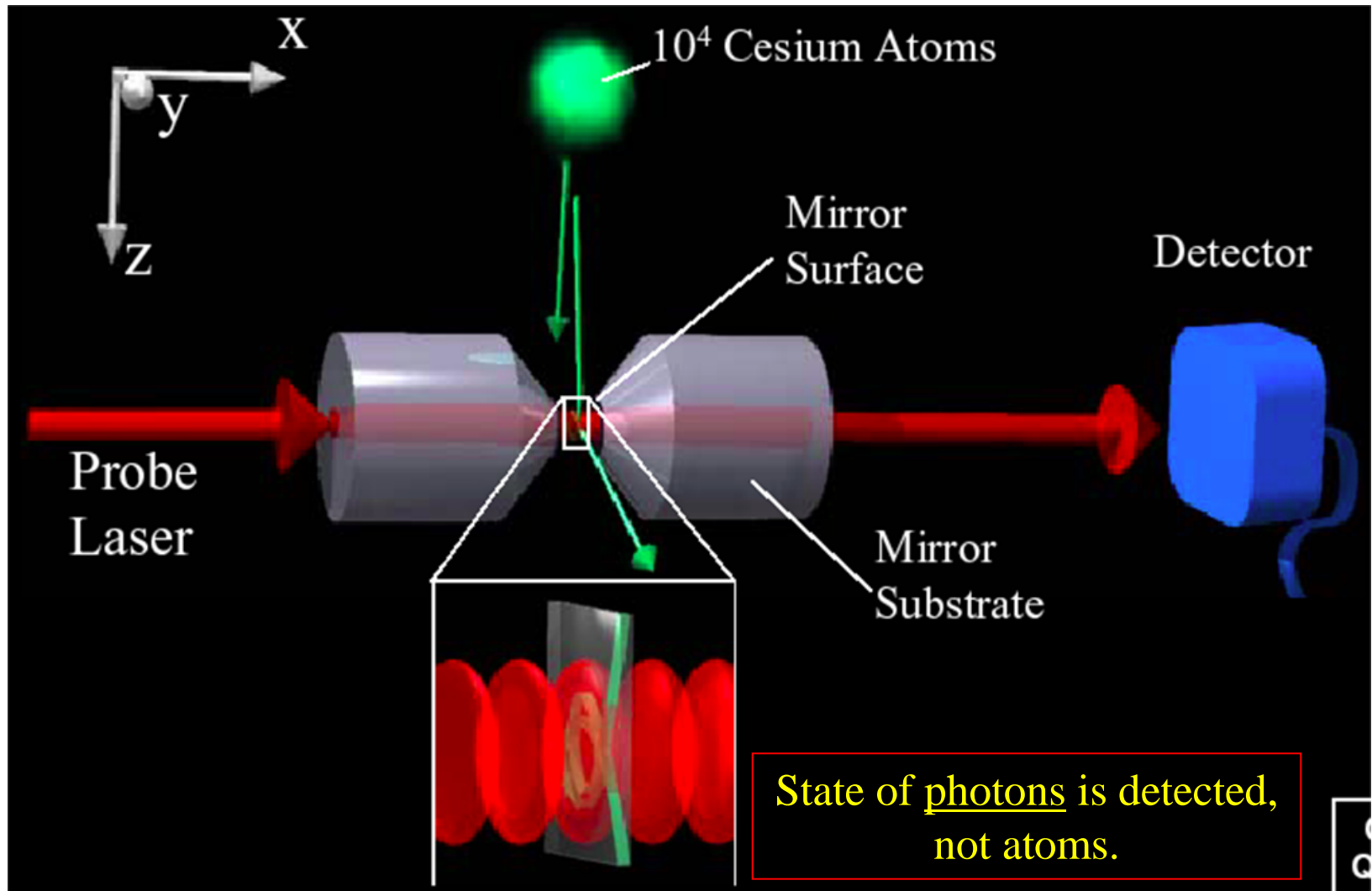
vacuum Rabi oscillations



observe dependence of atom final
state on time spent in cavity

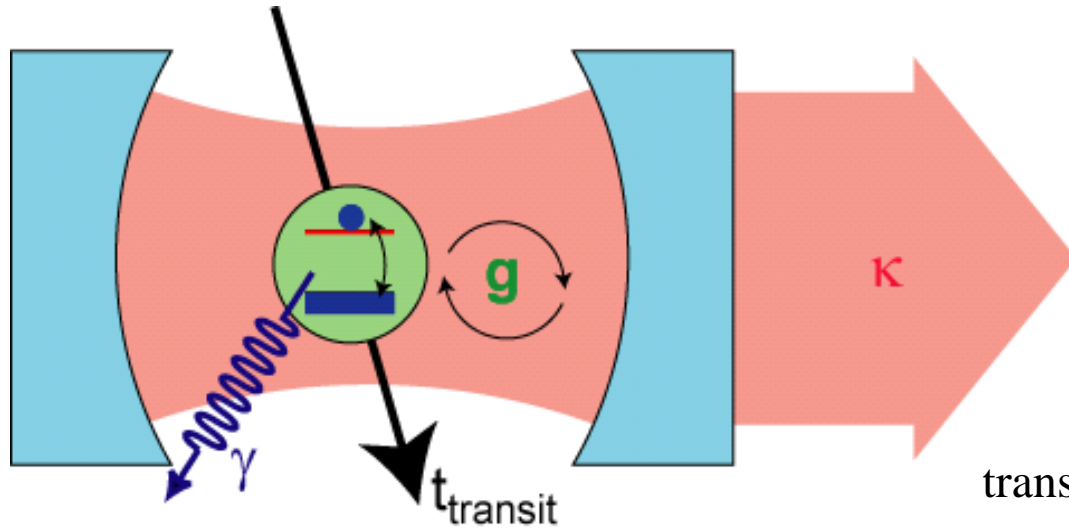
measure atomic state, or ...

cQED at Optical Frequencies



... measure changes in transmission of optical cavity

Cavity Quantum Electrodynamics (CQED)



$2g =$ vacuum Rabi freq.
 $\kappa =$ cavity decay rate
 $=$ “transverse” decay rate

Jaynes-Cummings Hamiltonian

transition dipole vacuum field

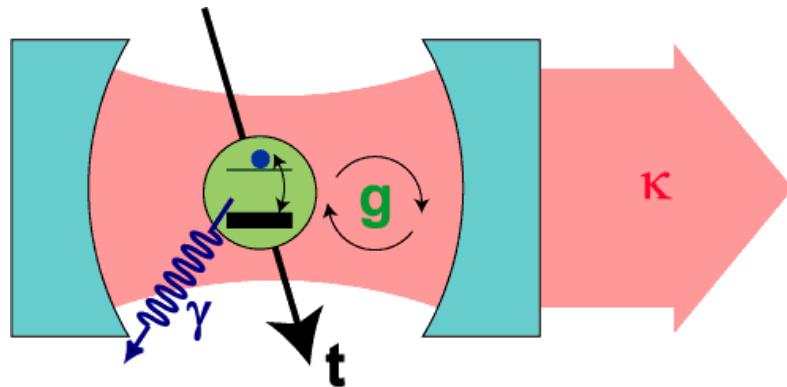
$$\hbar g \quad d E_{\text{RMS}}$$

$$E \quad E_{\text{RMS}} (a \quad a^\dagger)$$

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_a}{2} \sigma^z + \hbar g (a^\dagger \sigma^- + a \sigma^+) + H_\kappa + H_\gamma$$

strong coupling limit ($g = dE_0/\hbar > \gamma, \kappa, 1/t_{\text{transit}}$)

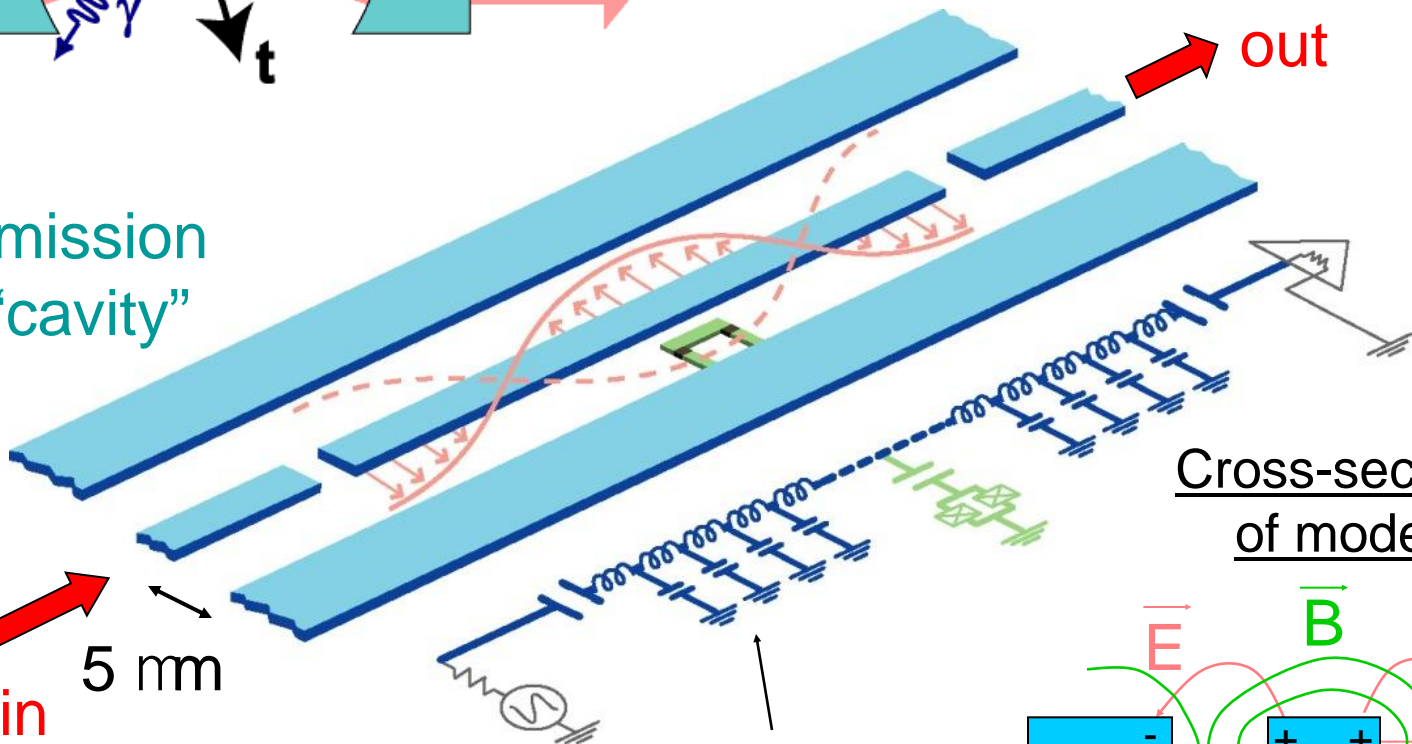
A Circuit Analog for Cavity QED



$2g$ = vacuum Rabi freq.
 κ = cavity decay rate
 = “transverse” decay rate

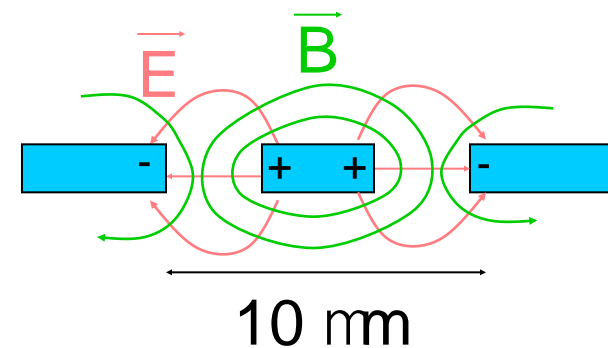
transmission
line “cavity”

DC + 6 GHz in
5 mm



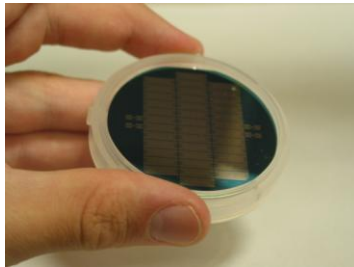
Lumped element
equivalent circuit

Cross-section
of mode:

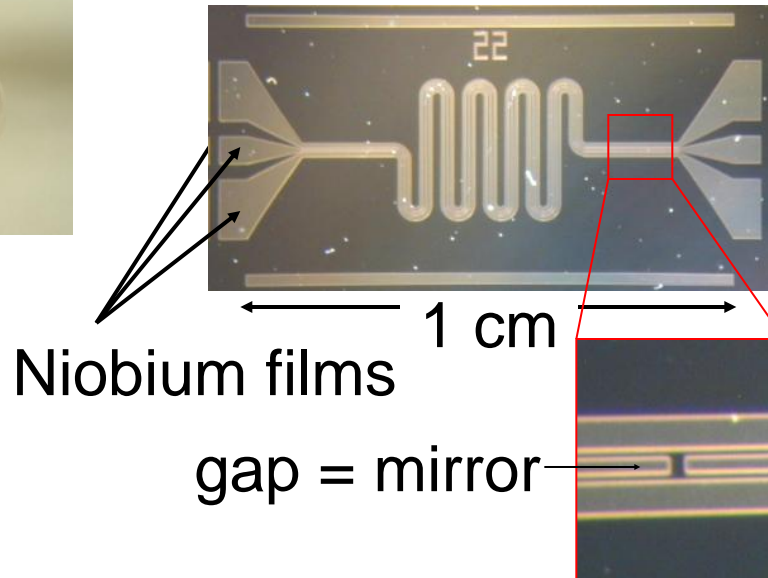


Implementation of Cavities for cQED

Superconducting coplanar waveguide transmission line



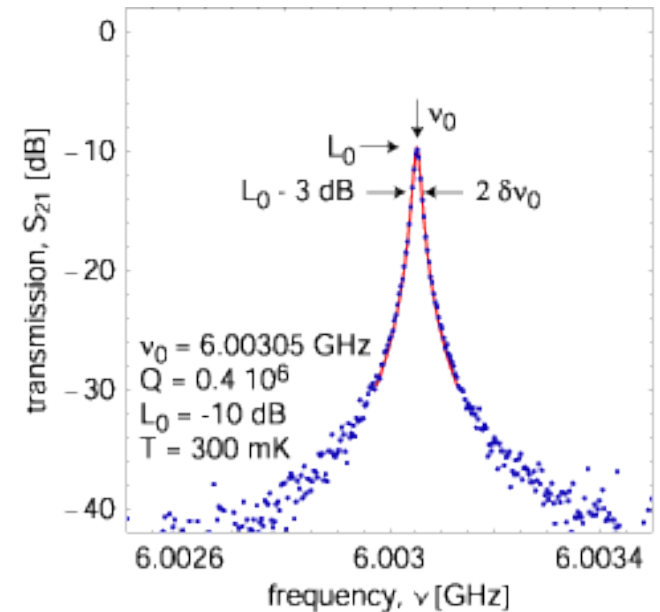
Optical lithography at Yale



Niobium films

gap = mirror

$Q > 600,000 @ 0.025 \text{ K}$

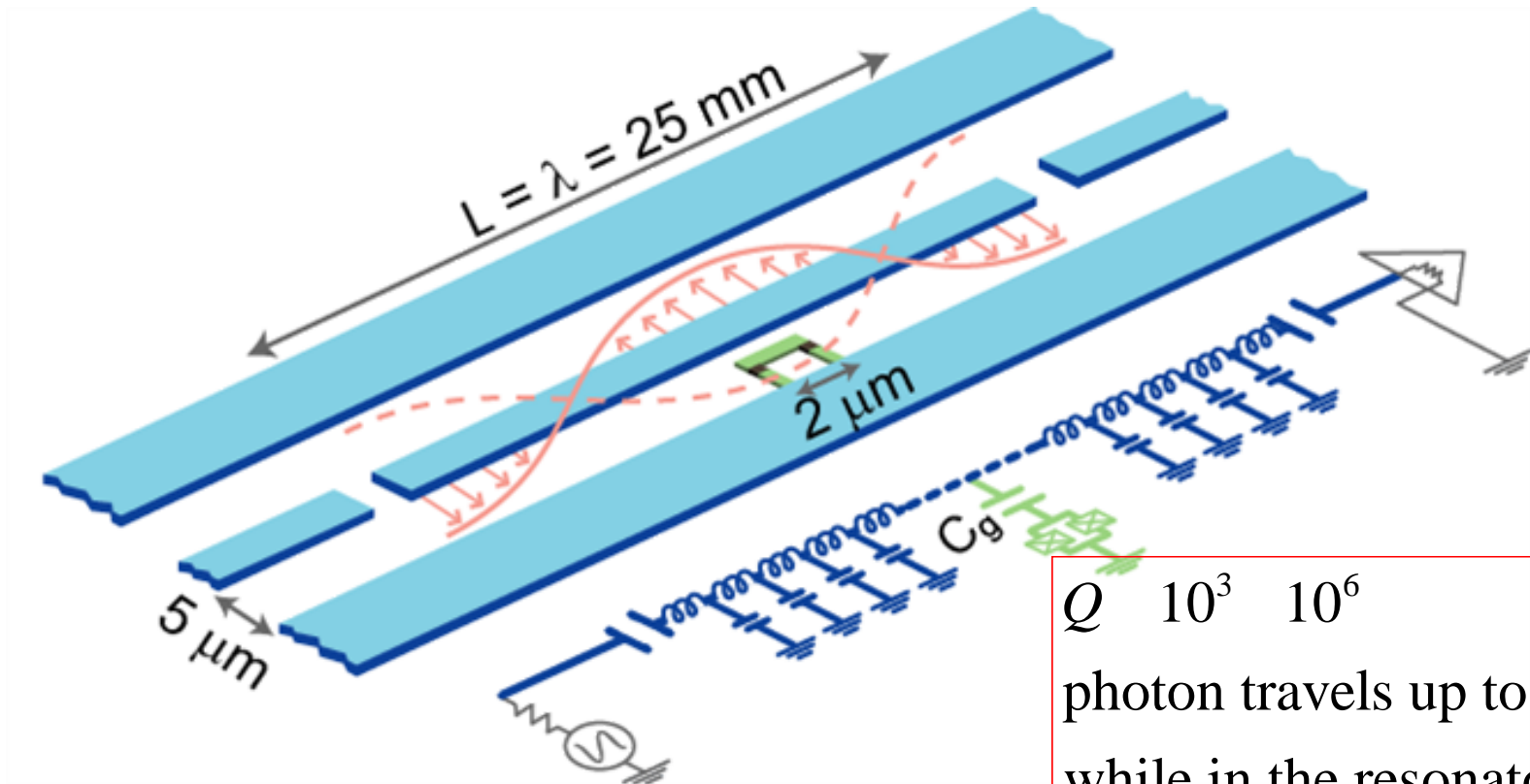


6 GHz:

$$\hbar \quad 300 \text{ mK} \quad \longrightarrow \quad \langle n \rangle \ll 1 \quad @ \quad 20 \text{ mK}$$

- Internal losses negligible – Q dominated by coupling

Circuit Quantum Electrodynamics



$Q \quad 10^3 \quad 10^6$
photon travels up to 10 km
while in the resonator!
CPW optical fiber

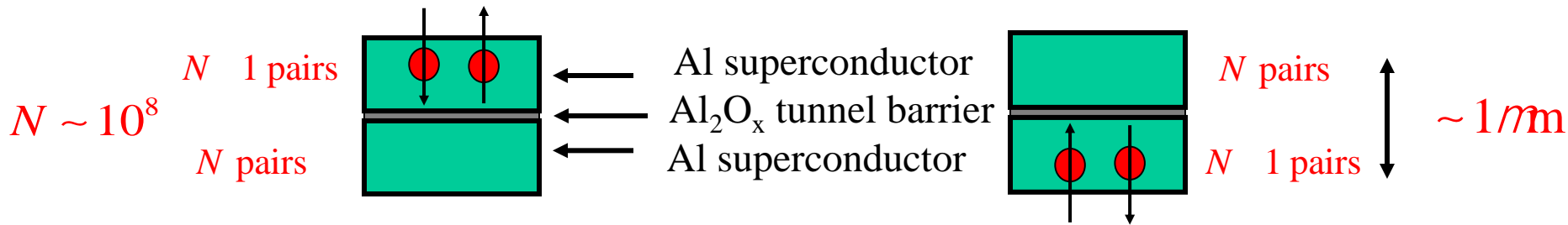
elements

- cavity: a superconducting 1D transmission line resonator
- artificial atom: a Cooper pair box (large d)

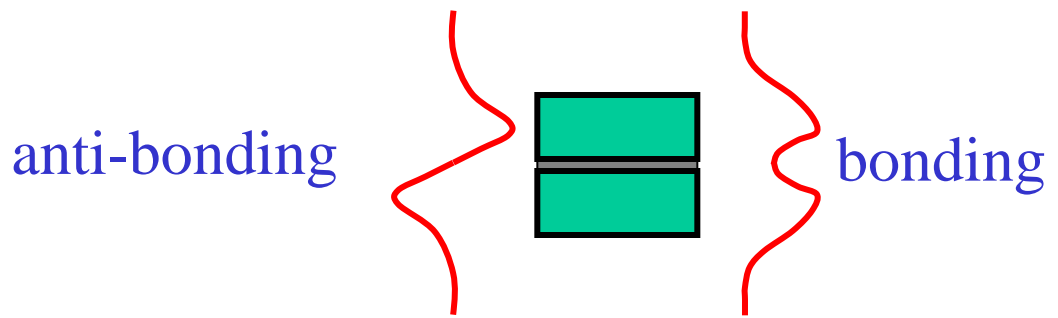
Artificial Atom

Superconducting Tunnel Junction

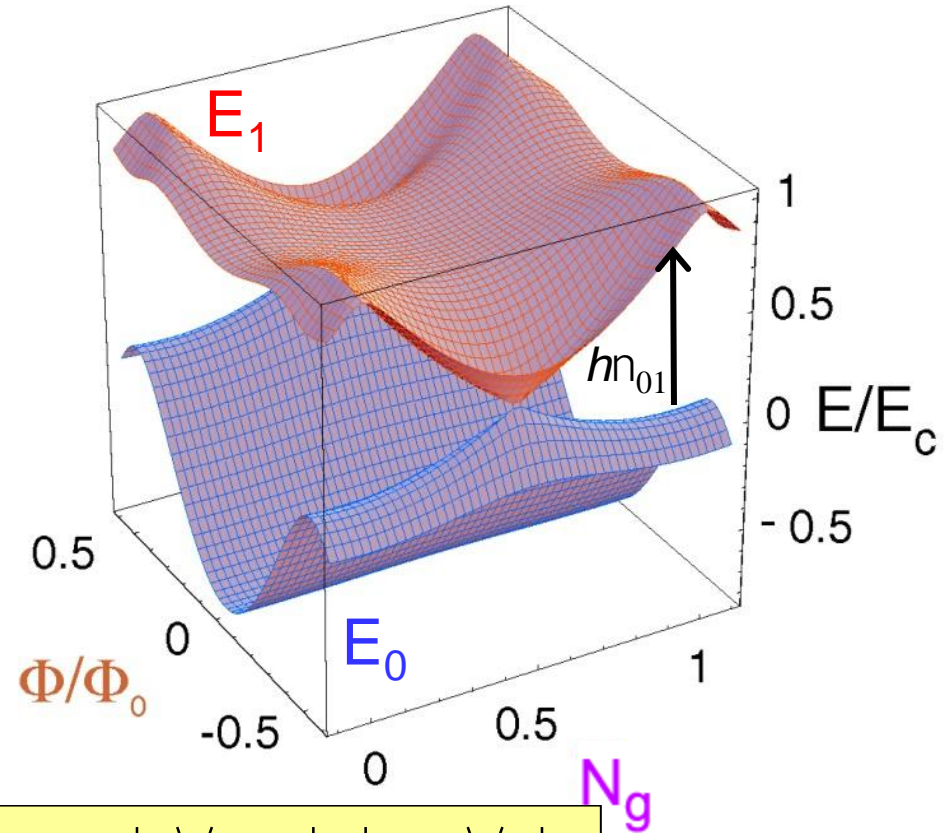
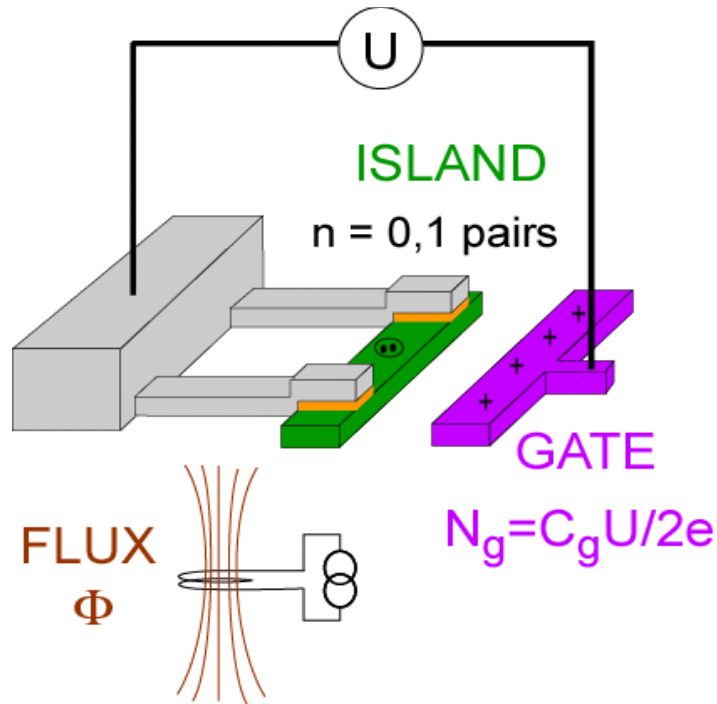
(The only non-linear dissipationless circuit element.)



Josephson Tunneling Splits
the Bonding and Anti-bonding „Molecular Orbitals“
Covalently Bonded Diatomic „Molecule“



SPLIT COOPER PAIR BOX QUBIT: THE “ARTIFICIAL ATOM” with two control knobs

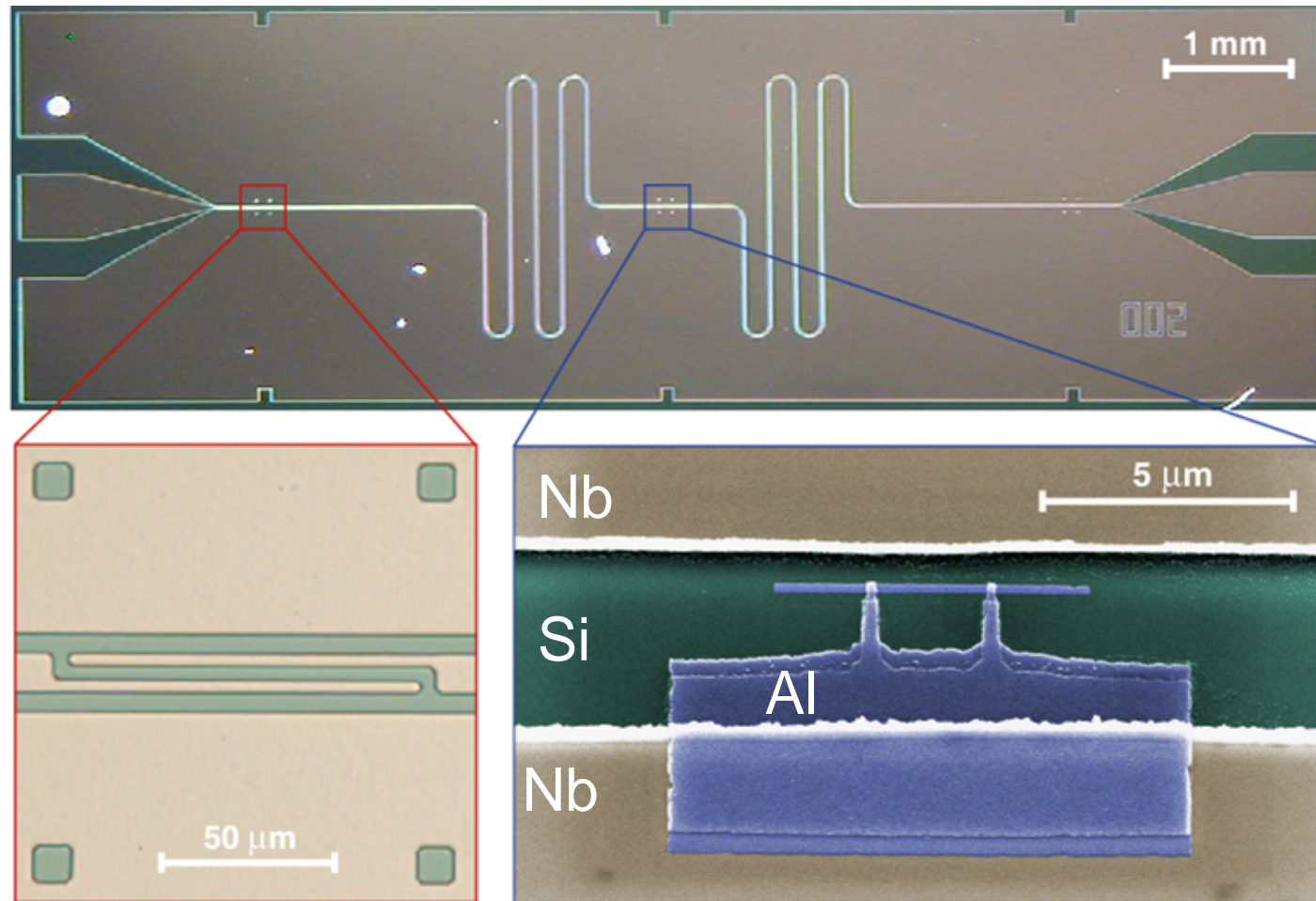


$$\hat{H} = \sum_n E_C (n - N_g)^2 |n\rangle\langle n| - E_j \cos \frac{\Phi}{\Phi_0} \frac{|n\rangle\langle n-1| + |n-1\rangle\langle n|}{2}$$

THE Hamiltonian

[Devoret & Martinis, QIP, 3, 351-380(2004)]

First Generation Chip for Circuit QED



First coherent coupling of solid-state qubit to single photon:
A. Wallraff, et al., *Nature (London)* **431**, 162 (2004)

Theory: Blais et al., *Phys. Rev. A* **69**, 062320 (2004)

Advantages of 1d Cavity and Artificial Atom

$$g = \vec{d} \cdot \vec{E}_{\text{RMS}} / \hbar$$

$$\hbar g = d E_{\text{RMS}} (a \quad a^\dagger)$$

quantized electric field

Vacuum fields:

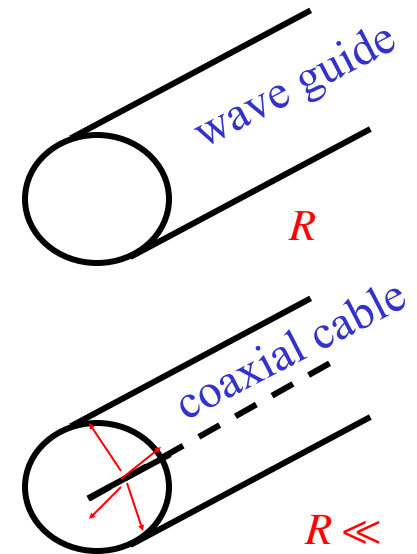
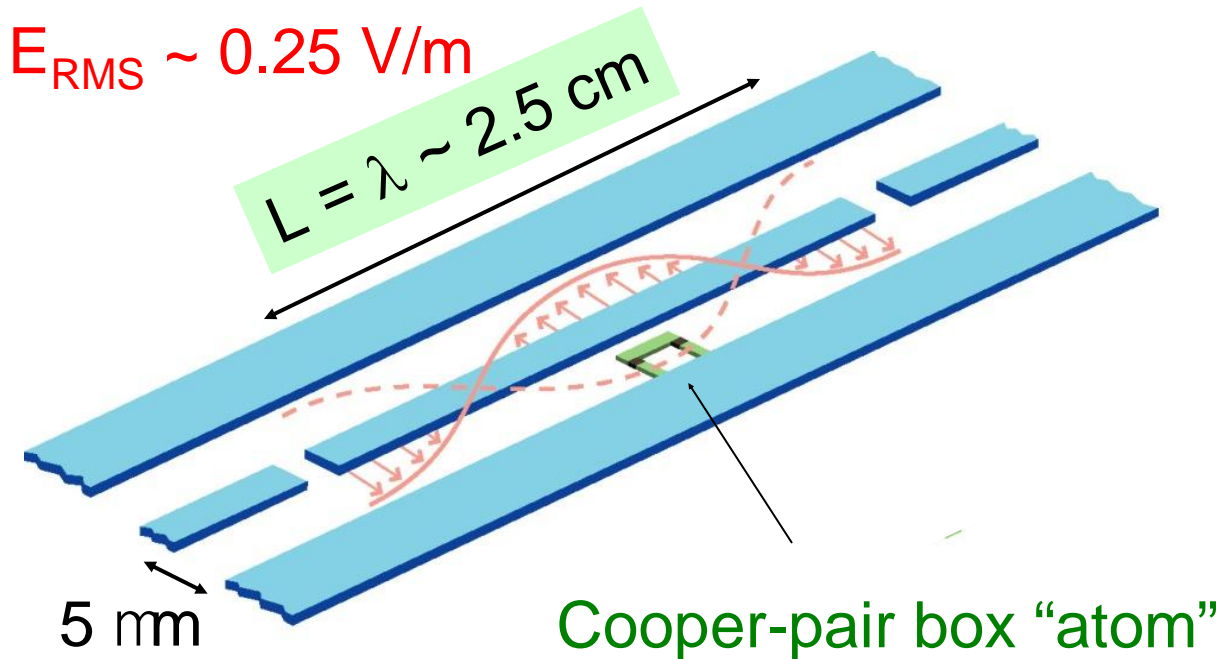
mode volume 10^6 ^3

zero-point energy density
enhanced by 10^6

Transition dipole:

$d \sim 40,000 ea_0$

$\sim 10d_{\text{Rydberg } n=50}$



Extreme Strong Coupling Limit



$$g = \vec{d} \cdot \vec{E}_{\text{RMS}} / \hbar$$

Maximum dipole moment: $2e \frac{d}{2} = ed$

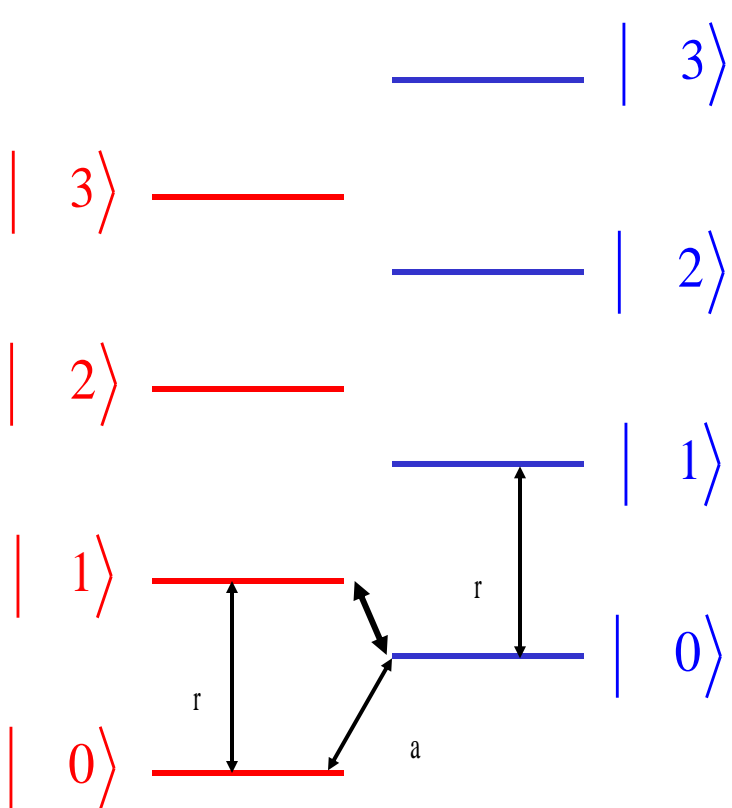
Vacuum electric field: $\frac{0_r}{2} E_{\text{rms}}^2$ (energy density) d^2 (mode volume) $\frac{1}{2} \frac{1}{2} \hbar$

Vacuum Rabi coupling: (independent of d !) $\frac{g}{\sqrt{r}} = 0.04$

Present experiments: $\frac{g}{2} \frac{100 \text{ MHz}}{5 \text{ GHz}} = 0.02$

Jaynes Cum m ings H am iltonian: “dressed atom ” picture

$$H = \underbrace{\hbar \omega_r a^\dagger a}_{\text{cavity}} + \underbrace{\frac{\hbar \omega_0}{2} \sigma_z}_{\text{qubit}} + \underbrace{\hbar g a^\dagger \sigma_+ + \hbar g a \sigma_-}_{\text{dipole coupling}}$$



Dispersive case:
Atom is strongly detuned
from the cavity

Photons do not cause „real“ transitions
in the qubit.. only level repulsion.

Cavity and atom frequencies shift.
Second order p.t. $\frac{g^2}{\Delta}$

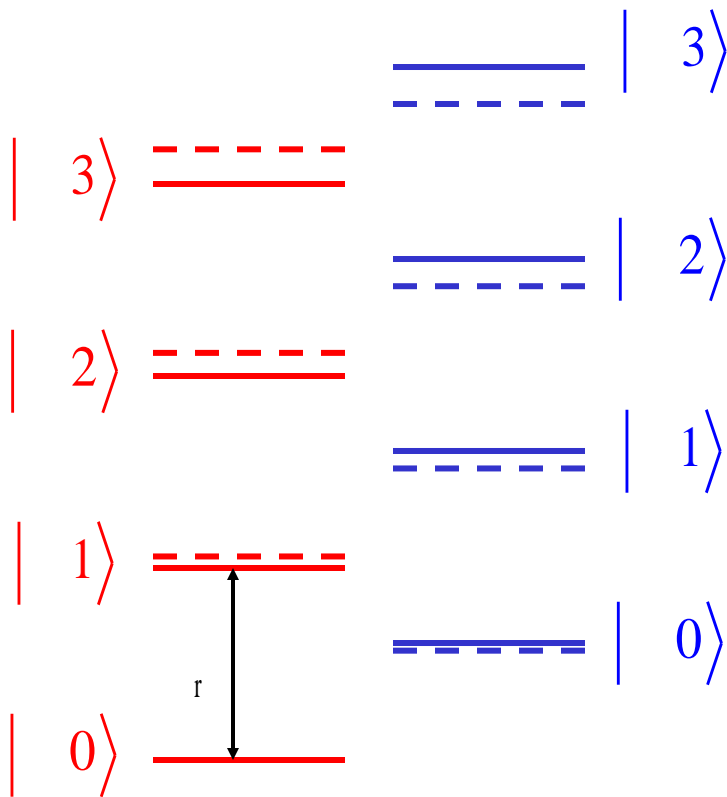
Qubit ground

Qubit excited

“dressed atom” picture: 2nd order perturbation theory

approximate diagonalization for $|\Delta| = |\omega_a - \omega_r| \gg g$

$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$



Cavity frequency shifts up
if qubit is in ground state;
down if qubit in excited state.

(0)

Qubit ground

Qubit excited

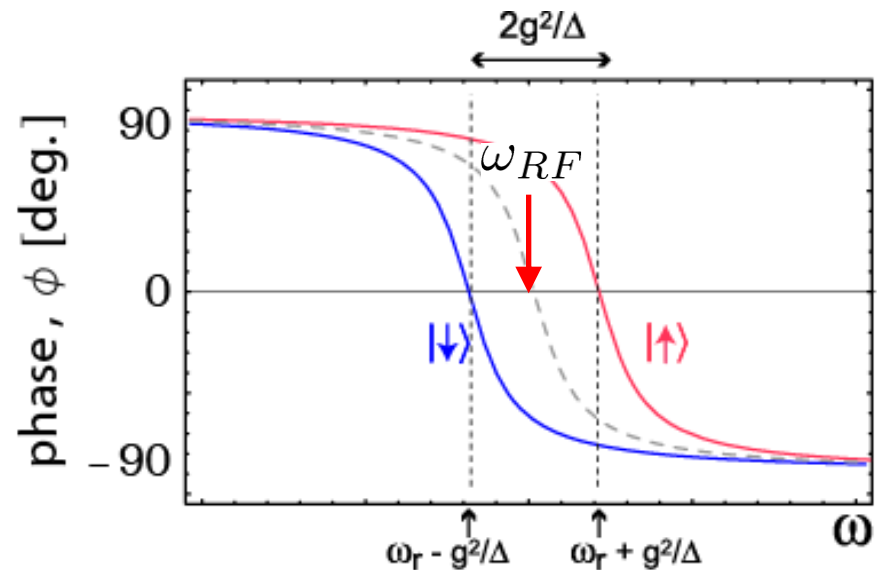
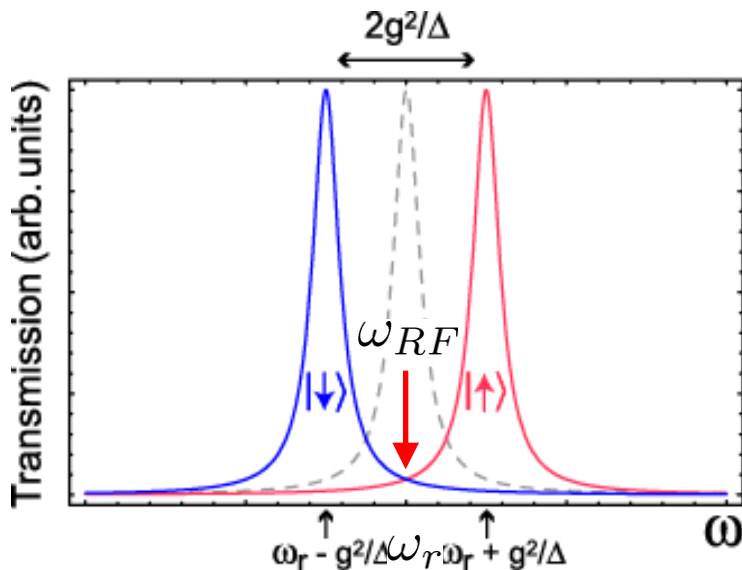
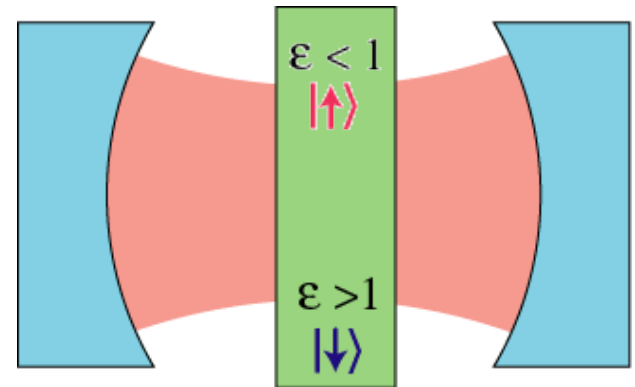
cQED Dispersive Measurement I: atom strongly detuned from cavity

approximate diagonalization for $|\Delta| = |\omega_a - \omega_r| \gg g$

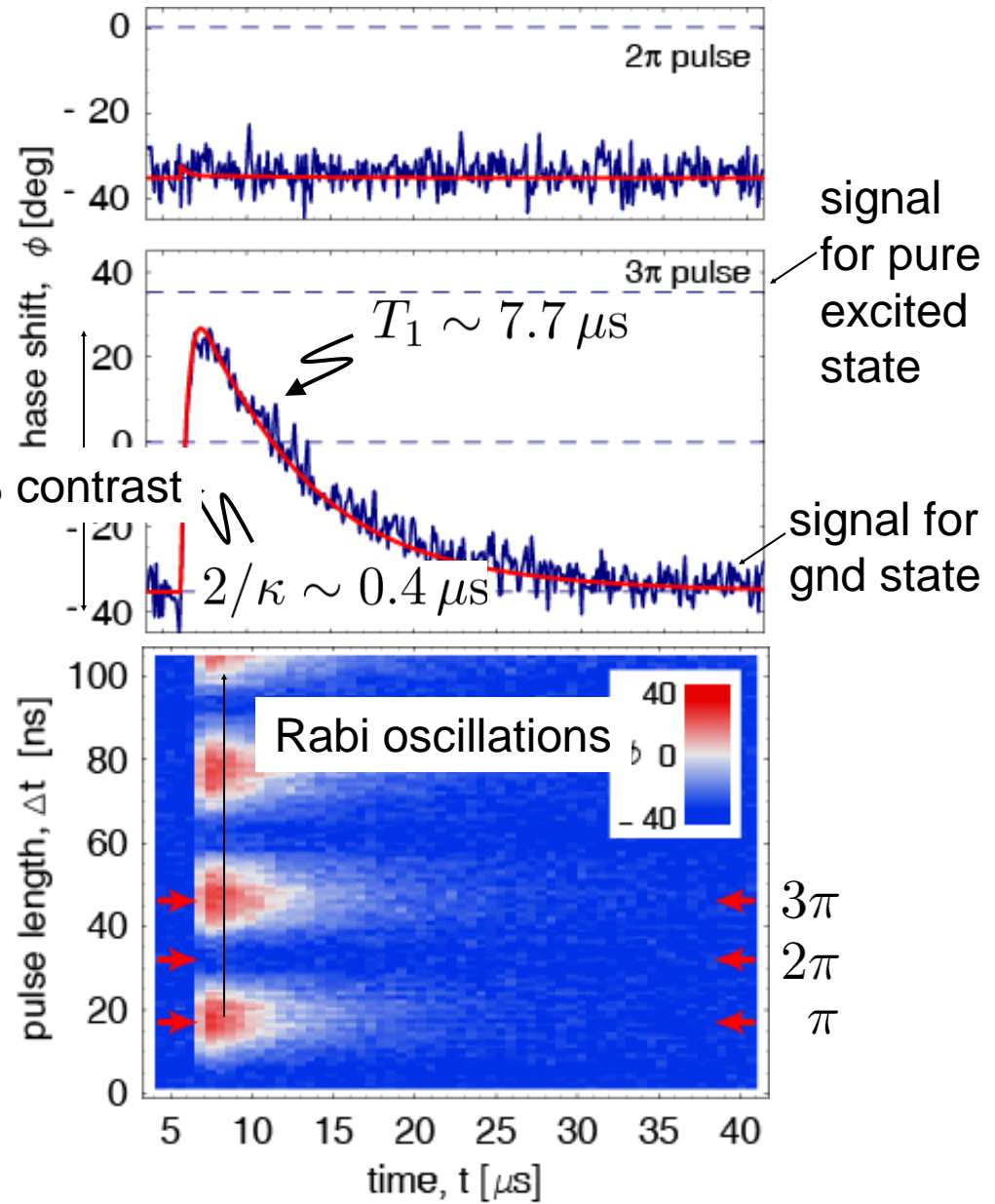
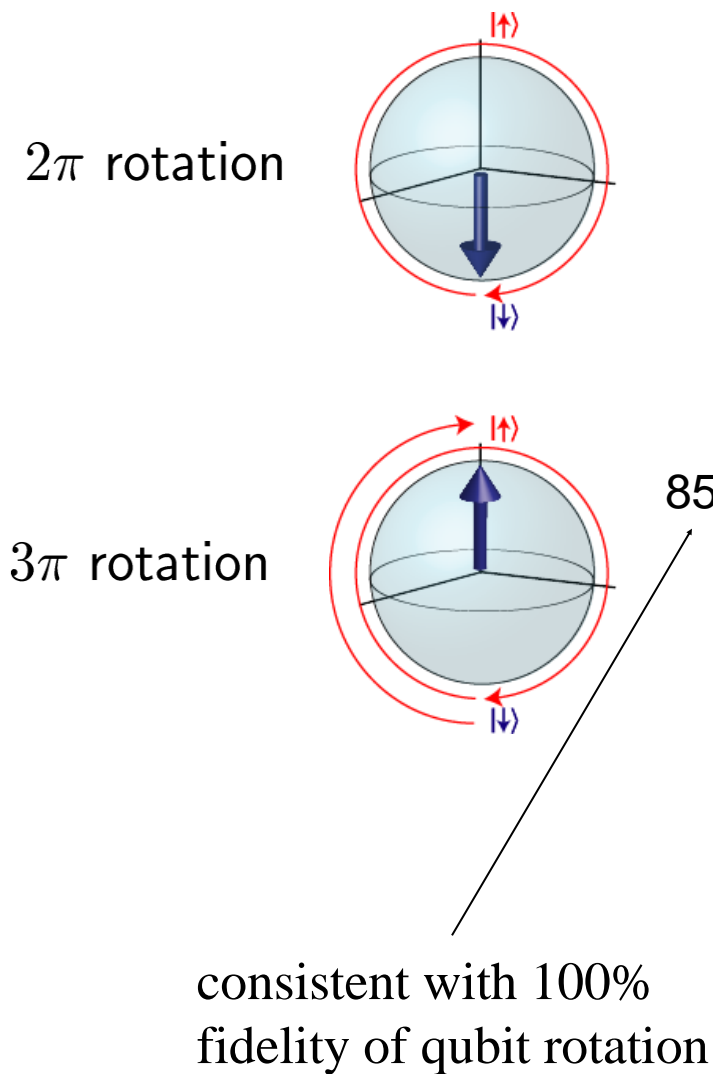
$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta} \sigma_z \right) a^\dagger a + \frac{1}{2} \hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

//
cavity frequency shift
and qubit ac-Stark shift

//
Lamb shift

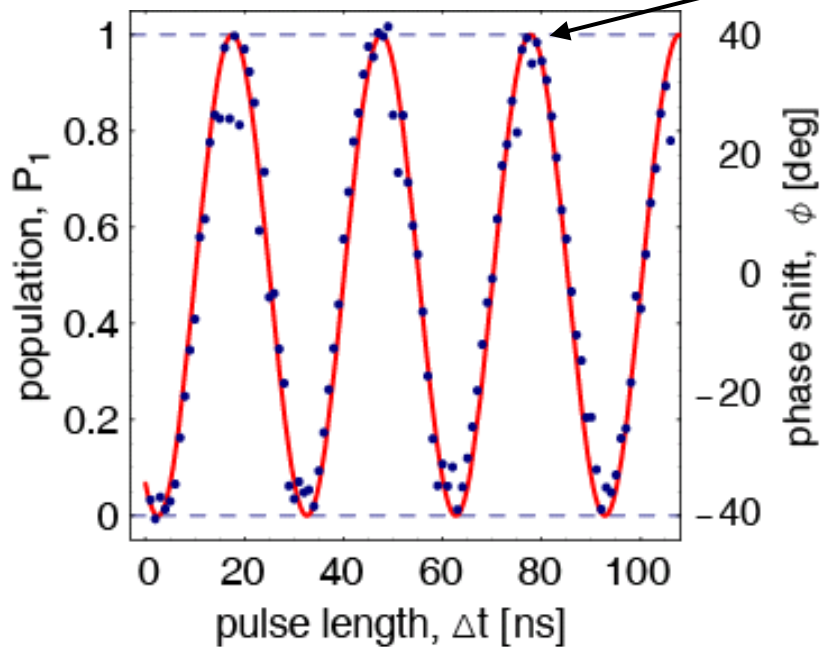


Coherent Control of Qubit in Cavity



High Visibility Rabi Oscillations

Rabi oscillations:



visibility $95 \pm 5\%$

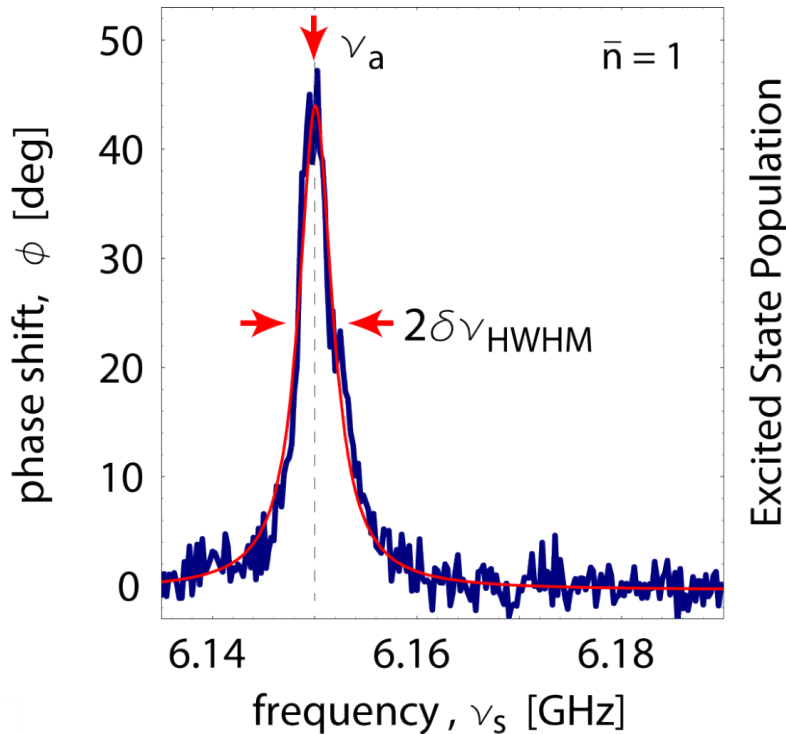
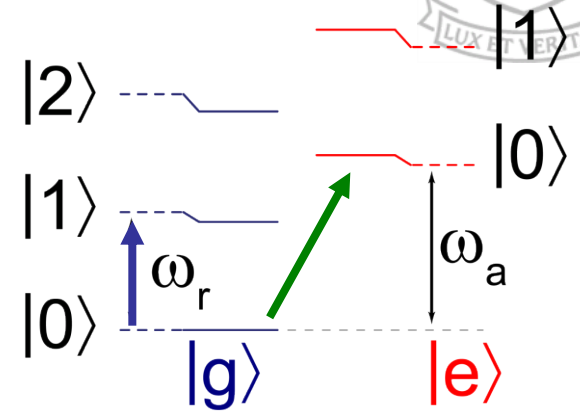
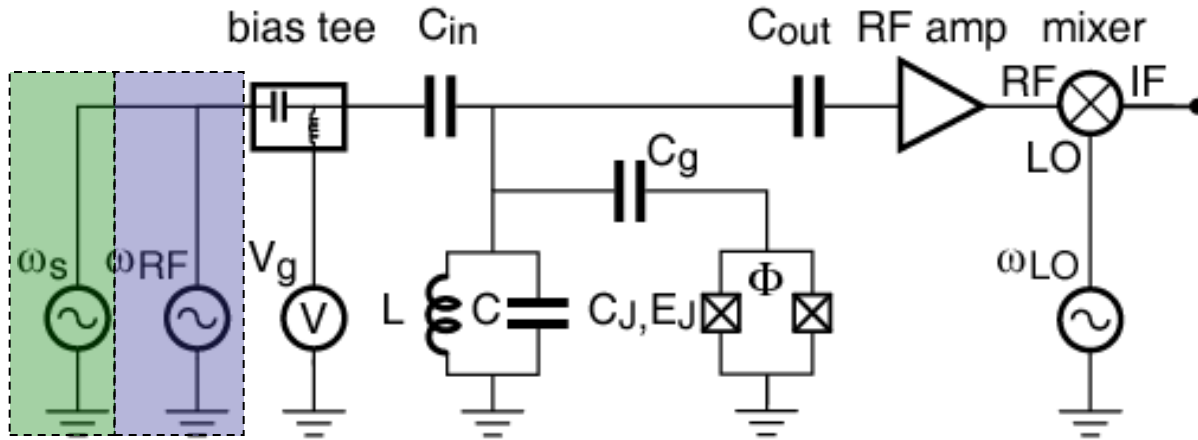
(i.e. inferred fidelity of operation)

for superconducting qubits:

- first high visibility
- well characterized and understood measurement
- good control accuracy

Indicates no undesired entanglement with environment during operations.

Spectroscopy of the qubit



When qubit is excited cavity responds

$n \approx 3$ MHz \sim dephasing rate

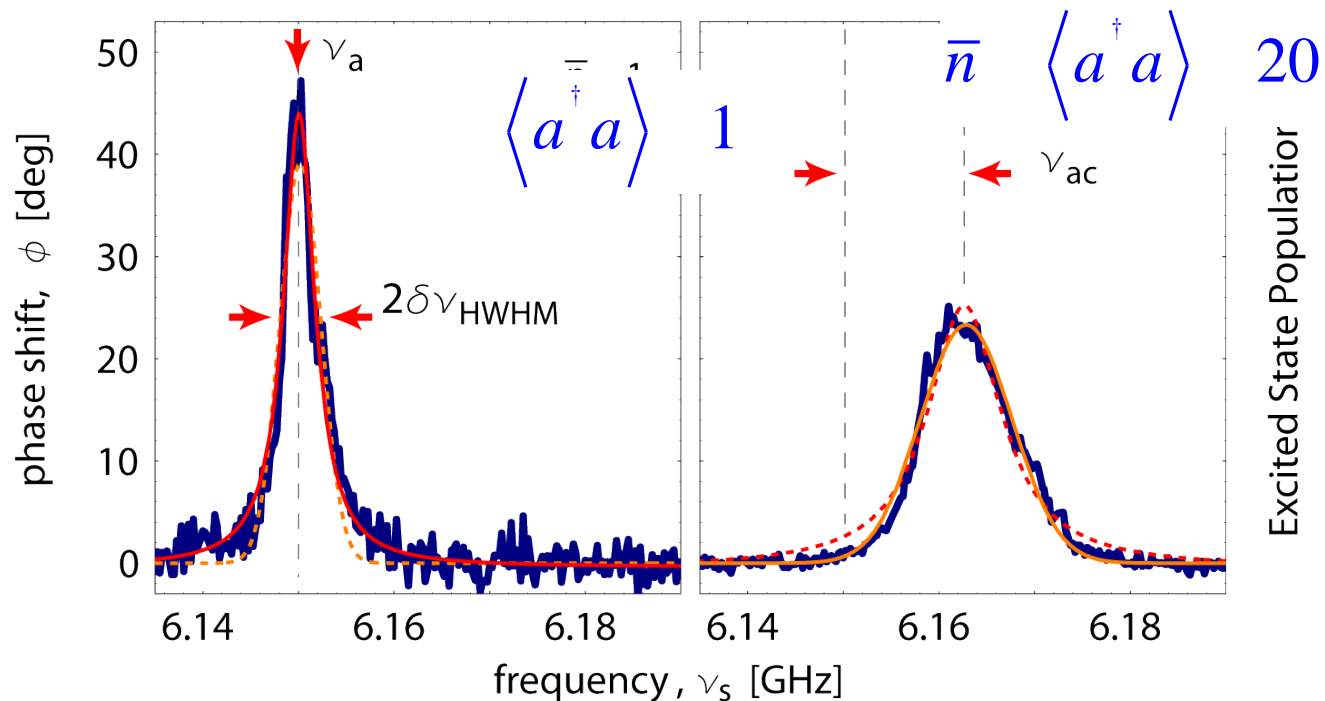
$Q > 20,000$!!!

Backaction of QND Measurement

$\gg g :$

$$H_{\text{eff}} = \hbar \nu_r a^\dagger a + \frac{\hbar}{2} \nu_{ac} \frac{g^2}{\omega - \nu_s} a^\dagger a$$

AC Stark shift of qubit by photons

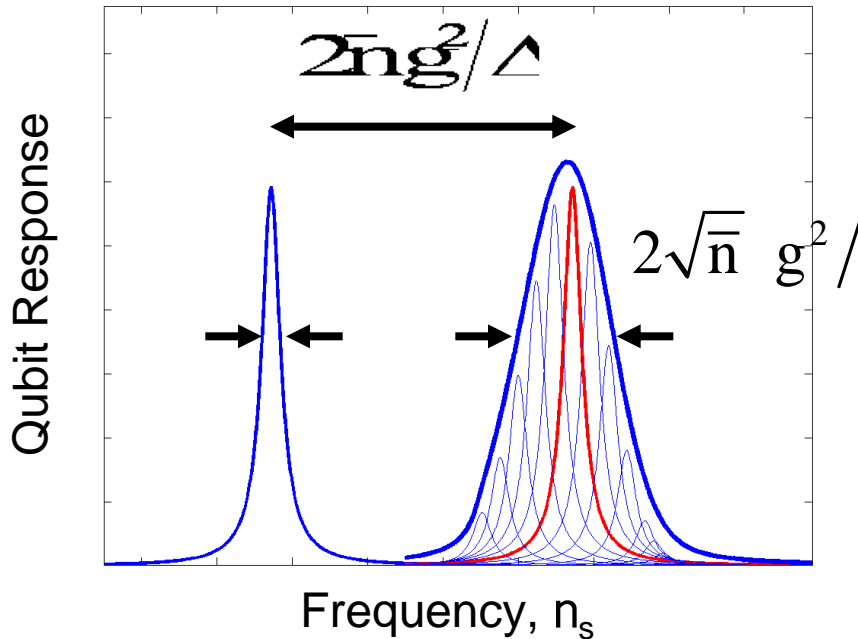


AC Stark measurements: Schuster et al., PRL 94, 123602 (2005).

Measurement induced dephasing (back action)

- Measurement dephasing from Stark random shifts
- Gaussian lineshape at strong coupling is sum of Lorentzians

$$H_{\text{eff}} = \frac{1}{2} \omega_0 a^\dagger a + \frac{g^2}{2\Delta} a^\dagger a$$



- Coherent state has shot noise
- Peaks are Poisson distributed

$$P(n) = \frac{(\bar{n})^n}{n!} e^{-\bar{n}}$$

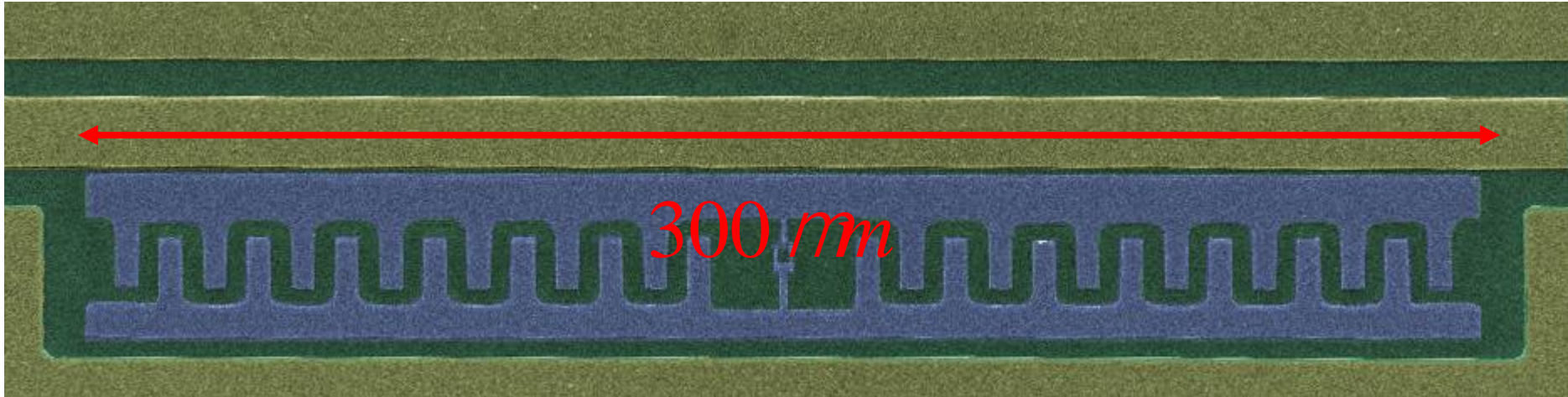
“Strong Dispersive Regime”:

What if $2g^2/\Delta > \dots$?

Need to increase coupling strength

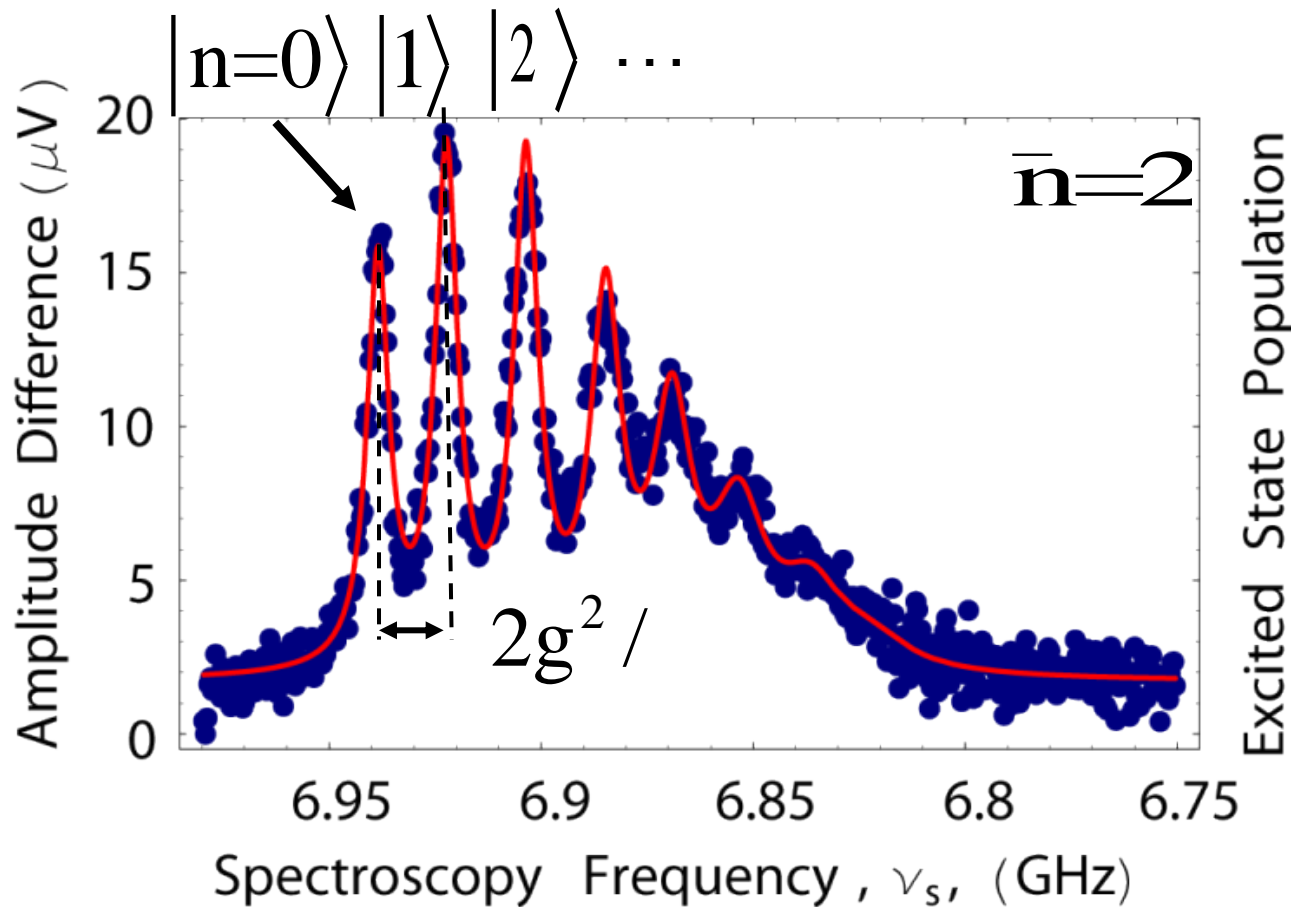


Want bigger coupling... Make a bigger atom!



	Old	New
Resonant	$g \ 12 \text{ MHz} > ,k$	$g \ 115 \text{ MHz} > ,k$
Dispersive	$\frac{g^2}{ k }$	$\frac{g^2}{ k }$

Resolving Photon States in a Circuit



$$g = 100 \text{ MHz}$$

$$2g^2/ = 20 \text{ MHz}$$

- # distribution visible in spectroscopy
- Peaks well separated $\frac{2g^2}{\hbar\omega} \gg 1$
- Well into dispersive limit $\frac{\hbar\omega}{g} \ll 1$

Resolving individual photon numbers using ac Stark shift of qubit transition frequency

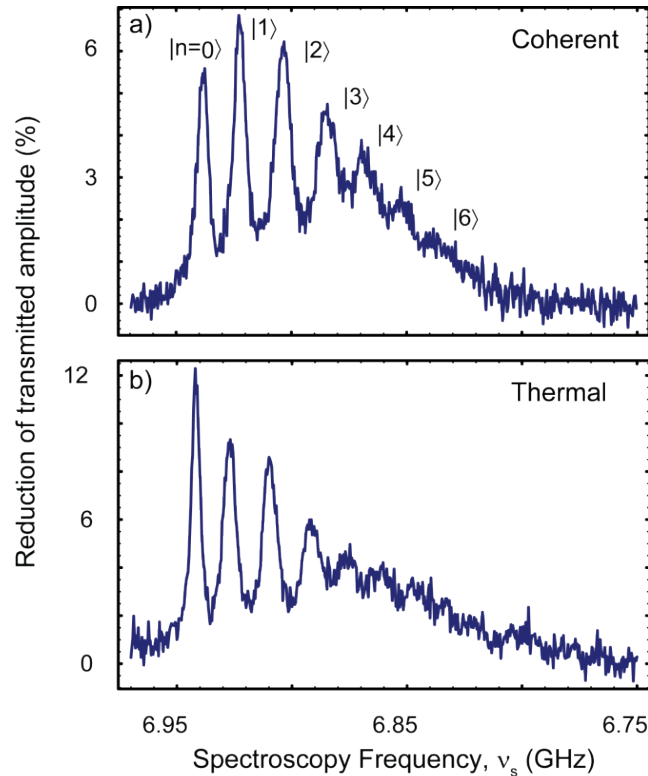
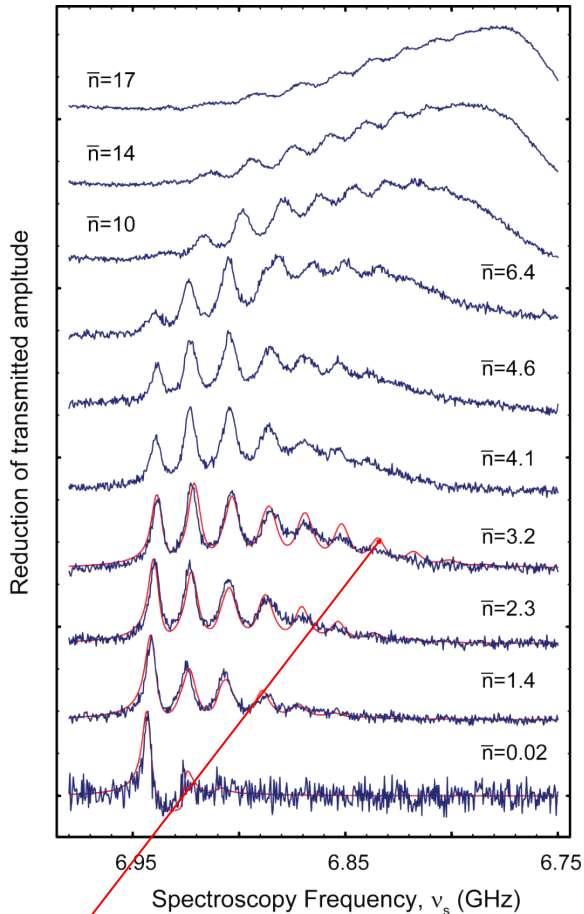
coherent input power = 10^{17} Watts

Coherent state
 $\bar{n} = 2$
 Poisson distribution

$$P(n) = \frac{(\bar{n})^n}{n!} e^{-\bar{n}}$$

Thermal state
 $\bar{n} = 2$
 Bose-Einstein distribution

$$P_{th}(n) = \frac{(\bar{n})^n}{\bar{n} (1 + \bar{n})^{n+1}}$$

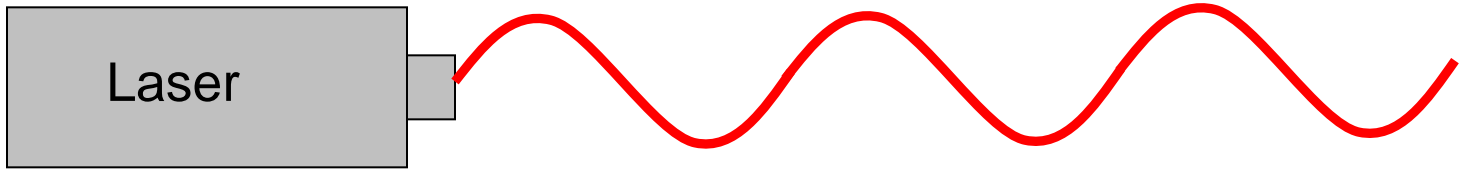
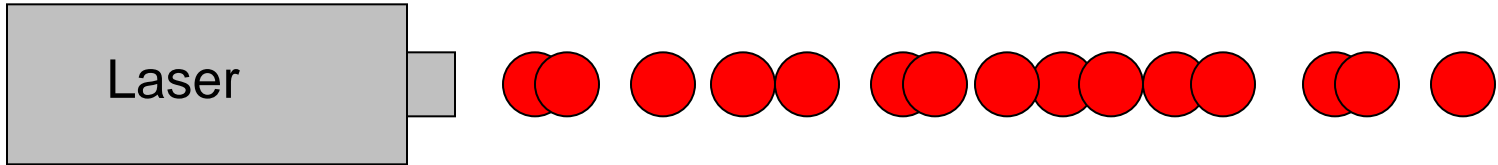


Master eqn. simulations show that homodyne signal is an approximate proxy for cavity photon number distribution.

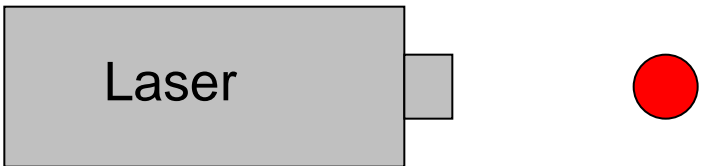
Master Eqn. Simulations

Waves and Particles

Coherent State: Many Photons



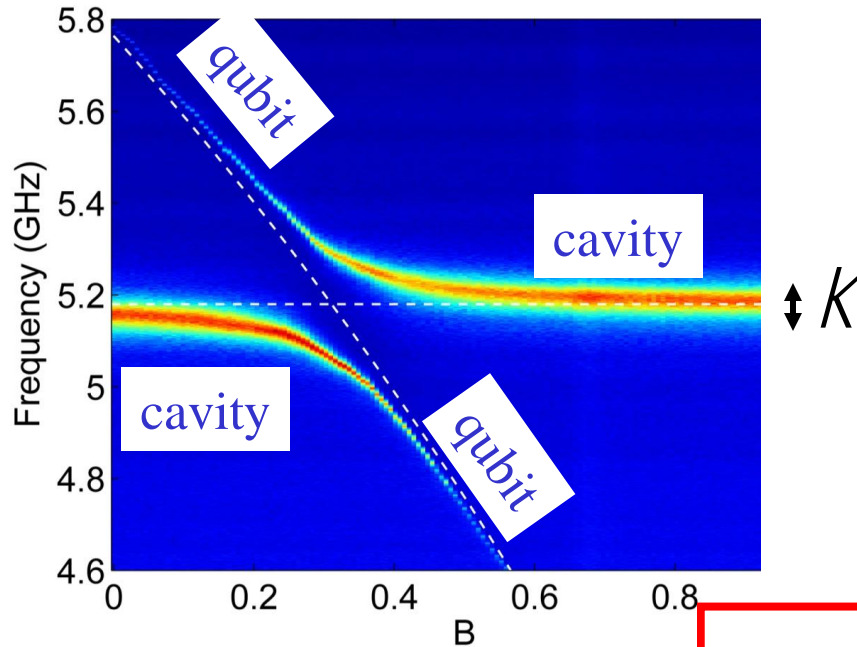
Single Photons



What is the electric field of a single photon?

Purcell Effect:

Low Q cavity can enhance rate of spontaneous emission of photon from qubit



Photon emission becomes dominate decay channel. Maps qubit state onto state of flying qubit (photon field):

$$\begin{array}{l}
 |\text{IN}\rangle \quad | \rangle \quad | \rangle |0\rangle \\
 |\text{OUT}\rangle \quad | \rangle \quad a^\dagger |0\rangle
 \end{array}$$

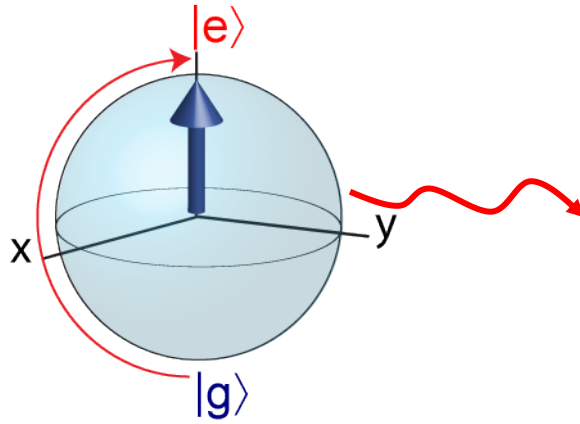
$$\frac{g^2}{k}$$

Intrinsic non-radiative decay rate

Cavity enhanced decay rate

Oscillations in Electric Field?

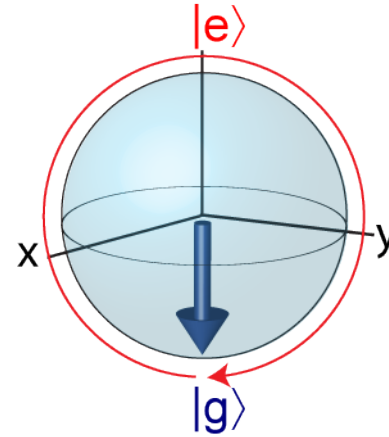
pulse



$$|0e\rangle \quad |1g\rangle$$

$$\langle 1|a^\dagger \quad a|1\rangle = 0$$

2 pulse



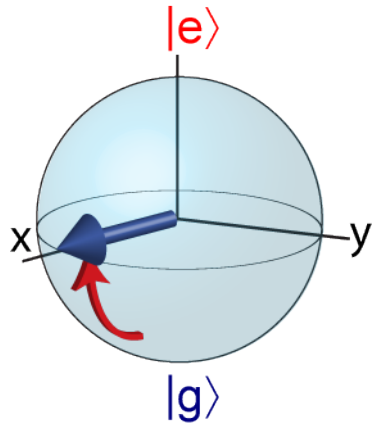
$$|0g\rangle \quad |0g\rangle$$

$$\langle 0|a^\dagger \quad a|0\rangle = 0$$

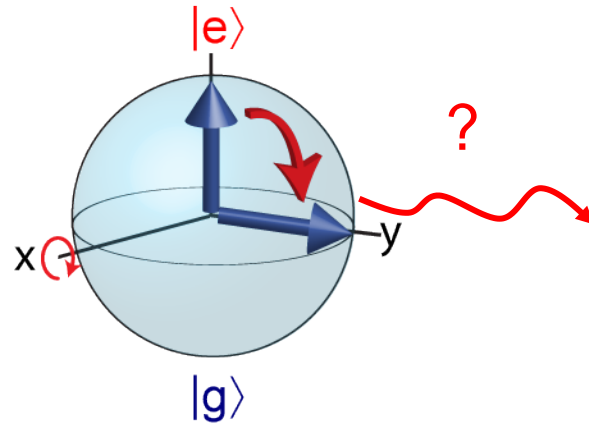
No average voltage for Fock states!
Phase completely uncertain!

Oscillations in Electric Field?

$\pi/2$ pulse



Arbitrary pulse



$$\frac{1}{\sqrt{2}} |0\rangle |g\rangle |e\rangle$$

$$|0g\rangle |0e\rangle$$

$$\frac{1}{\sqrt{2}} |0\rangle |1\rangle |g\rangle$$

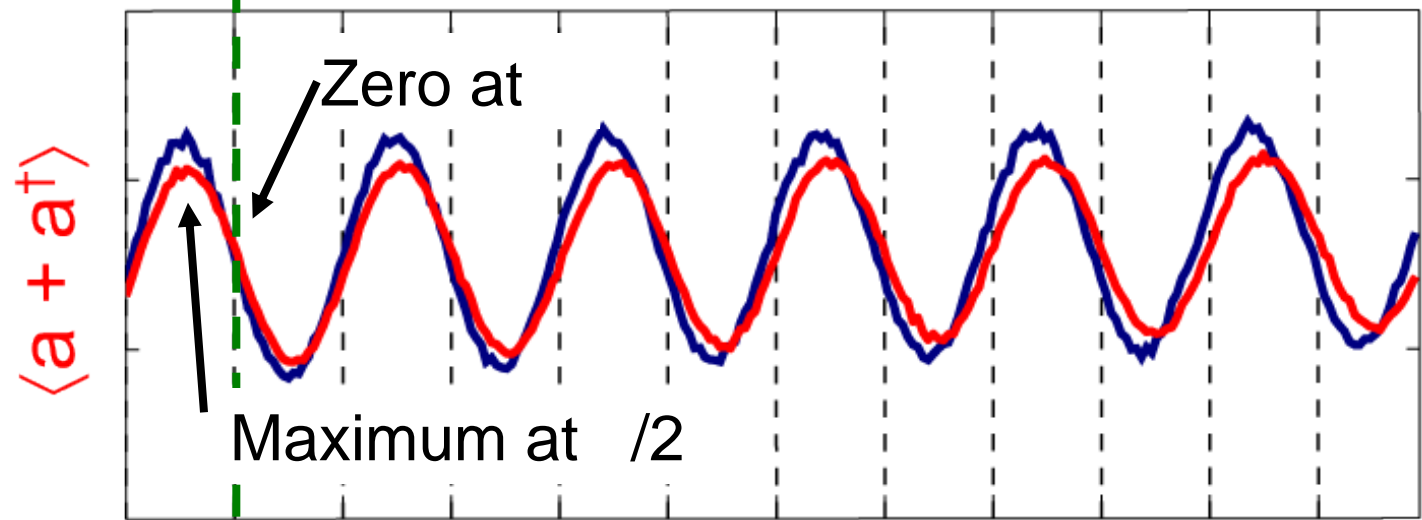
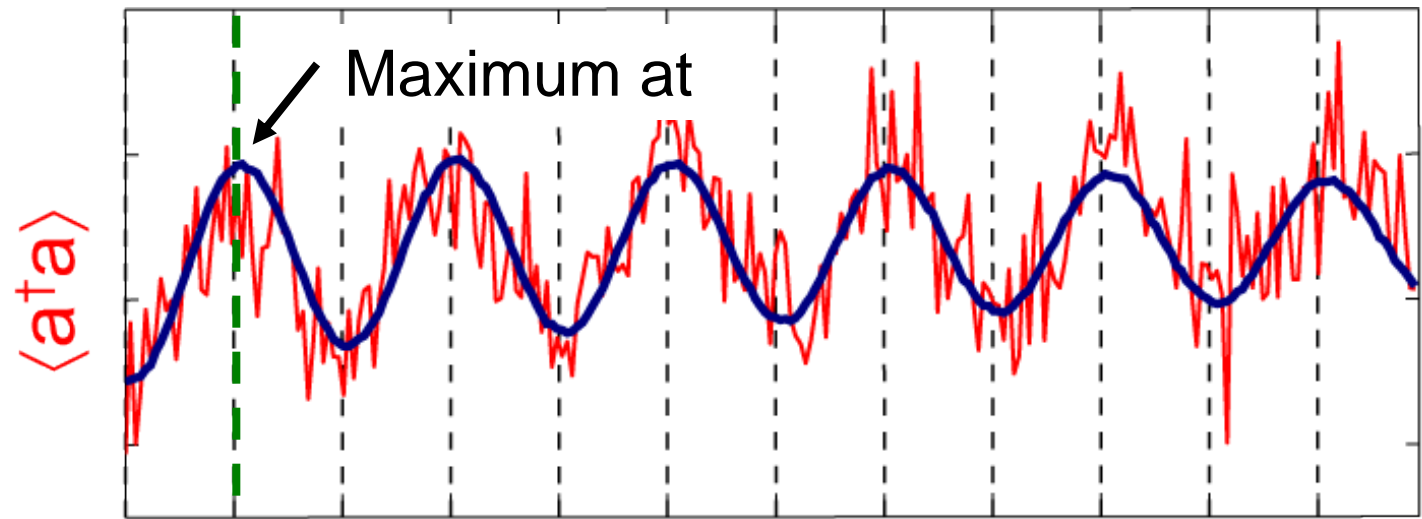
$$|0\rangle |1\rangle |g\rangle$$

$$\frac{1}{2} \langle 0 | a^\dagger a | 0 \rangle = \frac{1}{2} \langle a^\dagger a \rangle^* \quad * \quad * \quad \langle x \rangle$$

Mapping the qubit state on to a photon



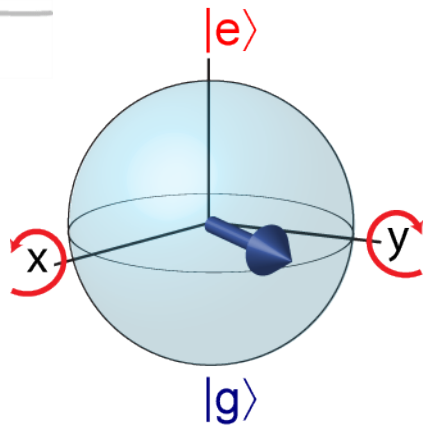
Measured photon state



0 2π 4π 6π 8π 10π
Rabi angle (rads)

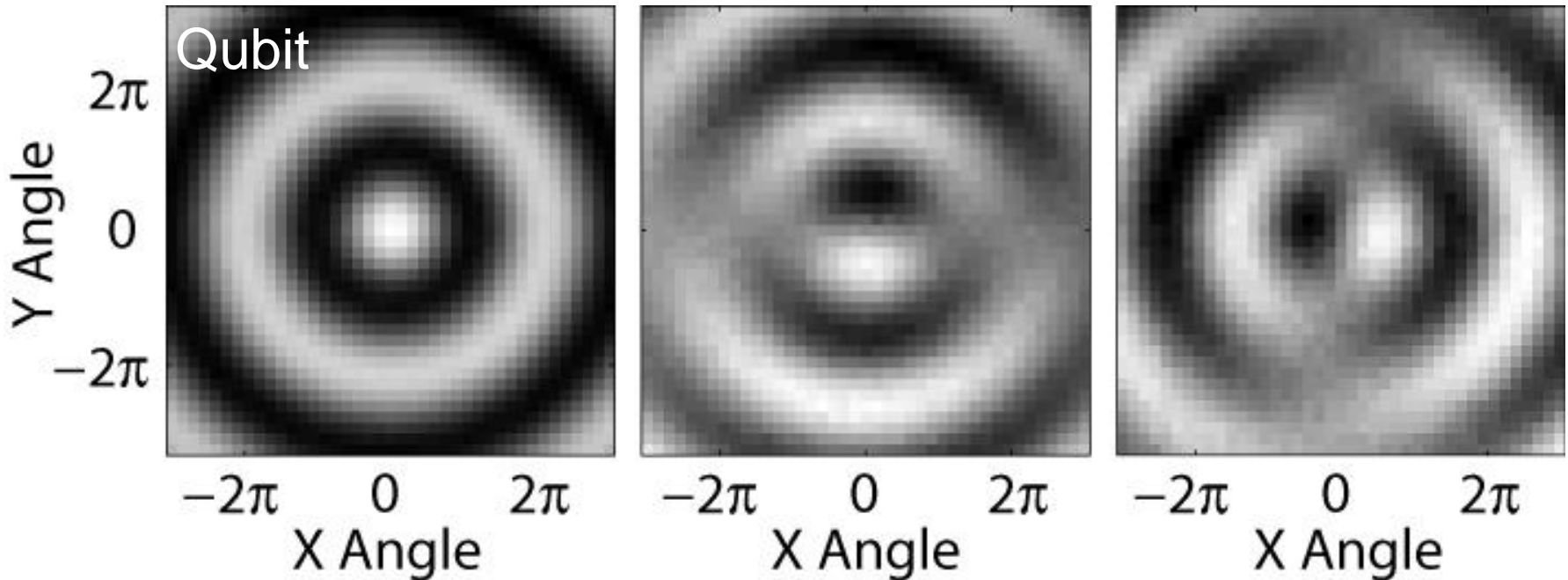
Measured qubit state
 $\langle \sigma_z \rangle$
 $\langle \sigma_x \rangle$

“Fluorescence Tomography”



- Apply pulse about arbitrary qubit axis
- Qubit state mapped on to photon superposition

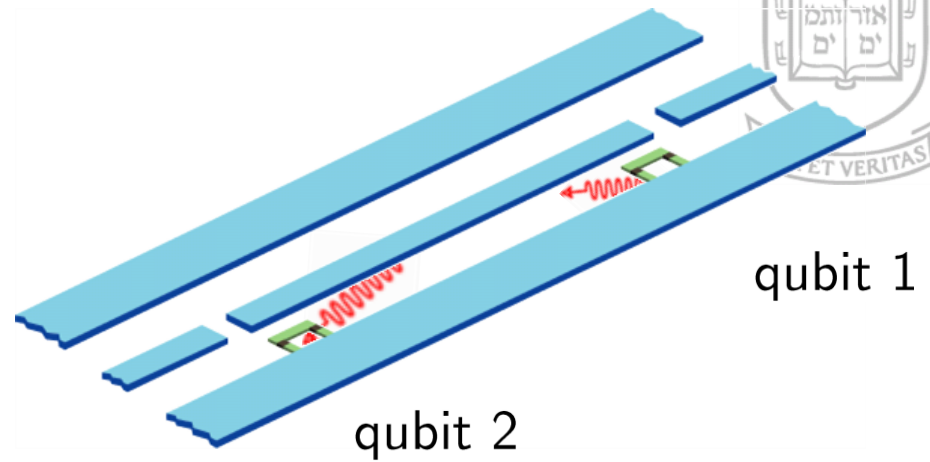
$$\langle \hat{z} \rangle \quad \langle a a^\dagger \rangle \quad \langle a a^\dagger \rangle$$



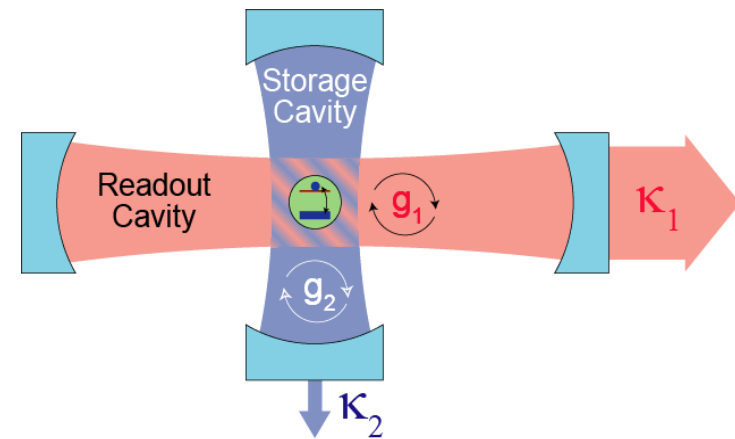
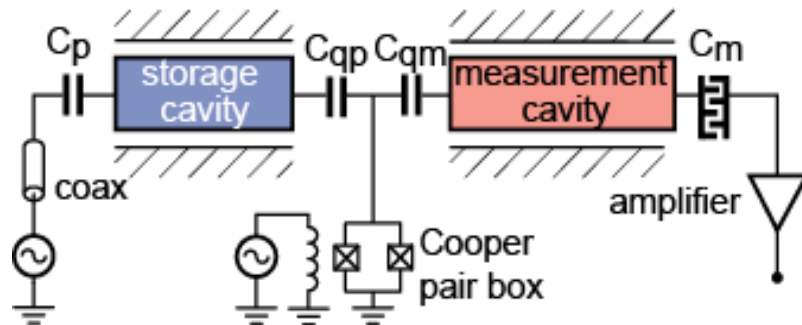
all of the above are data!

Future Possibilities

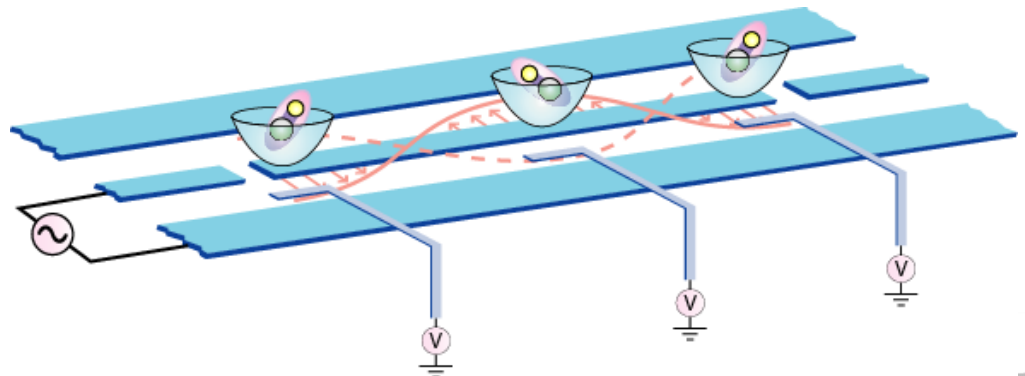
Cavity as quantum bus
for two qubit gates



High-Q cavity as quantum memory



Cavities to cool
and manipulate
single molecules?



SUMMARY

Cavity Quantum Electrodynamics



cQED



“circuit QED”

Coupling a Superconducting Qubit to a Single Photon

- first observation of vacuum Rabi splitting
- initial quantum control results
- QND dispersive readout
- detection of particle nature of microwave photons
- single microwave photons on demand

„Circuit QED Strong Coupling of a Single Photon to a Cooper Pair Box

<http://pantheon.yale.edu/~smg47>

<http://www.eng.yale.edu/rslab/cQED>

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- “Coherent Coupling of a Single Photon to a Superconducting Qubit Using Circuit Quantum Electrodynamics,” A. Wallraff et al., [Nature](#) **431**, 162 (2004).
- “AC Stark Shift and Dephasing in a Superconducting Qubit Strongly Coupled to a Cavity Field,” D. I. Schuster, et al., [Phys. Rev. Lett.](#) **94**, 123602 (2005).
- “Approaching Unit Visibility for Control of a Superconducting Qubit with Dispersive Readout,” A. Wallraff et al., [Phys. Rev. Lett.](#) **95**, 060501 (2005).
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