The Enigma of The Transition to Turbulence in a Pipe

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Motivation

Scientific interest:

all theory –> laminar flow the norm.

In practice most pipe flows are turbulent even at modest flow rates.

Practical interest:

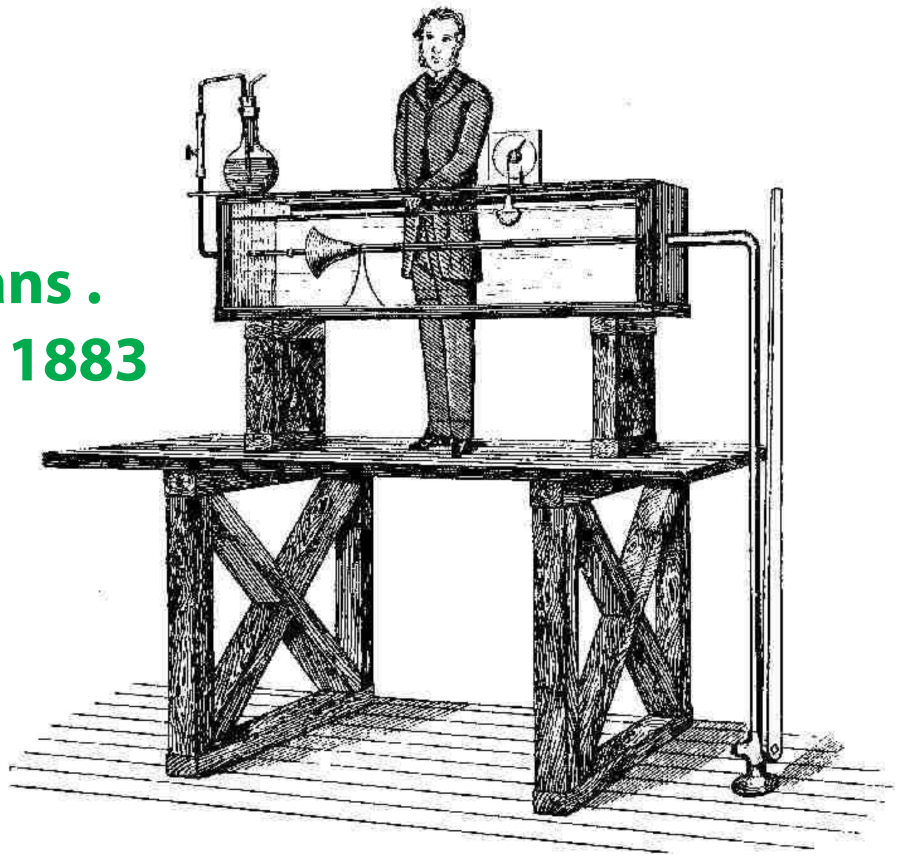
if flow could be maintained laminar –> tremendous energy saving.

Difficult problem, unresolved for ~ 125 years (Equations of motion known for even longer!)

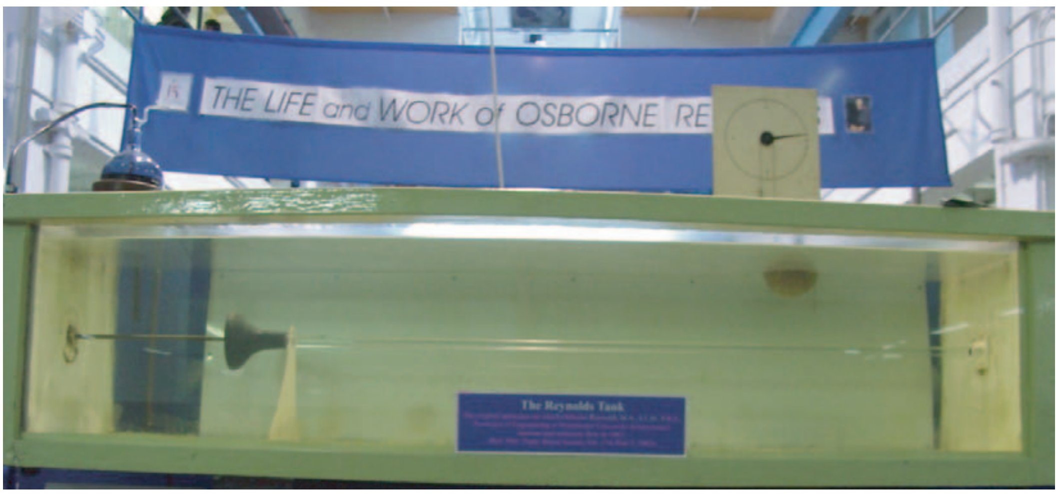
Use experimental physics approach with modern theory to try and resolve.

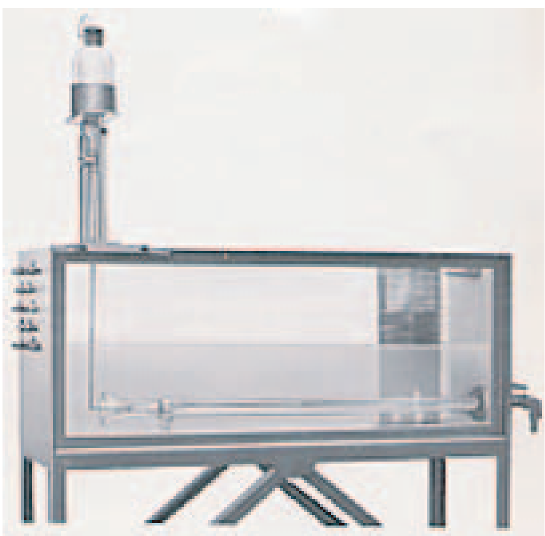
Reynolds' Experiment

Phil. Trans .
Roy. Soc. 1883



Manchester Engineering Dept.





Maruto Testing Company
Tokyo.

Facts About Pipe Flow

Fully developed circular pipe flow (Poiseuille flow) is linearly stable.

Single parameter: **Re = Ud/**ν.

Reynolds number: ratio of inertial to viscous terms in eqns. of motion.

When Re >2000 most pipe flows are turbulent.

BUT

Laminar Poiseuille flow can be created at Re~100,000 (Pfenniger 1961).

Suggests finite amplitude threshold required for transition.

Also global stability for Re < 2000.

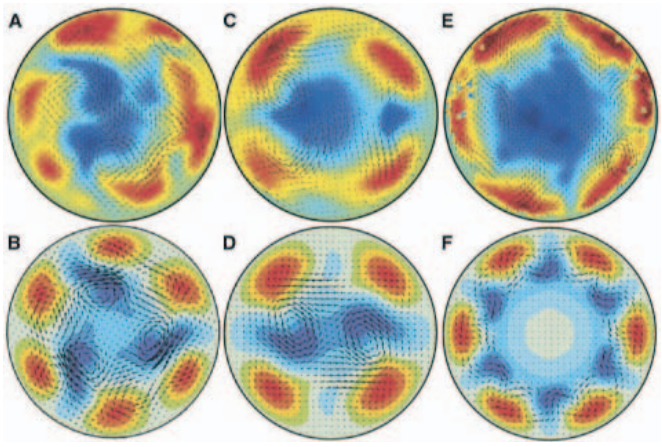
Modern Theoretical Developments

Kerswell Nonlinearity 18 (2005)

1. Existence of Nonlinear neutral solution.
   Smith & Bodonyi Proc. Roy. Soc A (1982)

2. Finite Amplitude Solutions Pipe : travelling waves.
   Wedin & Kerswell JFM (2004)
   Faisst & Eckhardt PRL (2003)

Poiseuille flow state: other travelling wave states NOT connected to it.



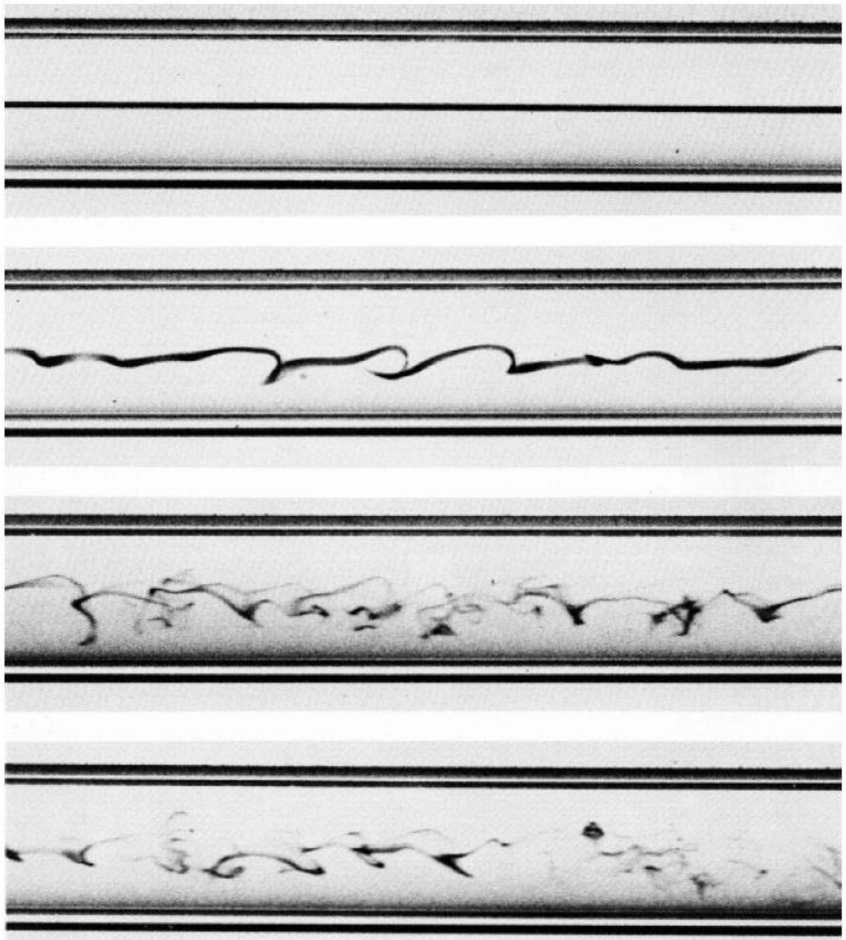
Hof et al Science (2004)

3. Transient growth of perturbations.

Butler, Farrell, Trefethen, Schmidt, Henningson 1990's.

Infinitessimal perturbation grows algebraically -> finite amplitude -> nonlinear effects take over.

Reynolds' Experiment



Van Dyke (1982) *An album of fluid motion, Parabolic press*

- Reynolds found : turbulence above

$$Re_c = 2000$$

- In careful experiments laminar flow up to

$$Re_c = 13000$$

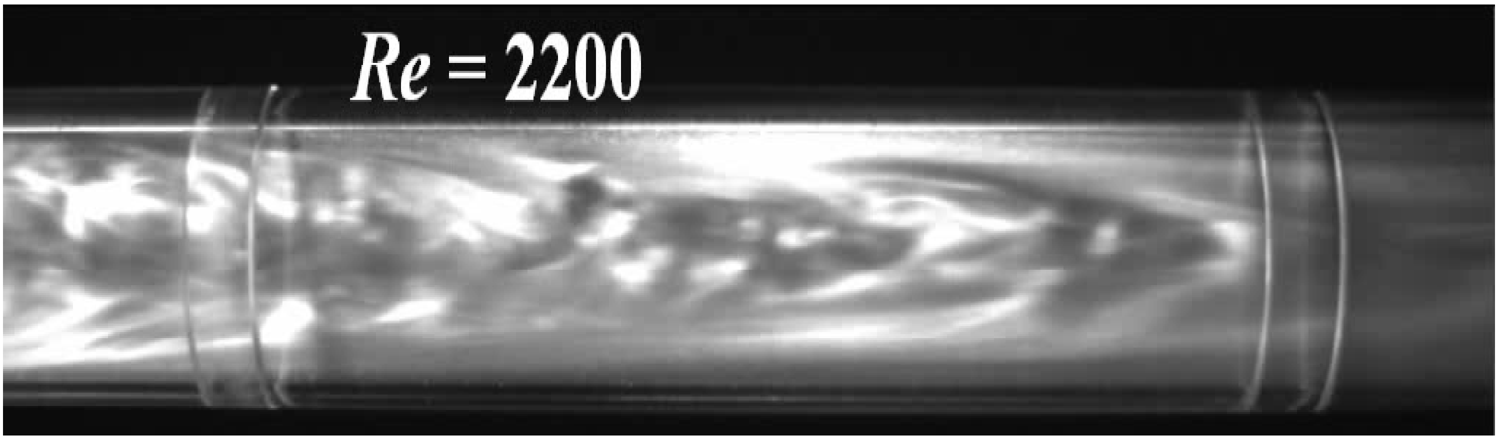
- $2000 < Re < 2700$ "flashes" "puffs"

$Re > 3500$ "slugs"

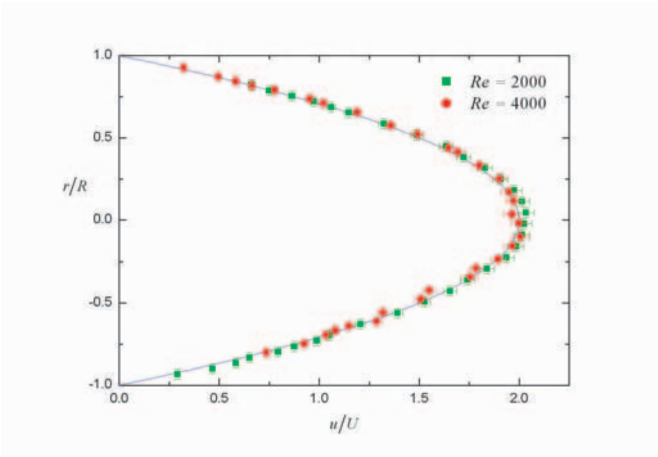
Wygnanski and Champagne, *J. Fluid Mech.*, 59, 281-351

`Turbulent' Puffs

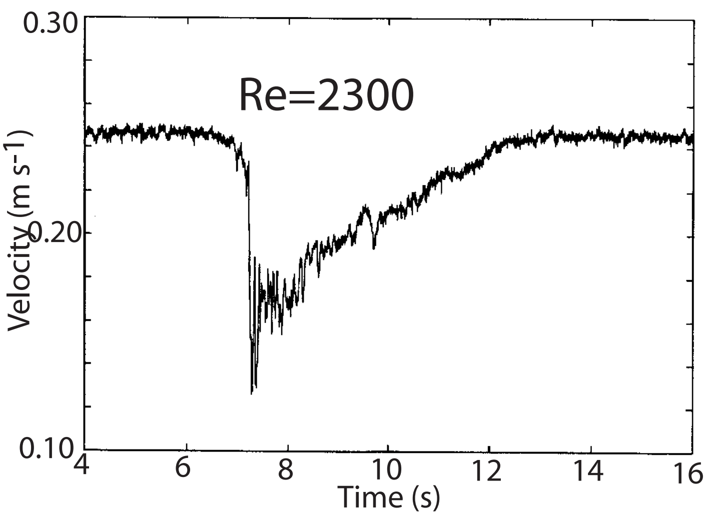
Exist in Re range ~1800 to 3000



Initial Profiles

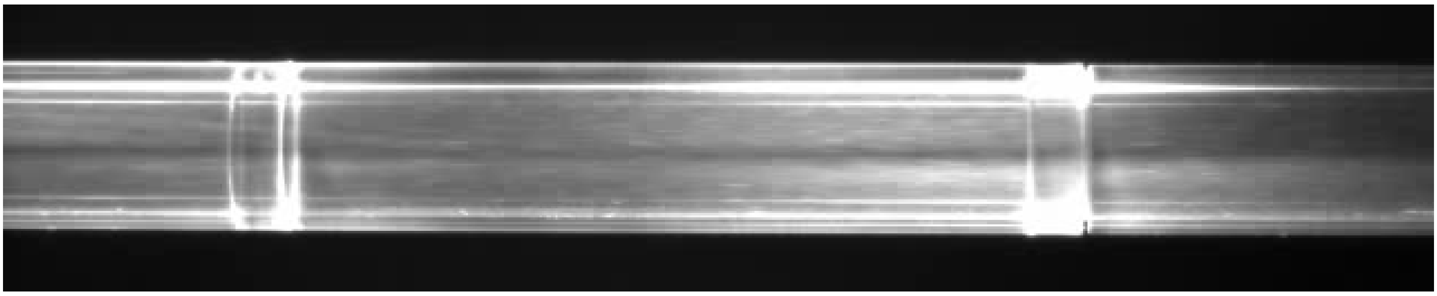


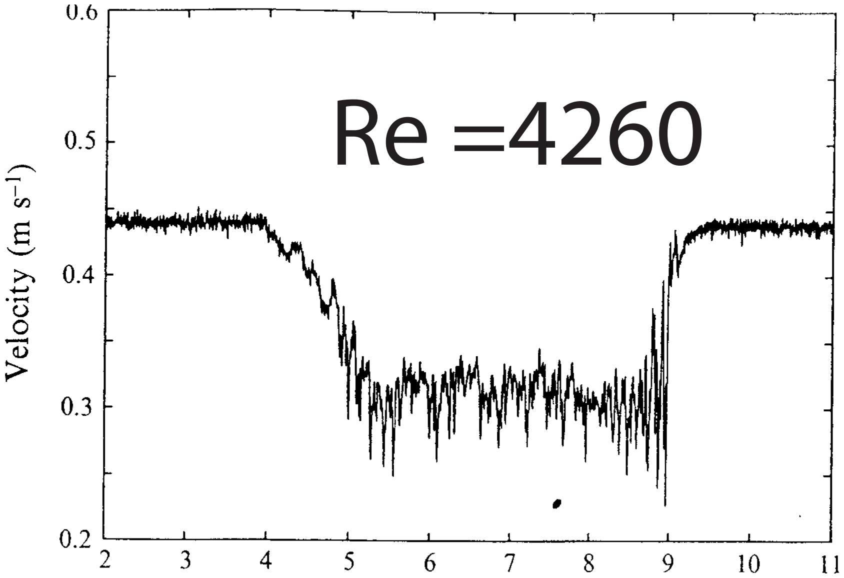
Stuart JT Exp Therm. Sci. (1996): Taylor diffusion.



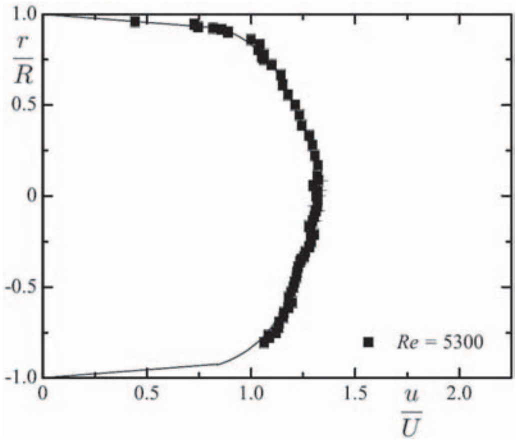
Travels at ~0.9u

Slugs (Re =4,000)

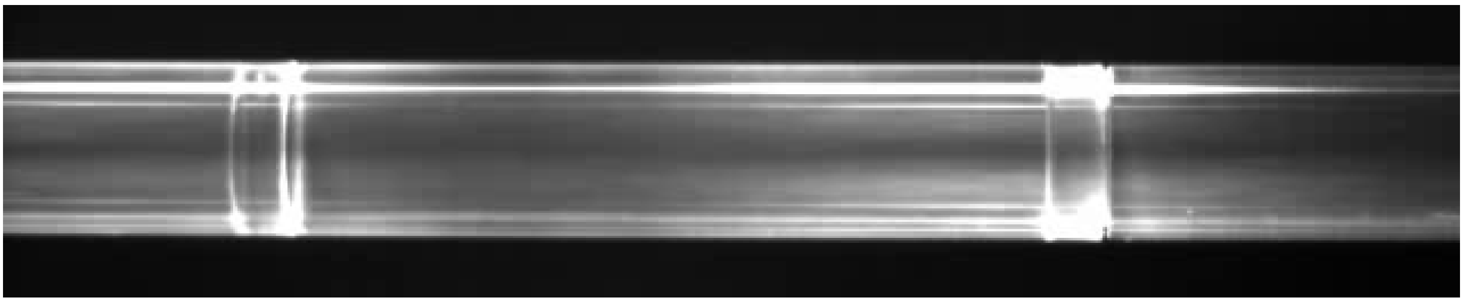




Axial velocity vs time (centre)



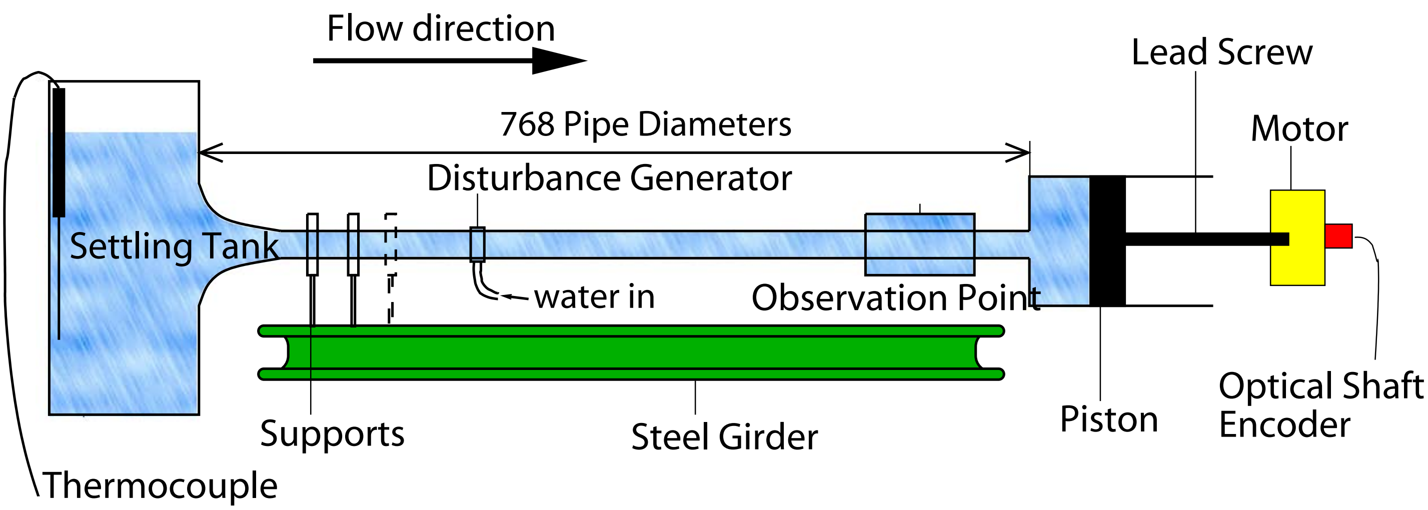
Mean Profile



Re = 10,000

Our Experiment

Constant Mass Flux Pipe i.e. Re fixed.



In most other experiments, pressure gradient drives the flow. On transition, flow rate will drop, hence Re will vary.

The long pipe

New large scale experimental facility:

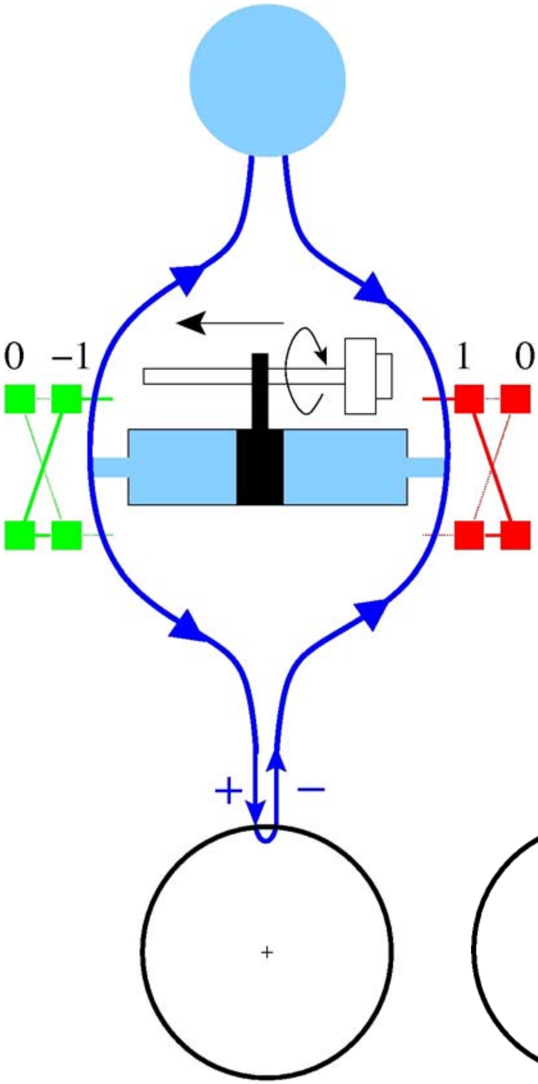
- 15.5 m or 768 pipe diameters long,
- temperature control,
- new perturbation, where a spread and amplitude of perturbation are decoupled.

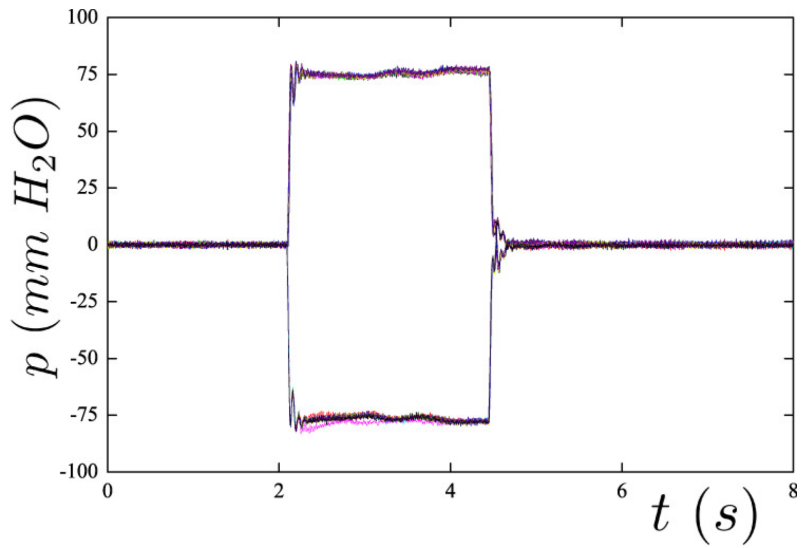
* Study of perturbed

Hagen-Poiseuille flow ($> 95\%$) for

up to $Re \simeq 20000$.

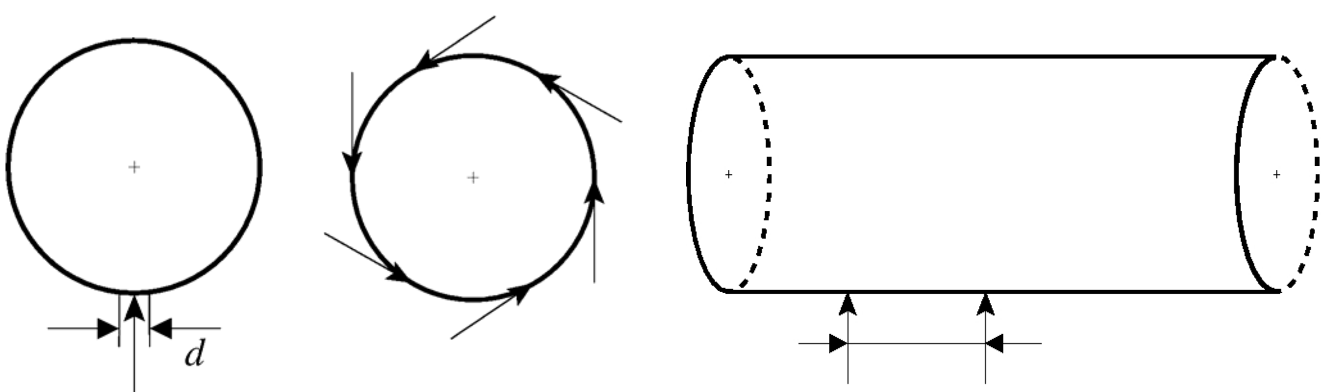
Perturbation Mechanism



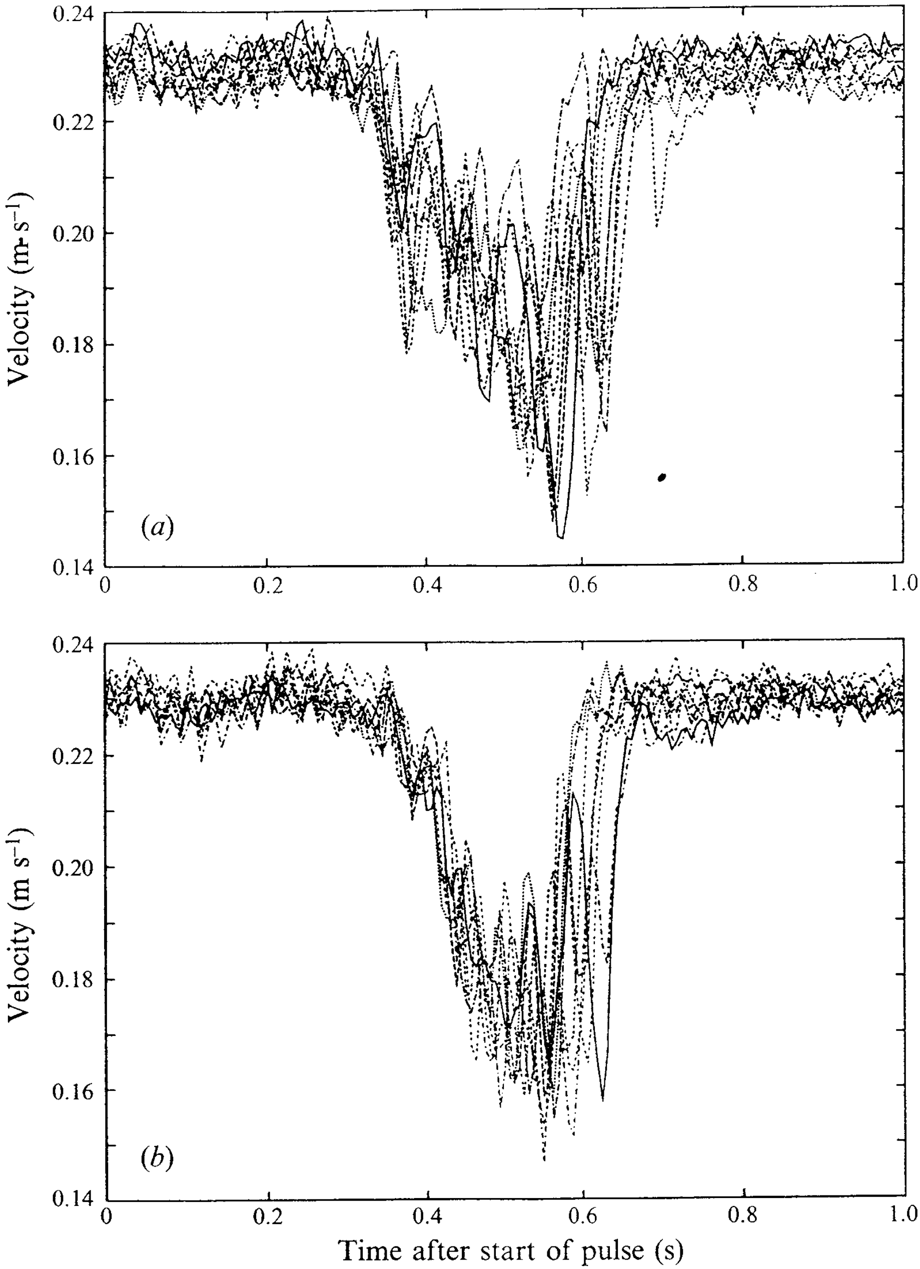


$$A = \frac{\Phi_{inj}}{\Phi_{pipe}}$$

$$L^* = \frac{Ut}{D}$$



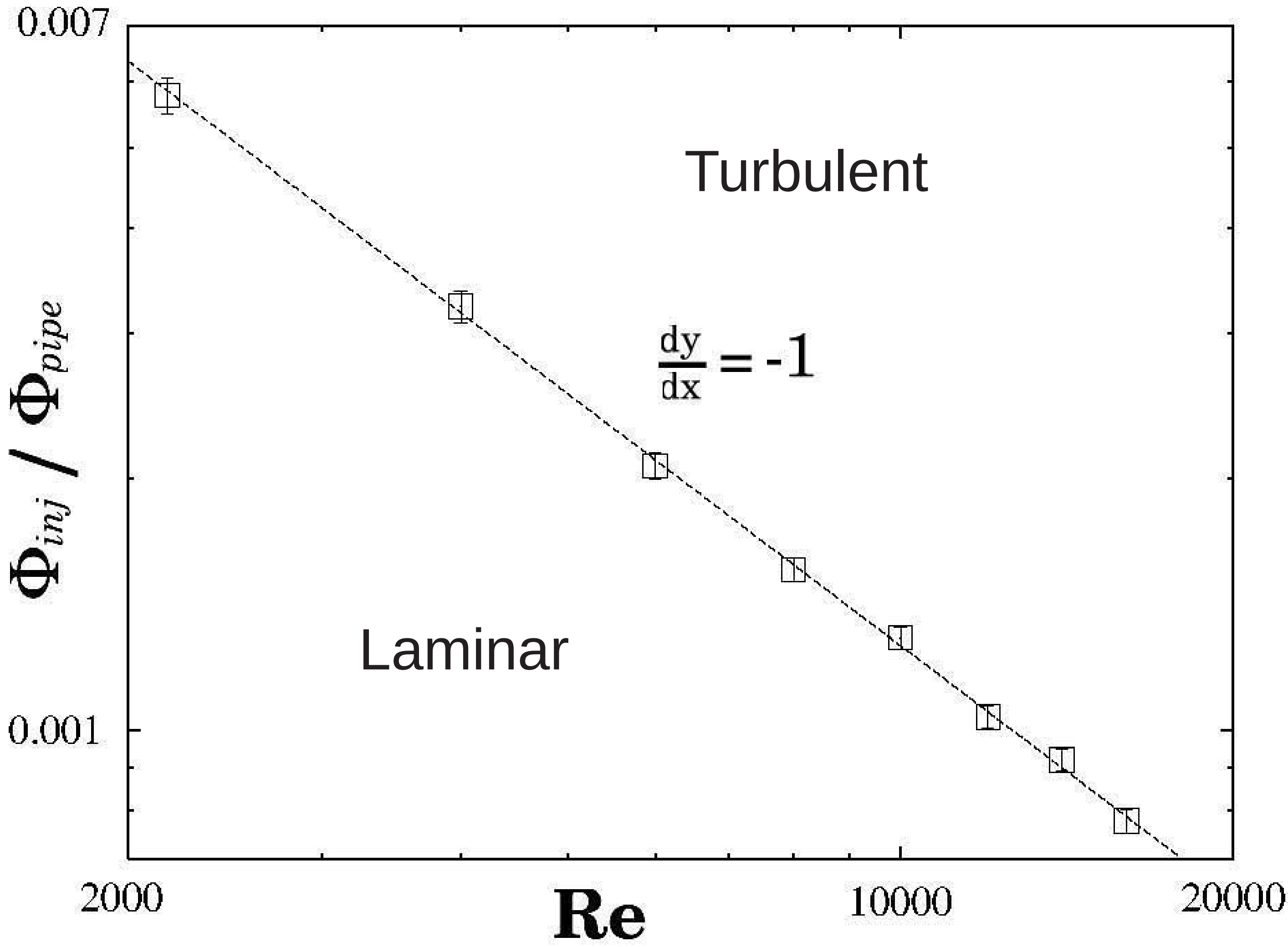
Impulsive Disturbance Applied



. Velocity time traces measured $2D_P$ from the single-jet inlet. (*a*) Disturbances that cause transition. (*b*) Disturbances that decay downstream.

Threshold Between Laminar and 'Turbulent' Flows.

Each data point requires 40 runs of the experiment.



Log-Log plot --> $\gamma \sim -1$

What happens below Re ~2,000?

All previous experimental work suggests that flow is globally stable.

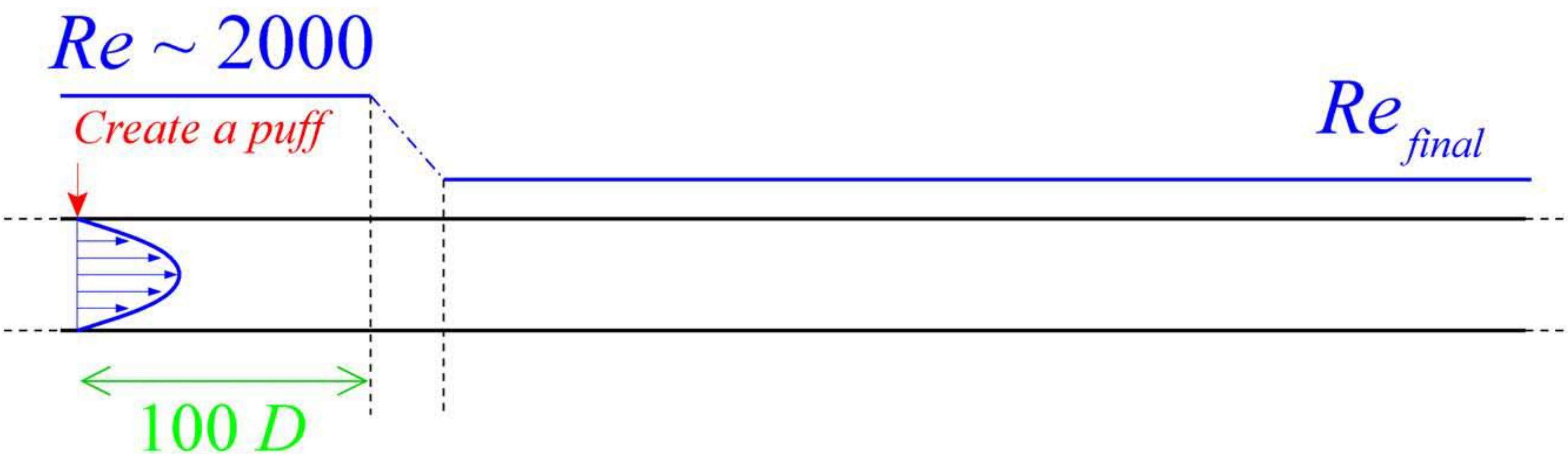
Re^{-1} can no longer hold.

Could add large perturbation and observe decay?

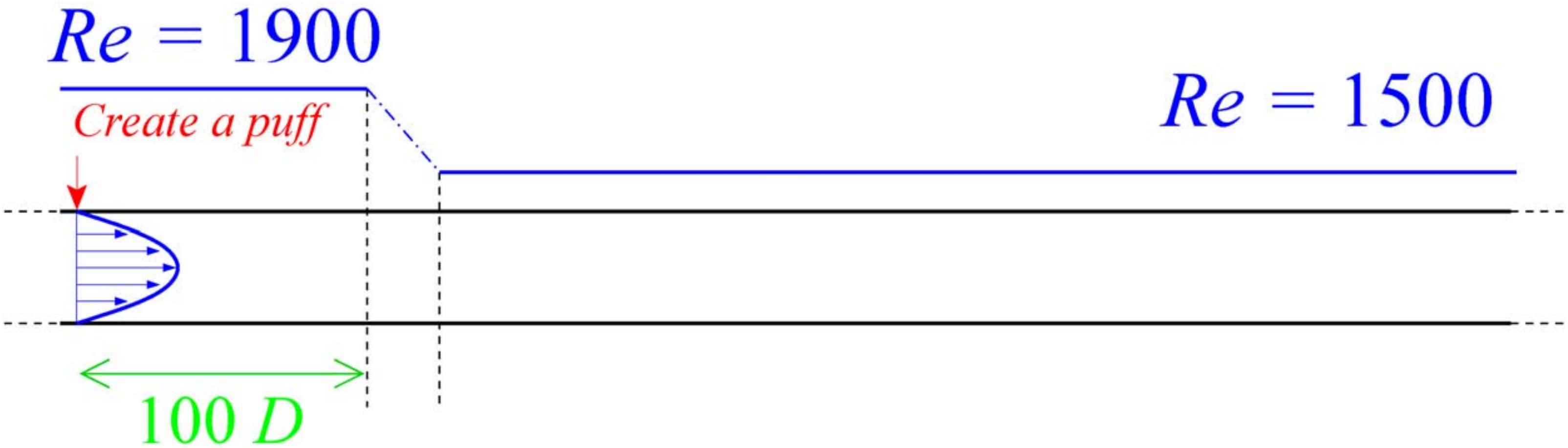
Binnie (1947) suggests -> long term transients.

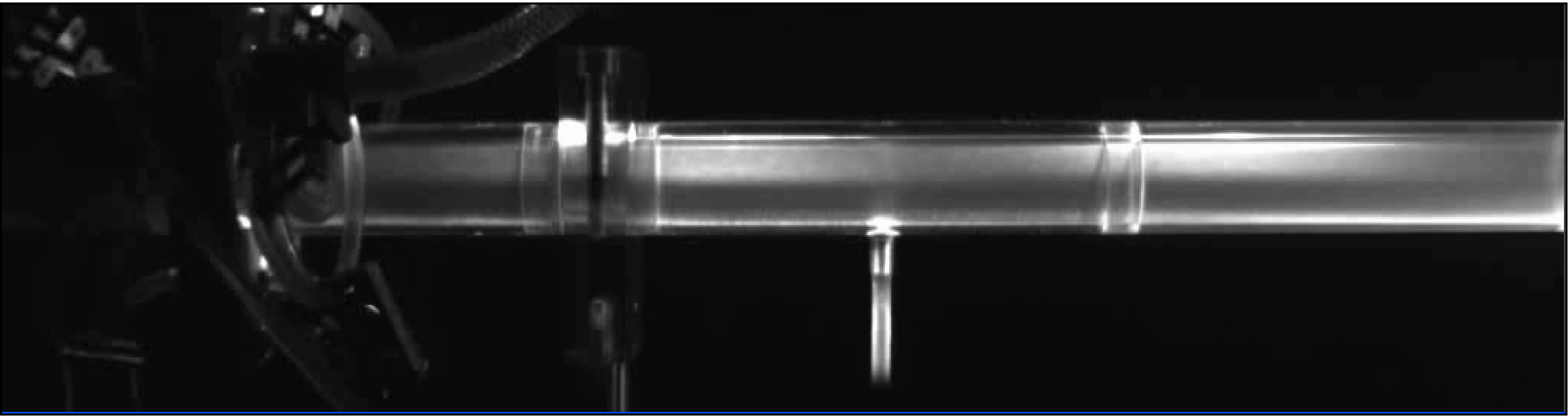
Direct Transition is Catastrophic

- Details of transition process not clear.
- Transition from Turbulence: **Laufer(1962), Sibulkin(1963). Sreenivasan(1980)**
- At low Re , transition proceed via an "equilibrium puff". Our idea is to study the stability of the equilibrium puff by reducing Re

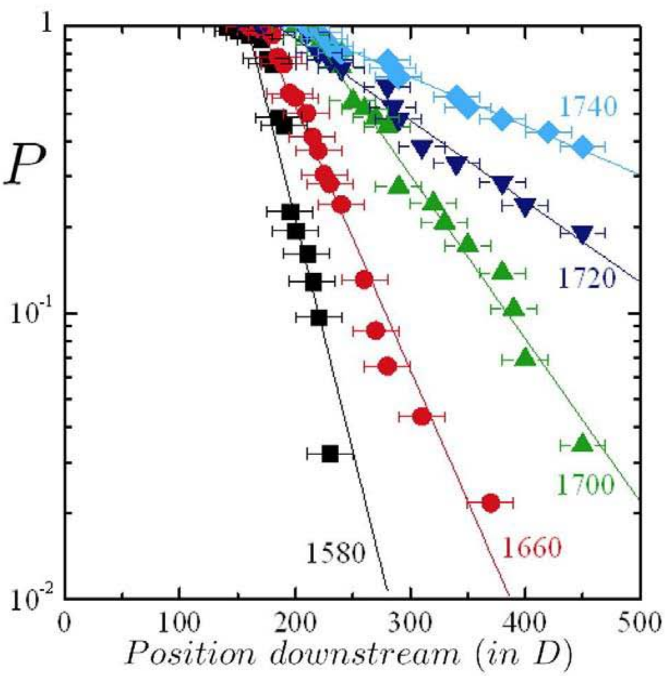


Transition *from* Turbulence



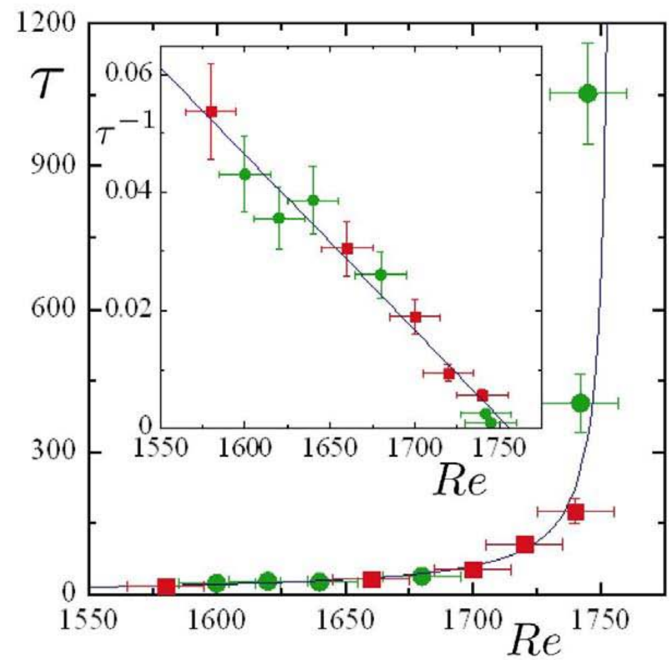


Transition *from* Turbulence



$$P(D) \propto \exp(-CD)$$

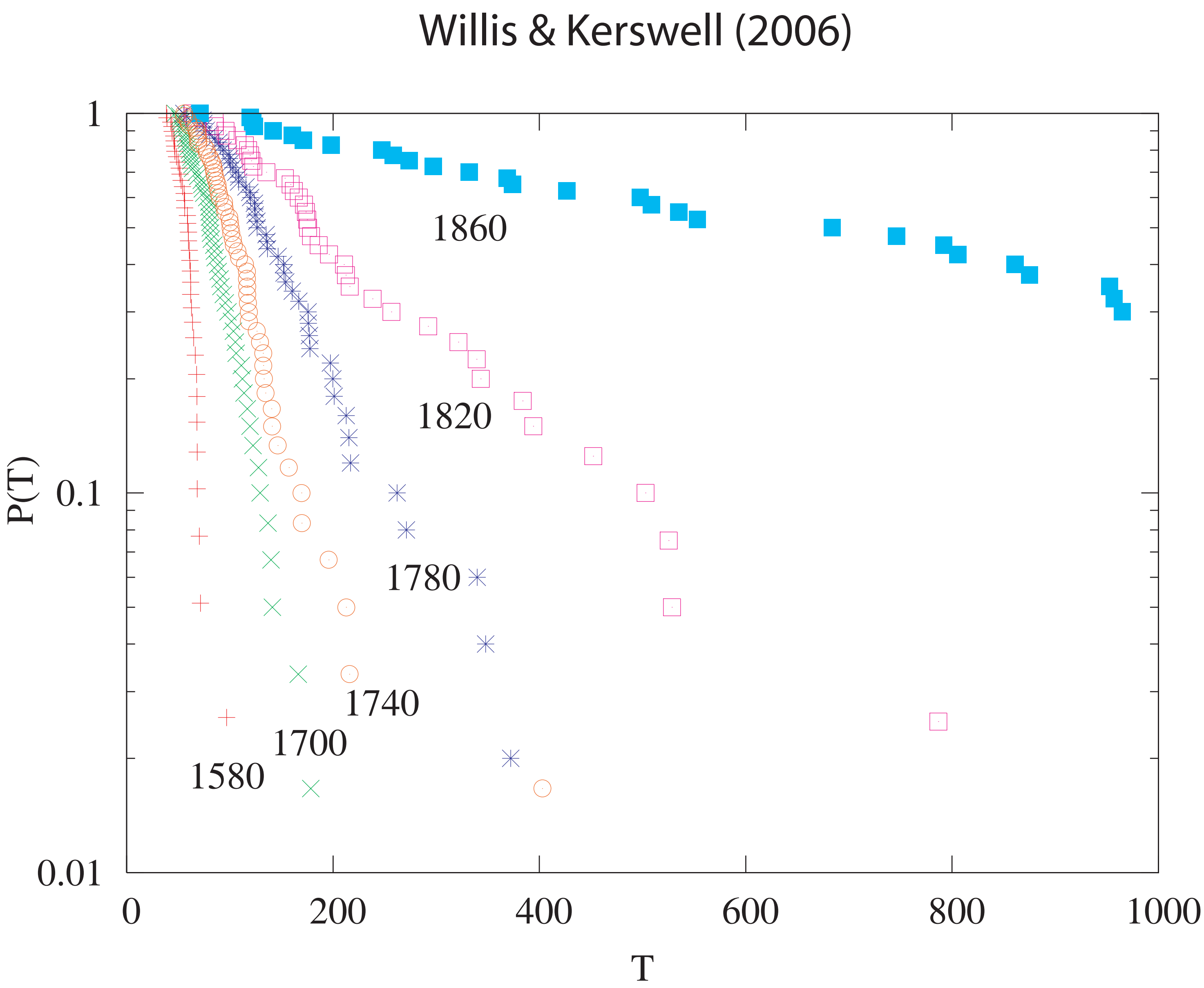
$$\tau = (\ln 2)/C$$



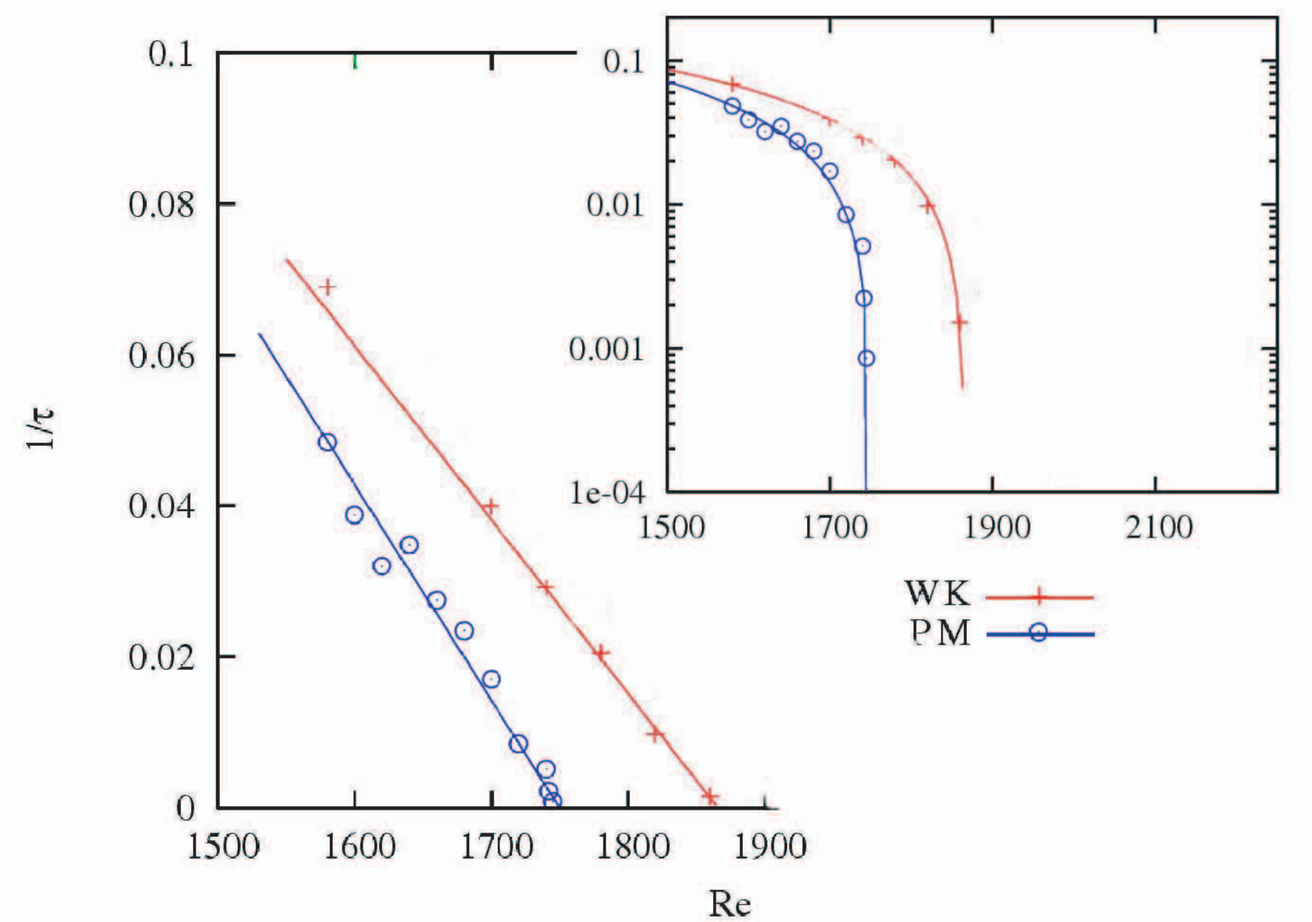
Additional points obtained with range of perturbations

$$Re_c = 1750 \pm 10$$

Numerical results for decay of puff at threshold.



Comparison between numerical and experimental results for decay of puff a lower threshold.



Numerics performed with PBC on pipe $16\,\pi$ diameters long.

A.P. Willis and R.R. Kerswell (PRL 014501, 2007)

Exponential decay in probability of observing puff downstream ----> Poisson process.

Divergence of timescales ---------> deterministic behaviour.

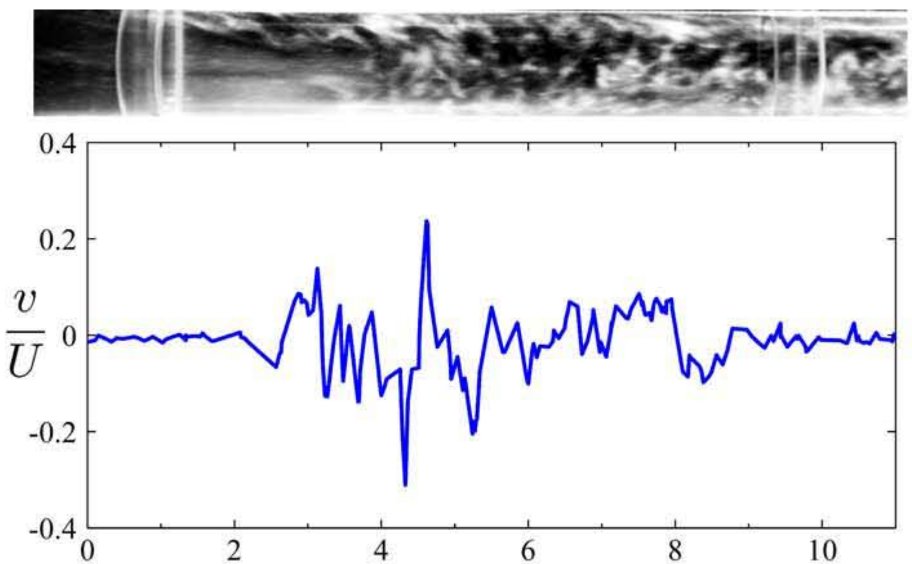
Qualitatively similar to boundary crisis of attractor Grebogi, Ott & Yorke (1986)

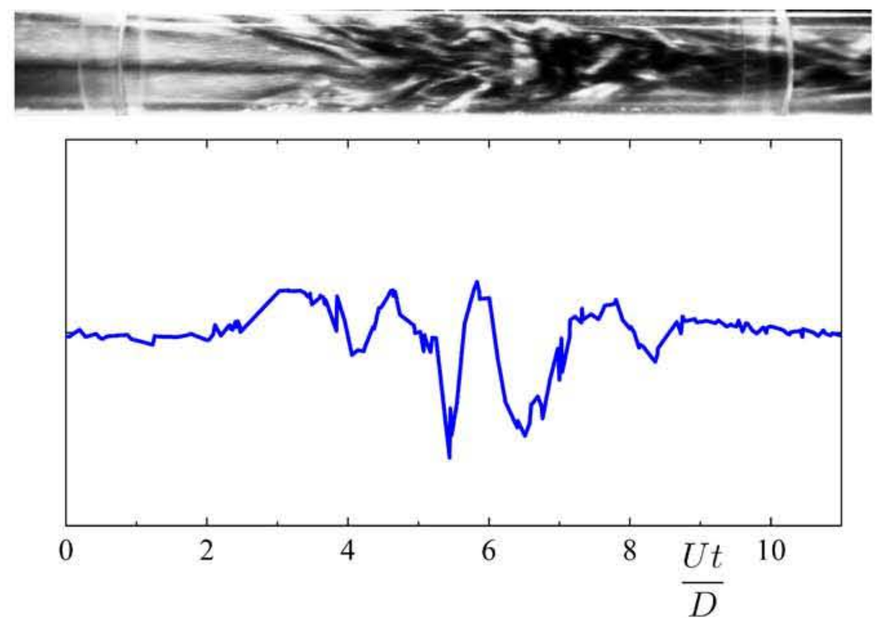
But low-d systems: exponents < 1

Wavy Patterns

Puff at Re = 1900

After reduction of Re down to 1750





Disordered signal

Contain wavelength of 1.5 D

Faisst&Eckhardt(2003)
Wedin &Kerswell(2004)

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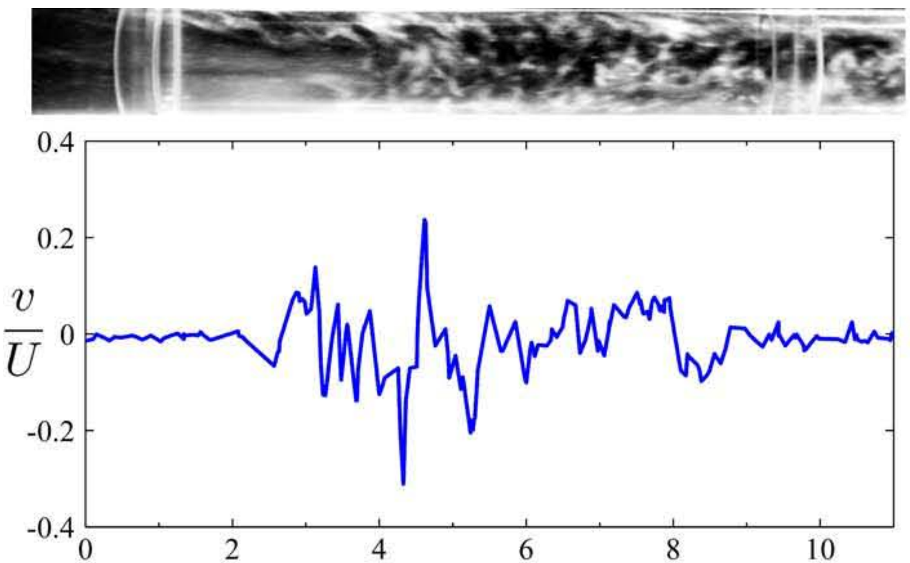

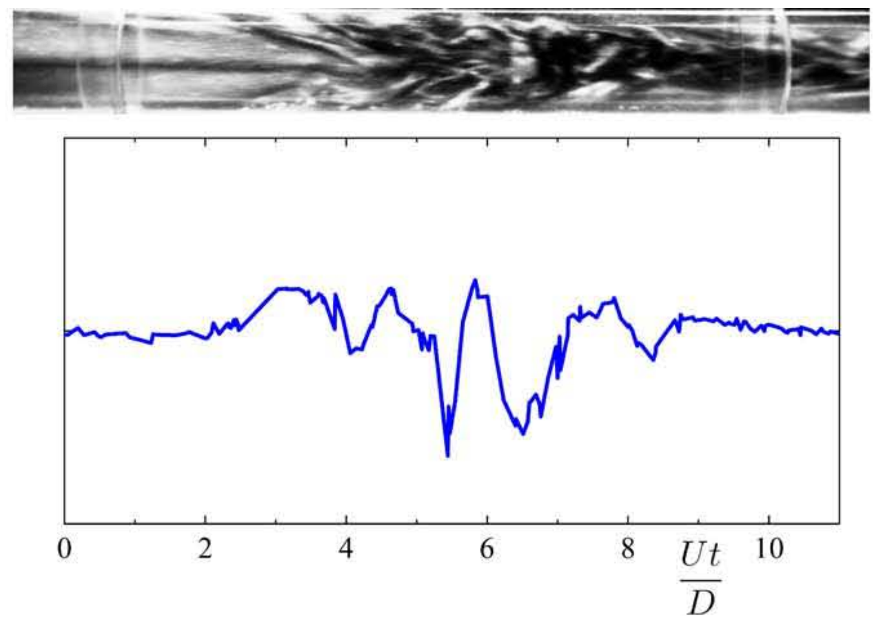

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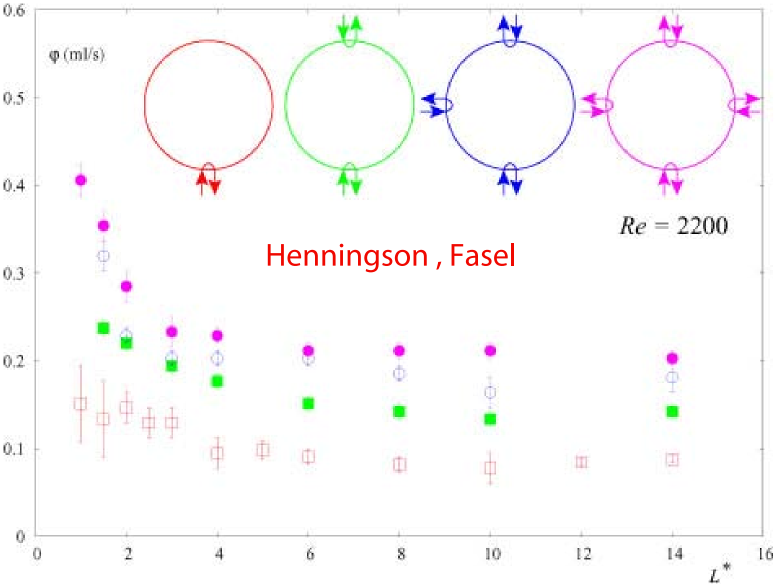
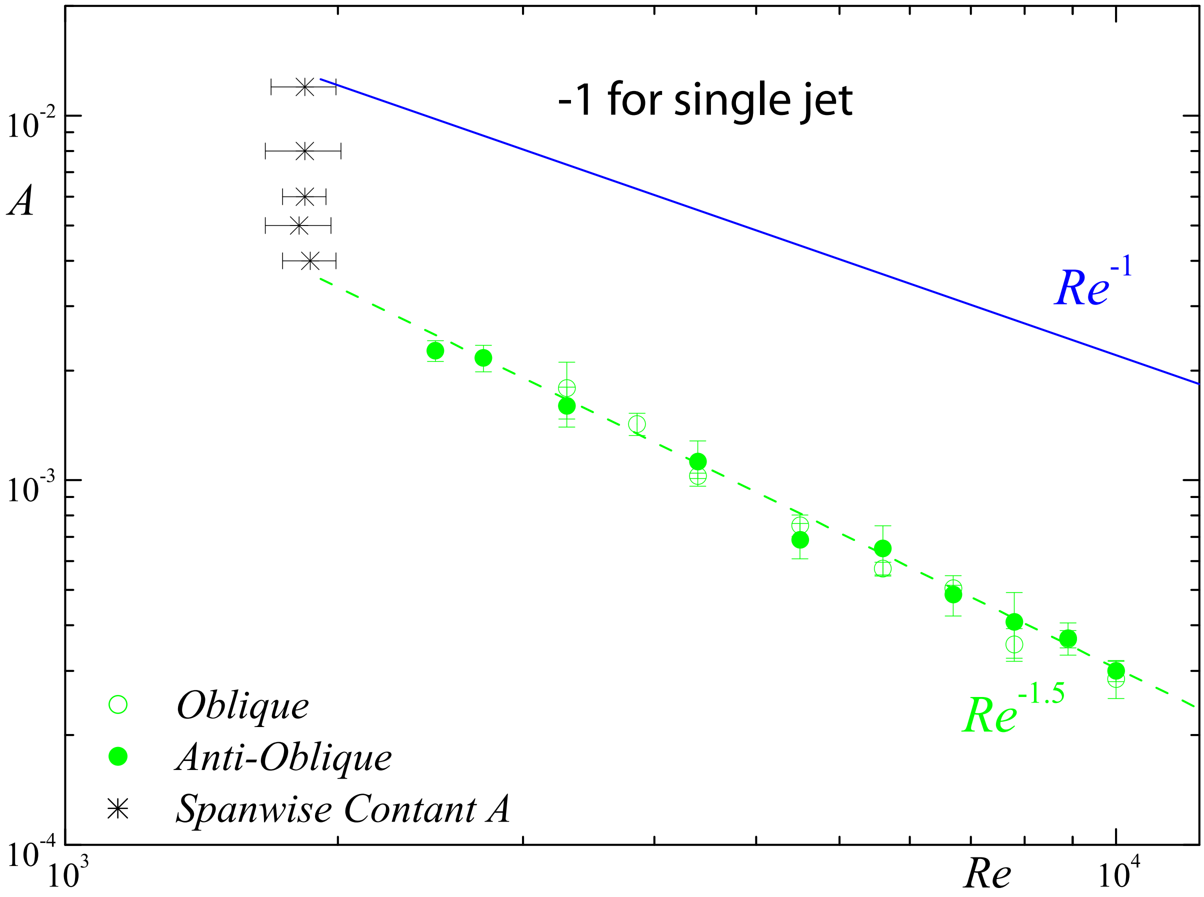
Four Push-Pull Disturbances



Threshold Curves with Two Different Perturbations.

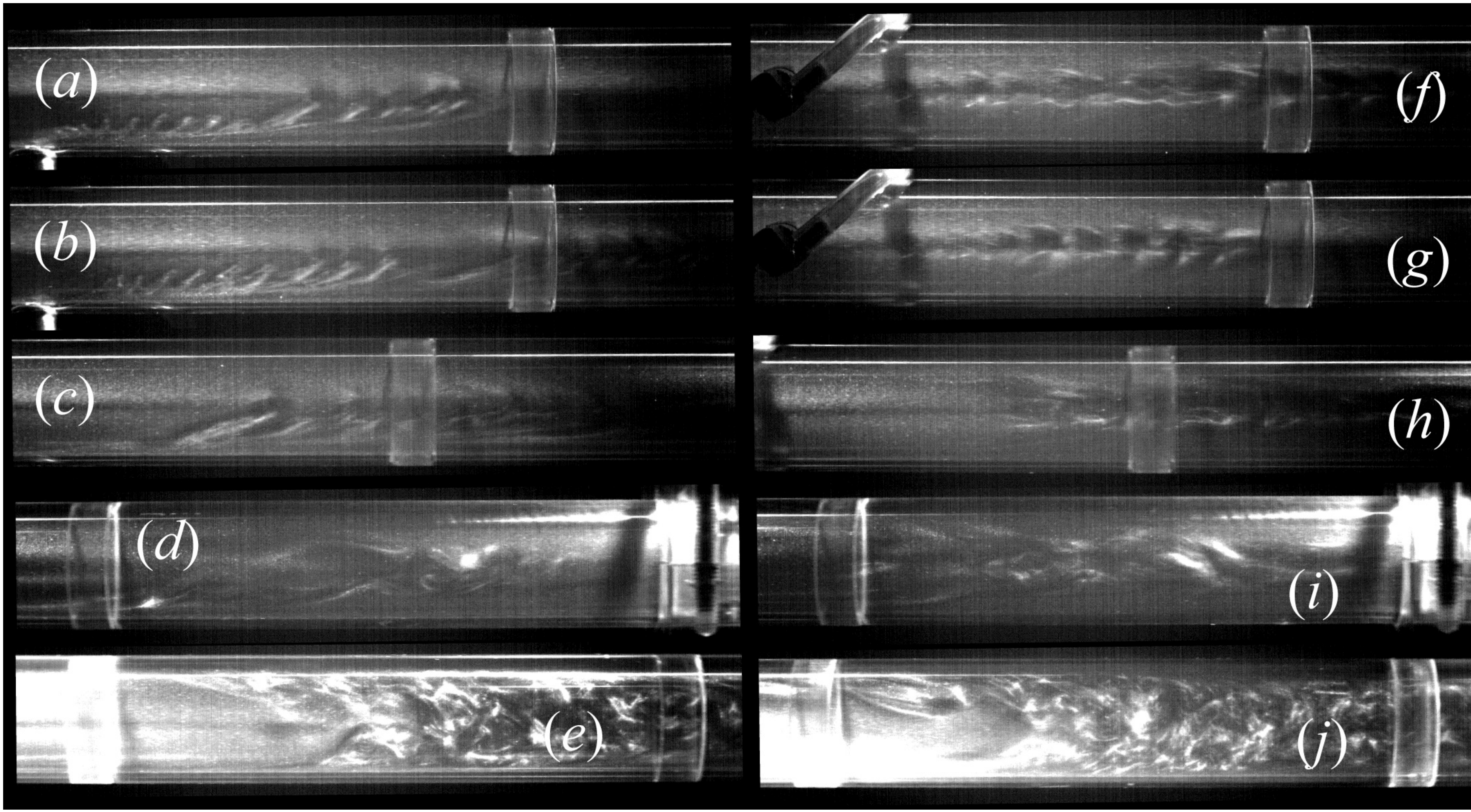


Localised push-pull slope -1.3

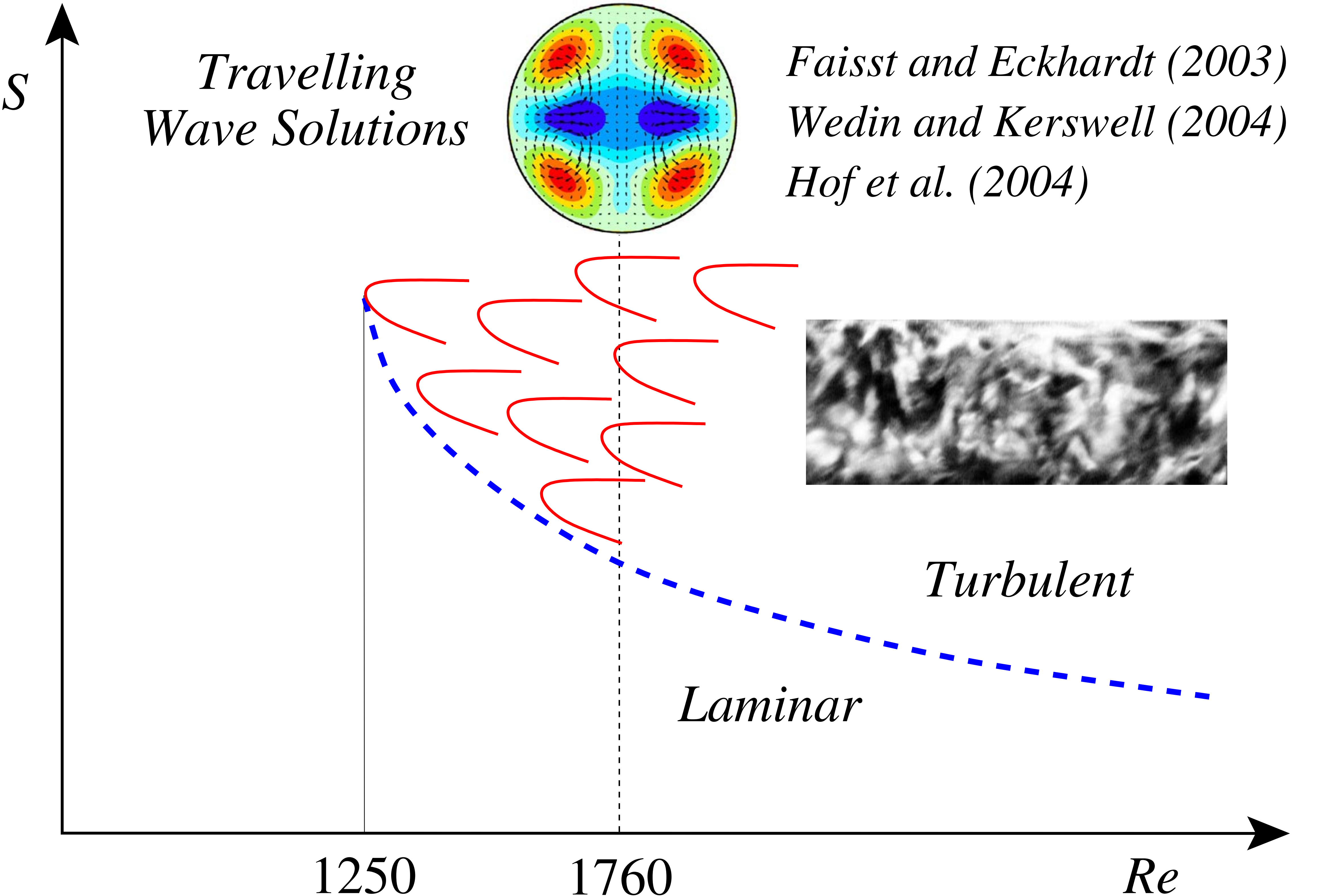
Note order of magnitude reduction in amplitude.

OBLIQUE PUSH-PULL DISTURBANCE

Side View Bottom View



$Re = 3000$

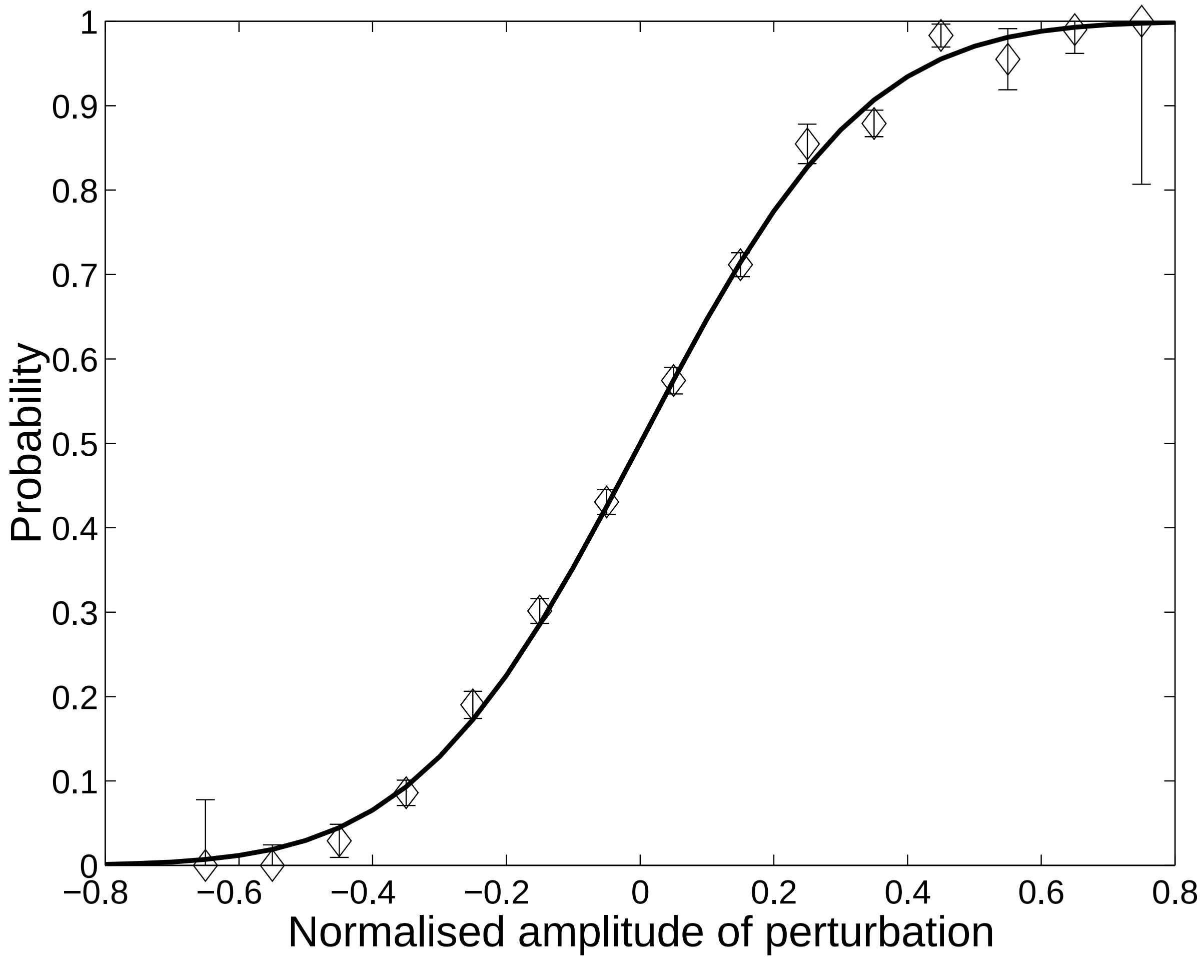

S
Travelling
Wave Solutions
Faisst and Eckhardt (2003)
Wedin and Kerswell (2004)
Hof et al. (2004)
Turbulent
Laminar
1250
1760
Re

Conclusions

- Scaling laws established for transition to turbulence in a pipe.
- -1 exponent --> balance of viscous and inertial terms.
- -1.3 --> possibility of transient growth.
- Slowing down suggests critical behaviour and waves during decay links with finite amplitude solutions.

PRL 91(2003) 244052, PRL (2006)
Proc. IUTAM Symp. Bangalore(2005)
Phys. Today (2004) Feb.

Transition probability



- Error function fit.
- Plot obtained from normalised data of 5000 experiments:
  1. Reynolds numbers of 2170, 3000 and 4000,
  2. Different location of perturbation along the developing flow.
- Threshold process?