The Wonderland at Low Temperatures!!

NEW PHASES AND QUANTUM PHASE TRANSITIONS

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- **physics of the very small:**

High energy physics & String theory

- **physics of the very large:**

Astrophysics & Cosmology

- **physics of the very complex:**

Condensed matter physics

Condensed Matter Physics

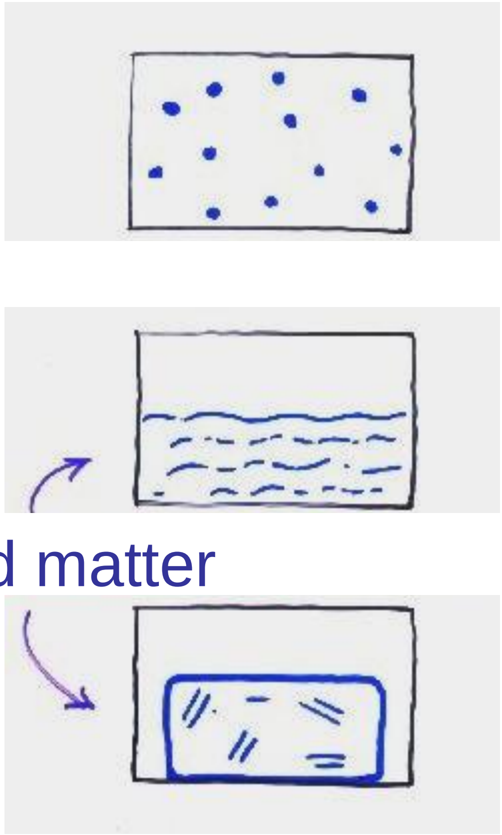
Complex behaviour of systems of
many interacting particles

Most Amazing: the complexity can often be understood as arising from simple local interactions

that of its constituents

TODAY'S TALK: MANY PARTICLES + QUANTUM MECHANICS

Emergent Properties



Phases and
Phase transitions

- **Rigidity**

- **Metallic behaviour**
- **Magnetism**
- **Superconductivity**

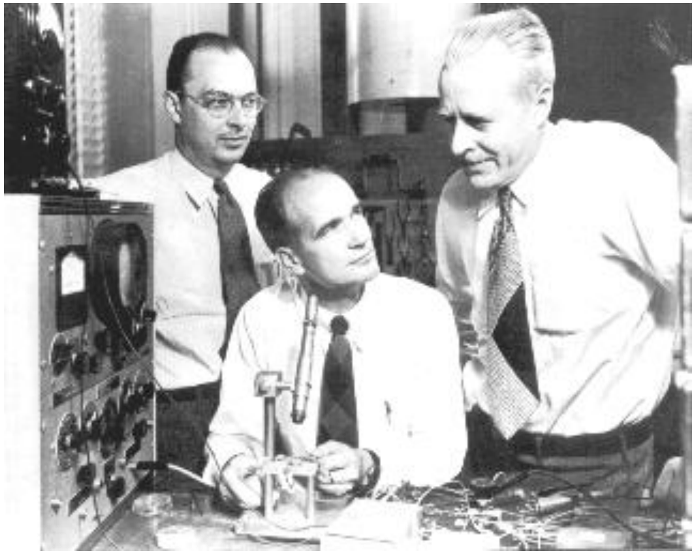
  ....

Many examples of emergent properties in biology!

L ife... .

Two facets of condensed matter physics

- Intellectual content
- Applications

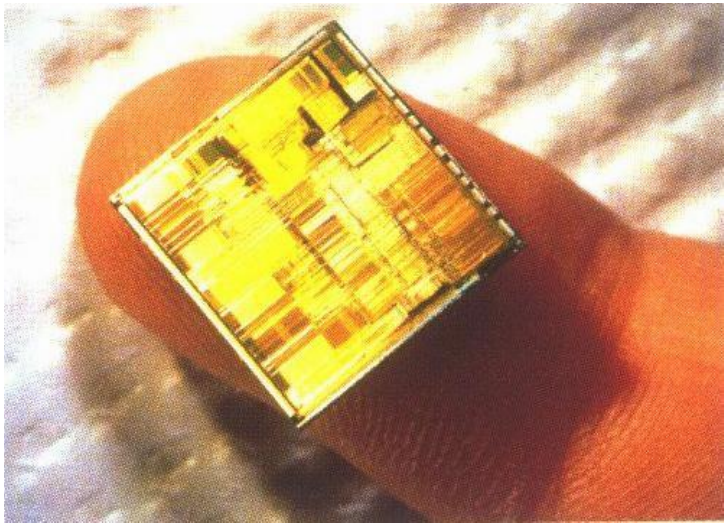


J. Bardeen, W. Shockley
& W. Brattain

The first transistor (1947)

ideas

technology



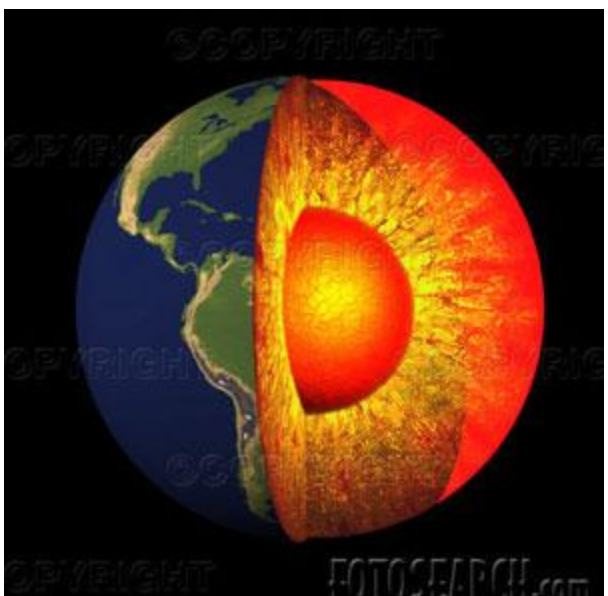
5 million transistors in a Pentium chip

The wonderland at low temperatures!!

Core $T \sim 10$ Million C

Surface $T \sim 6000C$

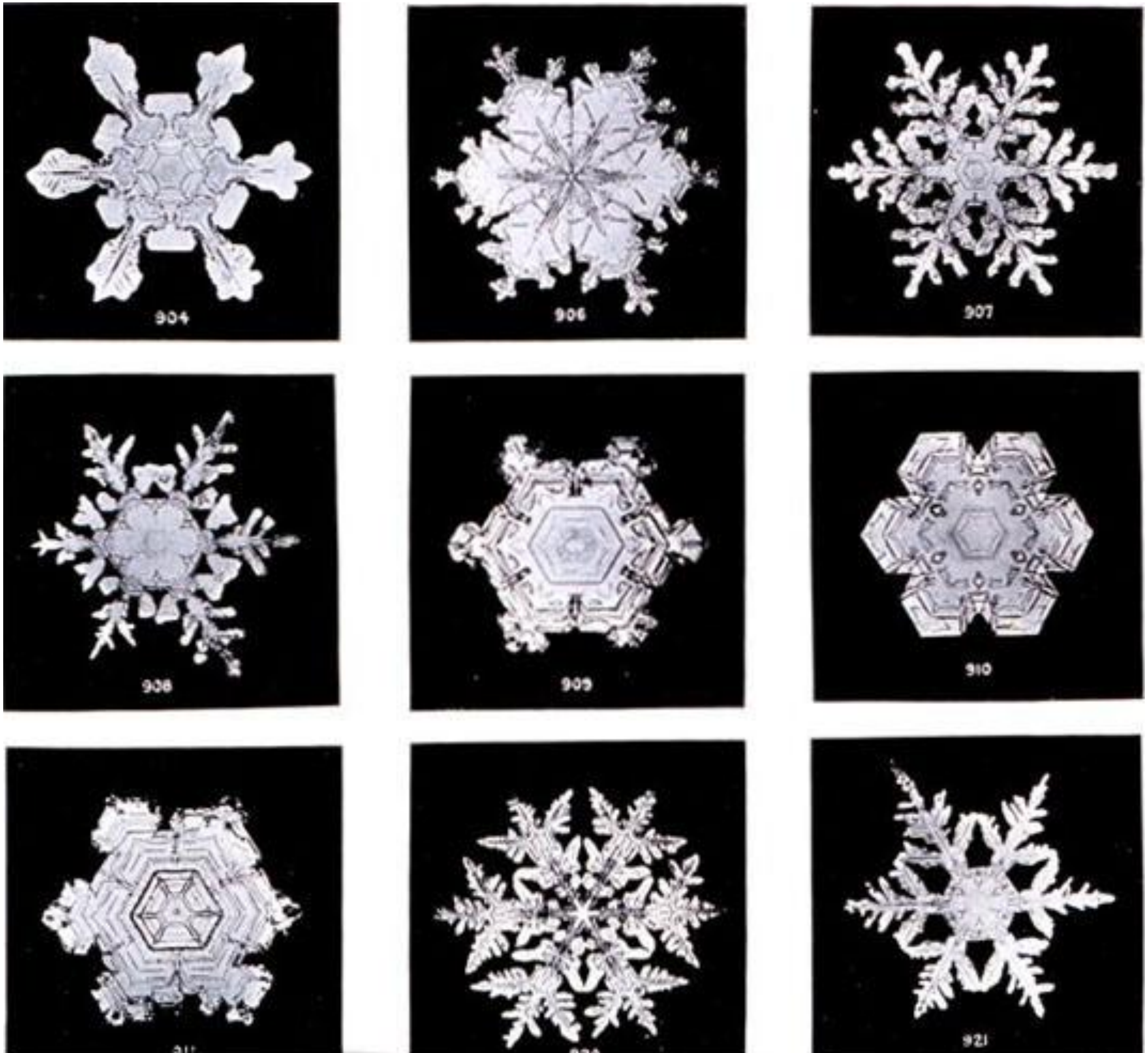
SUN



EARTH

Core $T \sim 7000C$

Surface $T \sim 15C(avg)$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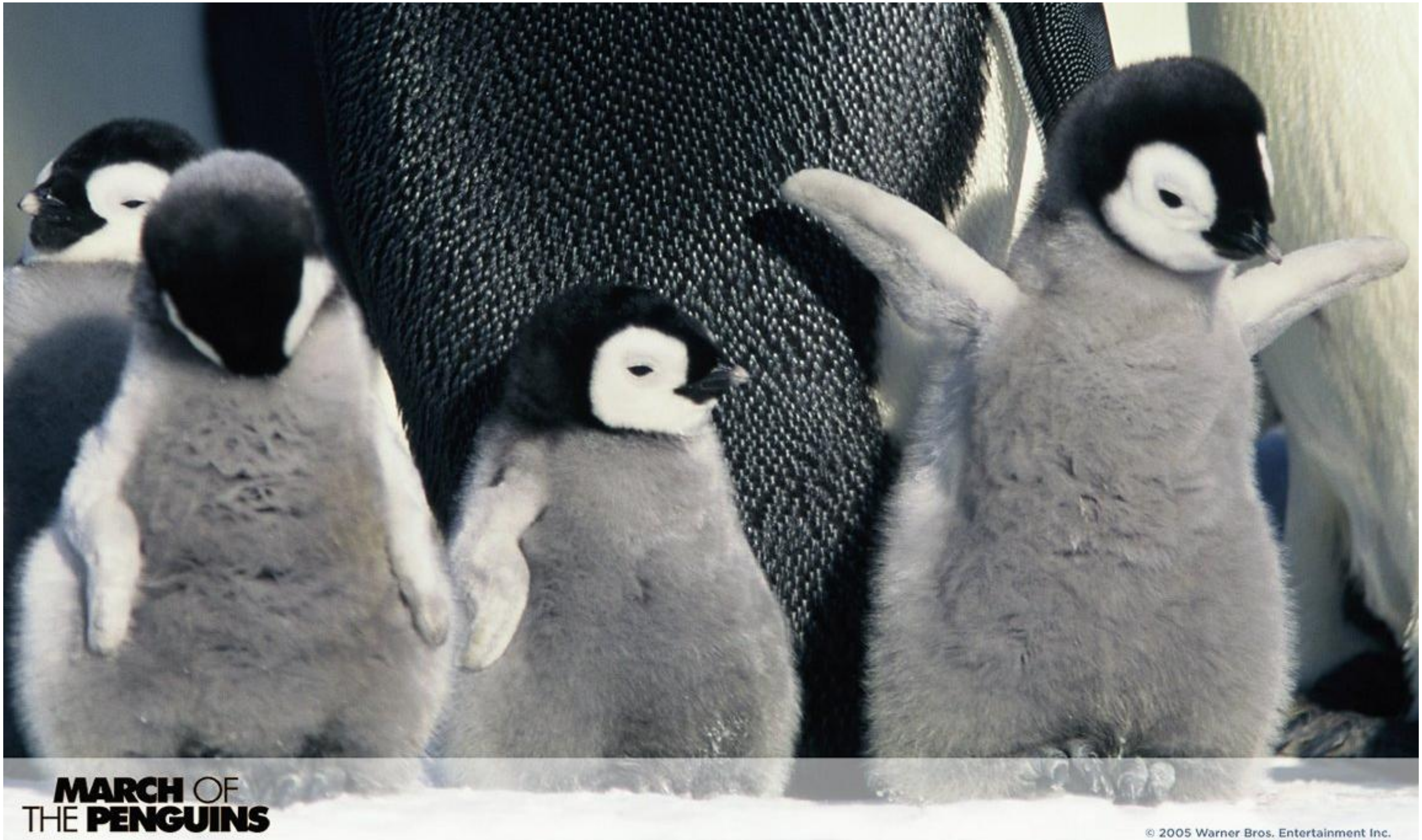
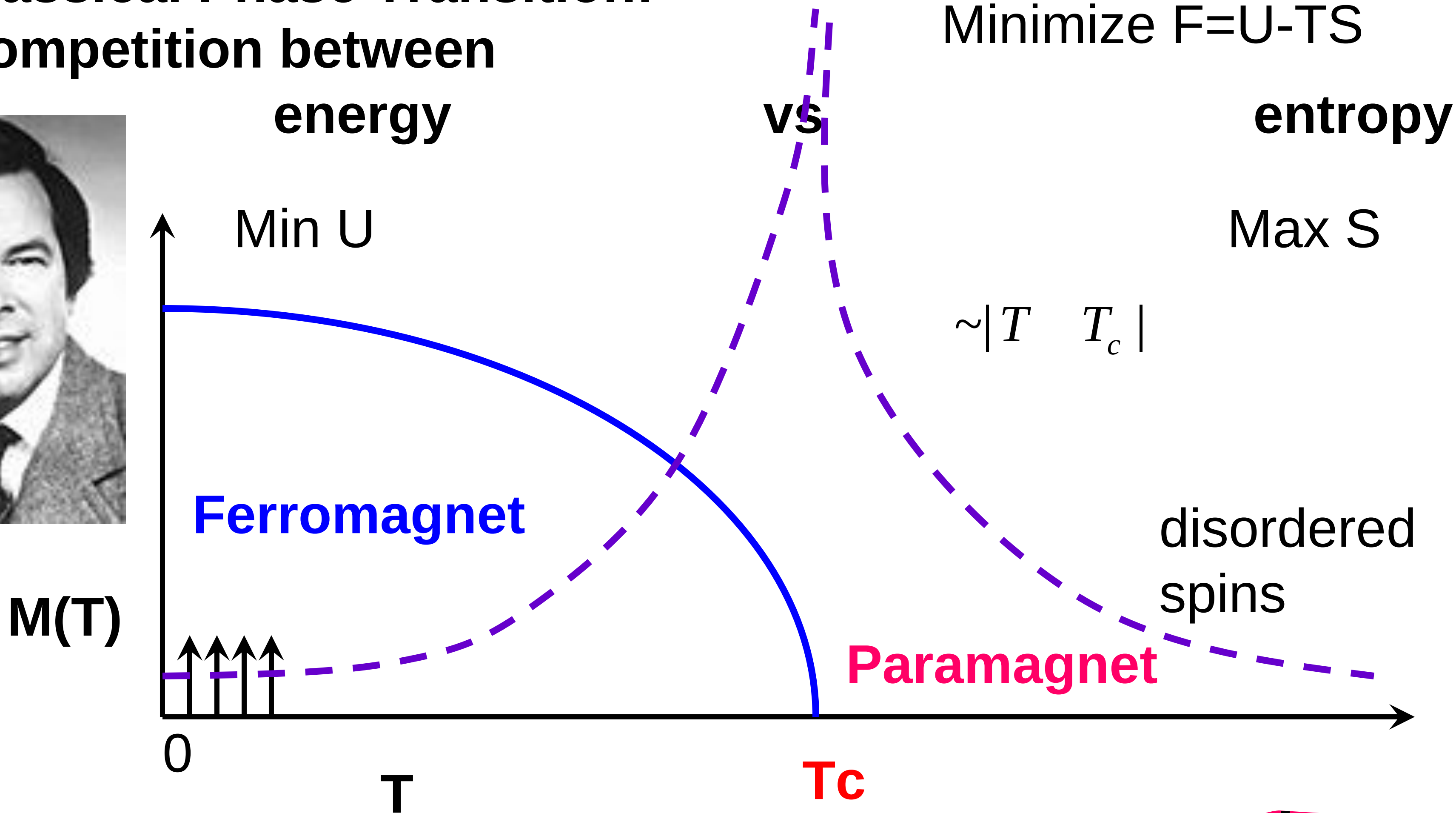


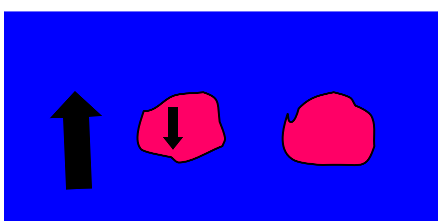
WHY do new phases occur at low temperatures?

F=E-TS

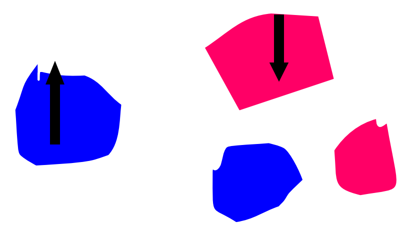
233K	(-40 C= -40 F)	
195K	(-78 C)	Sublimation of dry ice
77K	(-196 C)	Nitrogen liquefies
66K	(-207 C)	Nitrogen freezes
50K	(-223 C)	Surface temperature on Pluto
20K	(-253 C)	Hydrogen liquefies
14K	(-259 C)	Hydrogen solidifies
4.2K	(-268.8 C)	Helium Liquefies
2.73K	(-270.27 C)	***Interstellar space***
0 K		*ABSOLUTE ZERO*

Classical Phase Transition: Competition between



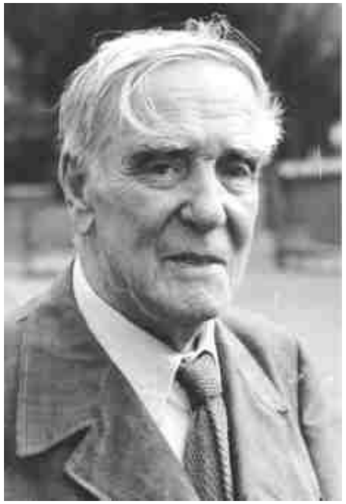


- **Broken symmetry**
- **Order parameter**



$\langle s_0 s_r \rangle \quad {}_{r} \qquad M^2 \quad \exp(\; r / \;) \qquad \langle s_0 s_r \rangle \quad {}_{r} \qquad \exp(\; r / \;)$

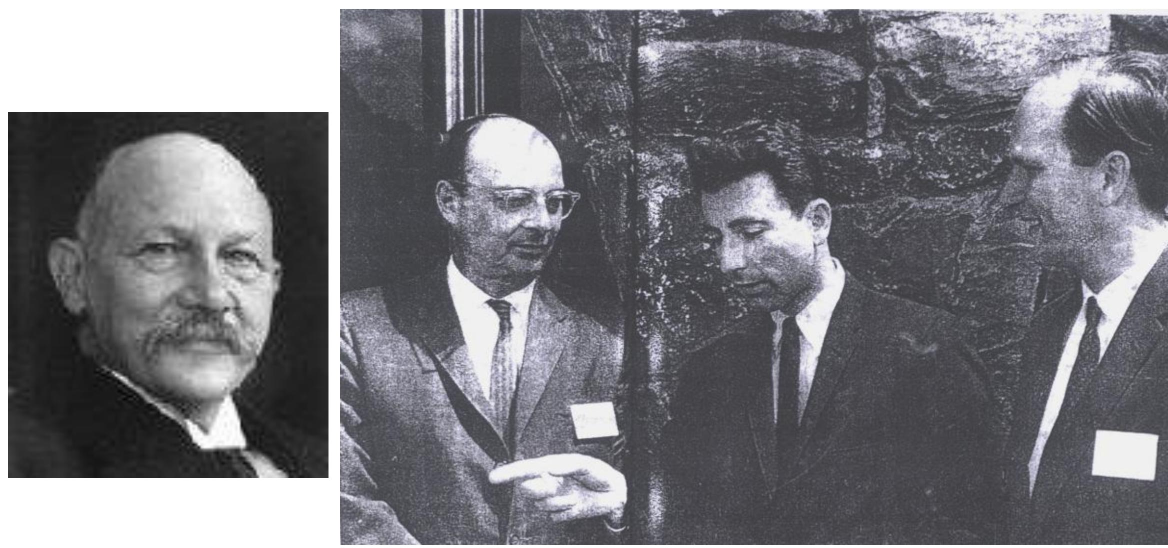
Quantum Mechanics rears its head



He4 does not solidify– superfluid

Bose-Einstein Condensation in alkali atoms

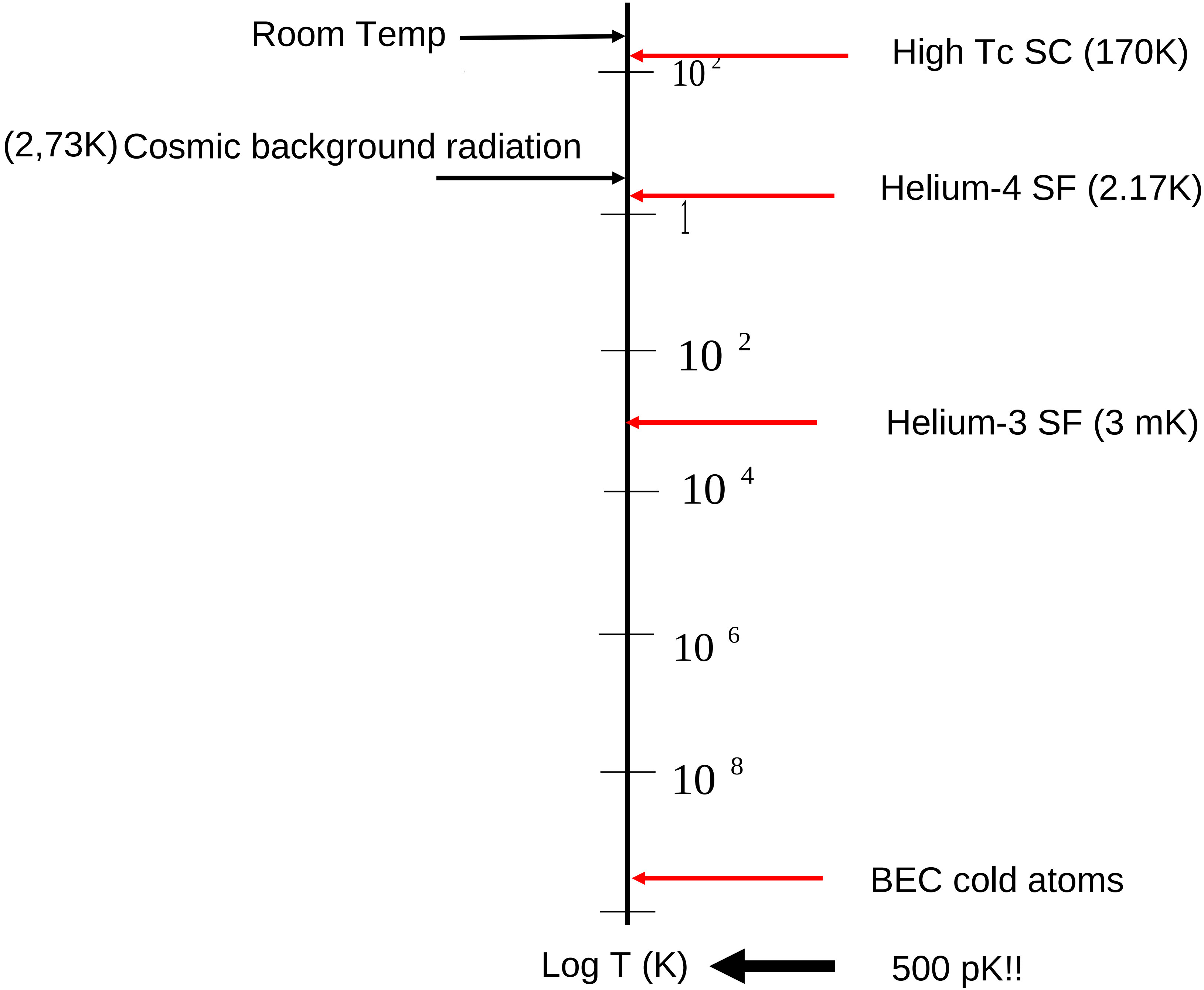
Electrons in metals



Superconductors

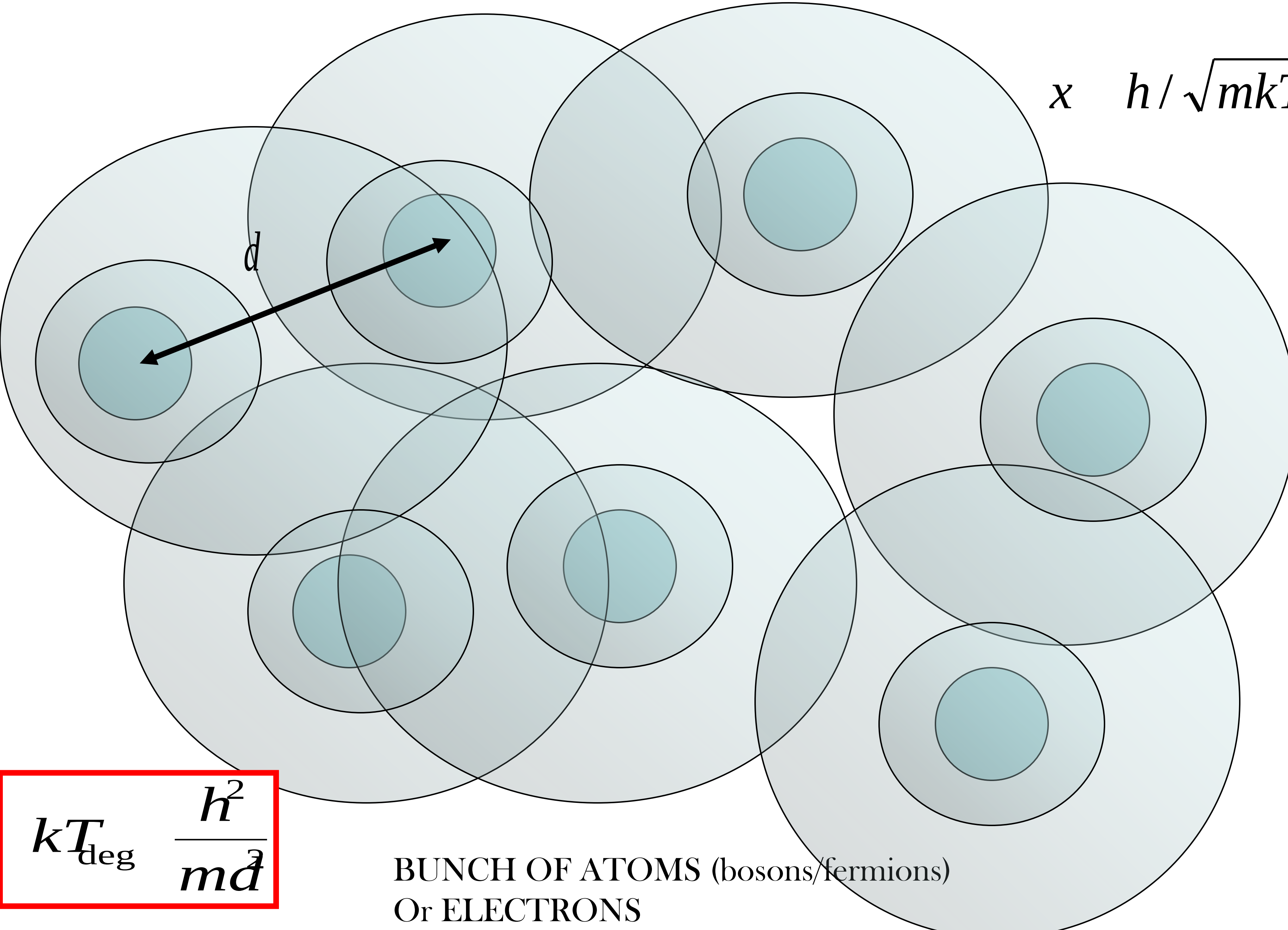
BCS@50

- 55 elements display SC at some combination of T and P
  Li under Pressure   Tc=20K
- Heavy fermions Tc 1.5 to 18.5 for $PuCoGa_5$
- Non cuprate oxides Tc 13-30 K $Ba_{1\ x}K_xBiO_3$
- MgB_2 (Tc 40 K)
- Graphite intercalation compounts CaC_6 4-11.5K
- Boron doped diamond Tc 11K
- Fullerides Cs_3C_{60} (40K under P)
- Borocarbides (16.5 K)
- Sr_2RuO_4 and some organic SC p wave pairing
- Copper oxides dwave


Room Temp
High Tc SC (170K)
10^{2}
(2,73K) Cosmic background radiation
Helium-4 SF (2.17K)
1
10^{2}
Helium-3 SF (3 mK)
10^{4}
10^{6}
10^{8}
BEC cold atoms
Log T (K)
500 pK!!

QUANTUM DEGENERACY

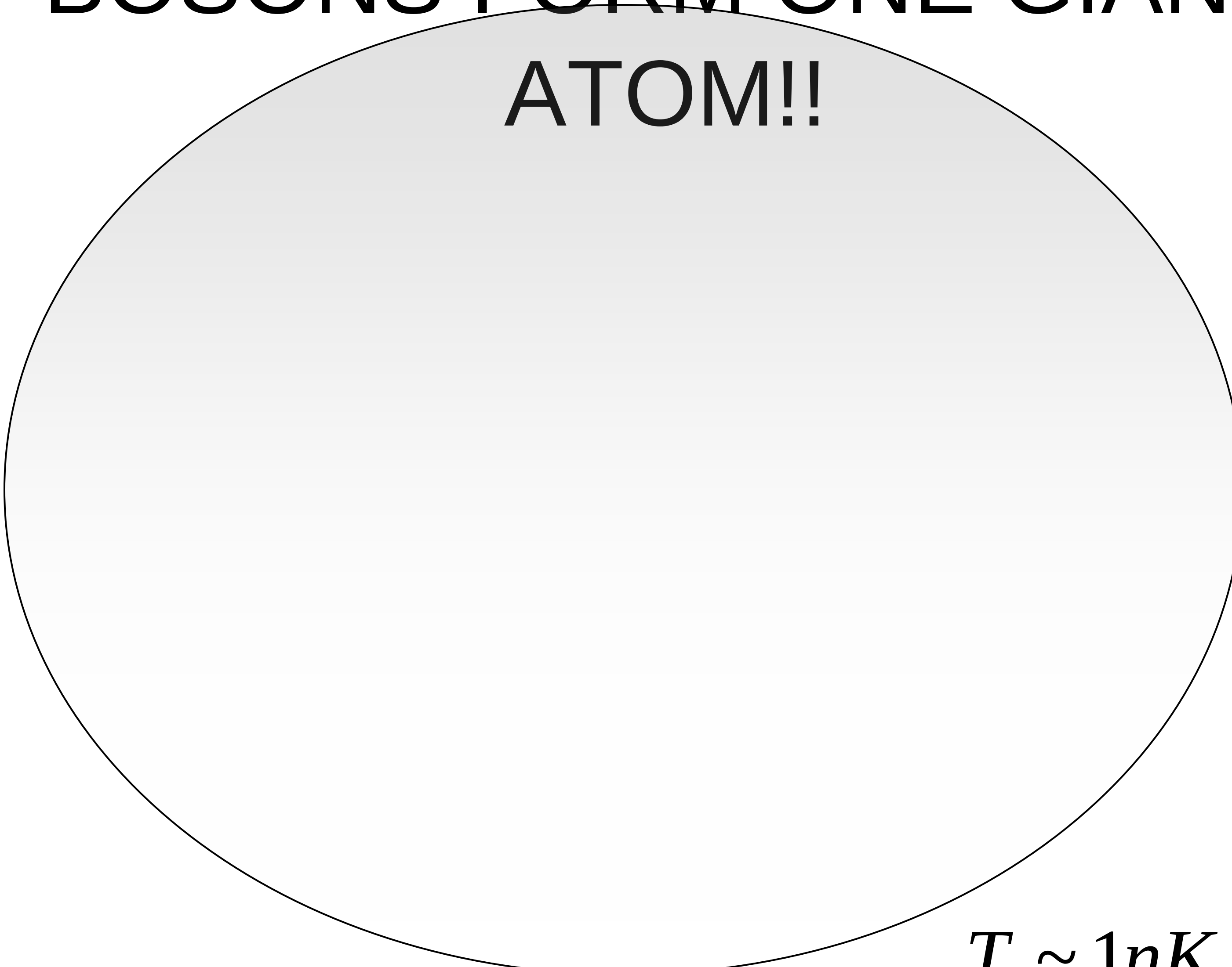
$$x \quad h/\sqrt{mkT}$$



$$kT_{\text{deg}} \quad \frac{h^2}{md^2}$$

BUNCH OF ATOMS (bosons/fermions)
Or ELECTRONS

BOSONS FORM ONE GIANT ATOM!!



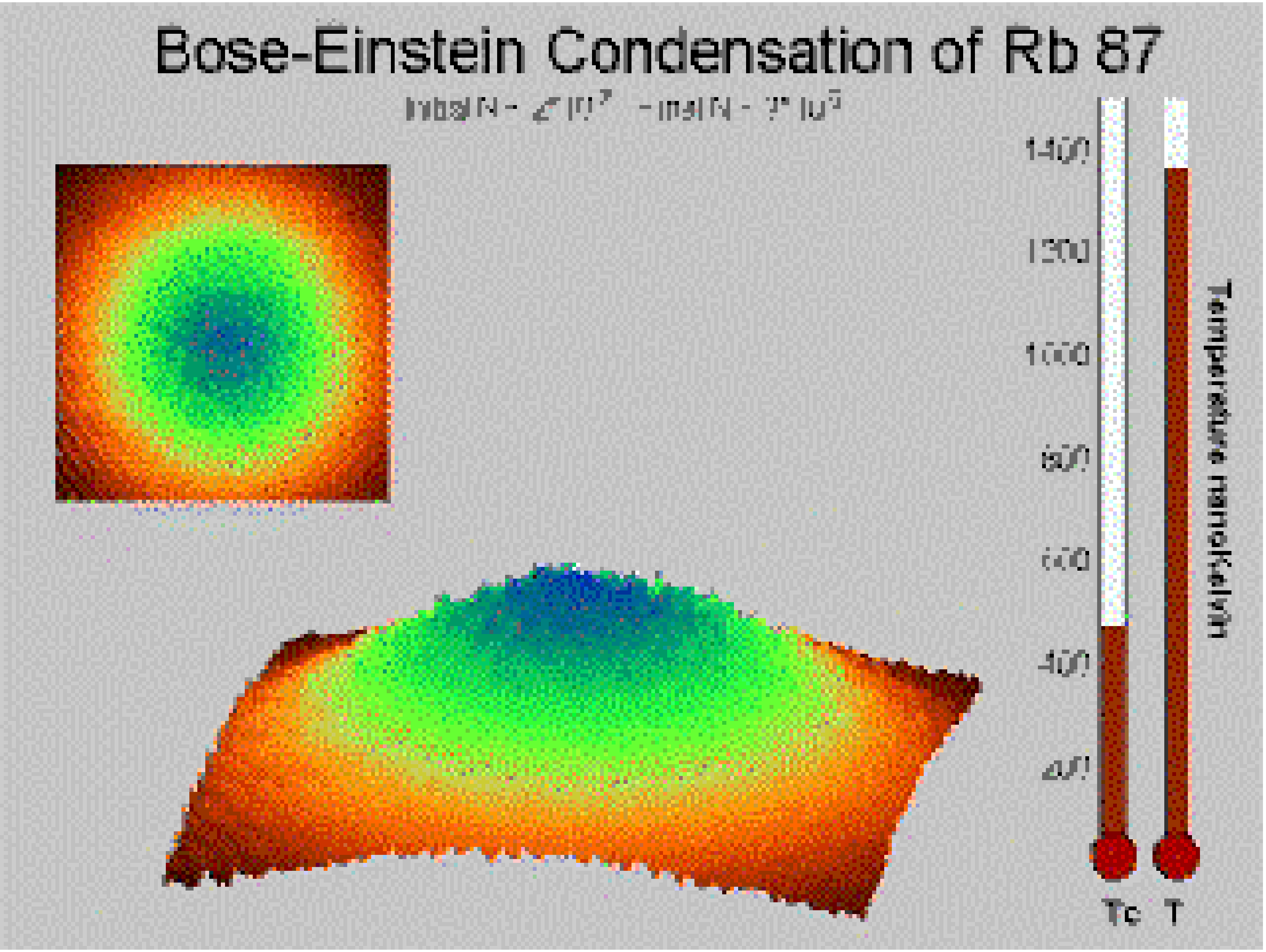
$T \sim 1nK \sim 10^{\ 9} K$

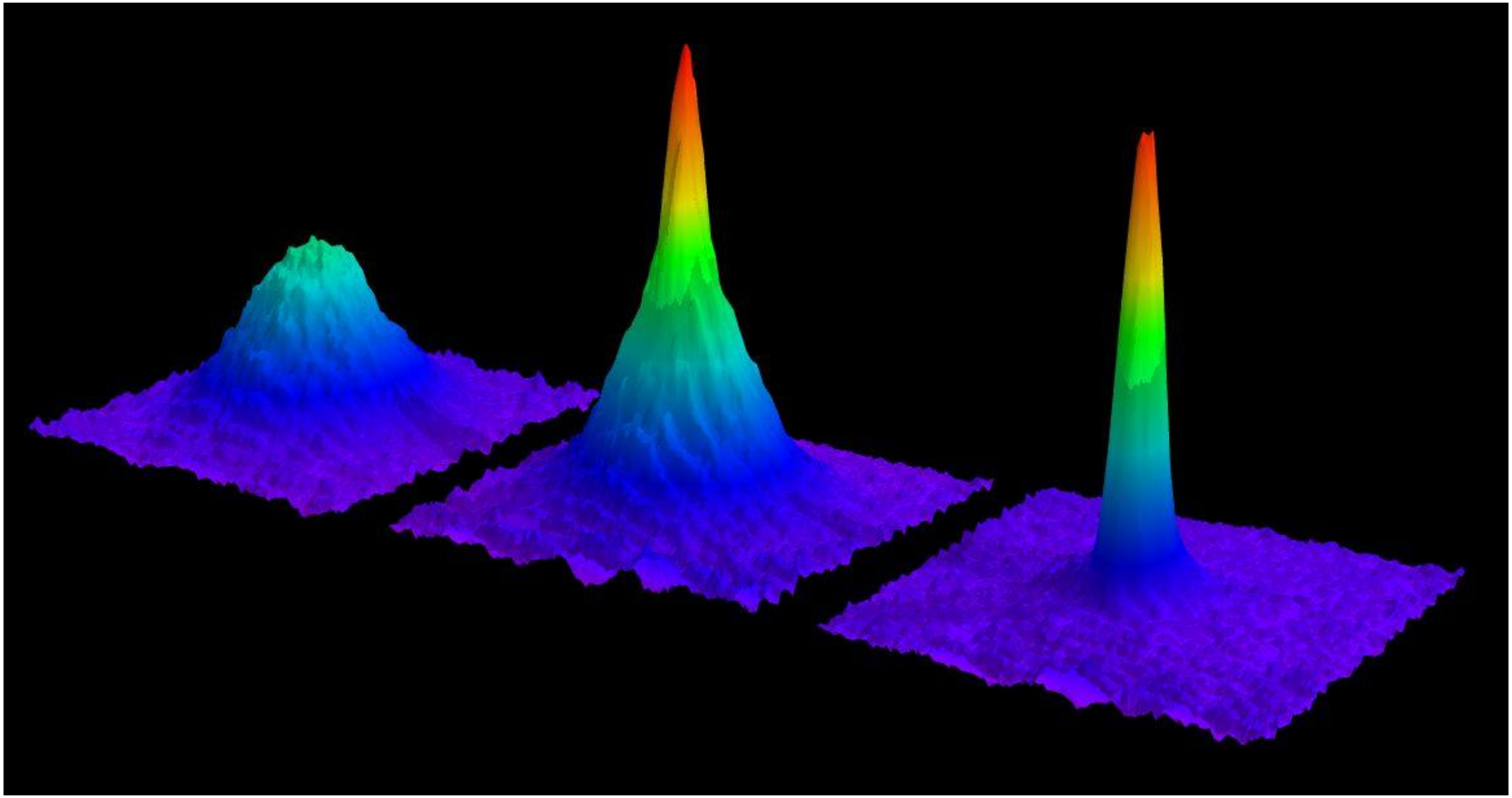
BOSE-EINSTEIN CONDENSATION

Bose Einstein condensate

Wonderland!!

http://www.colorado.edu/physics/2000/index.pl

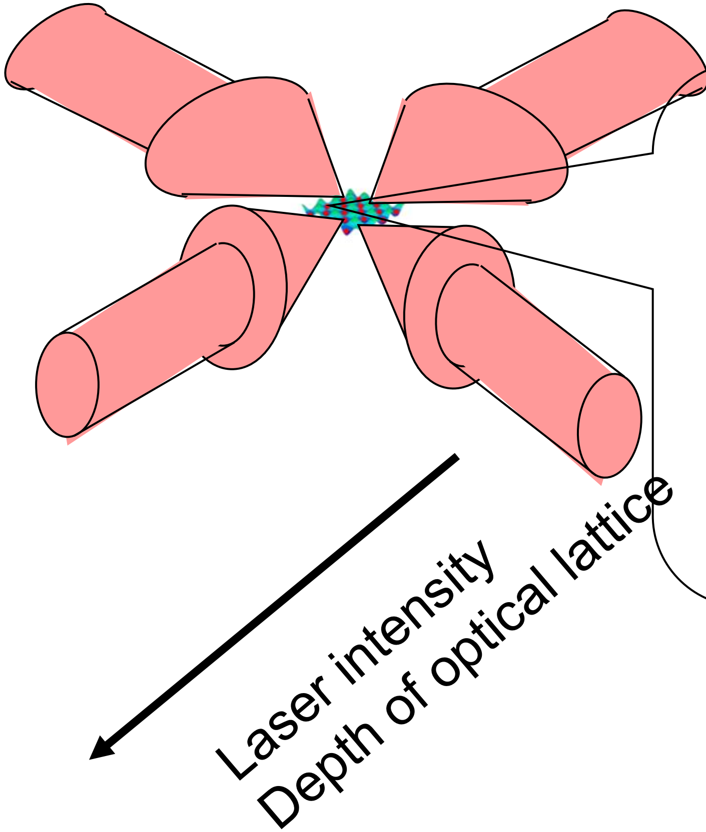

Bose-Einstein Condensation of Rb 87
1000
Temperature nanoKelvin
Tc
T

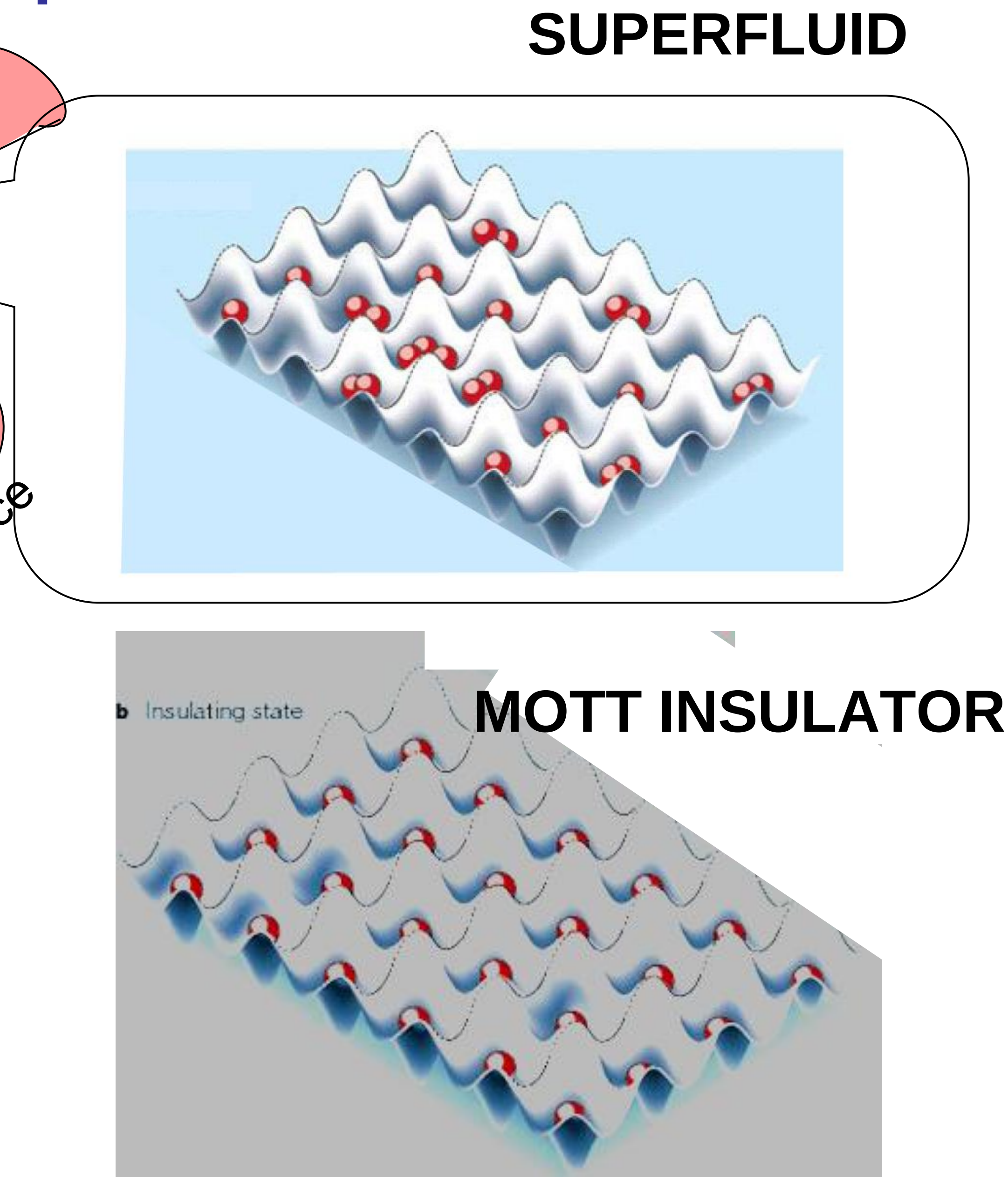


Temperature calculated by fitting to the
profile in the wings coming from thermal atoms

The Wonderland at Low Temperatures

Atoms in optical lattices

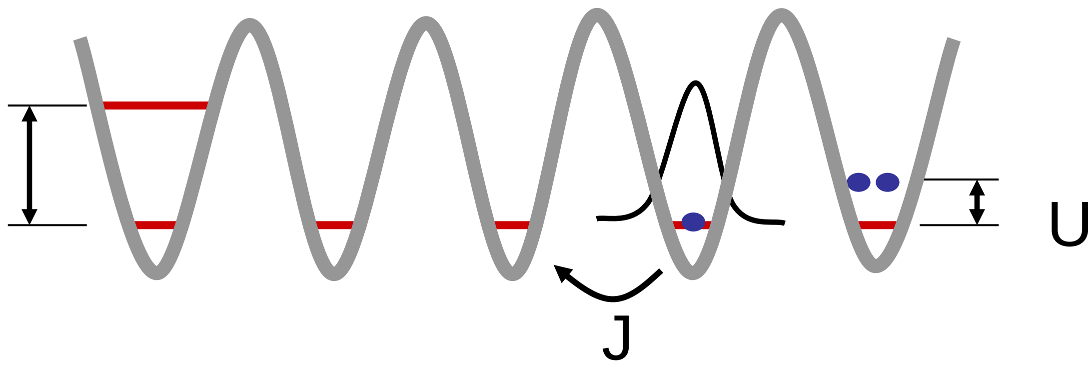




Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Phillips et al., J. Physics B (2002)
Esslinger et al., PRL (2004);

Bose Hubbard Model

t, U



$$H \qquad t \quad (a_i \;\; a_j \qquad h.c.) \;\; \frac{1}{2} U \quad n_i (n_i \quad 1) \qquad\qquad n_i$$
$$\qquad\qquad ij \qquad\qquad\qquad\qquad\qquad i \qquad\qquad\qquad\qquad i$$

t : tunneling of atoms between neighboring wells

U : repulsion of atoms sitting in the same well

M.P.A. Fisher et al.,
PRB40:546 (1989)

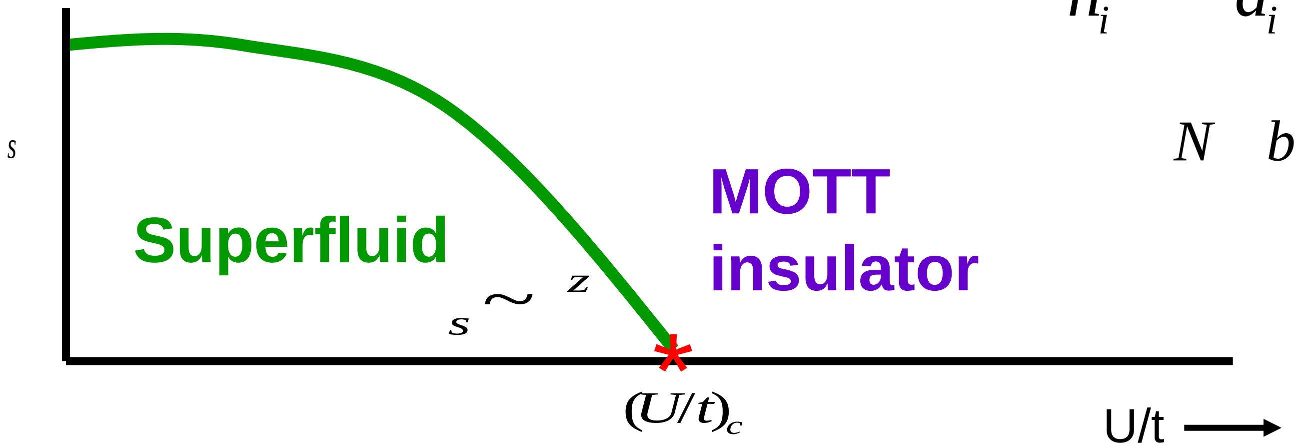
QUANTUM PHASE TRANSITION

$$H \quad t \sum_{i,j \; nn} a_i \; a_j \quad \frac{U}{2} \sum_i n_i (n_i$$

Bose Hubbard Model

$$n_i \quad a_i \quad a_i , a_i$$

$$N \quad b \quad o \quad s \quad o$$

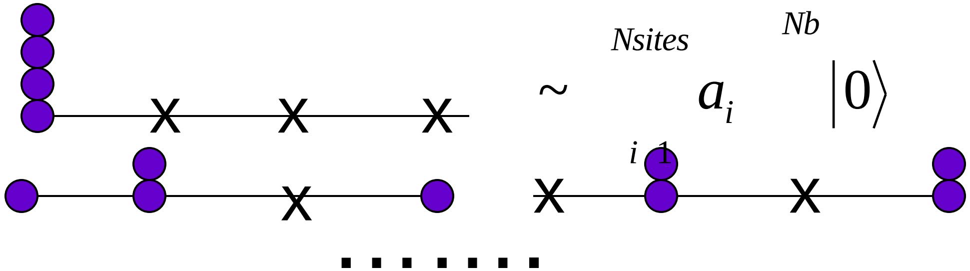


Continuous phase transition

FIXED PHASE (tunneling dominated)
NUMBER FLUCTUATIONS

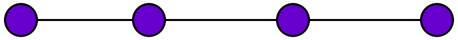
$$\left| \; _{SF} \right\rangle_{U \; 0} \sim a_{k \; 0}^{\;Nb} \left| 0 \right\rangle$$

$$\sim \left(\sum_{i \; 1}^{Nsites} a_i \right)^{Nb} \left| 0 \right\rangle$$



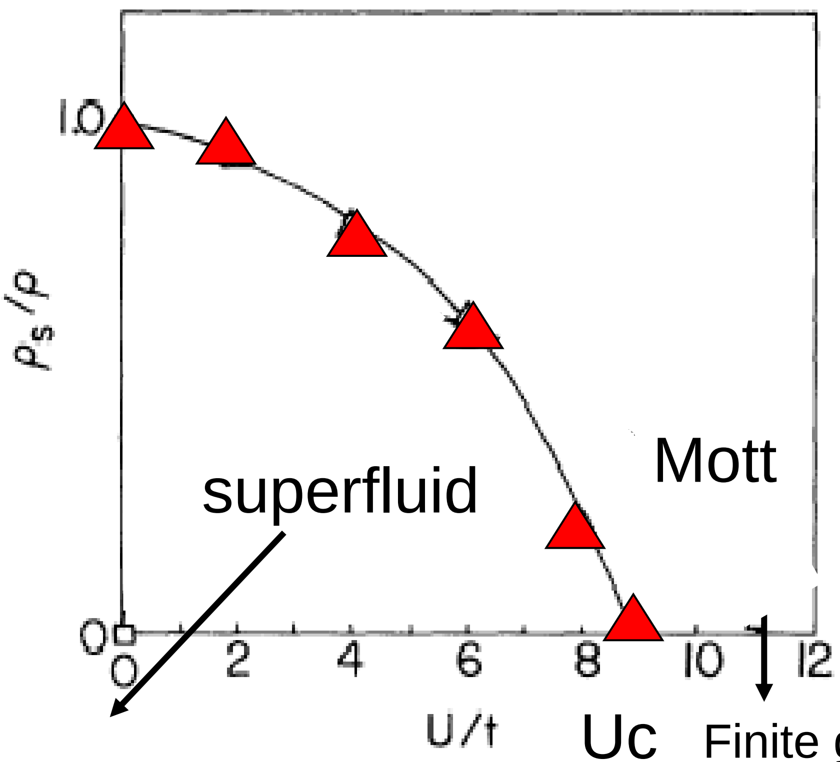
FIXED NUMBER (interaction dominated)
PHASE FLUCTUATIONS

$$\left| \; _{MOTT} \right\rangle_{t \; 0} \sim \prod_{i \; 1}^{Nsites} a_i^{\;Nb/Nsites} \left| 0 \right\rangle$$

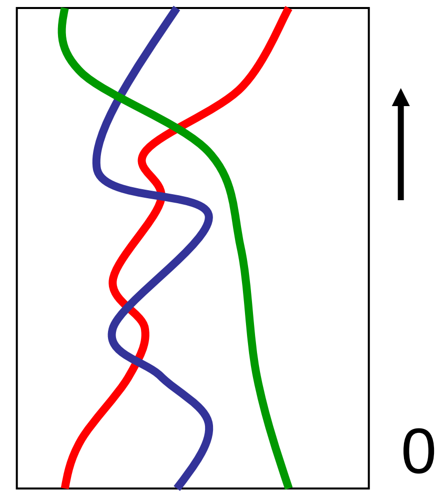


Bose Hubbard Model

$Z \quad Tre^{\quad H}$



paths
permutations



d-space

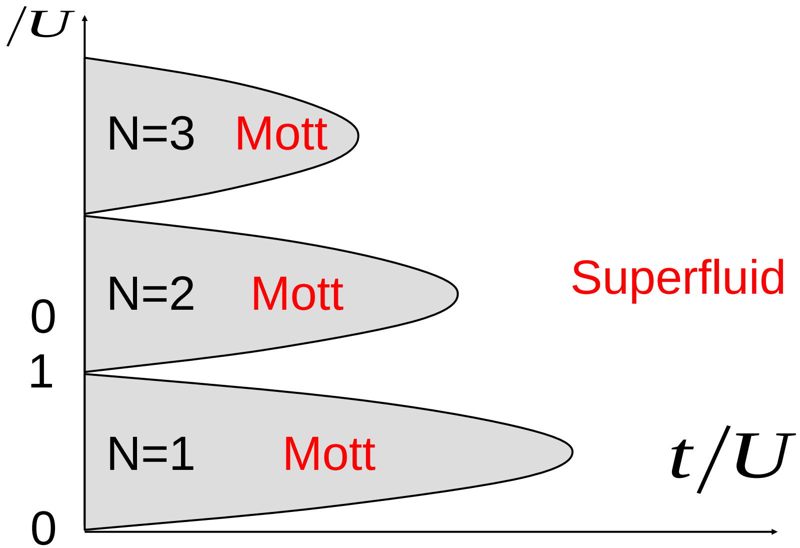
Finite gap
incompressible

Gapless excitations: phonons
compressible

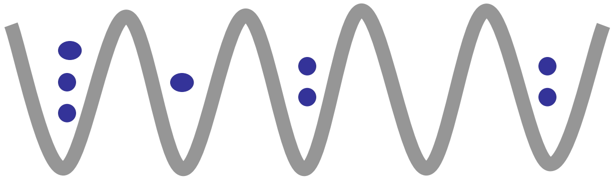
use Monte Carlo techniques to sum over important parts of phase space (Feynman path integral QMC)

Krauth and N. Trivedi, Euro Phys. Lett. 14, 627 (1991) QMC 2d

Bose Hubbard model. Mean-field phase diagram

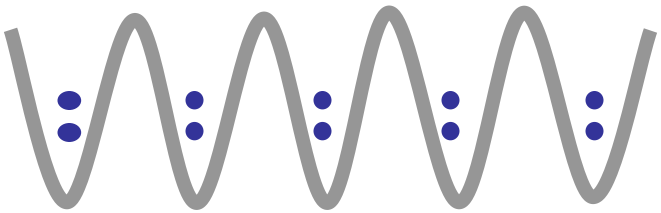


M.P.A. Fisher et al.,
PRB40:546 (1989)



$U \quad t$

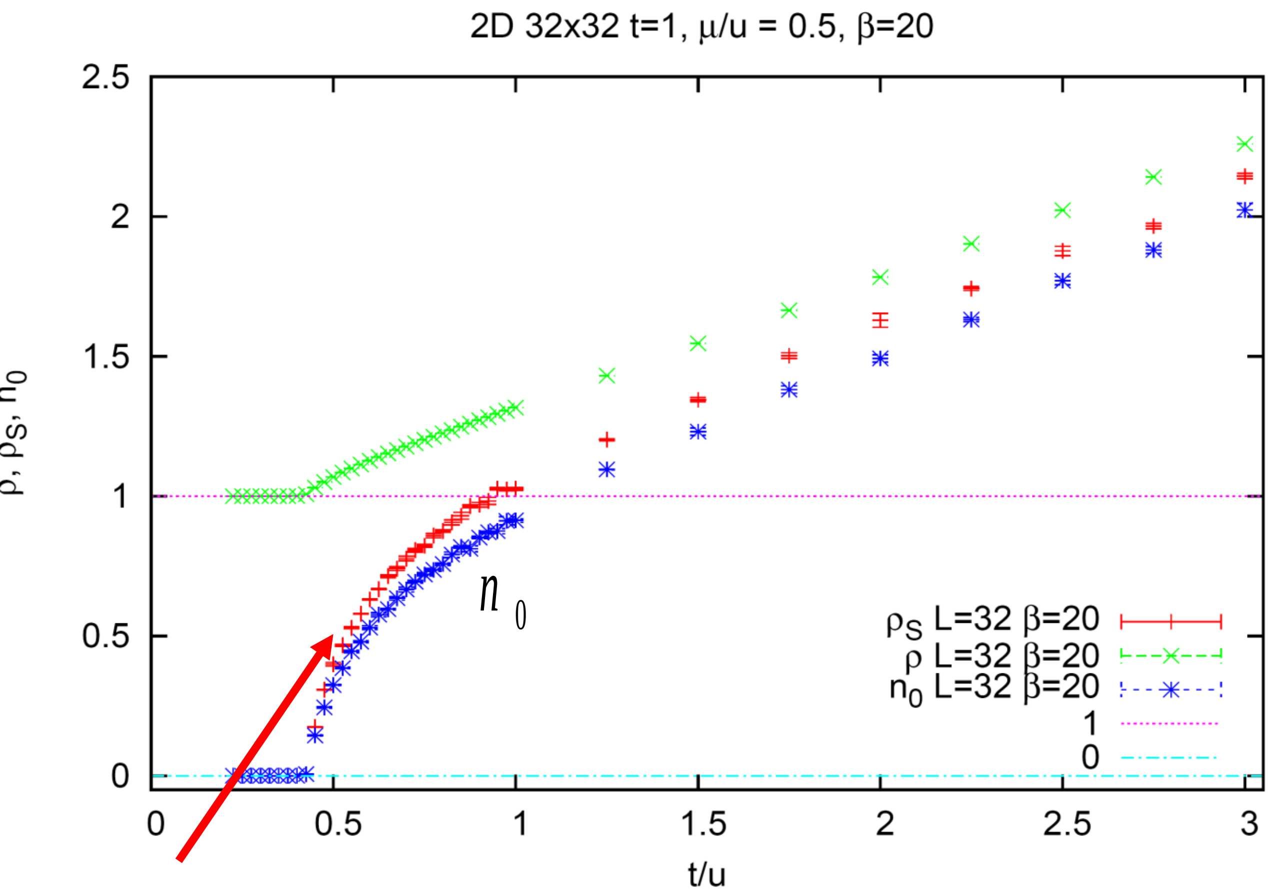
Superfluid phase
Weak interactions



$U \quad t$

Mott insulator phase
Strong interactions

QUANTUM PHASE TRANSITION



Diverging length scales

~

Diverging time scales

~ z ~ z

Energy

~ z

dynamics and statics linked by H

s ~ z

Universality class: (d+z) XY model

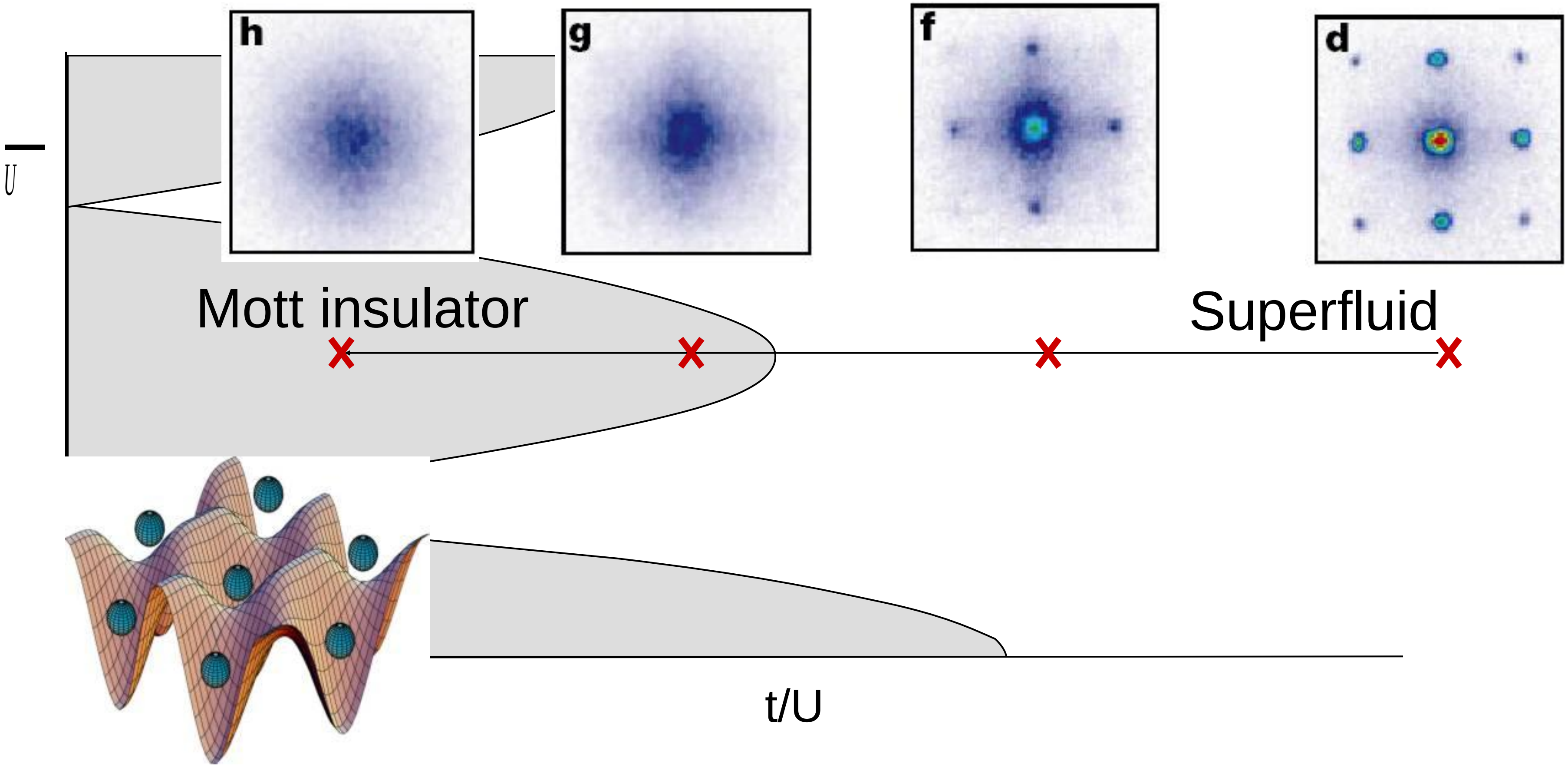
d=2; z=2 Mean field exponents: $1/4$

Fisher et al PRB 40, 546 (1989)

Yasuyuki Kato, Naoki Kawashima, N. Trivedi (unpublished)

Superfluid to insulator transition

Greiner et al., Nature 415 (2002)



Quantum statistical mechanics of many degrees of freedom at $T=0$

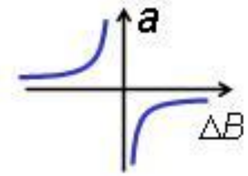
- New kinds of organisations (new phases) of the ground state wave function
- Phase transitions with new universality classes
- Tuned by interactions, density, pressure, magnetic field, disorder
- Phases with distinctive properties
- New applications

Atomic physics “knobs” to control many-body physics

Density 10^{11} to 10^{15} cm^{-3}

Temperature 500 pK to 1 mK

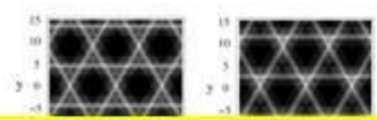
Interactions: scattering length a $-\infty$ to $+\infty$



Choice of hyperfine state(s): $|\uparrow\rangle$, $|\downarrow\rangle$; spinors

Optical traps and lattices: 1D, 2D systems

Optical lattices with different symmetries



Use the tools and precision of atomic physics to realize new phenomena (Hamiltonians) of many-body physics

Condensed-matter physics at ultra-low densities (100,000 times thinner than air)

Courtesy: Ketterle

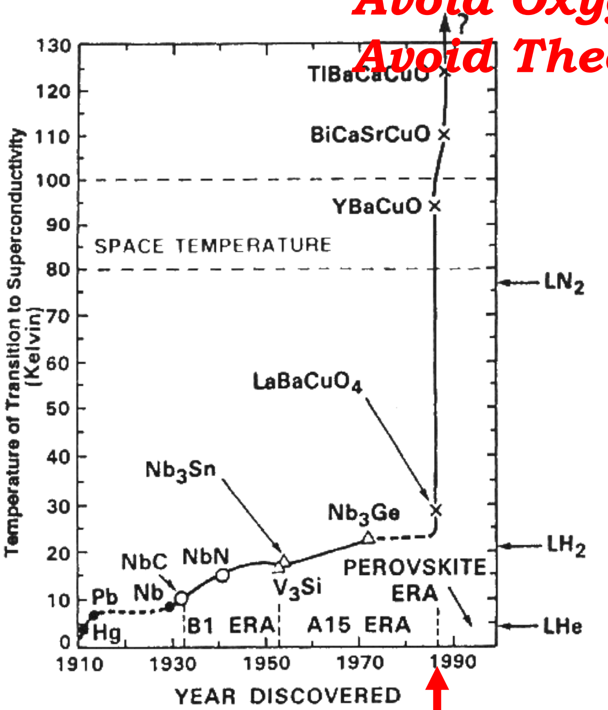
Electrons

CuO planes

Matthias' rules:

Avoid Insulators
Avoid Magnetism
Avoid Oxygen
Avoid Theorists

Tc



1986

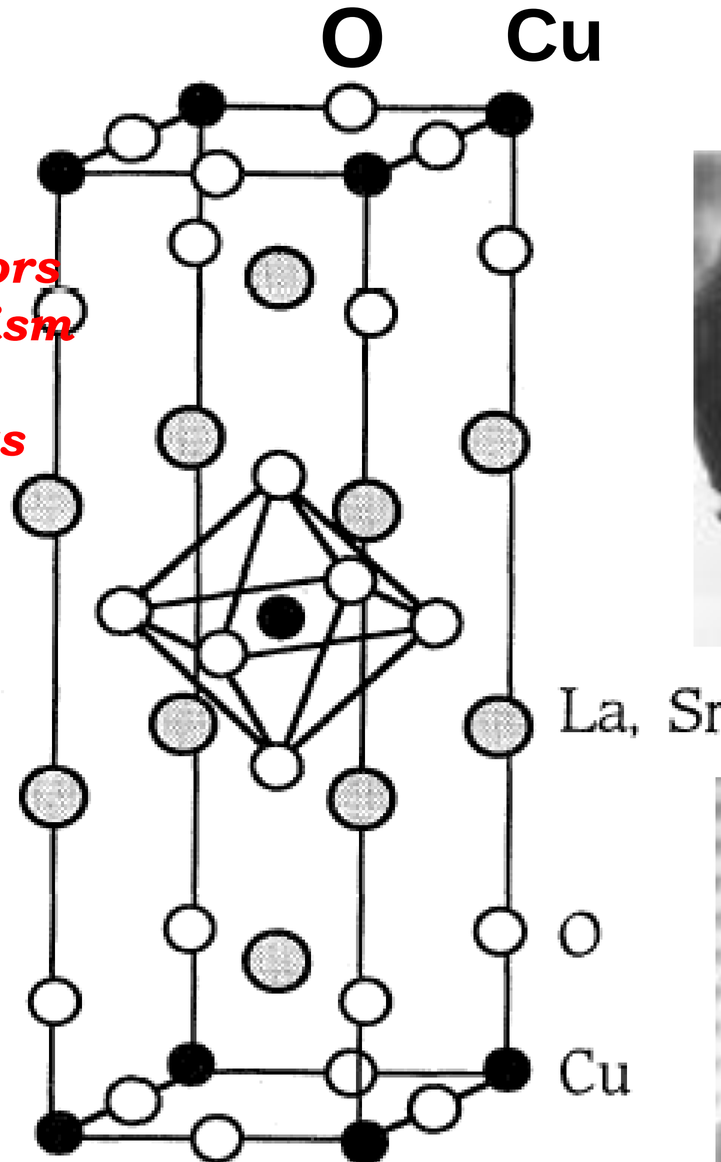
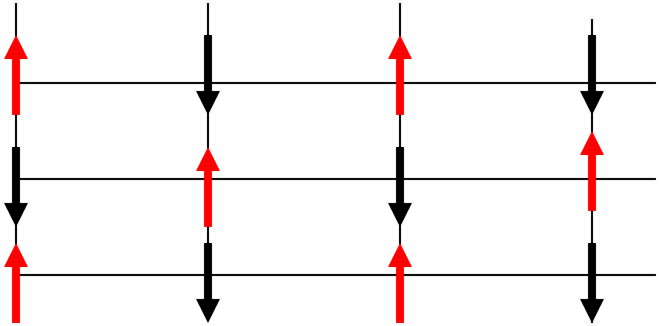


FIG. 1. Crystal structure of $La_{2-x}Sr_xCuO_4$ (T phase). Taken from Almasan and Maple (1991).

HIGH Tc Superconductivity: NEW PARADIGM

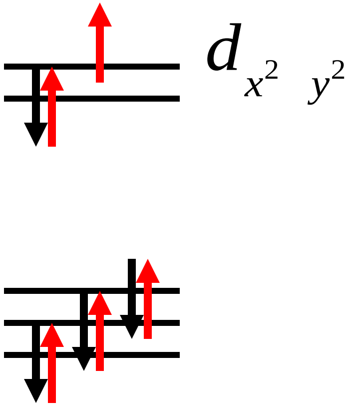
- SC found close to magnetic order and can coexist with it suggesting that spin plays a role in the pairing mechanism.
- Proliferation of new classes of SC materials, unconventional pairing mechanisms and symmetries of SC
- Exotic SC features well above the SC Tc
- Record breaking Tc
- Rich field

HUBBARD MODEL FOR ELECTRONS



<n>=1

La_2CuO_4

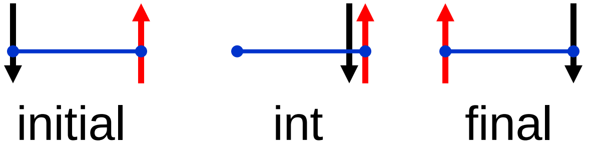


$$H \quad t \quad \,_{i,j} \quad c_{\ i}\, c_j \quad U \quad \,_{i} \quad n_i\, n_i$$

MOTT insulator: Finite gap in spectrum

Heisenberg Model

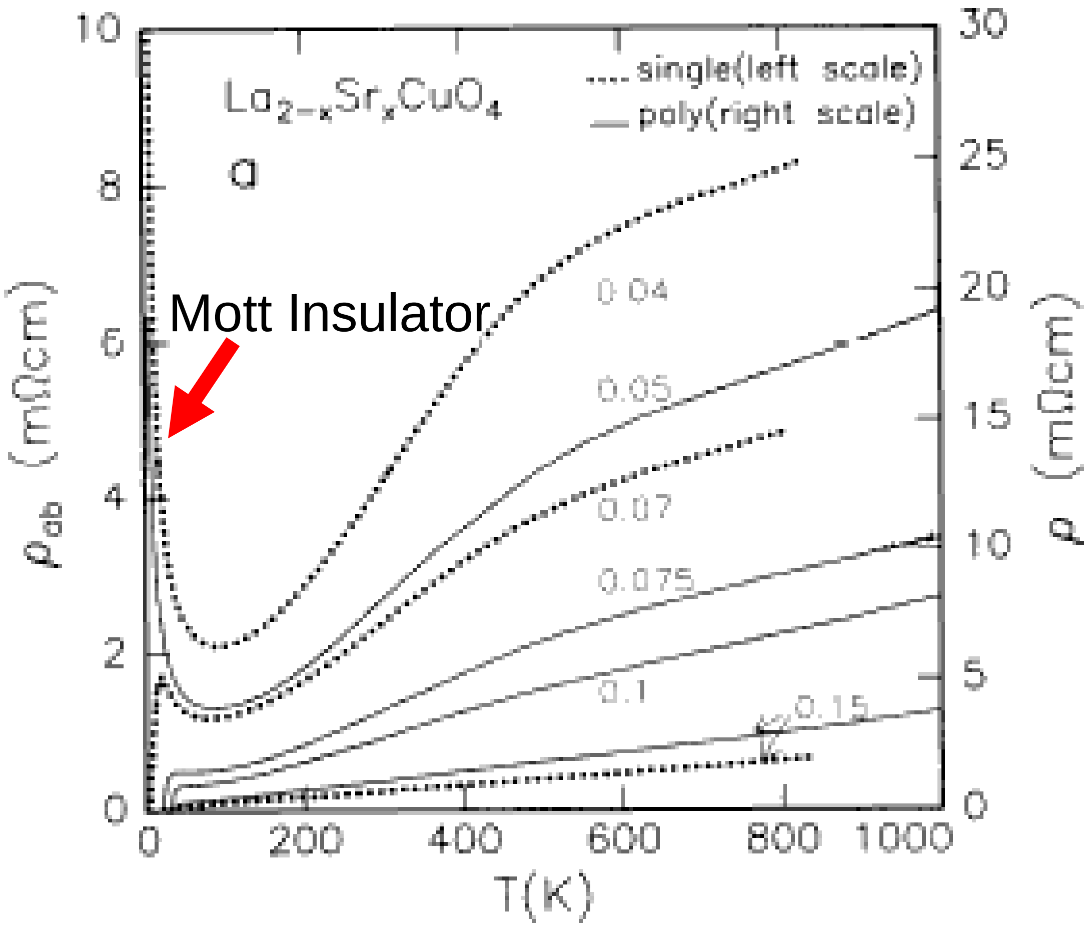
$$t/U \quad 1 \qquad H_{\ 1} \quad J \quad \,_{i,j} \quad \vec{S}_i \ \vec{S}_j$$



$$J \sim t^2/U$$

Antiferromagnetic long range order

Ignore interactions
metal



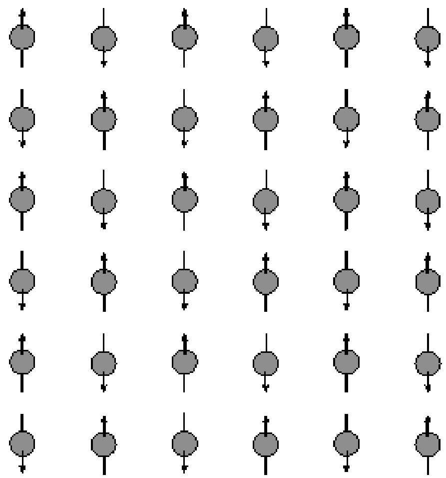
Strong Coulomb Interaction U

Half-filled in **r**-space: one el./site

Experiment:

$La_{\mathbf{2}}CuO_{\mathbf{4}}$

Insulator!

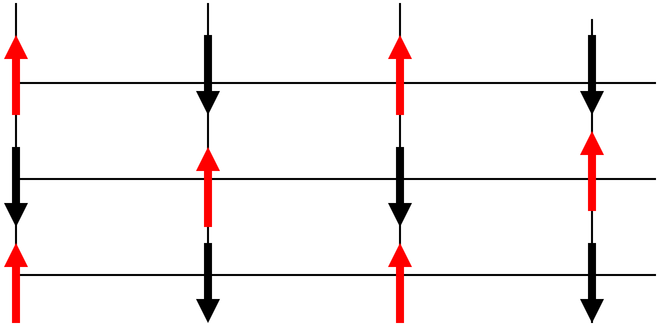


Mott Insulator:

Antiferromagnet

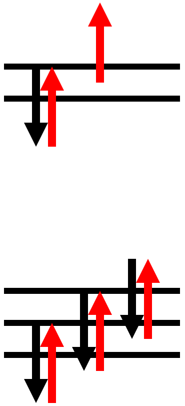
Gap ~U

HUBBARD MODEL FOR ELECTRONS



<n>=1

$$La_2CuO_4$$



$$H \quad t \underset{i,j}{\;} \quad c \;{}_i\; c_j \quad U \underset{i}{\;} \quad n_i \, n_i$$

MOTT insulator: Finite gap in spectrum

Heisenberg Model

$$t/U \quad 1 \qquad H_1 \quad J \underset{i,j}{\;} \quad \vec{S}_i \; \vec{S}_j$$

$$J \sim t^2/U$$

Antiferromagnetic long range order

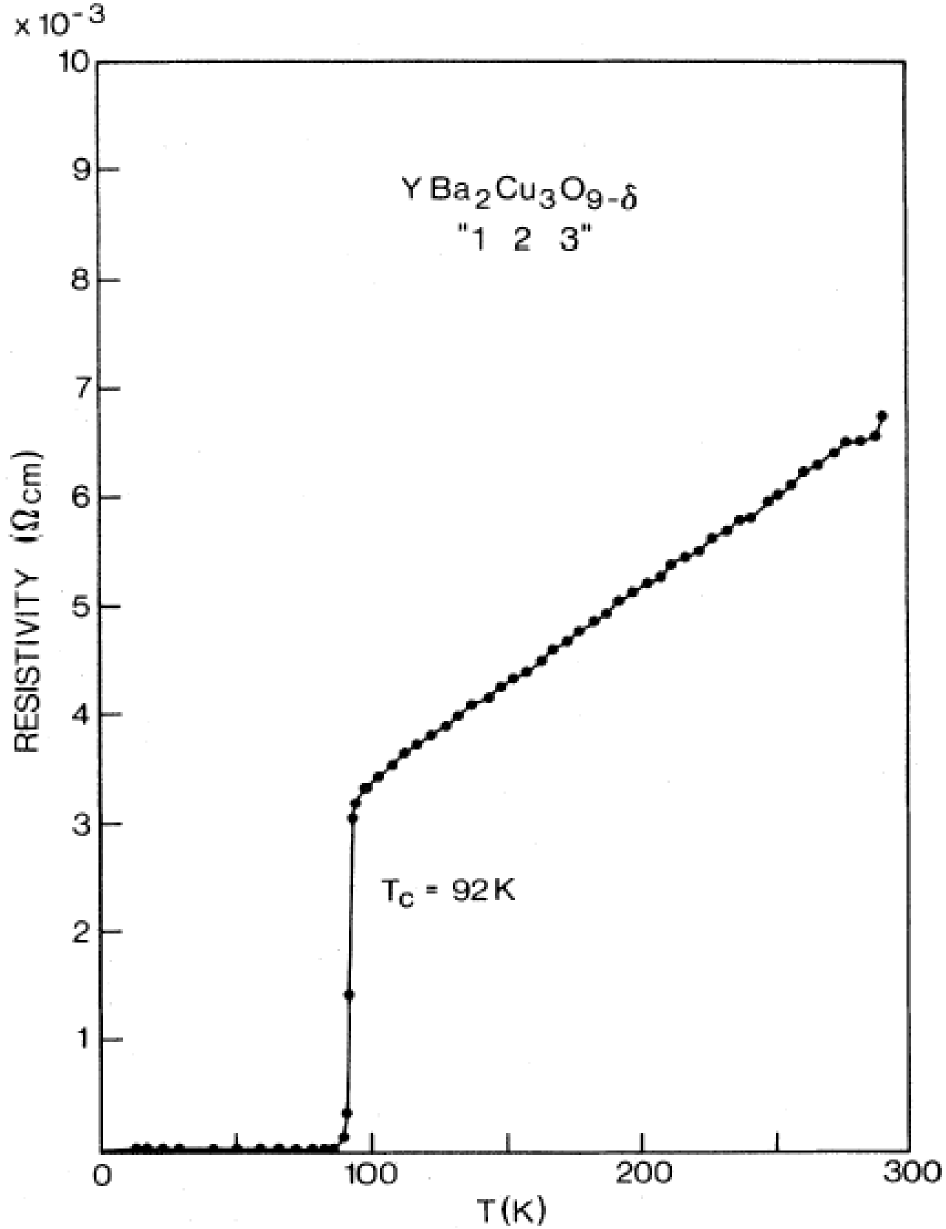
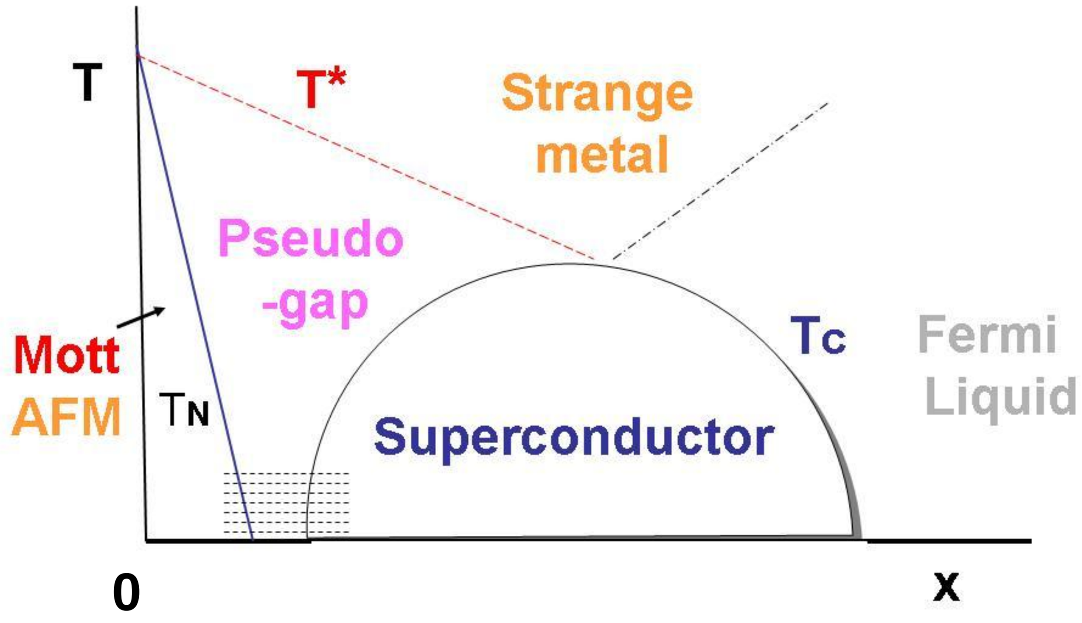


FIG. 14. Resistivity of a single-phase $YBa_2Cu_3O_7$ sample as a function of temperature.



focus <u>only</u> on <u>T=0</u>

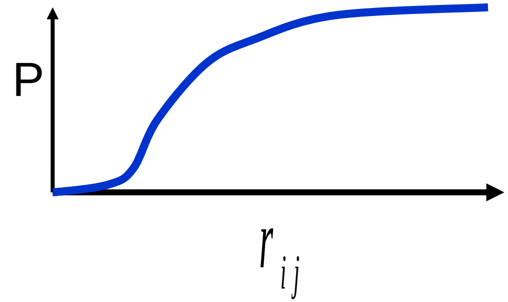
<u>ground state and low-lying excitations</u>

how do we construct wave functions for correlated systems?

$\left|\ {}_{bose}\right\rangle \qquad a_{k\ \ 0}^{\quad N}\left|0\right\rangle$ = uniformly spread out in real space

What is the w.f for bosons with repulsive interactions?

$\left|\psi_{bosons}\right\rangle = \prod_{i<j} \left[\quad\right]$



Correlation physics:
Jastrow factor

$\left|\ {}^{IQHE}_{1}\right\rangle$

$\left|\phi^{IQHE}_{2}\right\rangle =$

$\left|\ {}^{FQHE}_{1/3}\right\rangle \qquad \underset{i\quad j}{}(z_i \quad z_j)^2$

$\left|\ {}^{FQHE}_{2/5}\right\rangle \qquad \underset{i\quad j}{}(z_i \quad z_j)^2$

Jastrow correlation factor
Keeps electrons further apart

how do we construct wave functions for correlated systems?

$$|\;\;{}_{BCS}\rangle \qquad {}_{k} \qquad (k)c_{k}\;\; c_{k} \quad {}^{N/2}|0\rangle$$

$$|\;\;{}_{0}\rangle \qquad P|\;\;{}_{BCS}\rangle$$

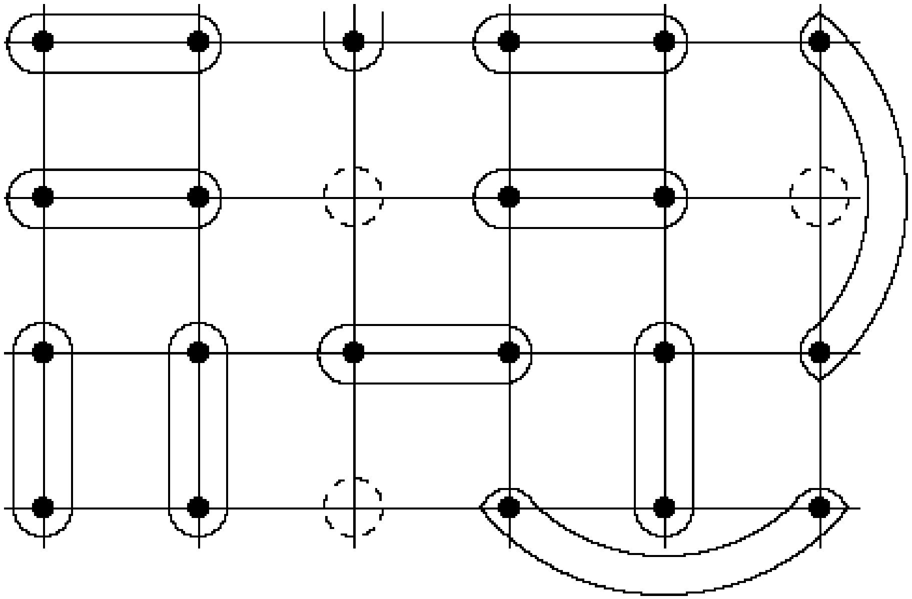
Explains the phenomenology of correlated SC in hitc

THE PROPERTIES OF $|\;\;{}_{0}\rangle$

ARE COMPLETELY DIFFERENT FROM THOSE OF $|\;\;{}_{BCS}\rangle$

Resonating valence bond wave function for High temperature superconductors

$$|\Psi_0\rangle \equiv \mathcal{P}|BCS\rangle = \mathcal{P}[\sum_{\mathbf{r},\mathbf{r}'} \varphi(\mathbf{r}-\mathbf{r}')c^\dagger_{\mathbf{r}\uparrow}c^\dagger_{\mathbf{r}'\downarrow}]^{N/2}|0\rangle$$



Projected SC $\equiv$ Resonating Valence Bond (RVB) liquid

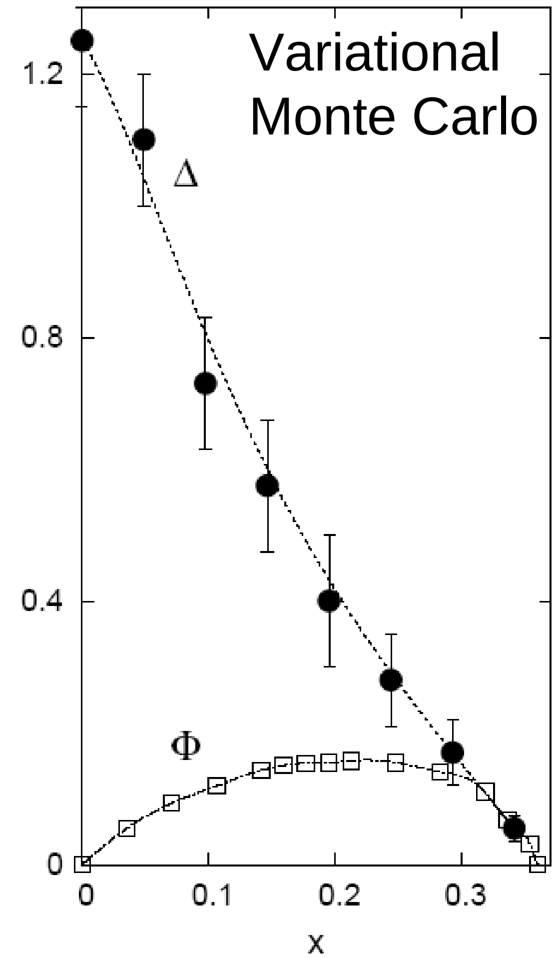
$$\mathrm{r} \; (\bullet \!-\!-\! \bullet) \; \mathrm{r}' = \frac{|\uparrow_\mathrm{r}\downarrow_{\mathrm{r}'}\rangle - |\downarrow_\mathrm{r}\uparrow_{\mathrm{r}'}\rangle}{\sqrt{2}} \; \varphi(\mathbf{r}-\mathbf{r}')$$

$$\varphi(\mathbf{r}-\mathbf{r}') = \sum_k \exp(i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}'))\,(v_\mathbf{k}/u_\mathbf{k})$$

P.W. Anderson,
Science 235, 1196 (1987)

Summary of work on RVB Projected wavefunctions:

- SC "dome" with optimal doping
- pairing and SC order have qualitatively different x-dependences.
- Evolution from large x BCS-like state to small x SC near Mott insulator
- x-dependence of low energy excitations & Drude weight



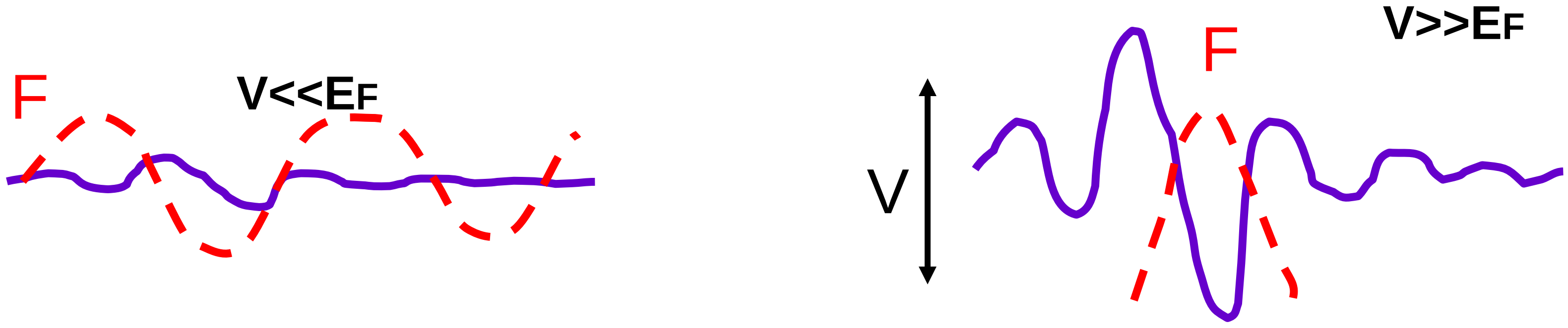
* A. Paramekanti, M.Randeria & N. Trivedi, PRL 87, 217002 (2001); PRB 69, 144509 (2004); PRB 70, 054504 (2004); PRB 71, 069505 (2005)

P.W. Anderson, P.A. Lee, M.Randeria, T. M. Rice, N. Trivedi & F.C. Zhang, J. Phys. Cond. Mat. 16, R755 (2004)

Simplest disorder driven quantum phase transition
Anderson Localization (1958)

non interacting electrons
in a random potential

$$H \quad p^2 / 2m \quad V(r)$$



3 dim

V (disorder) →

CONDUCTOR

ANDERSON INSULATOR

Extended wave function
Sensitive to boundary conditions

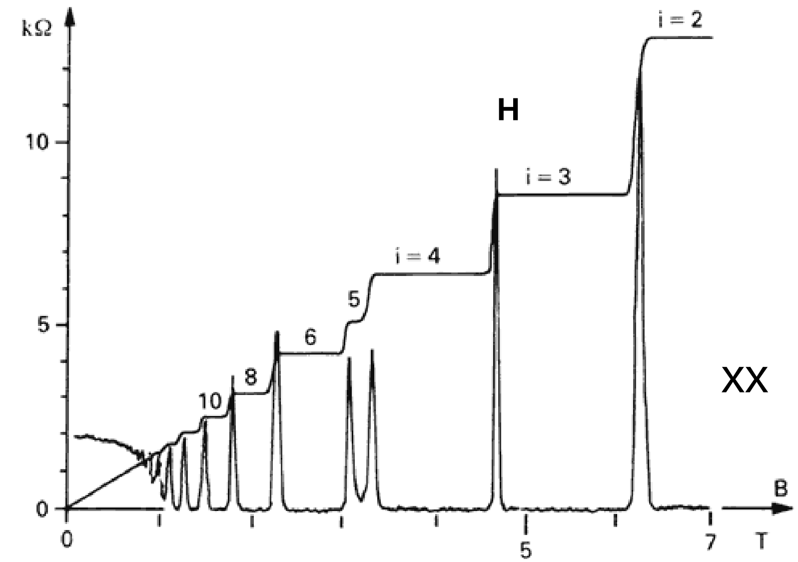
Localized wave function
Insensitive to boundaries

2d: All states are localized; No true metals in 2d
(Abrahams et.al PRL 1979)

DISORDER: yuch!!

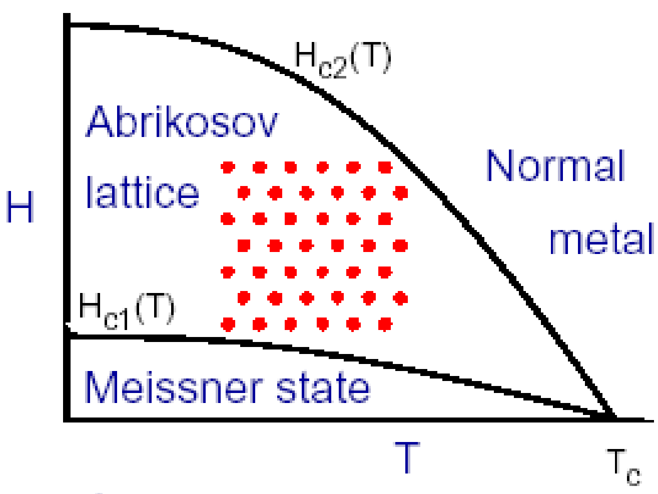
NEW PHENOMENA

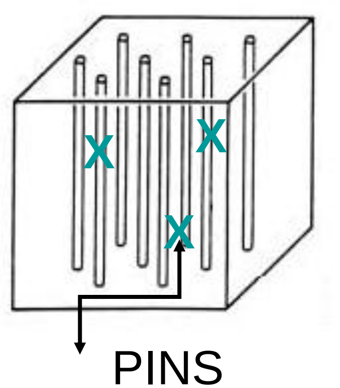
Quantum Hall Effect



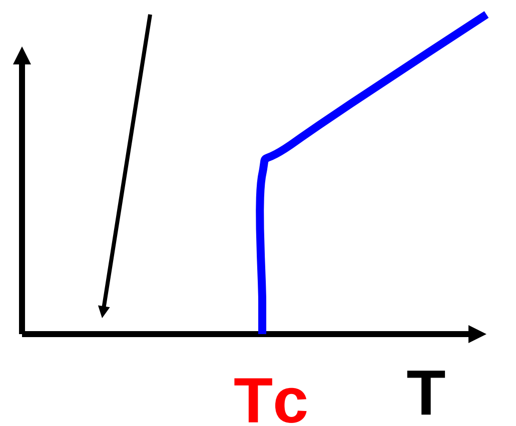
Quantization to 1 part in 10^8
ONLY if some disorder in sample

Superconductivity with vortices





=0 only if vortices are pinned



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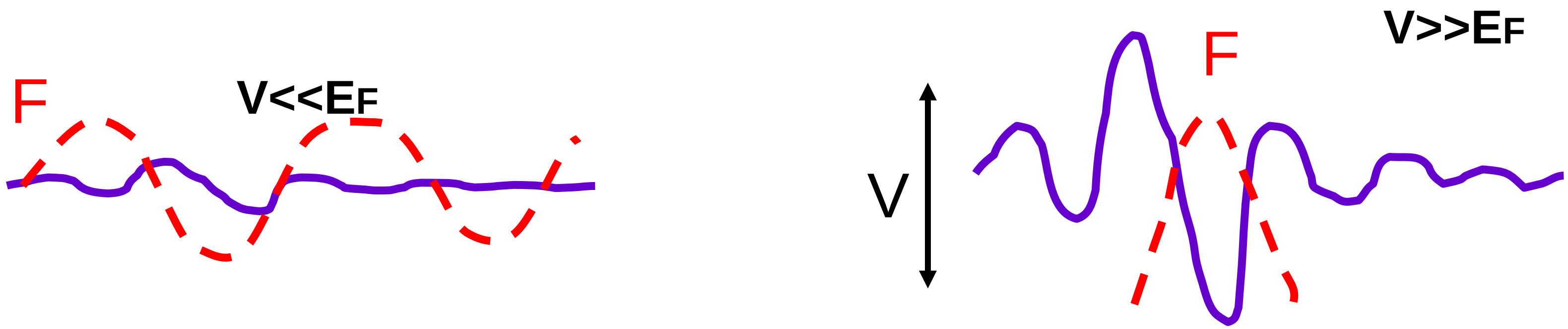

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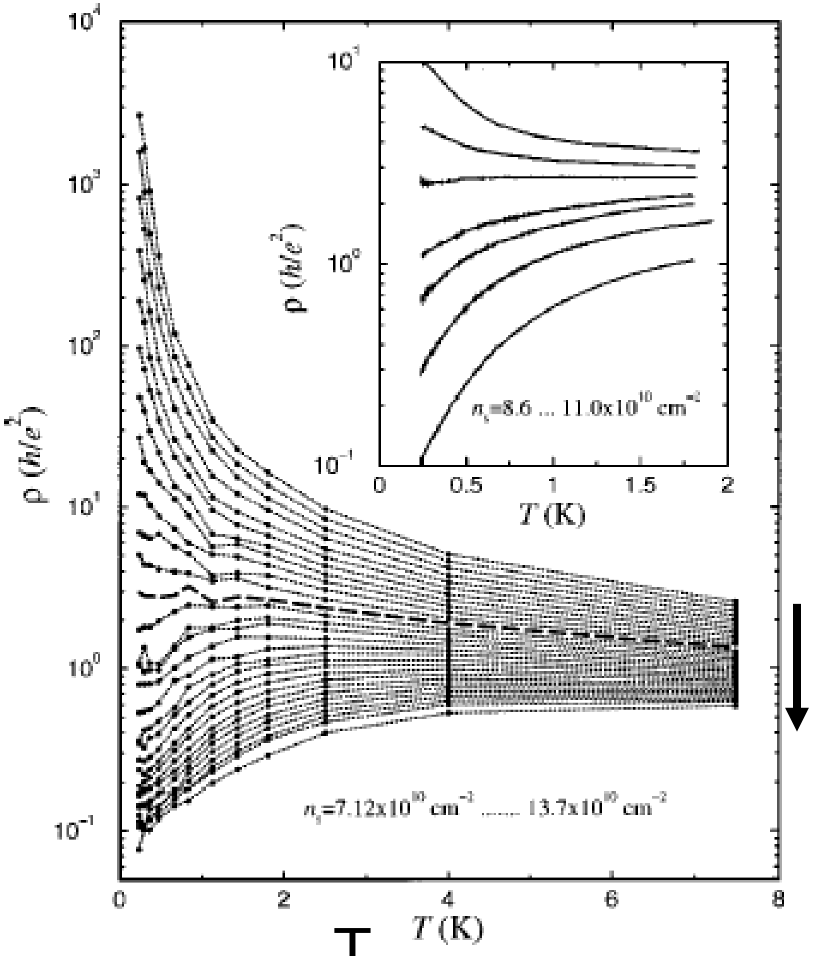
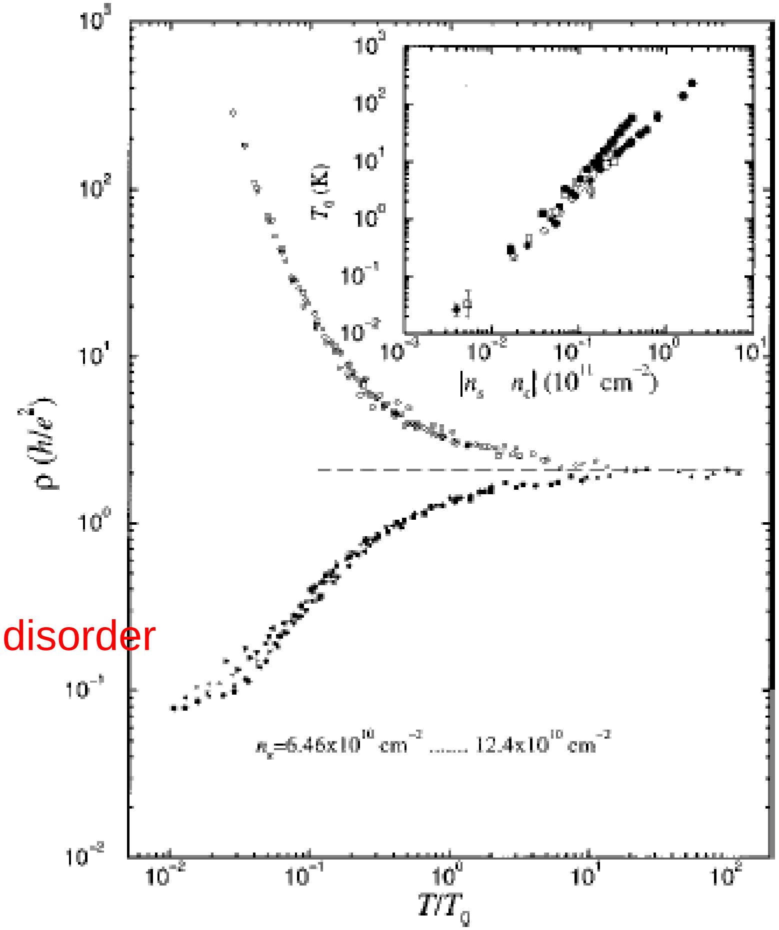
METALS IN 2D ?

INSULATOR



n

disorder



T

“METAL”

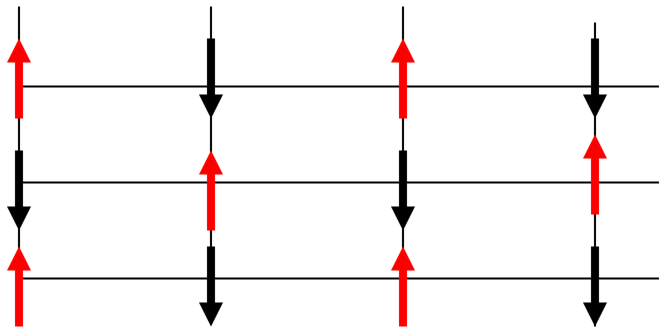
E. Abrahams, S. Kravchenko, M. Sarachik
Rev. Mod. Phys. 73, 251 (2001)

EXPERIMENTS

Could interactions and disorder cooperate to generate new phases

INTERPLAY OF INTERACTION AND DISORDER EFFECTS

MOTT INSULATOR



$$H \quad t_{i,j} \quad c_i \, c_j \quad U_i \quad n_i \, n_i$$

HUBBARD MODEL

<n>=1

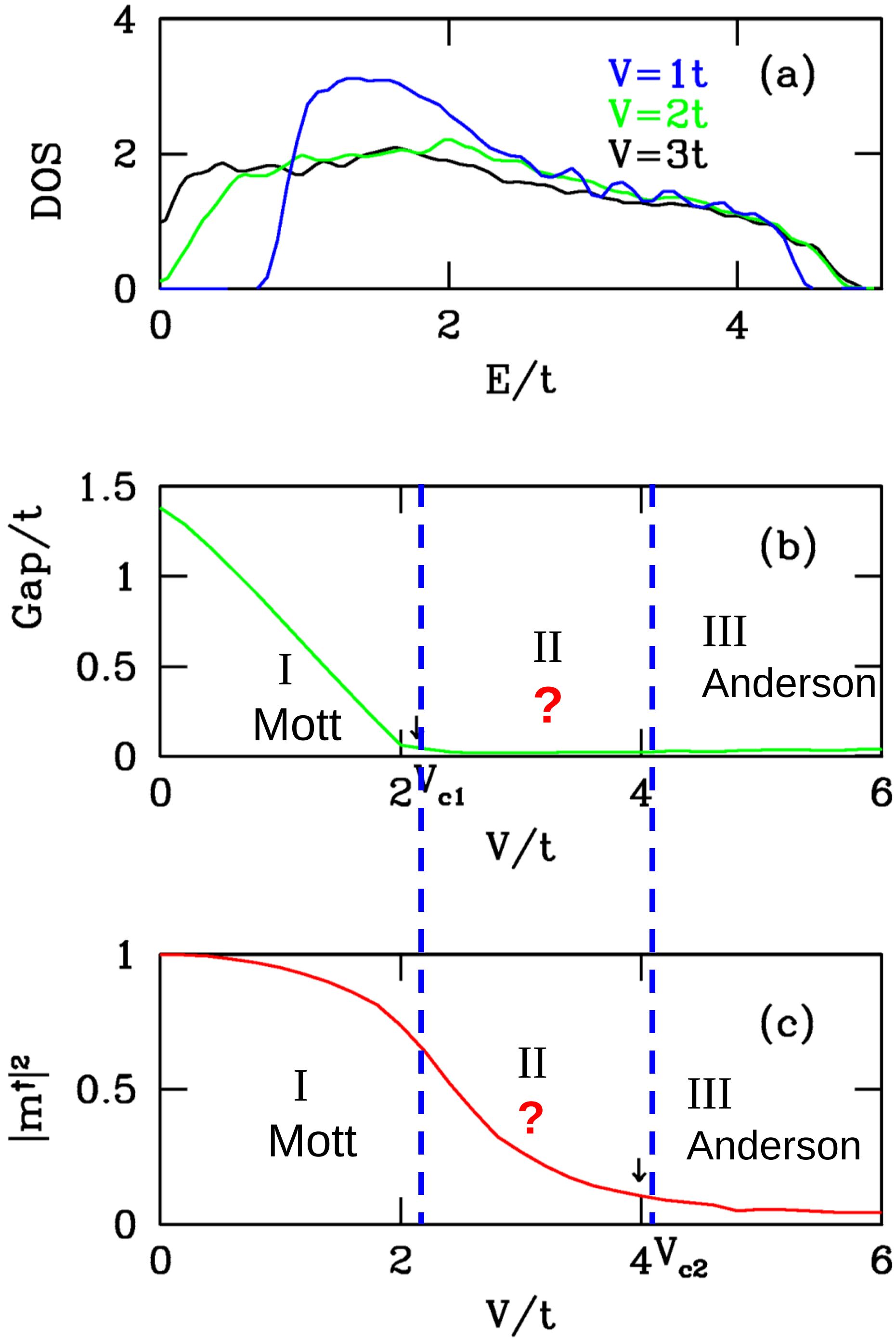
MOTT insulator: Finite gap in spectrum

Antiferromagnetic long range order

$$J \sim t^2 / U$$

MAIN QUESTION:

What is the effect of disorder
on AFM long range order J?
on charge gap U?
Which is killed first?
Or are they destroyed together…



Staggered magnetization

Gap ~ U

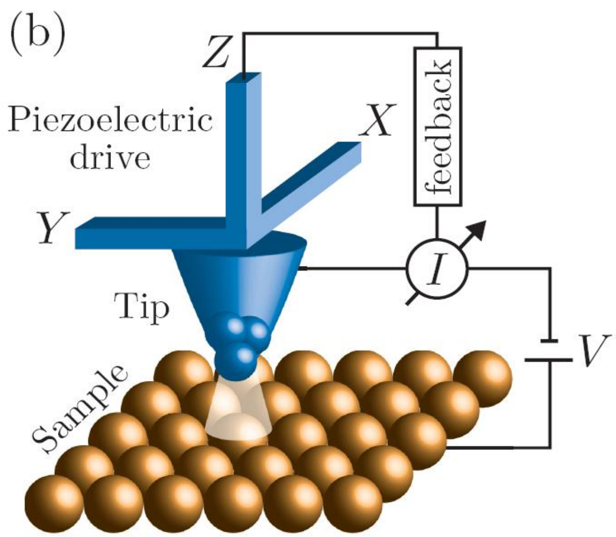
AFM J ~ $t^2/U \ll U$

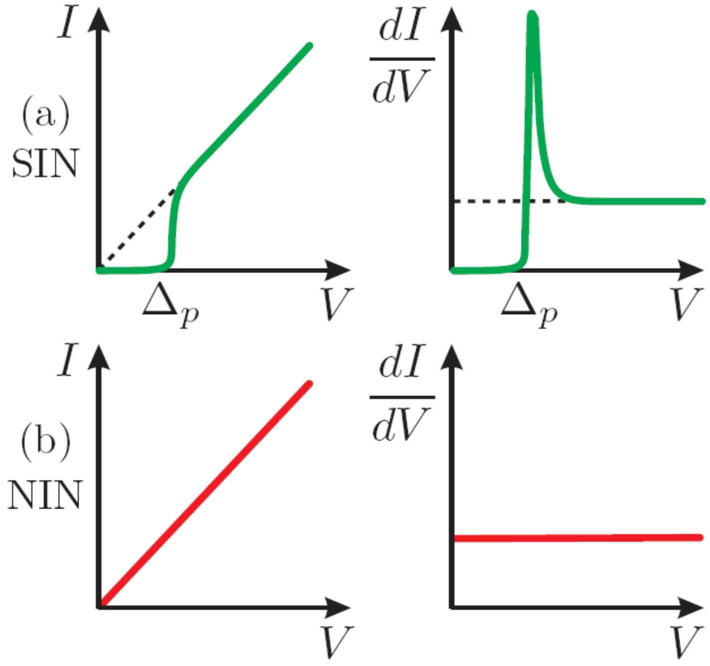
•**Why is the gap killed first?**

•**What is the ? phase?**

METAL

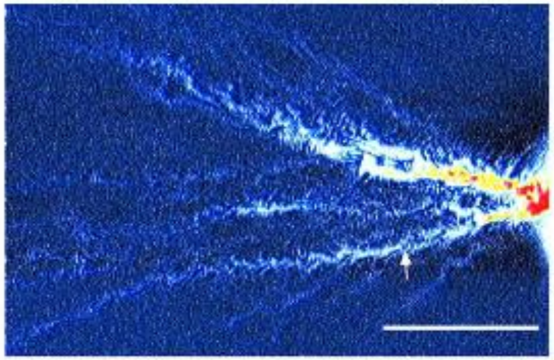
Scanning Tunneling Spectroscopy





Low-T Scanned Probes

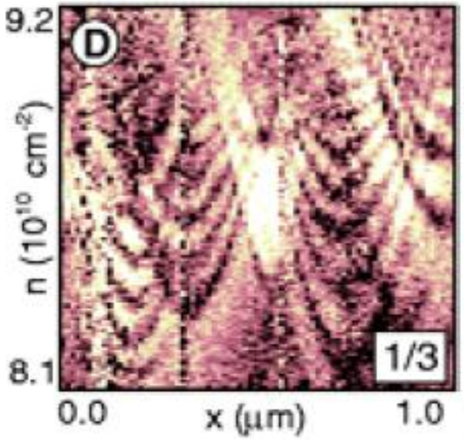
Electron trajectories in 2D electron gas:



SGM

Topinka (2001)

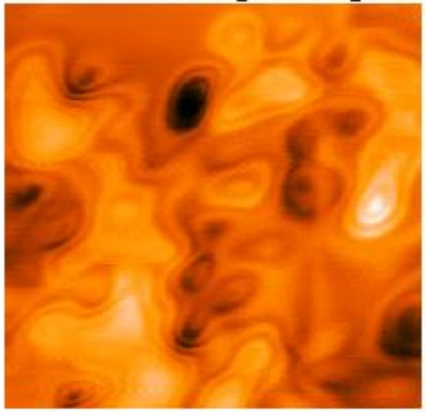
Localized integer & fractional charge state



S-SET

Ilani (2004)
Martin (2005)

Potential landscape of quantum Hall conductor:



SCM

Finkelstein (2001)

detect the presence of electrons by sensing electrostatic force

→ local compressibility

Carbon Nanotube: 1D conductor

Jun Zhu (Cornell)

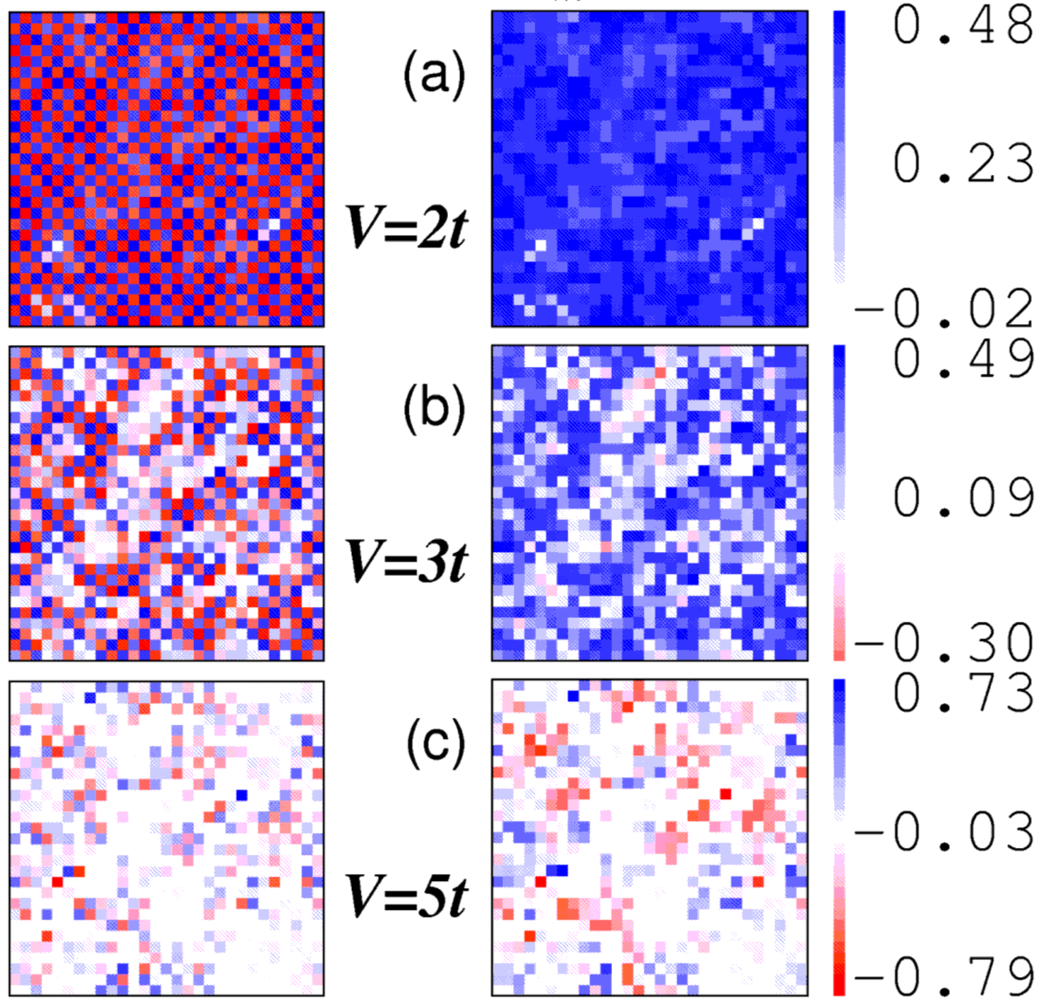
Also work by Ray Ashoori and A. Yacoby

DISORDERED HUBBARD MODEL AT HALF FILLING

$\langle S_z(i) \rangle$ $(\quad 1)^{i_x \quad i_y}_{m^\dagger} \langle S_z(i) \rangle$ **Staggered magnetization**

Local magnetization

N=24x24
U=4t



Disorder V:
uniform distribution
couples to density

As disorder strength increases the defected regions i.e. regions with suppressed checker board pattern grows

QM+Many Particles

- ***New phases emerge tuning some parameter***
  Quantum magnets; Spin Liquids; Superfluids+Superconductors;
  B=0 Wigner Crystals+Quantum Melting into electron liquids;
  B finite Wigner crystals + Quantum Hall liquids

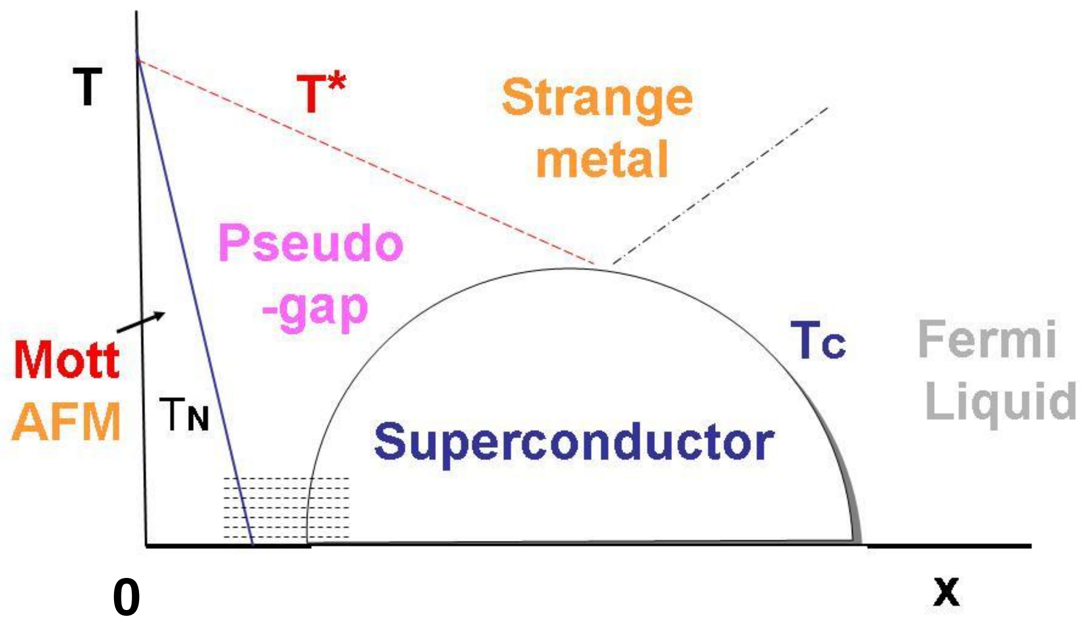
- ***reorganisation of degrees of freedom***
  new many body wave function often must be discovered by intuition rather than derived from a parent state (non-perturbative)

- ***Spontaneous symmetry breaking***

- Simple ***Hubbard-type models*** capture the physics
  quantum degeneracy+competition between different pieces of the hamiltonian

- different ***theoretical techniques***: path integrals and variational

- spectroscopy with ***local probes***: charge, spin and superconductivity

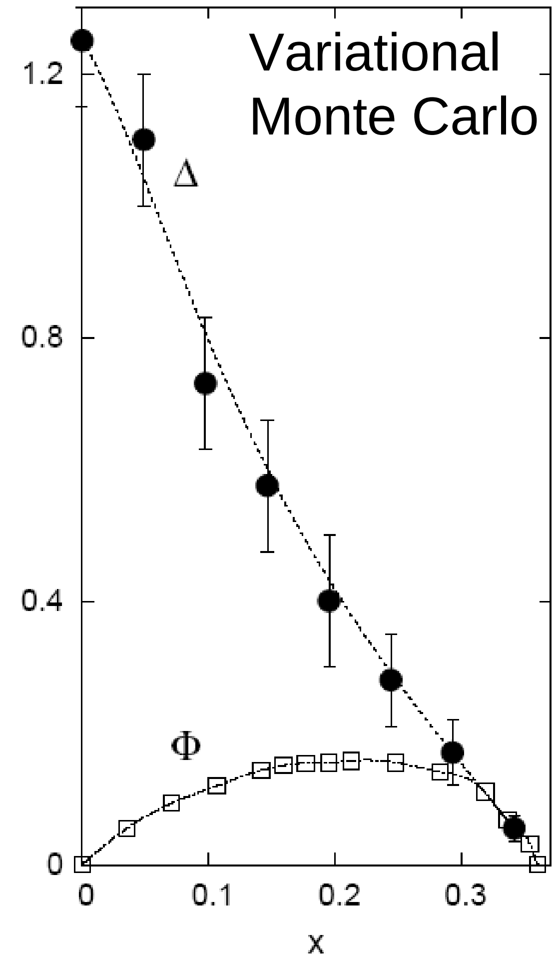


focus <u>only</u> on <u>T=0</u>

<u>ground state and low-lying excitations</u>

Summary of work on RVB Projected wavefunctions:

- SC "dome" with optimal doping
- pairing and SC order have qualitatively different x-dependences.
- Evolution from large x BCS-like state to small x SC near Mott insulator
- x-dependence of low energy excitations & Drude weight



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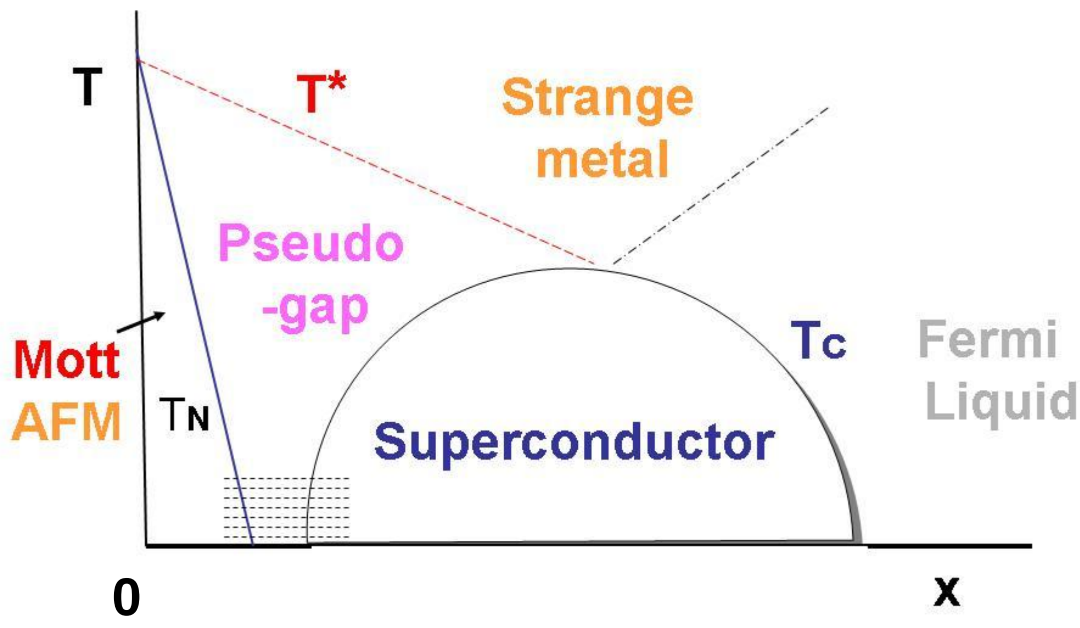

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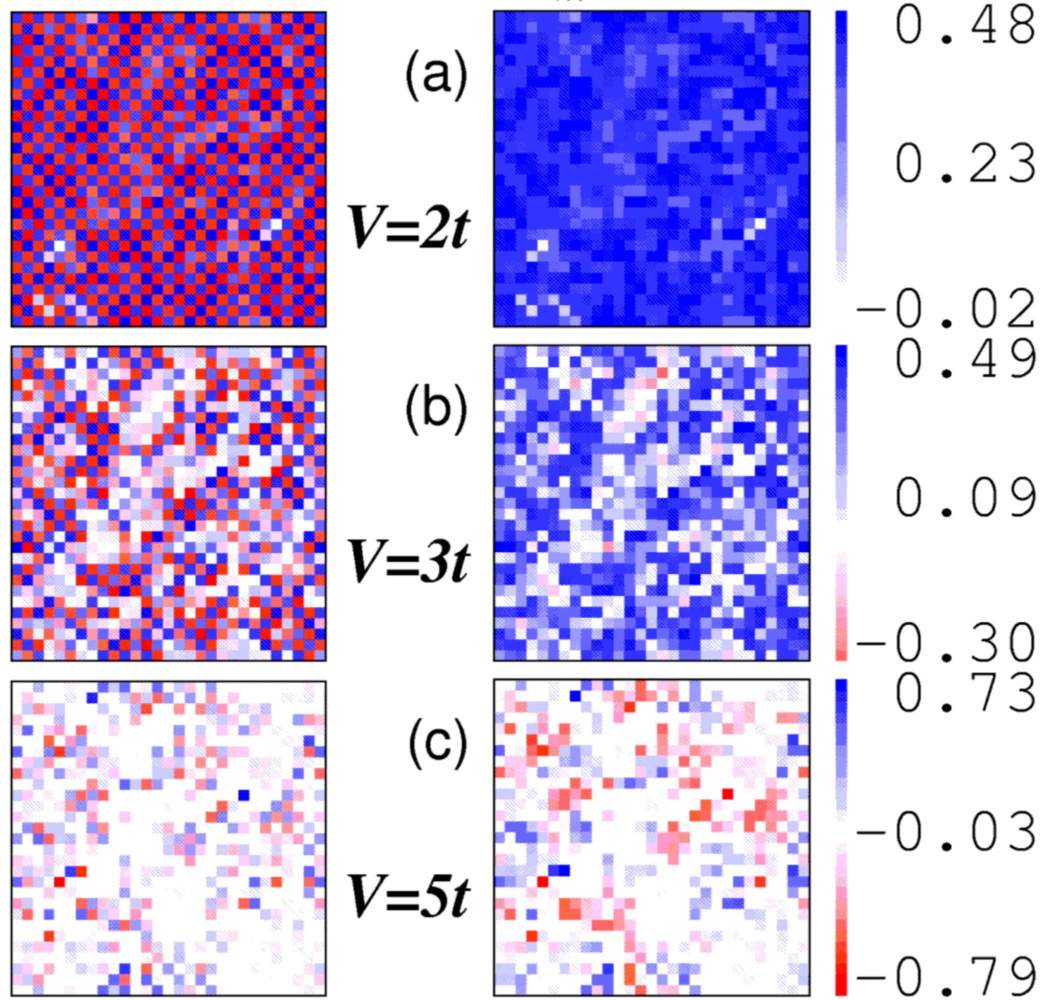

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