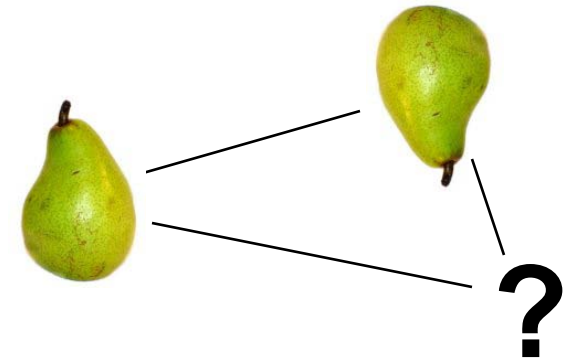
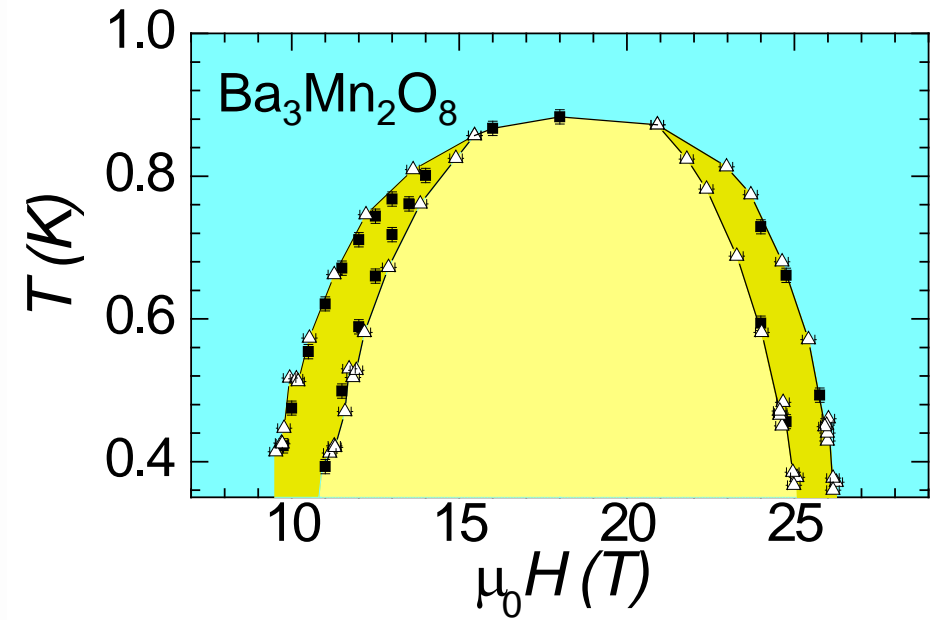
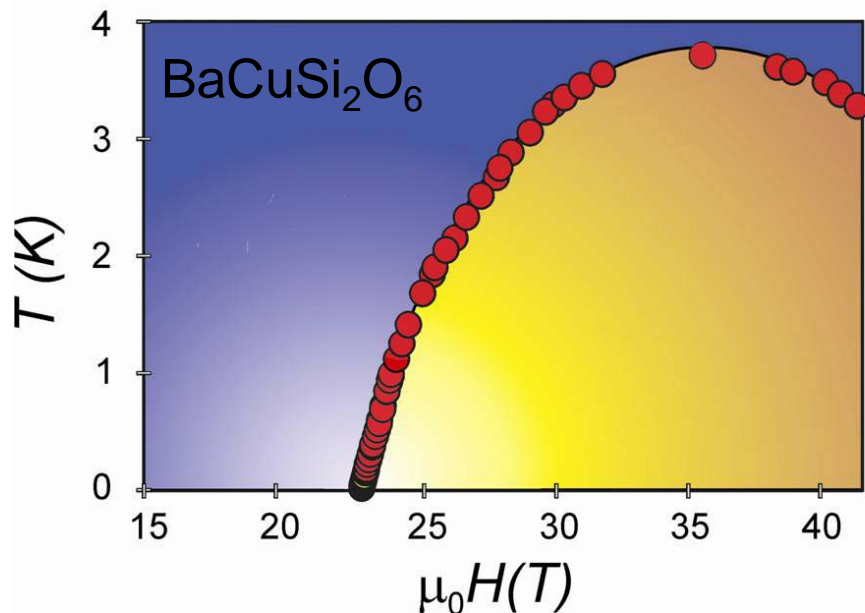


Frustrated pairs: magnon BEC in geometrically frustrated spin dimer compounds



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Acknowledgments

Stanford:

E. Samulon, S. Sebastian*

M. Shapiro, P. Brooks,

P. Tanedo, M. McCourt

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NHMFL:

M. Jaime, R. McDonald, N. Harrison,

L. Balicas, Y.-J. Jo, E. Palm, T. Murphy

S. Tozer

Theory:

C. Batista (LANL)

J. Schmalian (Ames)

Numerical simulations:

N. Kawashima (Tokyo)

S. Hass (USC)

Crystallography:

Z. Islam, J. Ilavsky (APS)

R. Kiyanagi (IPNS)

Neutron scattering:

C. Rugg, H. Ronnow,

D. McMorrow (UCL & PSI)

M. Stone, M. Lumsden (ORNL)

EPR:

S. Hill, S.-C. Lee, A. Wilson,

S. Kim (UF)

NMR:

S. Brown (UCLA)

R. Stern (Tallinn, Estonia)

C. Berthier, S. Kraamer,

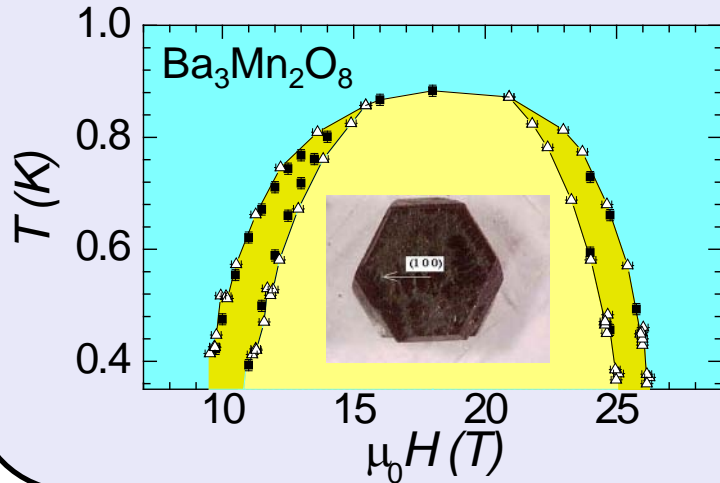
M. Horvatic (CNRS)



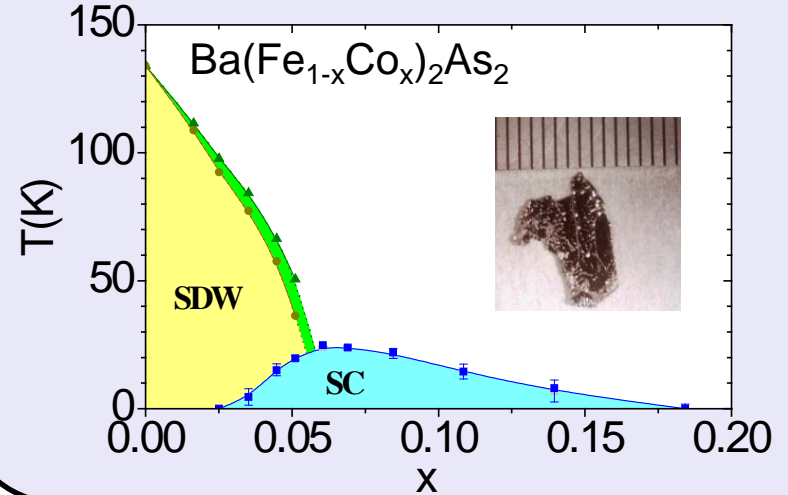
Research program:

New materials – unconventional magnetic & electronic ground states & phase transitions

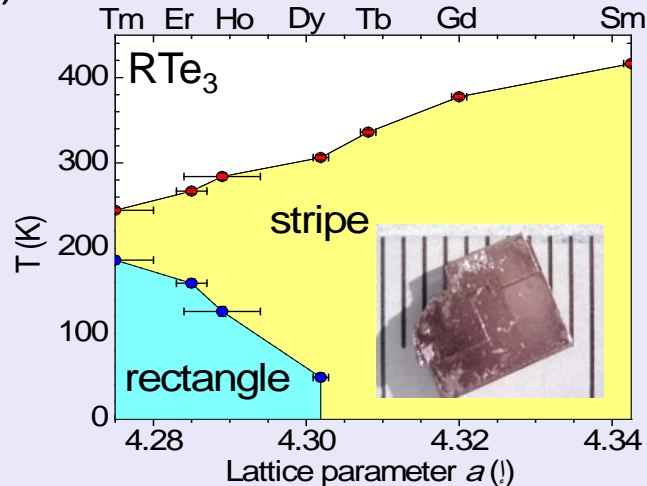
(1) Quantum magnetism:



(2) Superconductivity:



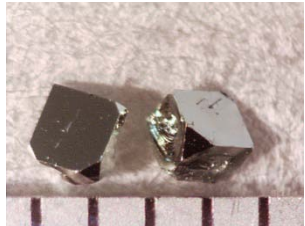
(3) Low dimensional materials:



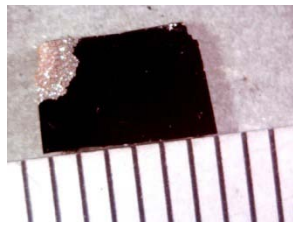
<http://www.stanford.edu/group/Fisher>

J. G. Analytis, J.-H. Chu, A. Erickson,
Y. Matsushita*, N. Ru*, E. Samulon,
S. E. Sebastian*, K. Y. Shin*
(* graduated)

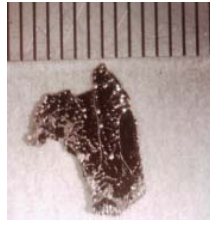
Crystal growth



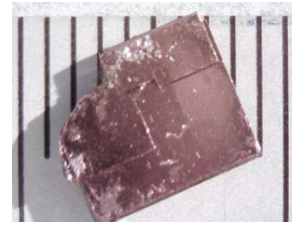
$\text{Pb}_{1-x}\text{Tl}_x\text{Te}$



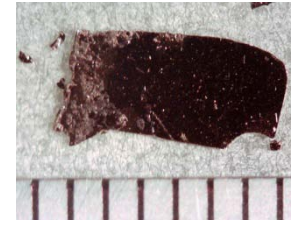
$\text{BaPb}_{1-x}\text{Bi}_x\text{O}_3$



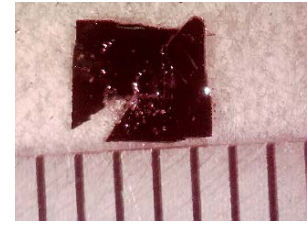
BaFe_2As_2



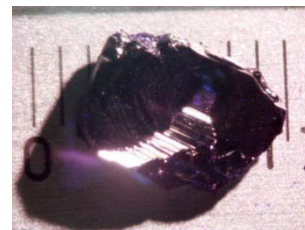
RTe_3



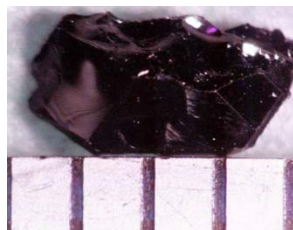
RTe_2



R_2Te_5



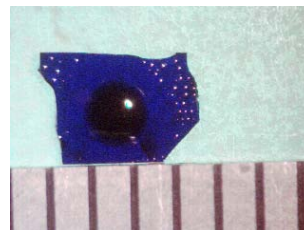
$\text{BaCuSi}_2\text{O}_6$



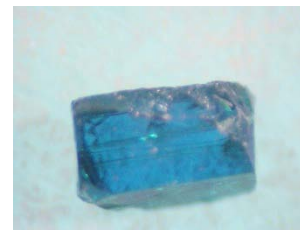
$\text{Sr}_2\text{Cu}(\text{BO}_3)_2$



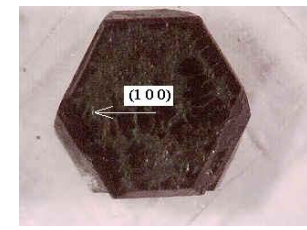
$\text{Ba}_2\text{Cu}(\text{BO}_3)_2$



$\text{BaCu}_2\text{Si}_2\text{O}_7$



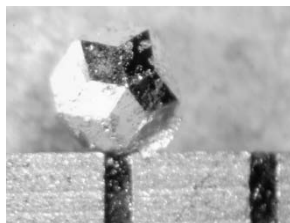
BaCuB_2O_5



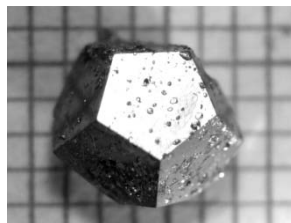
$\text{Ba}_3\text{Mn}_2\text{O}_8$



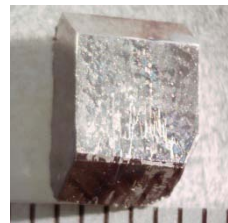
$\text{Ba}_2\text{OsNaO}_6$



R-Mg-Cd



Al-Pd-Re



$\text{R}_6\text{Mo}_4\text{Al}_{43}$

Outline

(1) A short introduction to spin dimer compounds:

- what they are, and why we should care

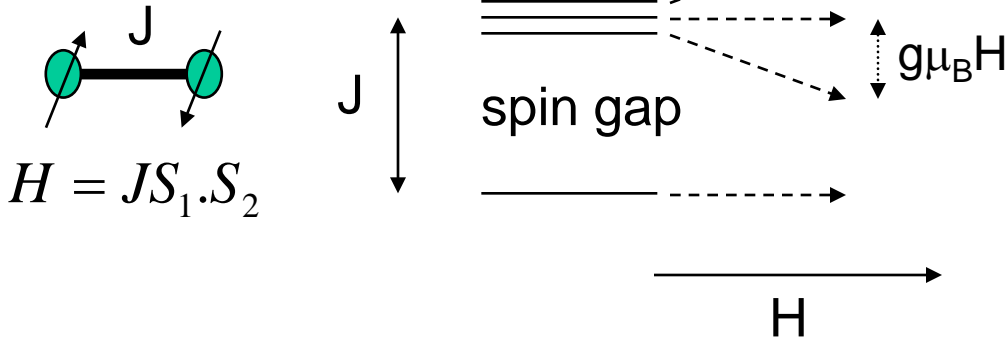
(2) “Frustrated pairs”: two neat examples for which frustration plays a crucial role...

- bct lattice ($\text{BaCuSi}_2\text{O}_6$)
- triangular lattice ($\text{Ba}_3\text{Mn}_2\text{O}_8$)

Properties of an isolated pair



eg $S = 1/2$:



Low field properties:

- thermodynamic distribution
- calculate C_p , M , χ ...

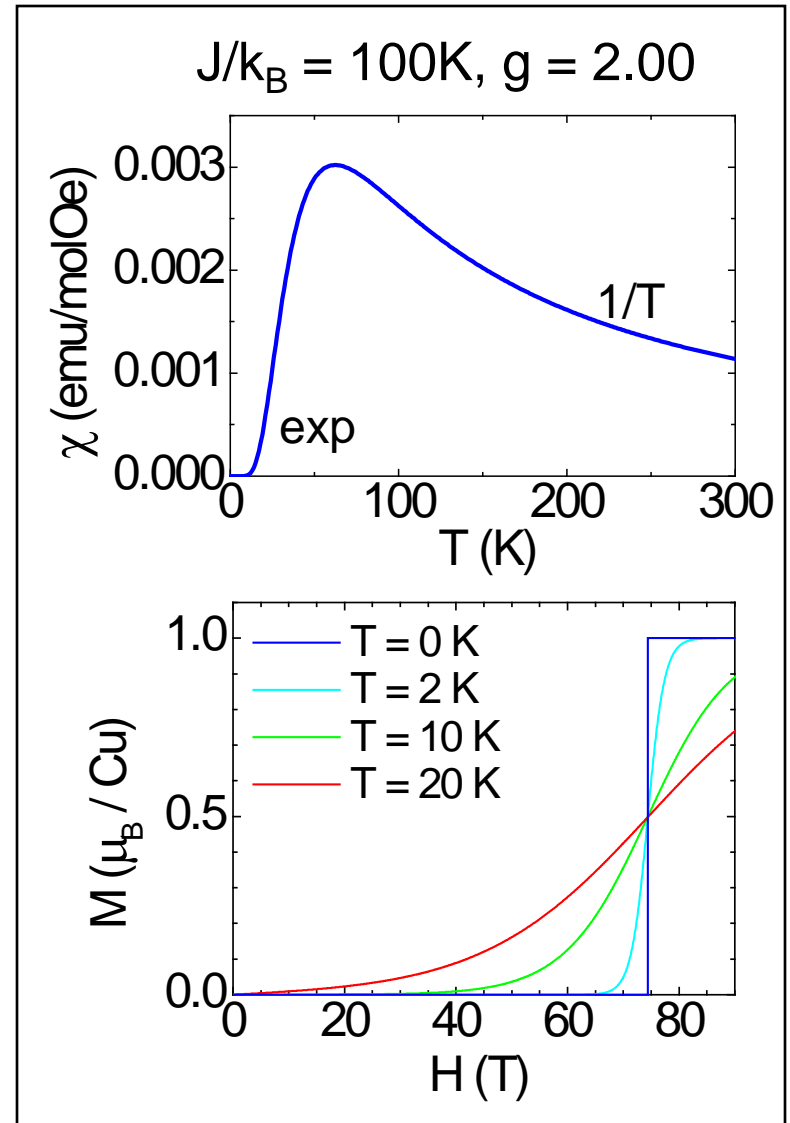
$$\chi = \frac{N(g\mu_B)^2}{k_B T (3 + \exp(J/T))}$$

High field properties:

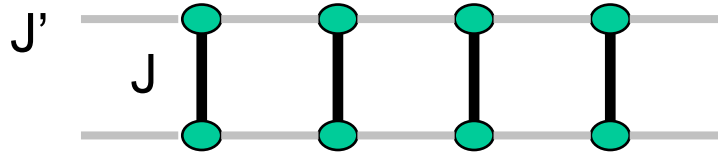
- $g\mu_B H_c = J$
- thermal broadening at finite T

Question:

- what happens for weakly interacting spin dimers?



Insight from the two leg ladder



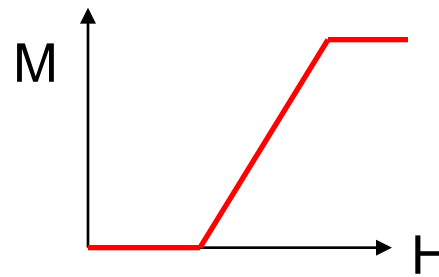
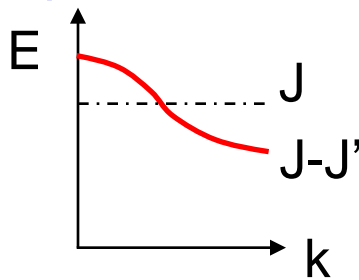
$$H = J \sum_{i=1}^N S_{1,i} \cdot S_{2,i} + J' \sum_{\alpha=1}^2 \sum_{i=1}^N S_{\alpha,i} S_{\alpha,i+1} - h \sum_{\alpha=1}^2 \sum_{i=1}^N S_{\alpha,i}^z$$

- for $J > J'$, treat perturbatively

$$S \equiv \left| \tilde{\downarrow} \right\rangle \quad T_- \equiv \left| \tilde{\uparrow} \right\rangle$$

$$H_{eff} = J' \sum_i \left[\tilde{S}_i^x \tilde{S}_{i+1}^x + \tilde{S}_i^y \tilde{S}_{i+1}^y + \frac{1}{2} \tilde{S}_i^z \tilde{S}_{i+1}^z \right] - \tilde{h}_{eff} \sum_i \tilde{S}_i^z + C$$

- delocalized triplet excitations (triplon)



$$H_{eff} \equiv t \sum_{i,\alpha} (b_{i+\hat{e}_\alpha}^\dagger b_i + h.c.) + V \sum_{i,\alpha} n_i n_{i+\hat{e}_\alpha} + \mu \sum_i n_i \quad \text{where } t = V = \frac{J'}{2} \quad \& \quad \mu = h - J$$

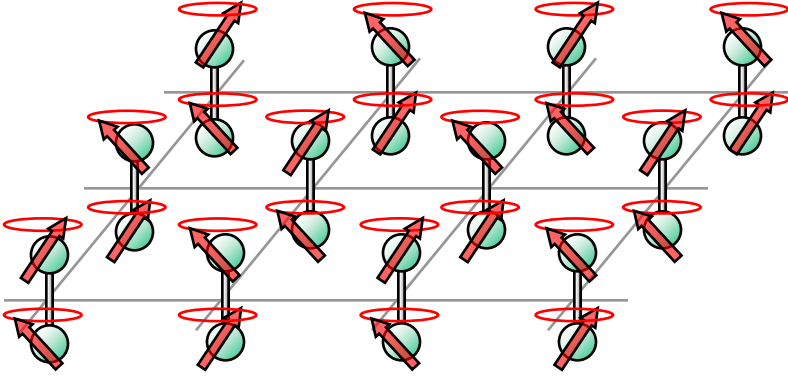
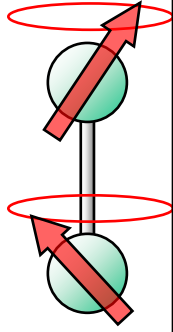
- i.e. realization of a lattice gas of hard core bosons, for which B controls μ

Ground states:

KE vs. PE

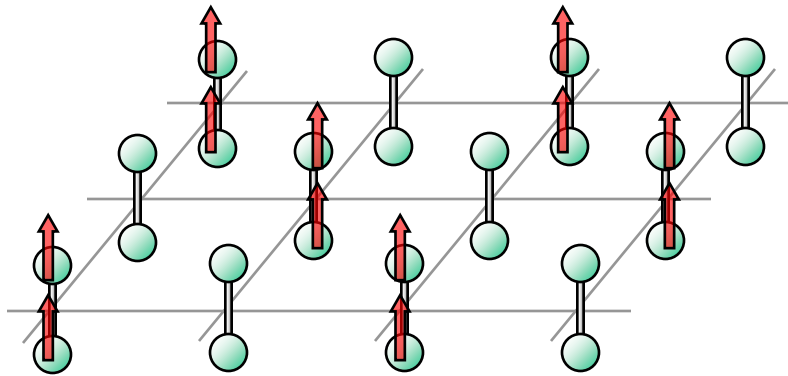
- Ordered state is a product of individual dimer terms of form:

$$|\psi_i\rangle = \cos \theta_i |s\rangle + \sin \theta_i e^{i\phi_i} |t\rangle$$

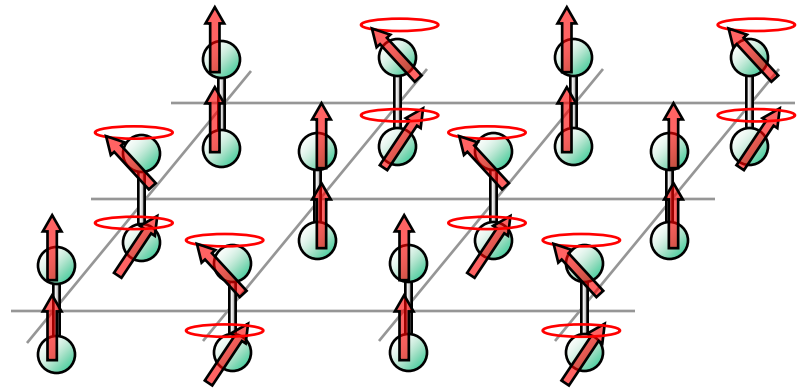


- macroscopic occupancy of minimum of triplet band $(\pi/a, \pi/a)$
- triplet density $\rho = \langle s_z \rangle = m_z$
- amplitude of order parameter = m_{xy} ;
phase = angle of spins in plane
- spontaneously broken U(1) symmetry

eg TiCuCl3



eg SrCu2(BO3)2

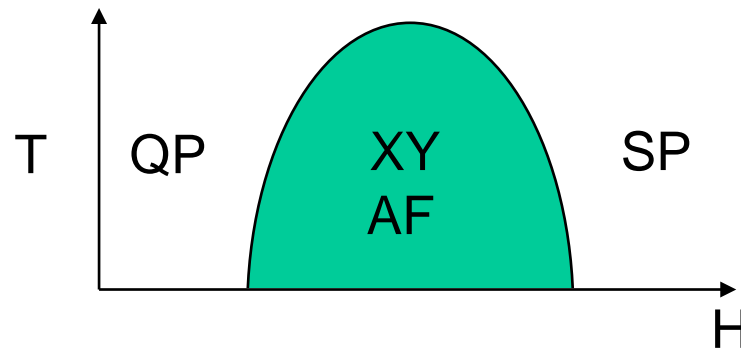


no candidate to date

So why study spin dimer compounds?

(1) Can access lattice gas models which would be unphysical for simple AFs:
i.e. in the limit of weak spin orbit coupling, can “engineer” Hamiltonian with highly anisotropic effective exchange...
... and potentially access to some rather unusual quantum phases of matter

(2) Tunable by magnetic field, so can explore entire quantum phase diagram:

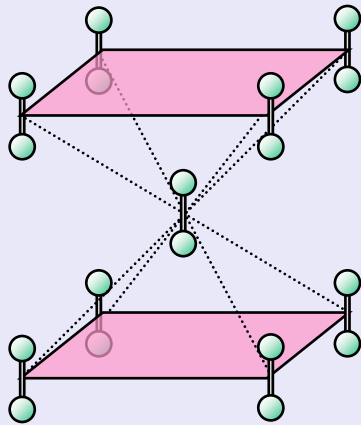
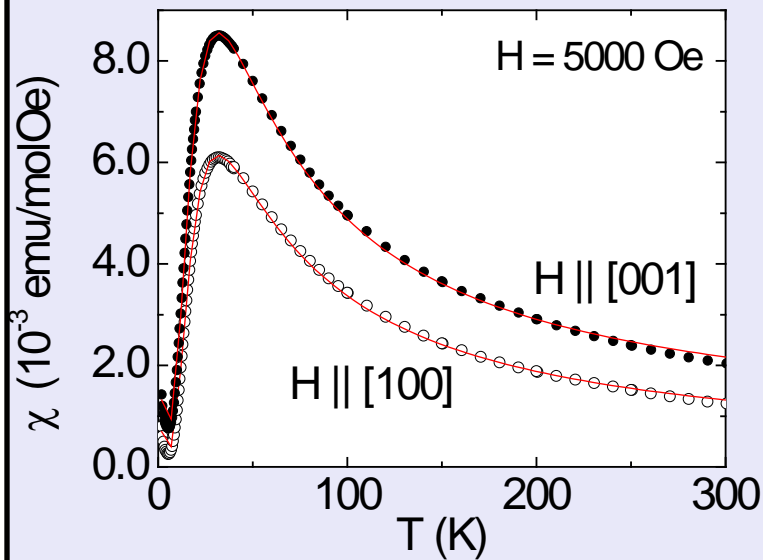


(3) “Protected” against symmetric anisotropies:
terms like DS_z^2 in effective Hamiltonian can only mix singlet and triplet states in second order \rightarrow terms which break axial symmetry are suppressed relative to a simple AF

(4) Relatively few candidate materials...

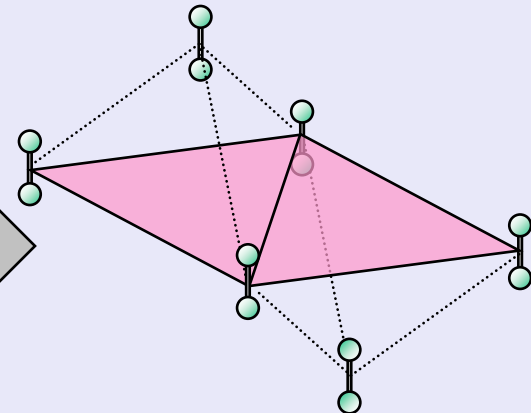
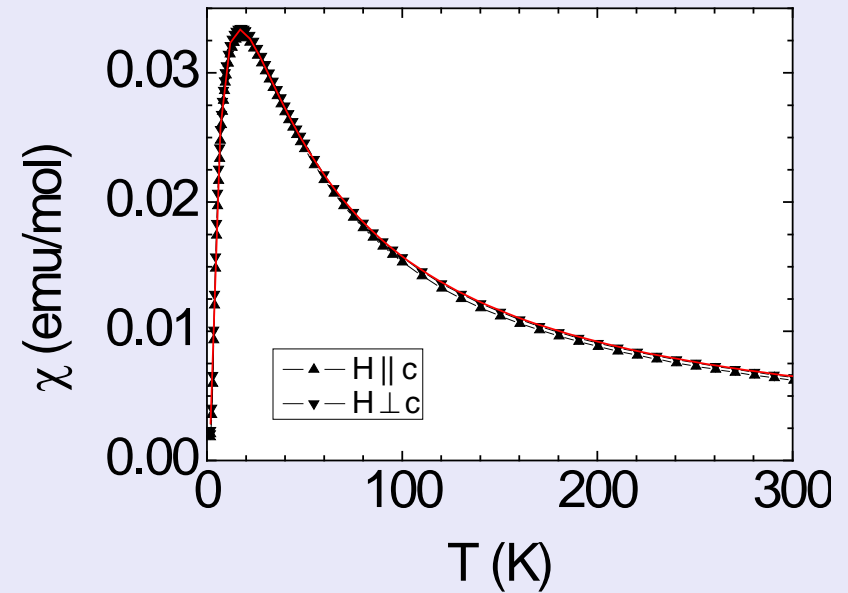
This talk: magnon BEC in two neat compounds with geometrically frustrated lattices

(1) $\text{BaCuSi}_2\text{O}_6$



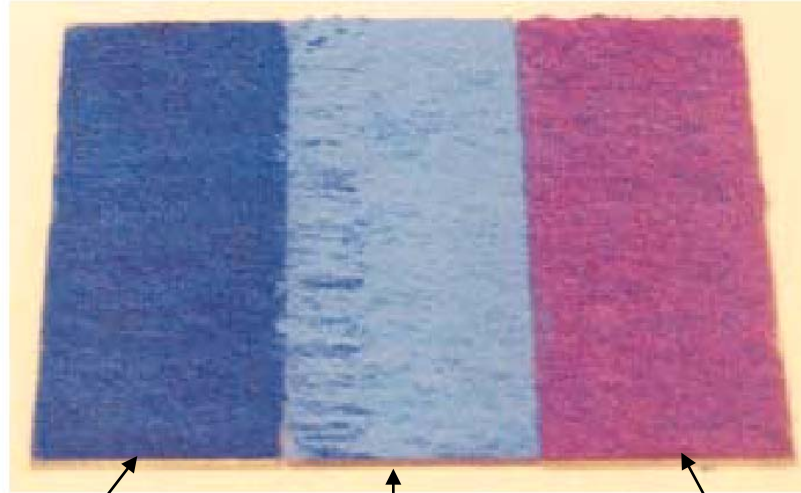
*more
frustrated*

(2) $\text{Ba}_3\text{Mn}_2\text{O}_8$



New materials?

Anthropogenic pigments:



Egyptian blue
 $\text{CaCuSi}_4\text{O}_{10}$

Chinese blue
 $\text{BaCuSi}_4\text{O}_{10}$

Han purple
 $\text{BaCuSi}_2\text{O}_6$

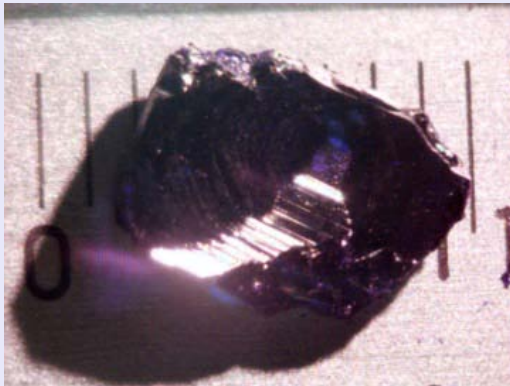
“Chemistry in ancient times:
the development of blue and purple pigments”
H. Berke, *Angew. Chem. Int. Ed.* **41**, 2483 (2002).

Crystal growth

(1) $\text{BaCuSi}_2\text{O}_6$

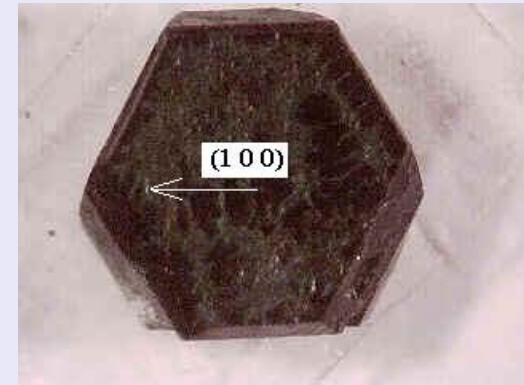
(a) Evidence for Pb in
archeological samples
- PbO flux works

(b) but LiBO_2 better

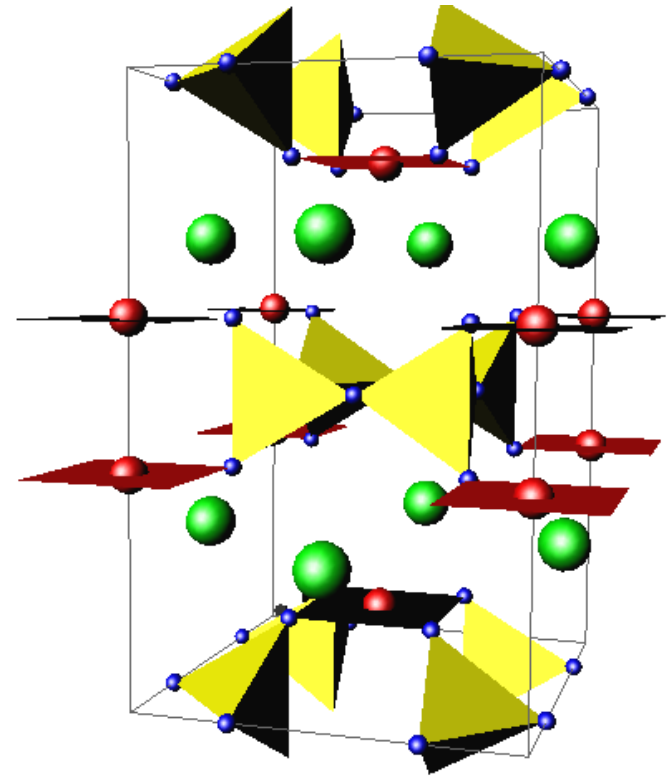
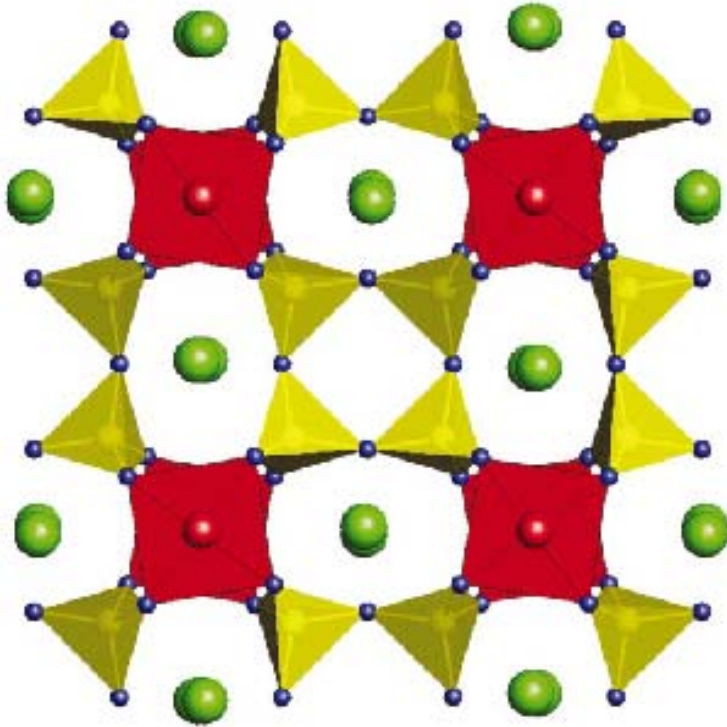


(2) $\text{Ba}_3\text{Mn}_2\text{O}_8$

Mn^{5+} valence unusual
Requires strongly oxidizing flux
NaOH works very well



Case 1: *bct* lattice - BaCuSi₂O₆

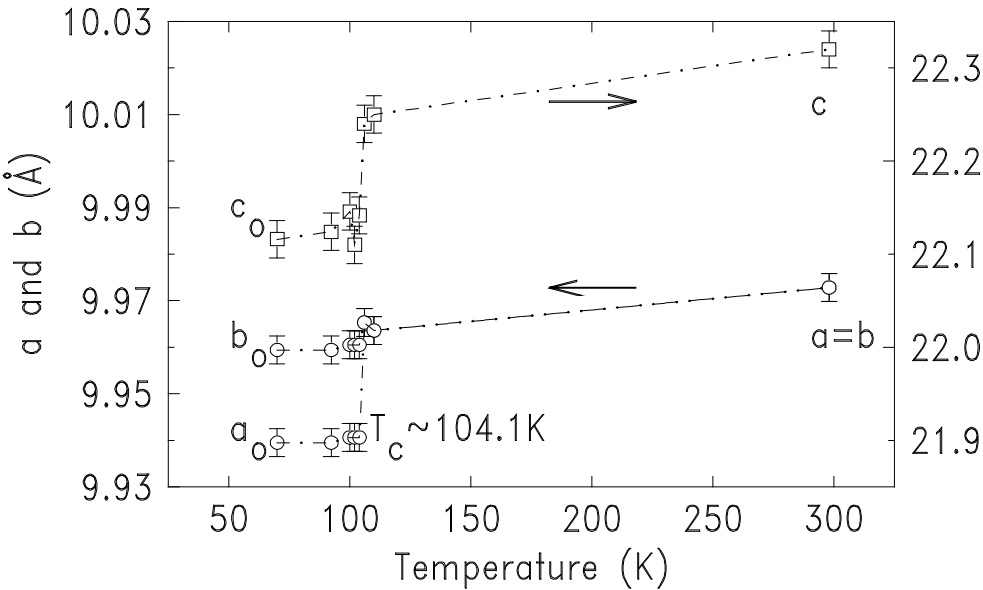


Inelastic neutron scattering (Ch. Ruegg):

- $J \sim 4.5$ meV; $J' \sim 0.5$ meV
- c-axis bandwidth finite but small (< 0.05 meV)
- spin gap $\Delta = 3.1$ meV $\rightarrow H_{c1} = \Delta/g\mu_B \sim 23.5$ T for $H \parallel [001]$

Structural phase transition

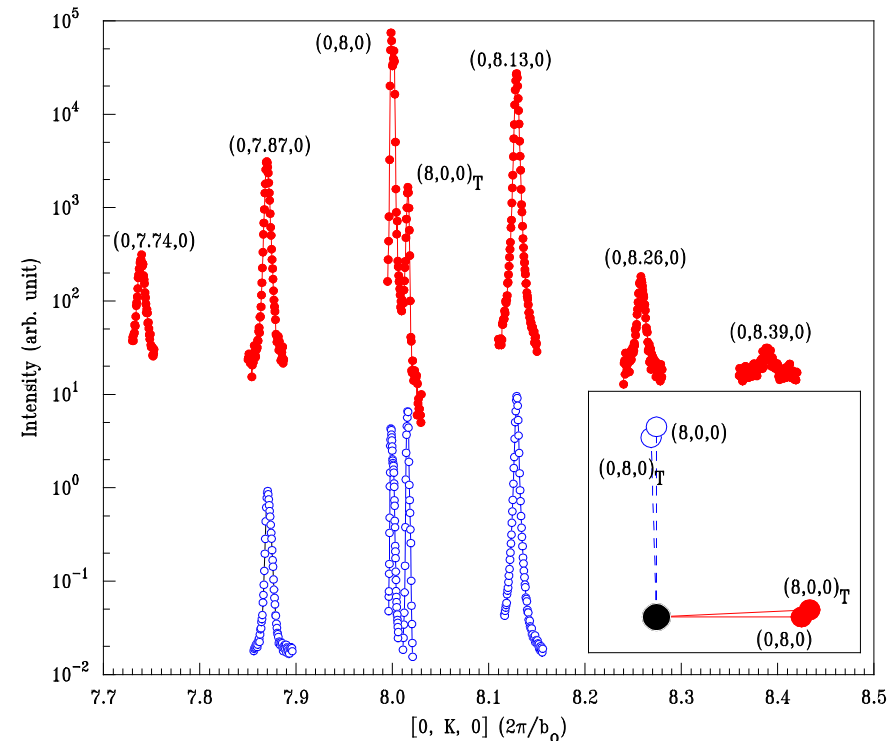
(with Z. Islam, APS)



- incommensurate lattice modulation
- $q_{IC} = 0.129 b^*$
- presumably driven by rigid rotation of SiO_4 tetrahedra
- lack a complete structural model
- subtle effect, and doesn't affect subsequent analysis

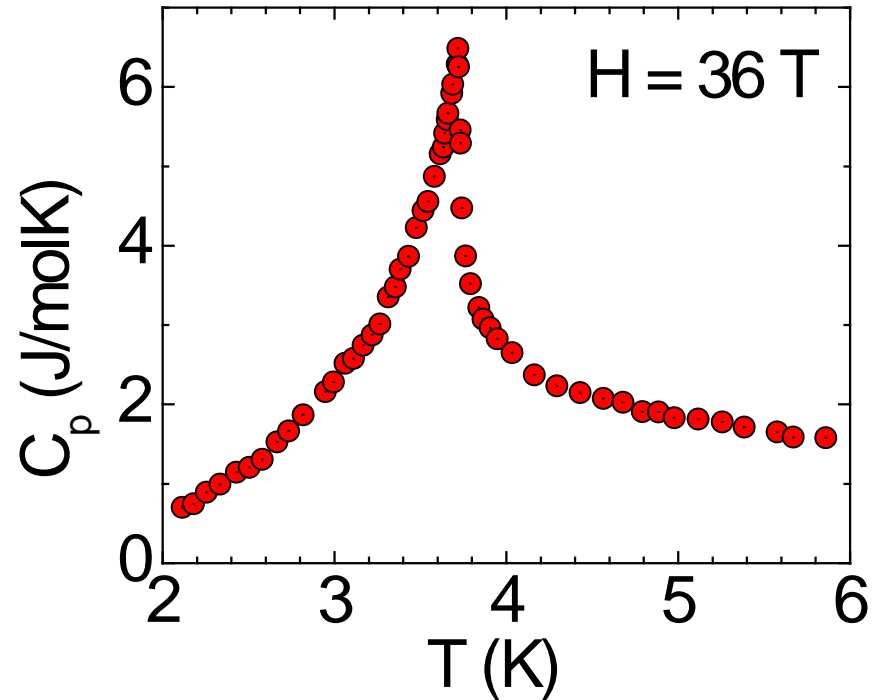
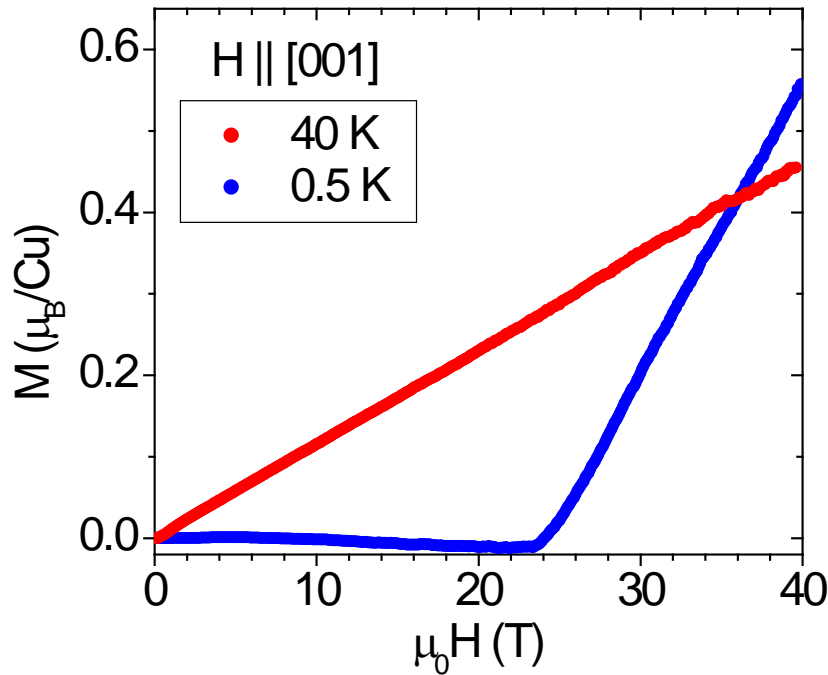
- weak orthorhombic distortion at ~ 100 K

$$\frac{b-a}{\frac{1}{2}(b+a)} = 0.2\%$$



High field behavior

(with M. Jaime & N. Harrison, NHMFL)



- does this correspond to BEC of triplons as conceived theoretically?
- address by looking at critical scaling (characteristic of universality class)
- thermal phase transition: $C_p \sim (T - T_c)^{-\alpha}$
- quantum phase transition: $T_c \propto (H - H_{c1})^\nu$
- BEC universality class: $\nu = 2/d$
- 3D Ising: $\nu = 1/2$

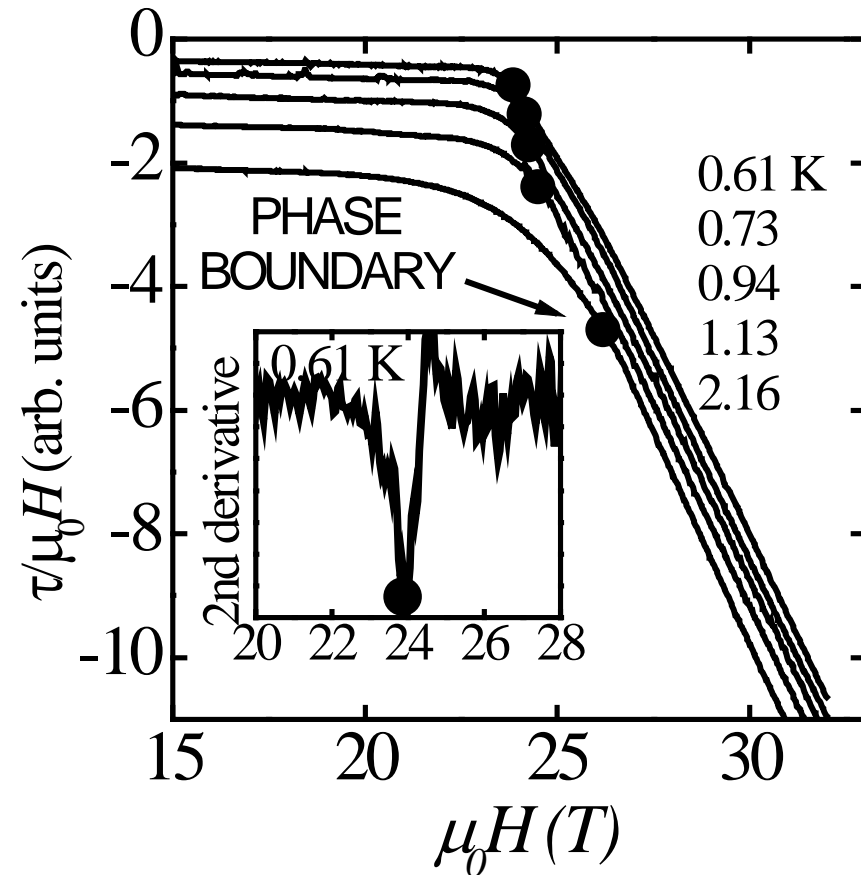
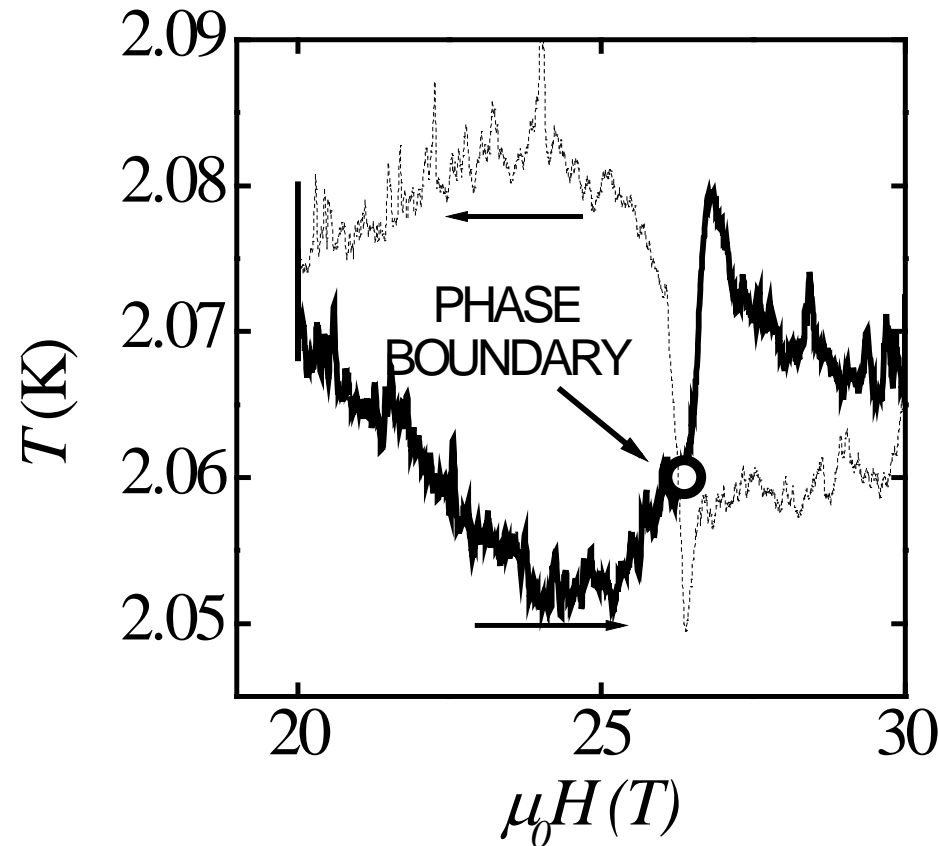
Experimental determination of $T_c(H)$ close to the QCP

(a) Magneto-caloric effect: (with Marcelo Jaime, NHMFL)

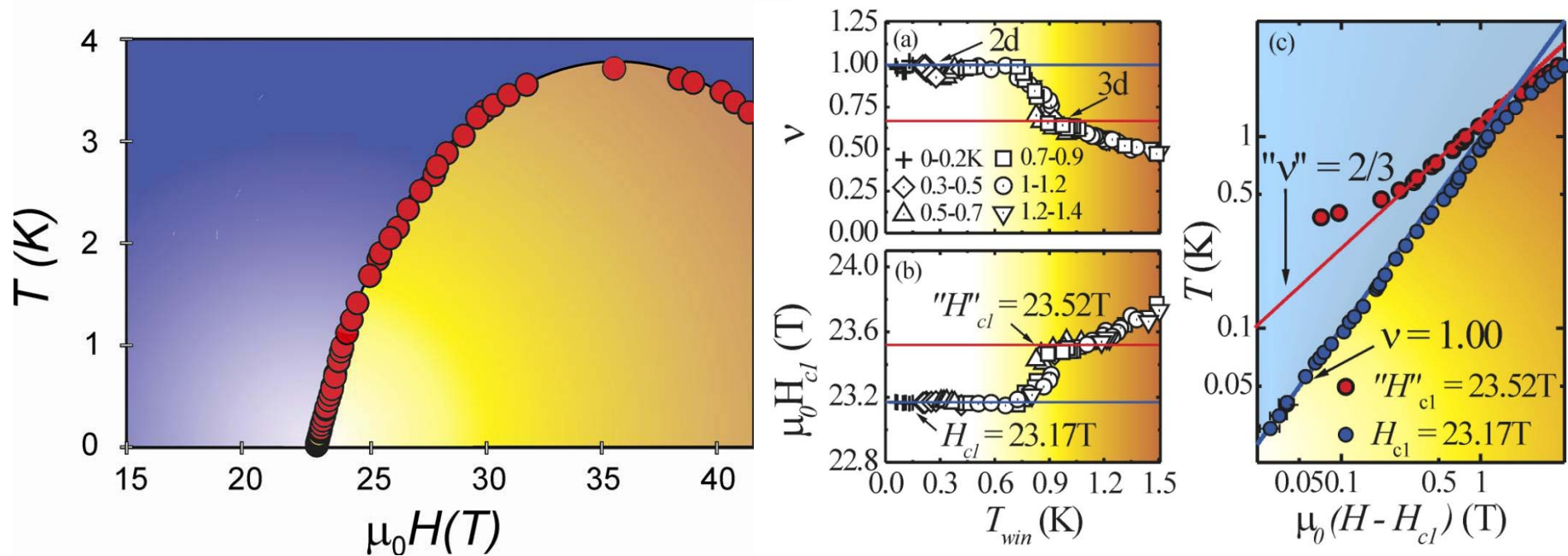
- jump in T of sample on entering/leaving ordered state

(b) Cantilever torque magnetometry: (with N. Harrison, L. Balicas, NHMFL)

- divergence in 2nd derivative of magnetization



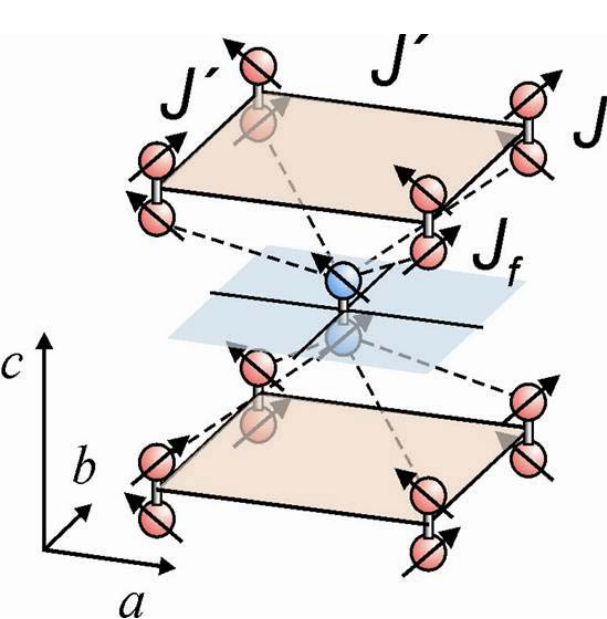
Phase diagram & critical scaling analysis



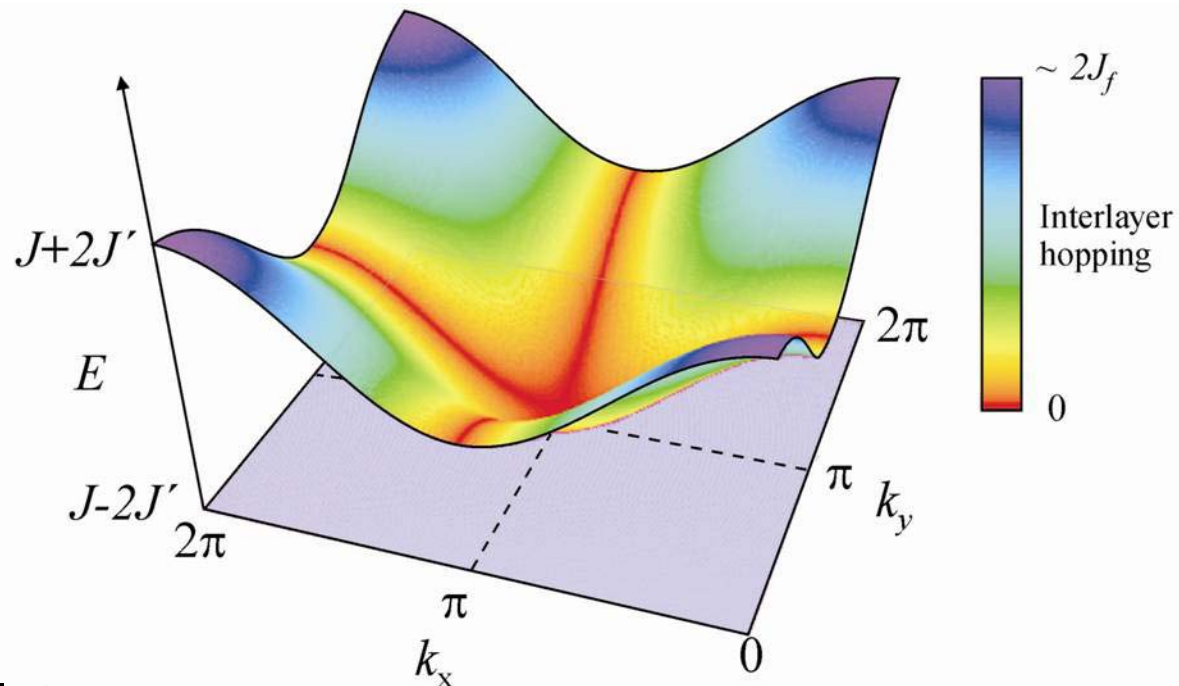
- Two parameter fit to $T_c \propto (H - H_{cl})^\nu$ with a sliding window
- BEC scaling exponents observed ($\nu = 2/d$)
- Surprise - cross-over to 2D exponent approaching the QCP!

Question: Individual triplets can move in 3D, so why is the collective behavior at the QCP in just 2D?

Dispersion relation for perfect *bct* lattice



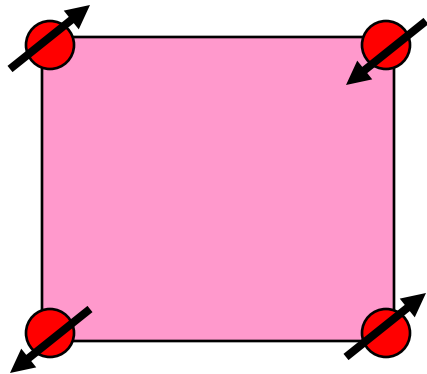
$$E = J'(\cos k_x + \cos k_y) + 2J_f \cos \frac{k_x}{2} \cos \frac{k_y}{2} \cos k_z$$



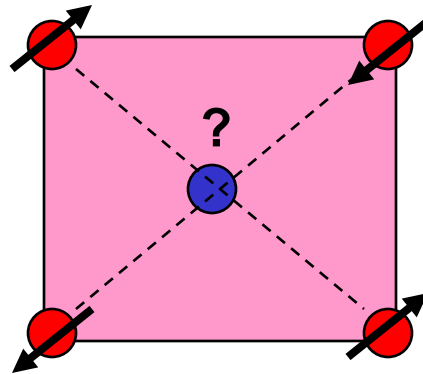
Case of non-interacting triplets:

- At $T = 0$ all particles will be in the condensate at $\mathbf{k} = (\pi, \pi)$
- But there is no interlayer hopping for $\mathbf{k} = (\pi, \pi)$!
- \rightarrow independent 2d condensates

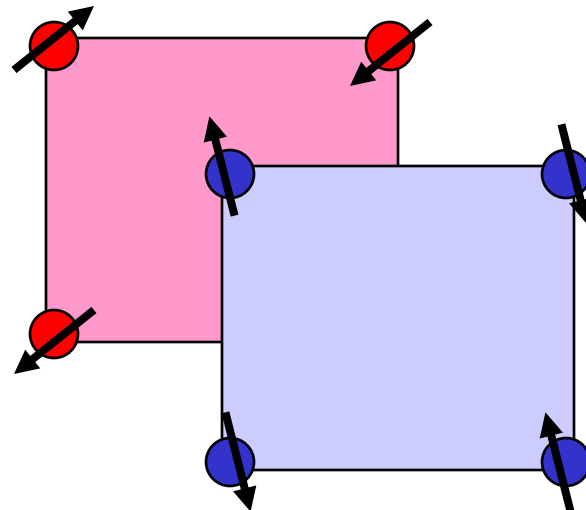
Origin of vanishing dispersion: geometric frustration



- in-plane ordering wave-vector = (π, π)



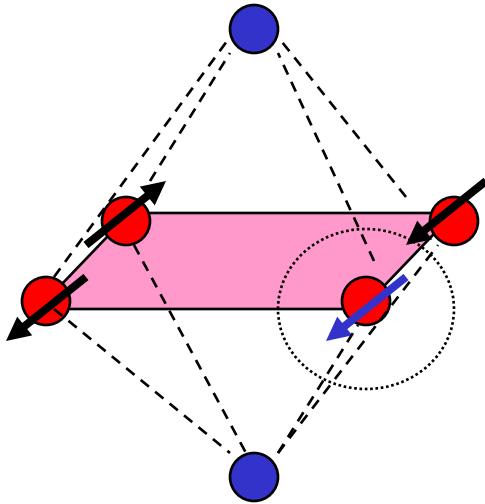
- body centered tetragonal lattice
- geometric frustration
- adjacent planes decoupled!



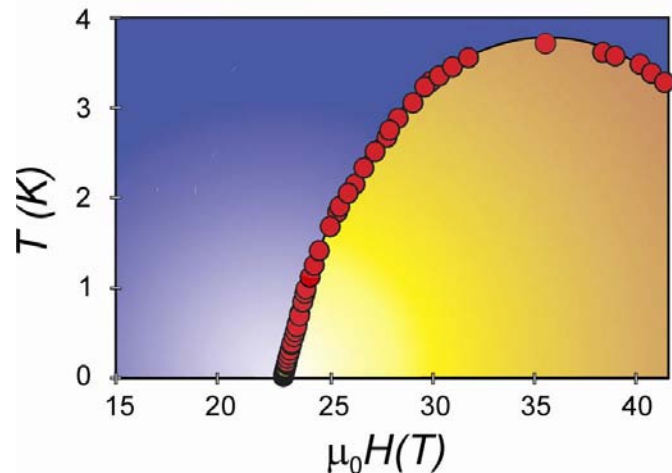
Order from disorder?

Theory: C. Batista, LANL
& J. Schmalian, Ames

- Must consider the effect of phase fluctuations on this delicate frustration...



- leads to an effective unfrustrated biquadratic interlayer hopping / coupling $K \sim \rho^2$
- restores 3D phase coherence for finite triplet concentrations ρ

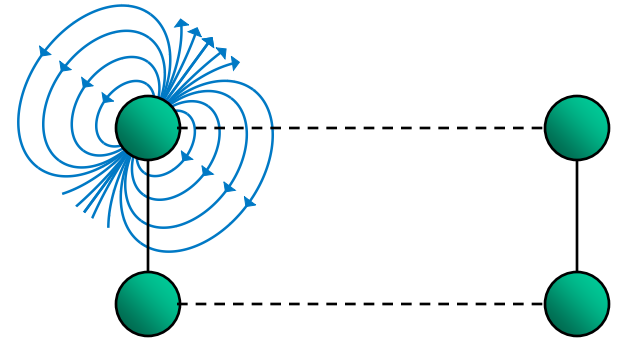


- but at the QCP, $\rho \rightarrow 0$ so $K \rightarrow 0$
- 2D fixed point determines universal scaling

U(1) symmetry?

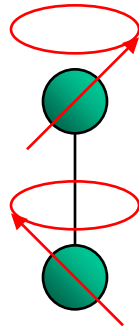
- BEC implies a spontaneously broken axial symmetry
- What effects might break this apparently delicate symmetry?

- For $\text{BaCuSi}_2\text{O}_6$, it turns out that the largest effect is from dipolar interactions...

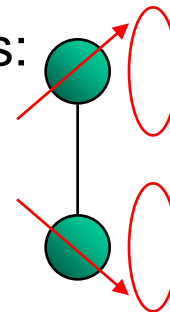


- Dipolar energy $\sim 1/r^3$, so consider first the intradimer dipolar coupling...

H || dimer axis:
(no anisotropy)

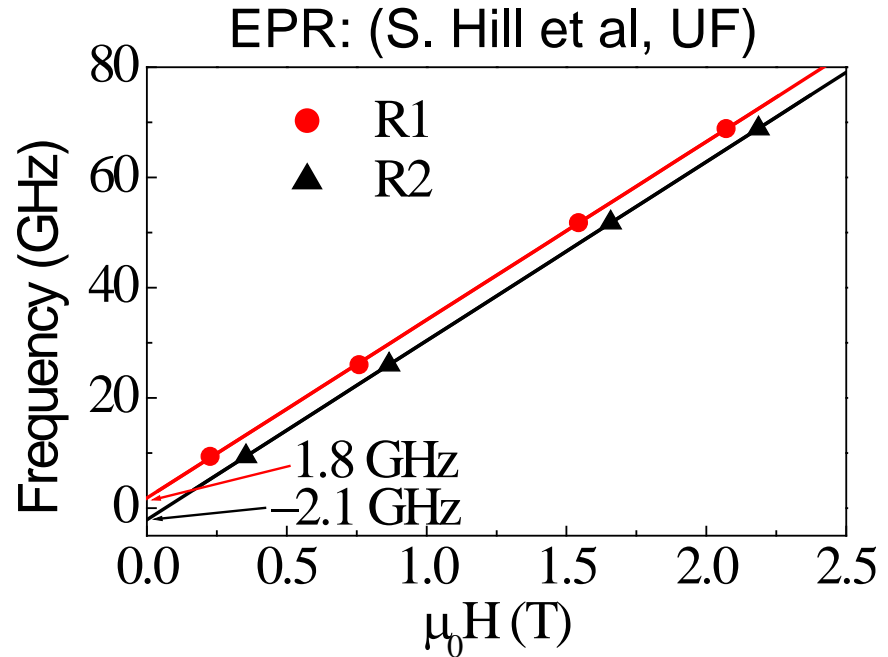
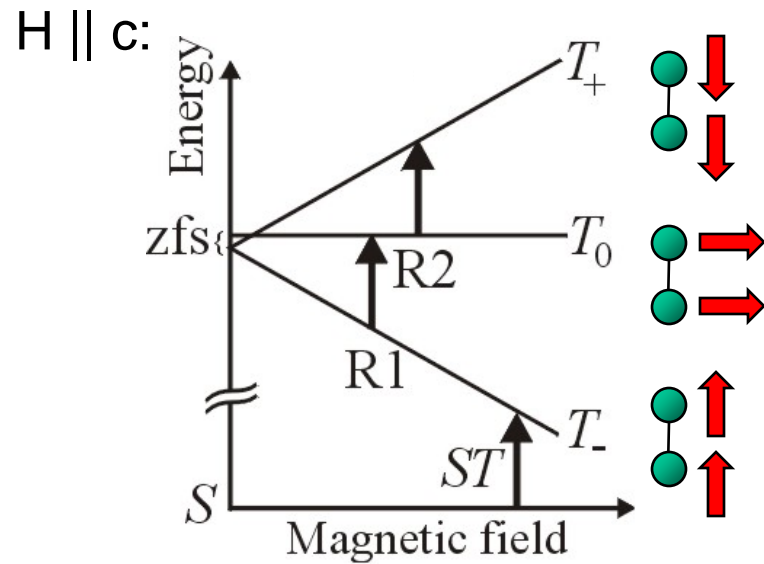


H \perp dimer axis:
(anisotropy)



U(1) symmetry?

- Intradimer dipolar coupling causes zfs of the triplet...



S. Sebastian et al
PRB 74, 180401(R) (2006).

$$zfs = D = \frac{\mu_0}{16\pi r^3} (2g_{\parallel}^2 + g_{\perp}^2) \mu_B^2 \cong 0.11 K \quad H_{dip} = DS_z^2 \quad D_{obs} = 0.10(1) K$$

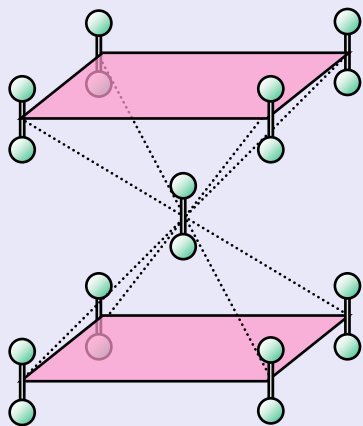
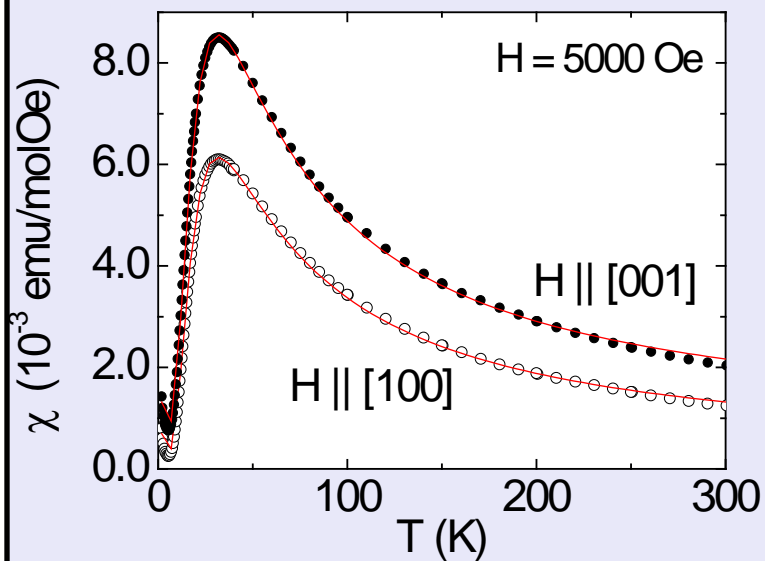
- For H \perp dimer axis \rightarrow effective anisotropy energy (gap to “Goldstone” mode)

$$\approx \frac{DJ'}{J} \approx 10 mK$$

- i.e. the axial symmetry required for BEC is remarkably robust (“protected”)

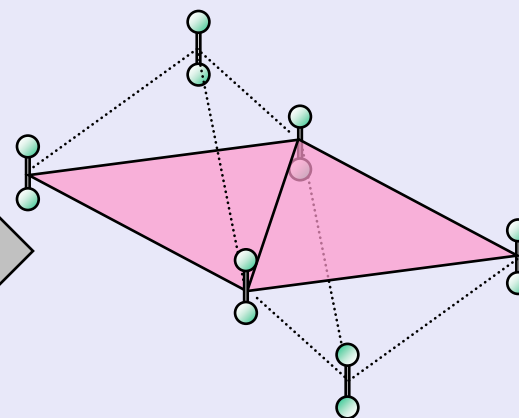
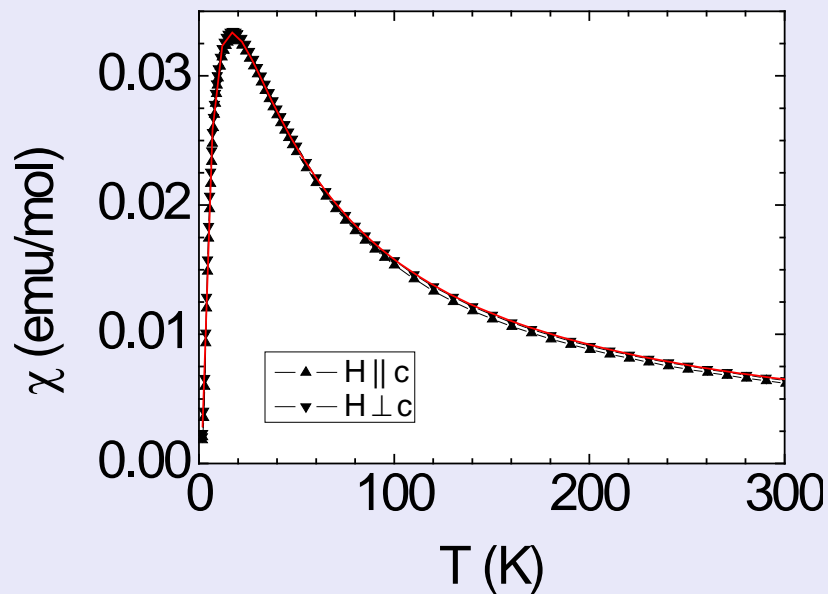
Case 2: triangular lattice

(1) $\text{BaCuSi}_2\text{O}_6$



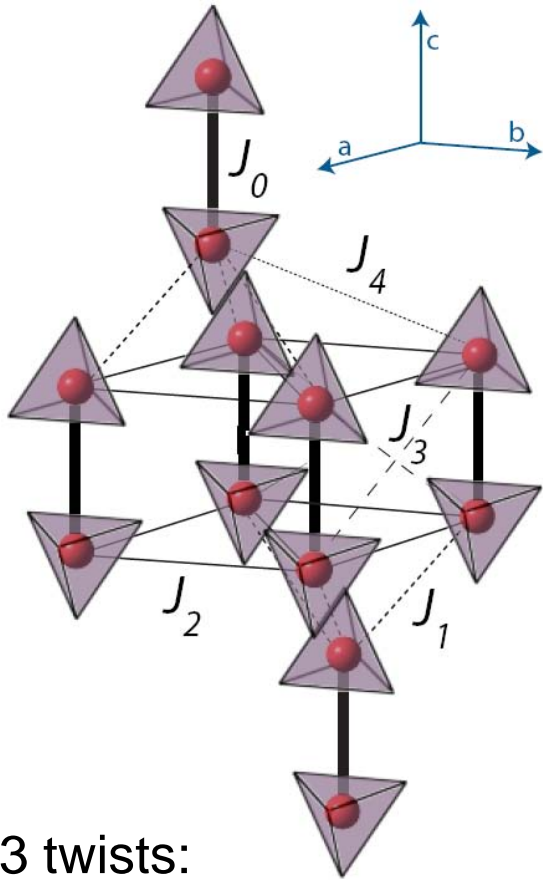
*more
frustrated*

(2) $\text{Ba}_3\text{Mn}_2\text{O}_8$



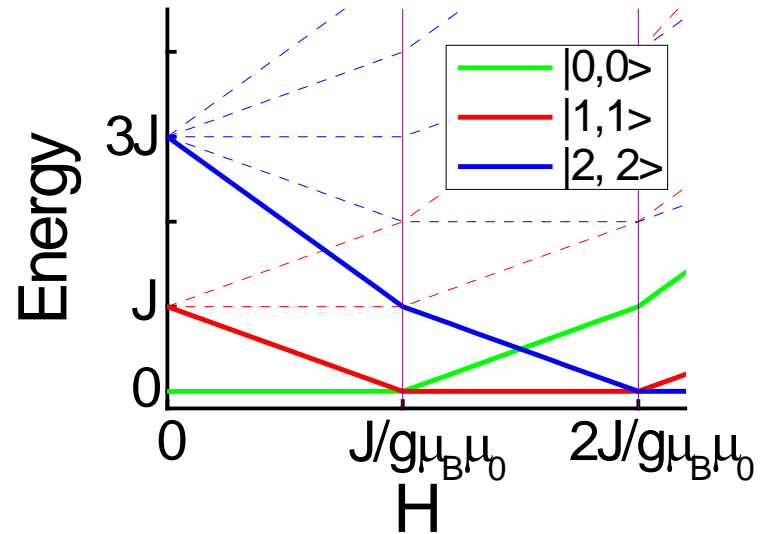
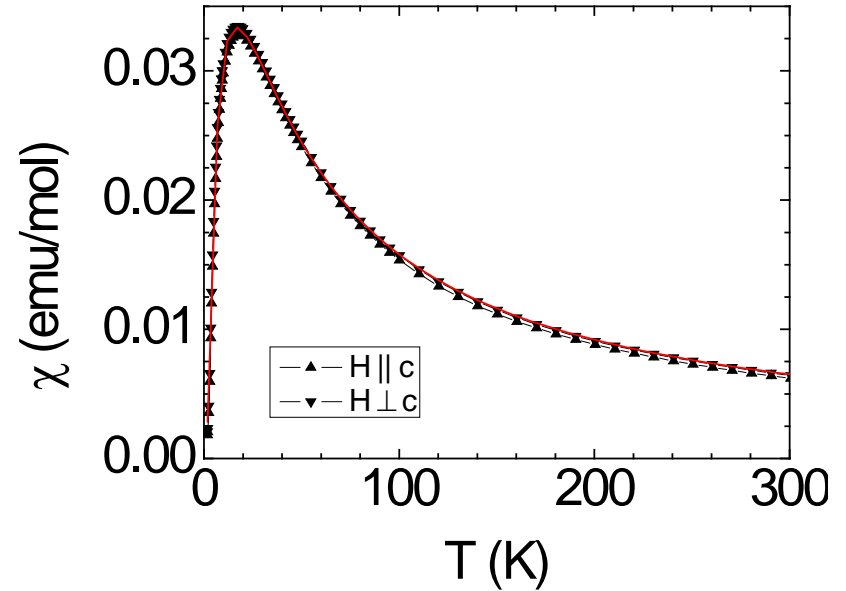
Ba₃Mn₂O₈

- $R\bar{3}m$
- Mn⁵⁺, 3d², s=1 dimers



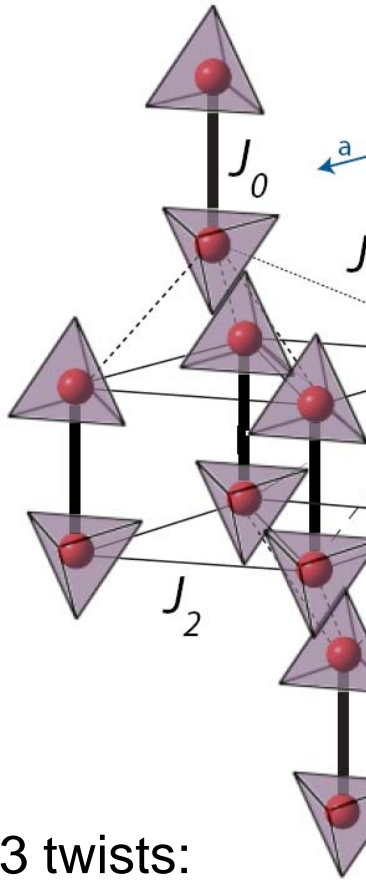
3 twists:

- in-plane frustration
- single ion anisotropy
- triplet and quintuplet condensation



Ba₃Mn₂O₈

- $R\bar{3}m$
- Mn⁵⁺, 3d², s=1 dimers

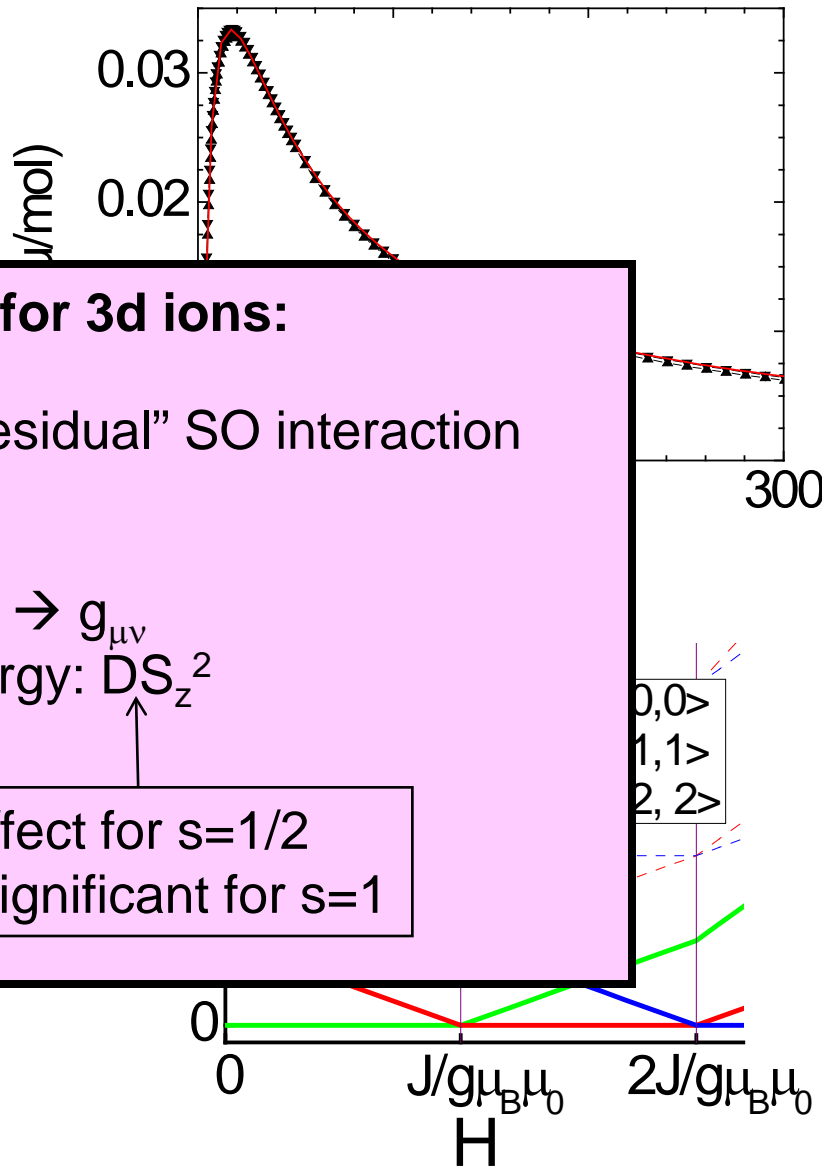


Spin-orbit coupling for 3d ions:

L is quenched, but “residual” SO interaction leads to...

- g anisotropy: $g \rightarrow g_{\mu\nu}$
- anisotropy energy: $D S_z^2$

no effect for s=1/2
but significant for s=1

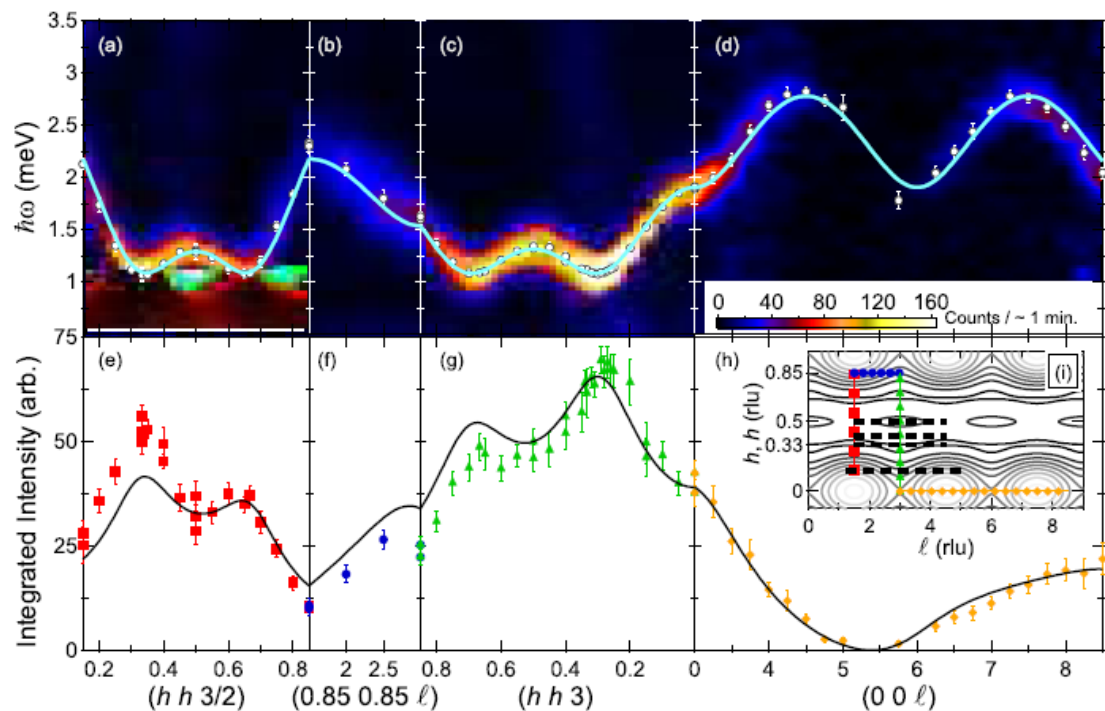
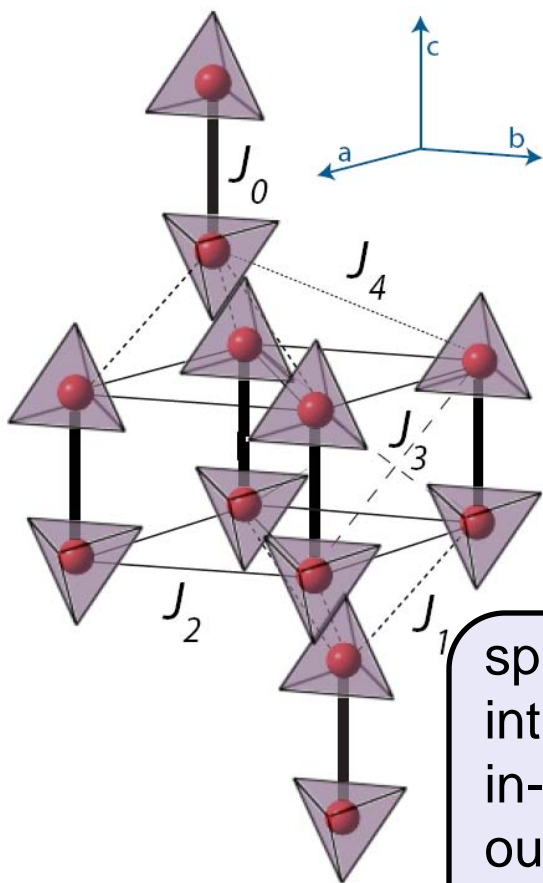


3 twists:

- in-plane frustration
- single ion anisotropy
- triplet and quintuplet condensation

Triplon dispersion

(M. Stone & M. Lumsden, ORNL)



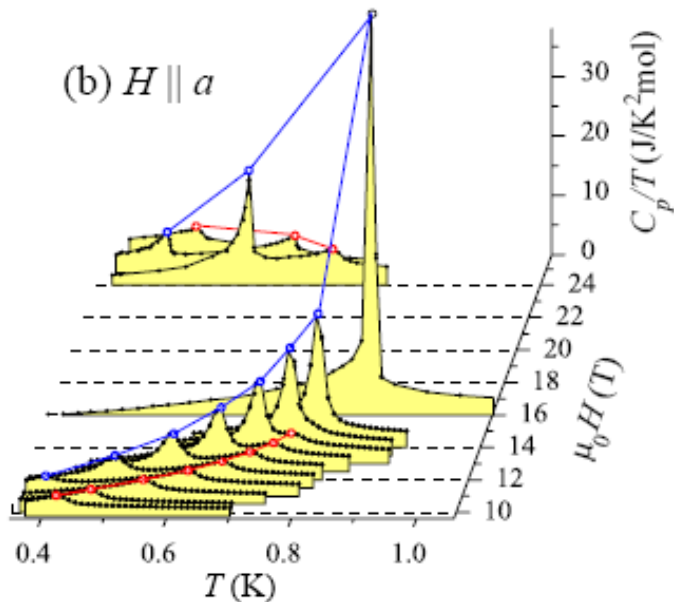
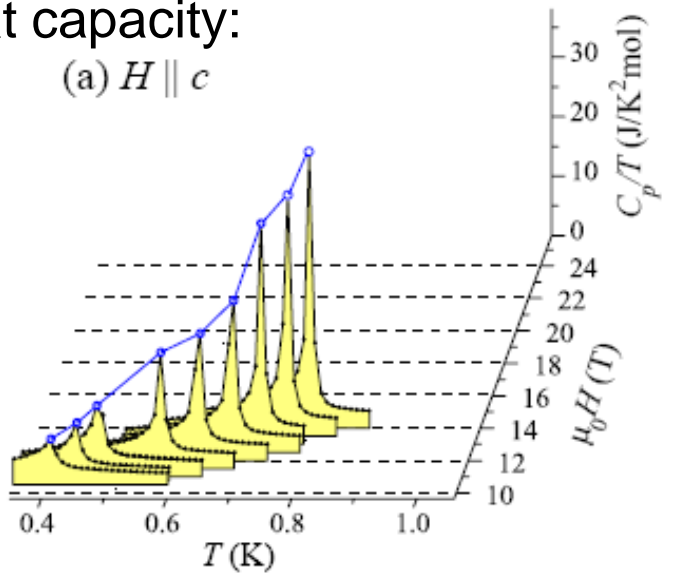
spin gap: $\Delta = 1.081$ meV
 intradimer: $J_0 = 1.642$ meV
 in-plane interdimer: $J_2 = 0.256$ meV (& $J_3 = 0.142$ meV)
 out-of-plane interdimer: $J_1 = -0.118$ meV (& $J_4 = -0.037$ meV)
 single ion anisotropy: $D = -0.032$ meV (from EPR)

M. Stone et al, PRB 77, 134406 (2008).
 & PRL 100, 237201 (2008).

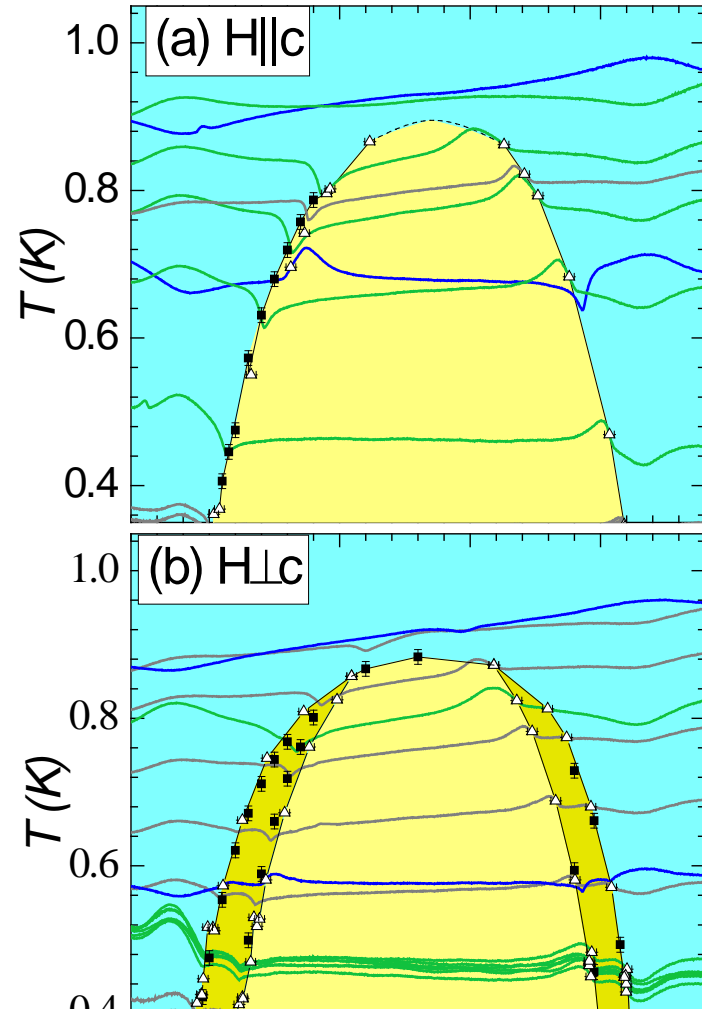
- i.e. quasi-2D material in which planes of vertical dimers arranged on triangular layers interact weakly in the perpendicular direction

High field behavior: triplet ordered states

Heat capacity:

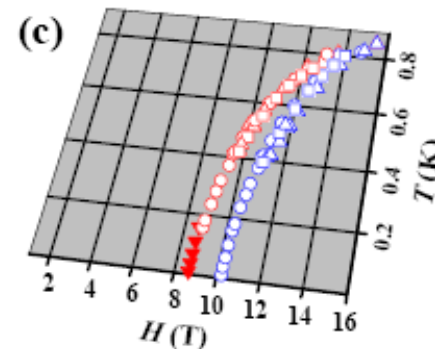
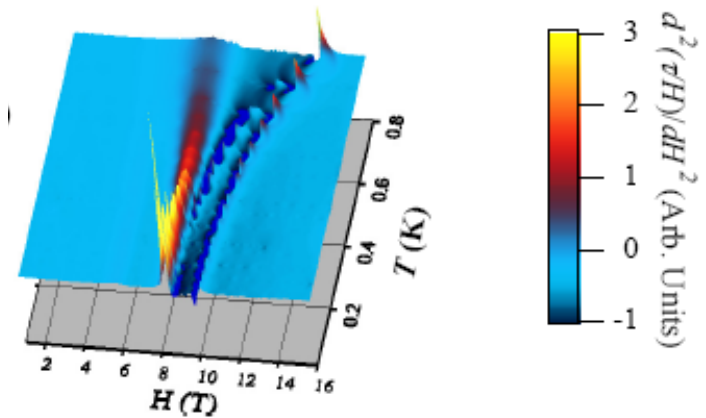
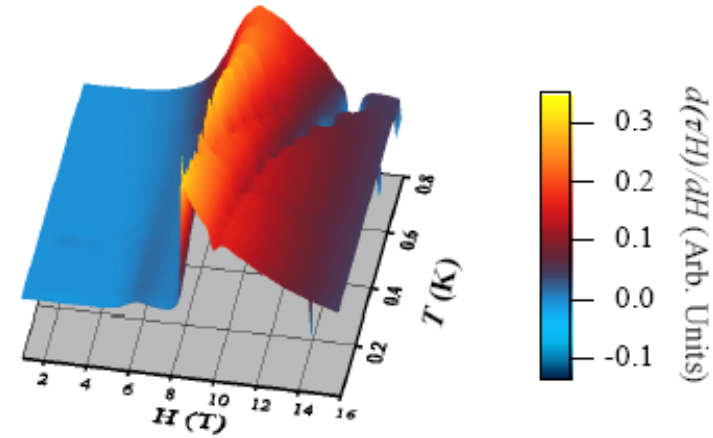
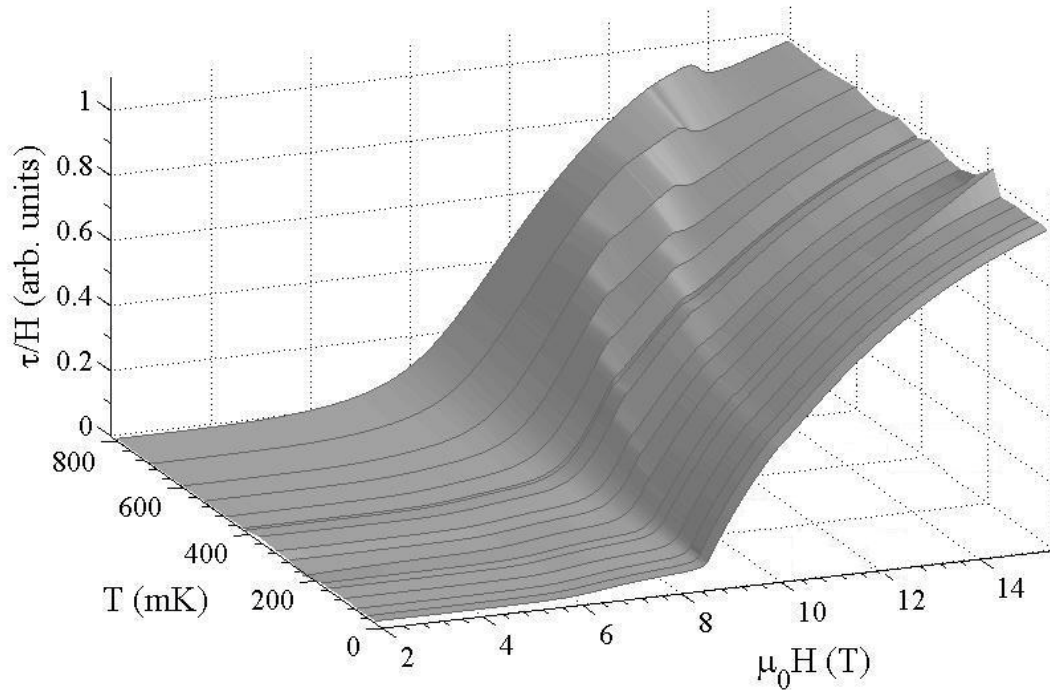


MCE (with M. Jaime, NHMFL):



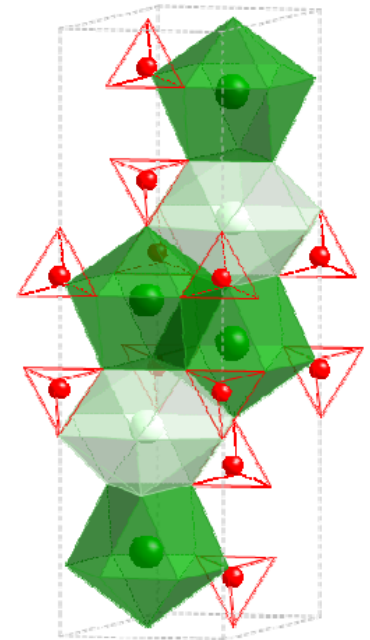
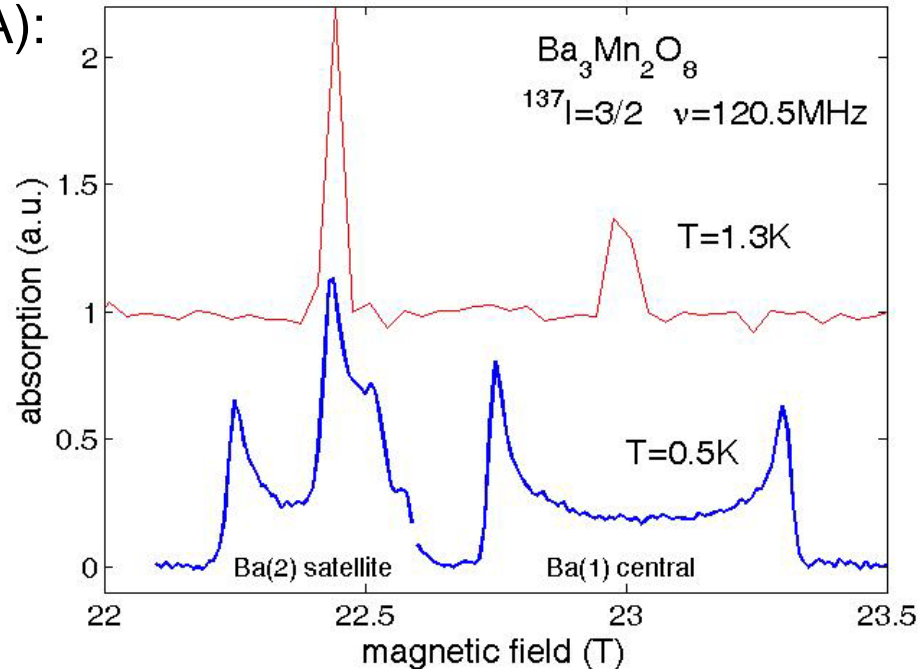
High field behavior: triplet ordered states

Torque (with L. Balicas & Y.-J Jo, NHMFL):



Magnetic structure?

(a) NMR ($H||c$):
(S. Brown, UCLA):



(b) Neutron scattering ($H||a$):
(M. Stone & M. Lumsden, ORNL)

- in-plane wave vectors are both very close to $(2\pi/3a, 2\pi/3a)$
- in absence of other information, resort to an analysis of the spin Hamiltonian for a qualitative understanding of the ground states...

Spin Hamiltonian for Ba₃Mn₂O₈ Theory: Cristian Batista (LANL)

For Ba₃Mn₂O₈ the minimal spin Hamiltonian is:

$$H = \sum_{i,j,\mu,\nu} \frac{J_{i\mu j\nu}}{2} \mathbf{S}_{i\mu} \cdot \mathbf{S}_{j\nu} + D \sum_{i,\nu} (S_{i\mu}^\eta)^2 - g_{\alpha\alpha} \mu_B H \sum_{i\mu} S_{i\mu}^z$$

Where i, j designate coordinates of dimer and $m, n = \{1, 2\}$ denote each of the two spins on a given dimer
 $D = -0.032$ meV (EPR: S. Hill)

The resulting effective Hamiltonian is:

$$H = \frac{4J_1}{3} \sum_{l \langle\langle i, j \rangle\rangle} \left[\mathbf{s}_{i,l} \cdot \mathbf{s}_{j,l+1} - \frac{13}{16} s_{i,l}^z s_{j,l+1}^z \right] + \frac{8J_2}{3} \sum_{l \langle i, j \rangle} \left[\mathbf{s}_{i,l} \cdot \mathbf{s}_{j,l} - \frac{13}{16} s_{i,l}^z s_{j,l}^z \right] +$$

$$+ J_1 \alpha(\eta) \sum_{l \langle\langle i, j \rangle\rangle} (s_{i,l}^x s_{j,l+1}^x - s_{i,l}^y s_{j,l+1}^y) + 2J_2 \alpha(\eta) \sum_{l \langle i, j \rangle} (s_{i,l}^x s_{j,l}^x - s_{i,l}^y s_{j,l}^y)$$

$$- B \sum_{l,i} s_{i,l}^z$$

Where l is the layer index, $\langle i, j \rangle$ indicates NN on same layer, and $\langle\langle i, j \rangle\rangle$ denotes NN on adjacent layers,
 $B = g_{\alpha\alpha} \mu_B H - J_0 - 3J_2/2 - 3J_1/4 - D \delta_{\eta\pi}/6$
 $a(x) = -8D/[3(3J_0 - 2g_{\alpha\alpha} \mu_B H)]$ and $a(z) = 0$

i.e. anticipate canted AF, with xy components similar to triangular Heisenberg AF

Form approximate eigenstates from linear combination of singlet and triplet:

$$|\psi_{i,l}\rangle = \cos \theta_{i,l} |00\rangle + \sin \theta_{i,l} e^{i\phi_{i,l}} |11\rangle \longrightarrow |\psi_{\vec{r}}\rangle = \sqrt{\frac{1 \mp \xi(\vec{r})}{2}} |00\rangle + \sqrt{\frac{1 \pm \xi(\vec{r})}{2}} e^{i\phi_{\vec{r}}} |11\rangle$$

Where $\xi(\vec{r}) = \sqrt{\cos^2 2\theta + \sin^2 2\theta \sin^2 \gamma \sin^2 \vec{q} \cdot \vec{r}}$ & $\tan \phi_{\vec{r}} = \cos \gamma \tan \vec{q} \cdot \vec{r}$
 & $\cos(\gamma)$ describes anisotropy in xy plane

Determine the ground states by minimizing E with respect to each parameter 30

Spin Hamiltonian for Ba₃Mn₂O₈ Theory: Cristian Batista (LANL)

For Ba₃Mn₂O₈ the minimal spin Hamiltonian is:

$$H = \sum_{i,j,k} J_{ij} \vec{S}_i \cdot \vec{S}_j + \dots$$

Aside: Classical ground state for a Heisenberg AF on a triangular lattice

For a single triangular plaquette:

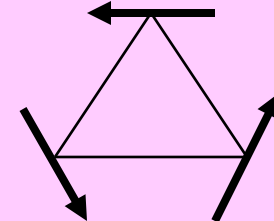
$$E = J(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_1) \equiv \frac{J}{2} (\vec{S}_1 + \vec{S}_2 + \vec{S}_3)^2 - \frac{3}{2} JS^2$$

From which the energy is minimized when...

$$\vec{S}_1 + \vec{S}_2 + \vec{S}_3 = 0$$

... corresponding to a 120° structure:

$$\mathbf{q} = (2\pi/3, 2\pi/3)$$



Where $\xi(\vec{r}) = \sqrt{\cos^2 2\theta + \sin^2 2\theta \sin^2 \gamma \sin^2 \vec{q} \cdot \vec{r}}$ & $\tan \phi_r = \cos \gamma \tan \vec{q} \cdot \vec{r}$
& $\cos(\gamma)$ describes anisotropy in xy plane

Where i, j designate coordinates of dimer
each of the two
S. Hill)

index, $\langle i, j \rangle$ indicates
and $\langle\langle i, j \rangle\rangle$ denotes
 S_i
[2-3J₁/4 - D δ_{nz}/6
μ_BH] and a(z)=0

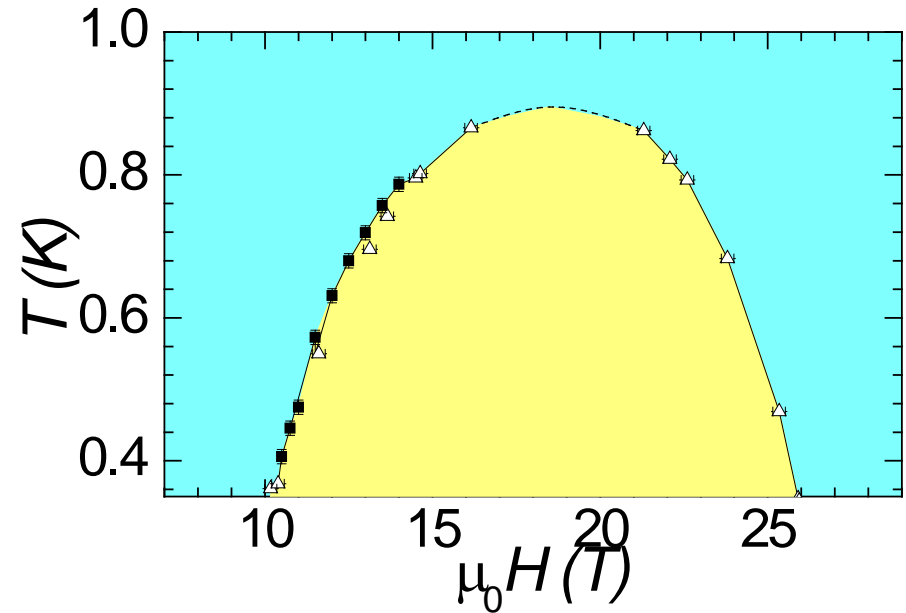
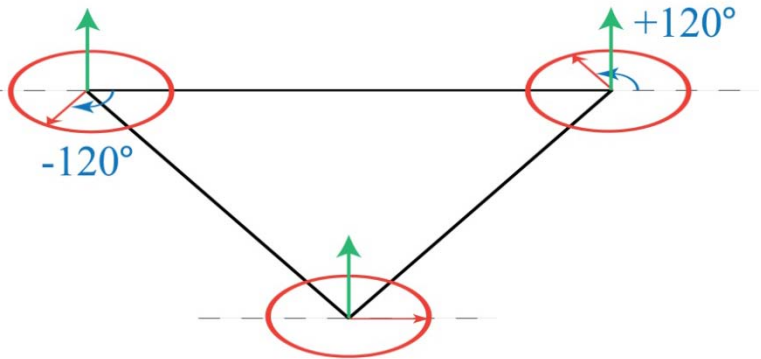
Heisenberg AF
triplet:

$|\phi_r | 11 \rangle$

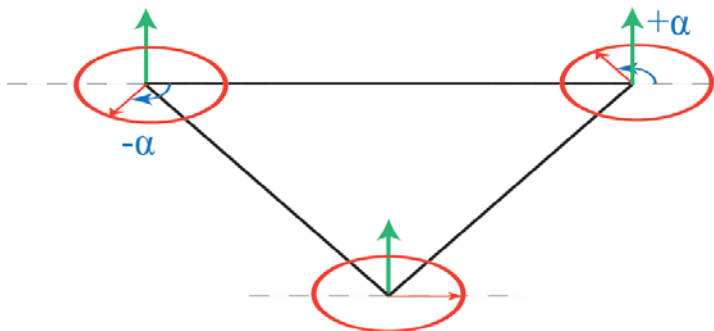
Determine the ground states by minimizing E with respect to each parameter 31

Classical groundstates: $H \parallel c$

(a) no interlayer coupling ($J_1 = 0$):



(b) finite interlayer coupling ($J_1 > 0$):

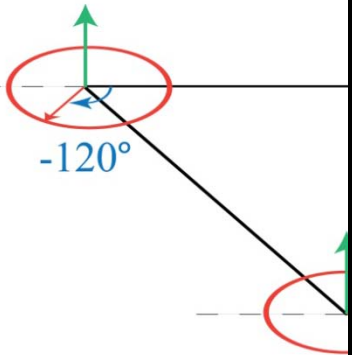


ΔE due to deviation from 120° :
 in-plane coupling: even (quadratic)
 out-of-plane coupling: odd (linear)

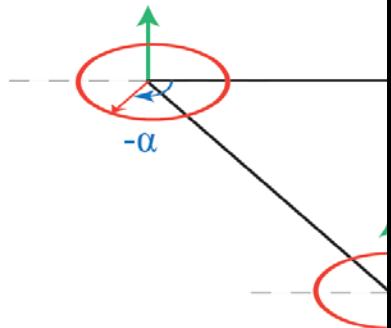
$$\alpha = \cos^{-1} \left(-\frac{1}{2} + \frac{J_1}{4J_2} \right) \approx 117^\circ$$

Classical groundstates: $H \parallel c$

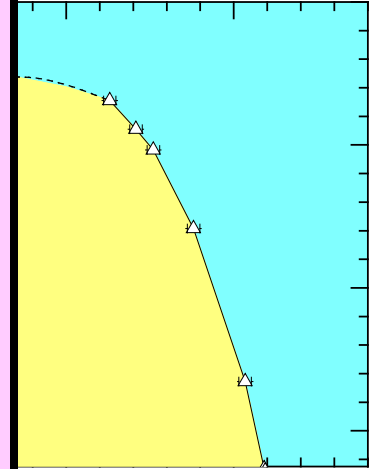
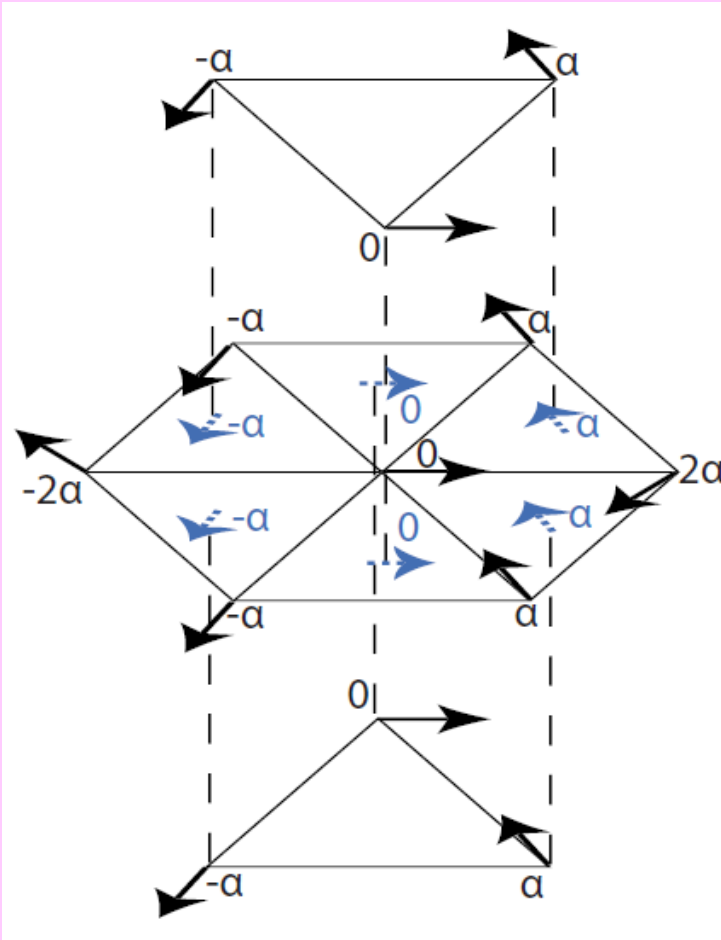
(a) no interlayer



(b) finite interlayer



Ferromagnetic interlayer coupling
 \rightarrow in-plane spiral structure:



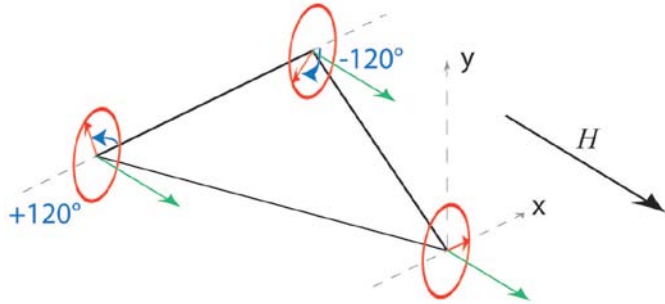
20
 (T)
 25

from 120° :
 even (quadratic)
 odd (linear)

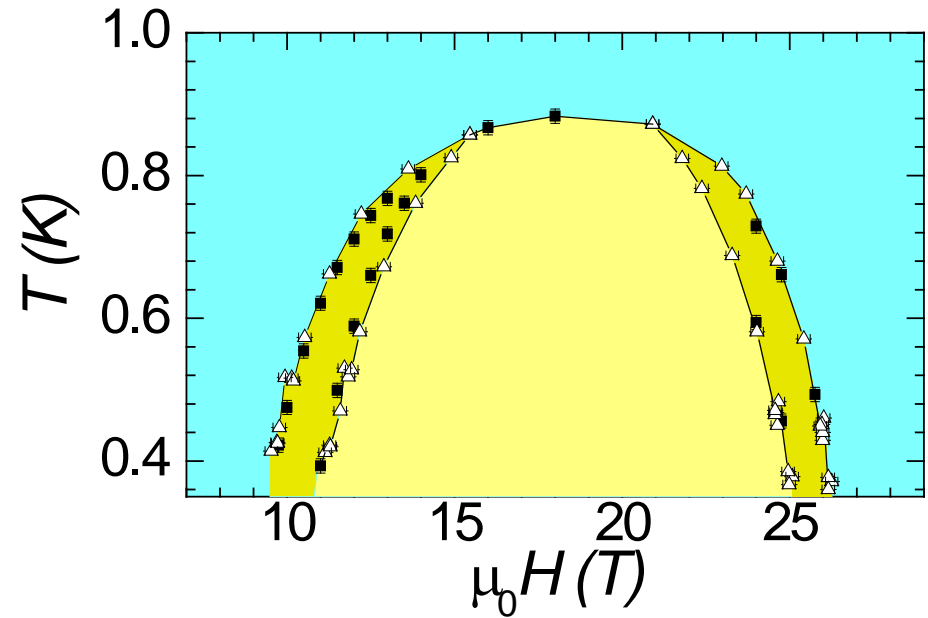
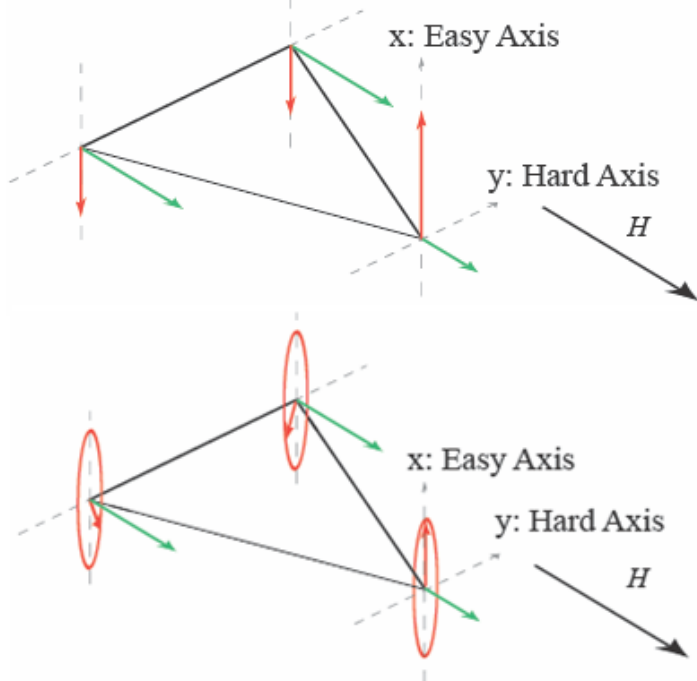
$$\left(\frac{1}{J_2} \right) \approx 117^\circ$$

Classical groundstates: $H \perp c$

(a) $D = 0, J_1 = 0$:



(b) $D < 0, J_1 > 0$:

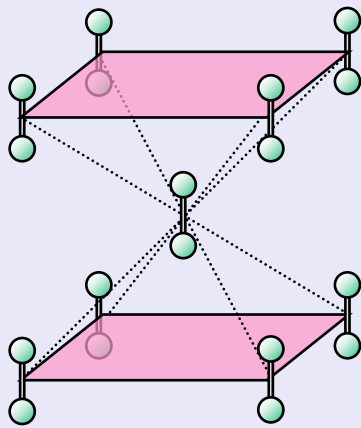


- easy axis in x-y plane
- sum of xy components ~ 0
- \rightarrow modulated structure
- but modulation costs energy...

$$\frac{8J_2}{3} \sum_{l \langle i, j \rangle} \left[\mathbf{s}_{i,l} \cdot \mathbf{s}_{j,l} - \frac{13}{16} s_{i,l}^z s_{j,l}^z \right]$$

What about dimensional reduction?

(1) $\text{BaCuSi}_2\text{O}_6$

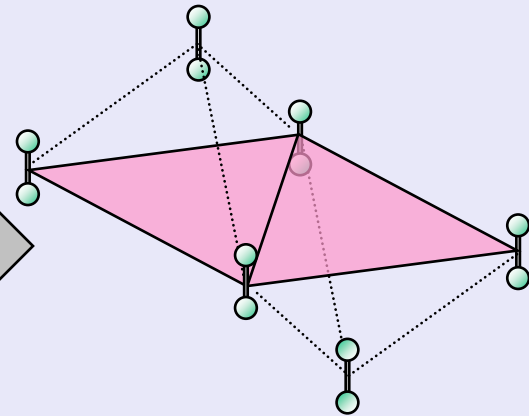


- $q = (\pi/a, \pi/a)$
- frustrates interlayer coupling

*more
frustrated*

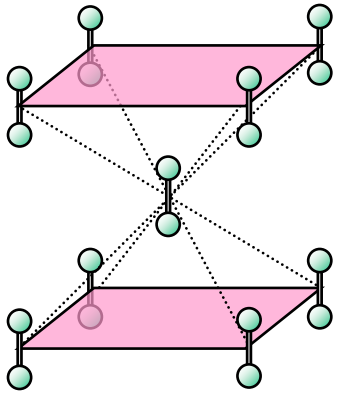
?

(2) $\text{Ba}_3\text{Mn}_2\text{O}_8$



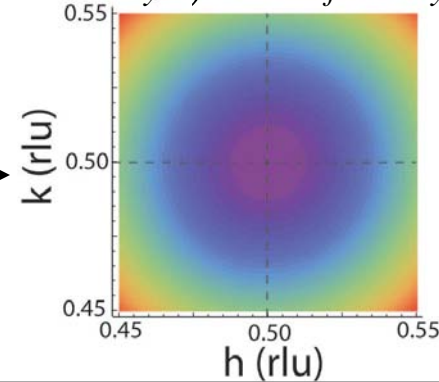
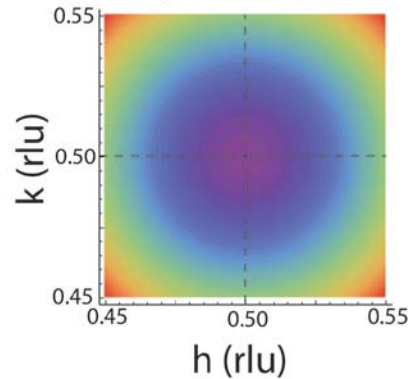
- if $q = (2\pi/3a, 2\pi/3a)$, then interlayer coupling is also perfectly frustrated
- but $q \neq (2\pi/3a, 2\pi/3a) \dots$

BaCuSi₂O₆ vs Ba₃Mn₂O₈: symmetry

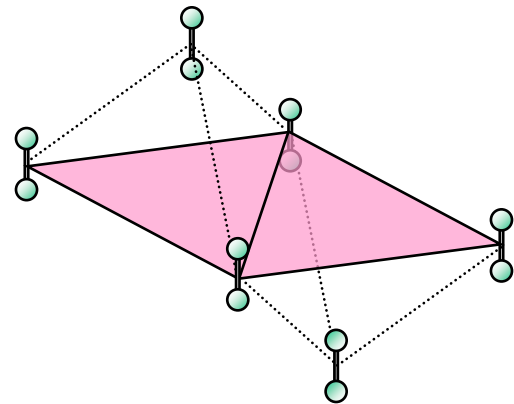


- Planar inversion symmetry about $Q = (\pi/a, \pi/a)$
- Interlayer coupling can only introduce terms quadratic in k

$$E(Q+k) \approx J'(k_x^2 + k_y^2) + 2J_f k_x k_y$$

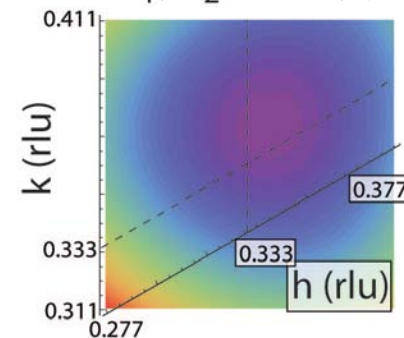
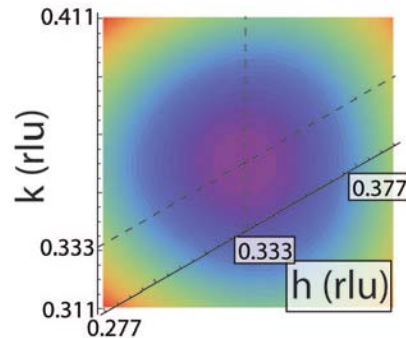


$Q = (\pi/a, \pi/a) \Rightarrow$
layers decouple



- No inversion symmetry about $Q = (2\pi/3a, 2\pi/3a)$
- Interlayer coupling introduces terms *linear* in k

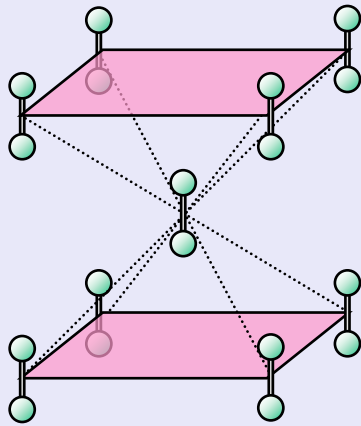
$$E(Q+k) \propto J_2(-3 + k_x^2 + k_y^2 + k_x k_y) + \frac{\sqrt{3}}{2} J_1(k_x + k_y)$$



$Q \neq (2\pi/3a, 2\pi/3a) \Rightarrow$
3D spiral

So which lattice is more frustrated?

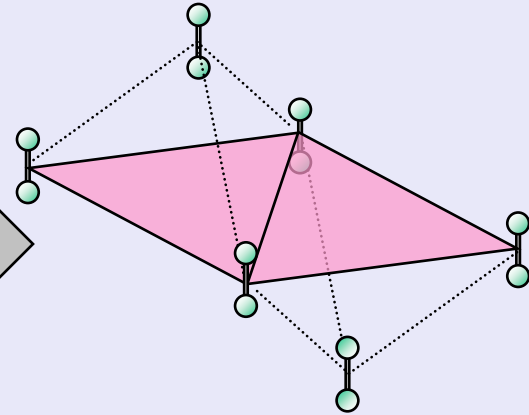
(1) $\text{BaCuSi}_2\text{O}_6$



robust interlayer frustration
→ dimensional reduction at $T = 0$

*less
frustrated* !

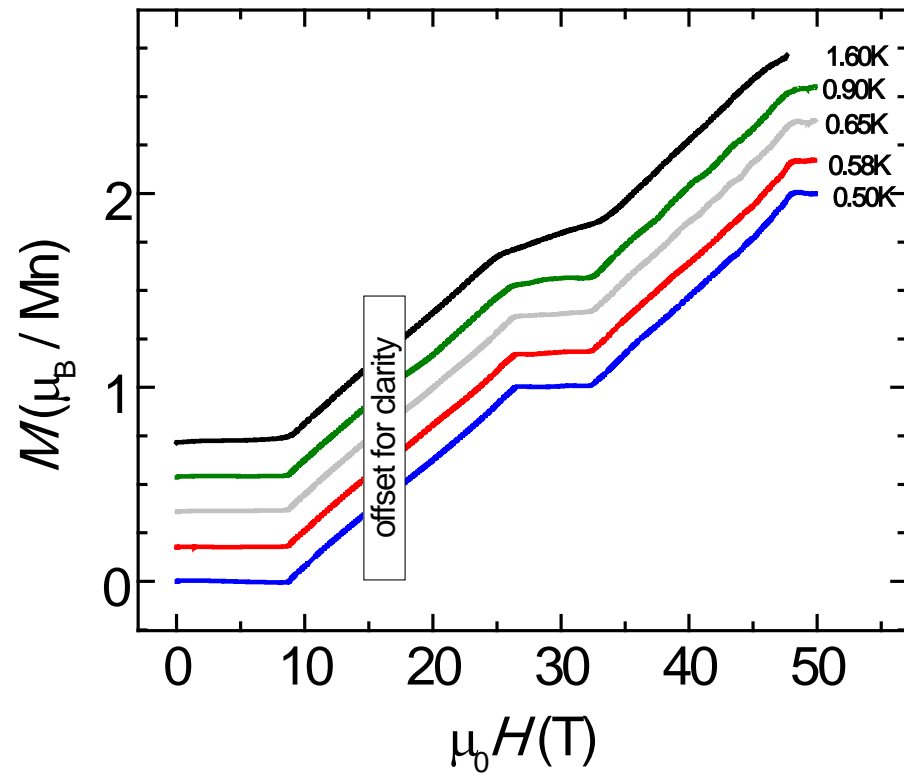
(2) $\text{Ba}_3\text{Mn}_2\text{O}_8$



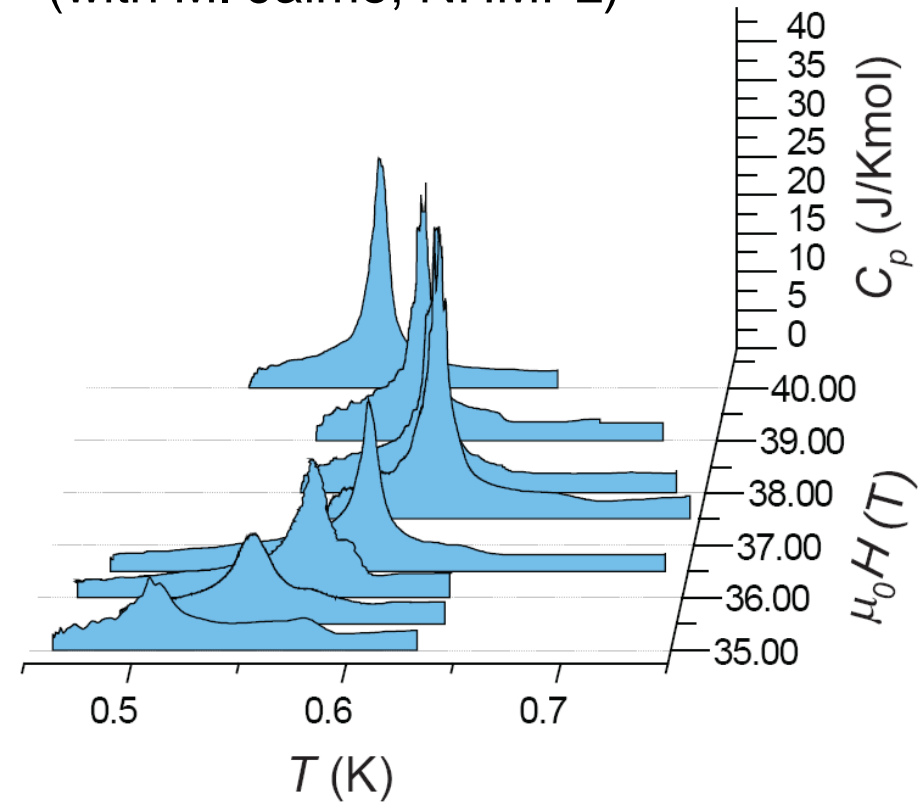
perfect frustration is not protected
→ 3d incommensurate spiral

What about the quintuplet states of $\text{Ba}_3\text{Mn}_2\text{O}_8$?

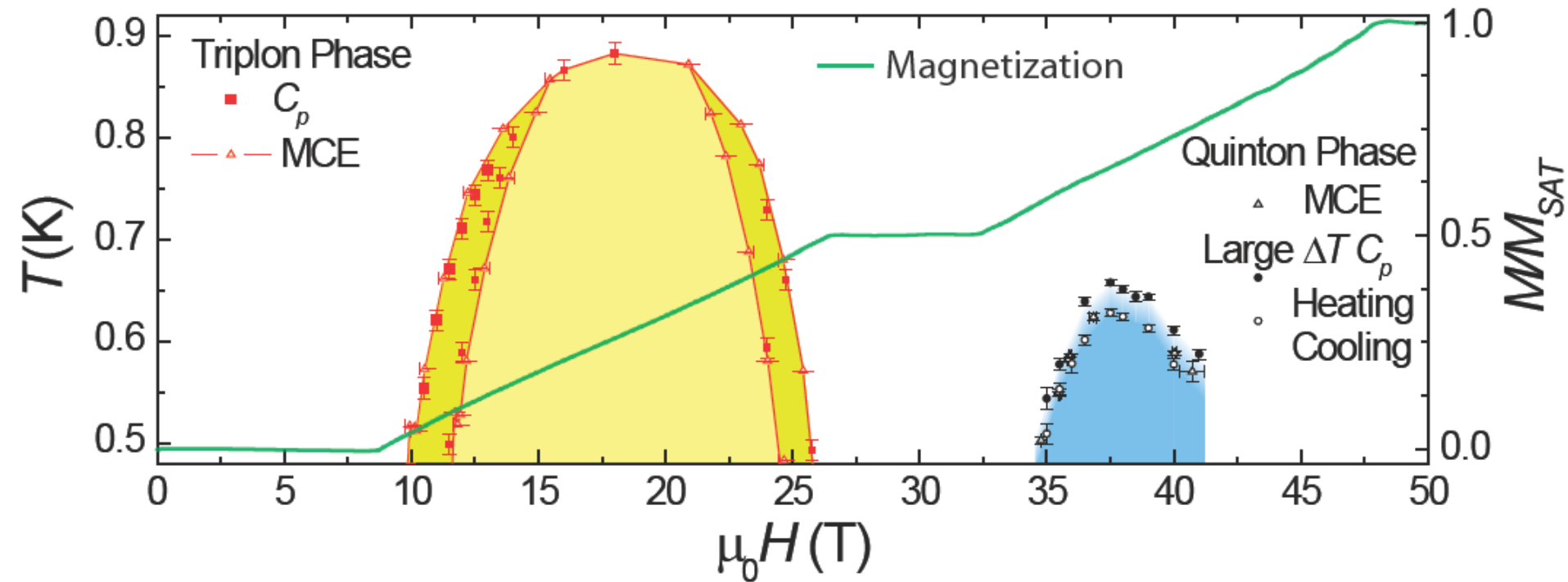
Magnetization:
(with R. McDonald, NHMFL)



C_p & MCE:
(with M. Jaime, NHMFL)



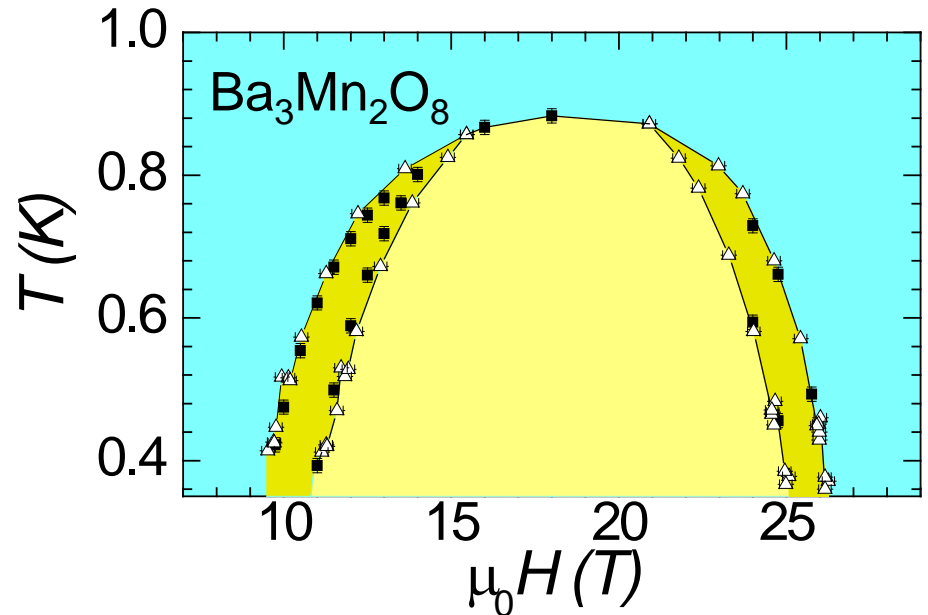
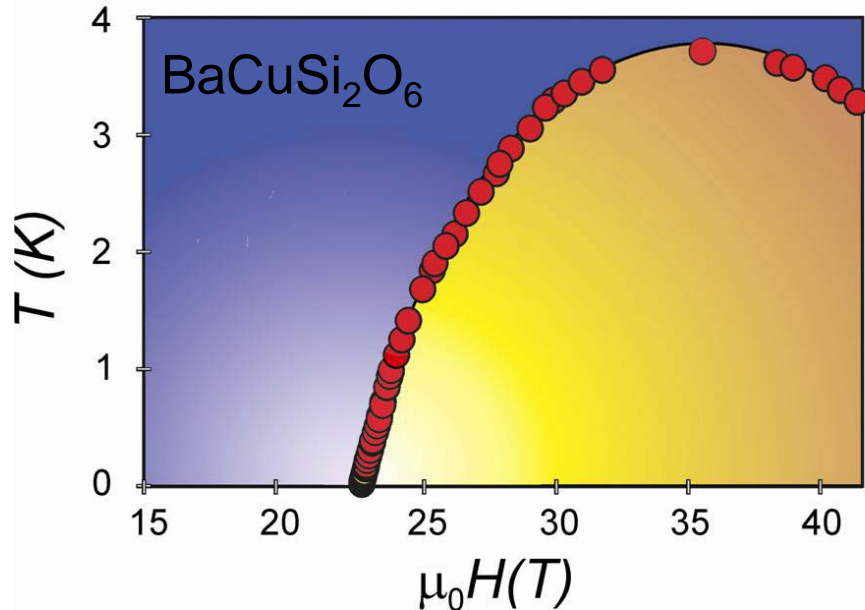
What about the quintuplet states of $\text{Ba}_3\text{Mn}_2\text{O}_8$?



- Second region of LRMO
- Can be understood following a similar treatment of the effective Hamiltonian

Summary

Spin dimer compounds provide access to some exciting physics and some beautiful magnetic structures...



- S. Sebastian *et al.*, PRB. **72**, 100404(R) (2005).
- E. Samulon *et al.*, PRB **73**, 100407(R) (2006).
- S. E. Sebastian *et al.*, PRB **74**, 180401(R) (2006).
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- M. Stone *et al* PRB **77**, 134406 (2008).
- E. Samulon *et al* PRB **77**, 214441 (2008).
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