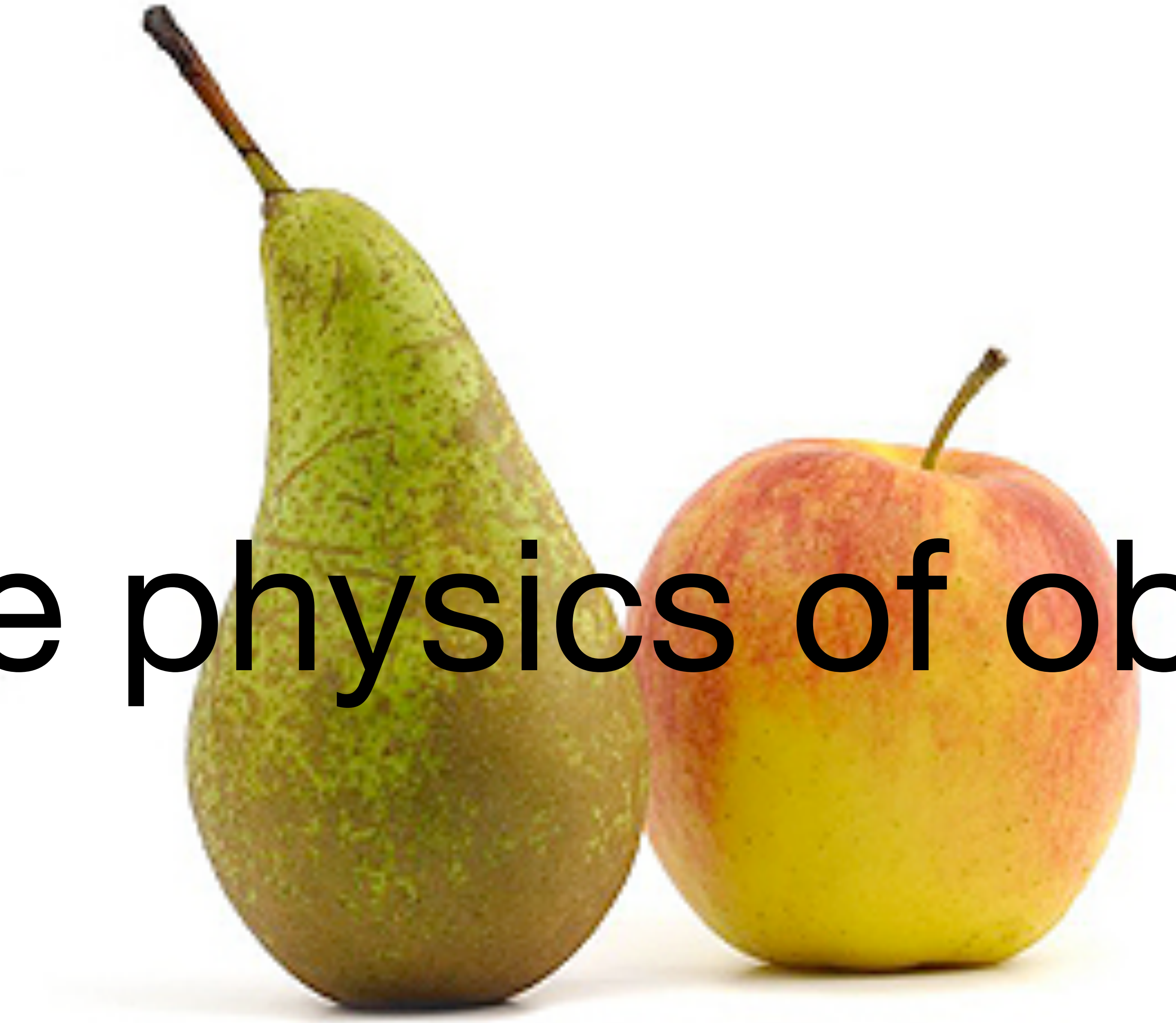
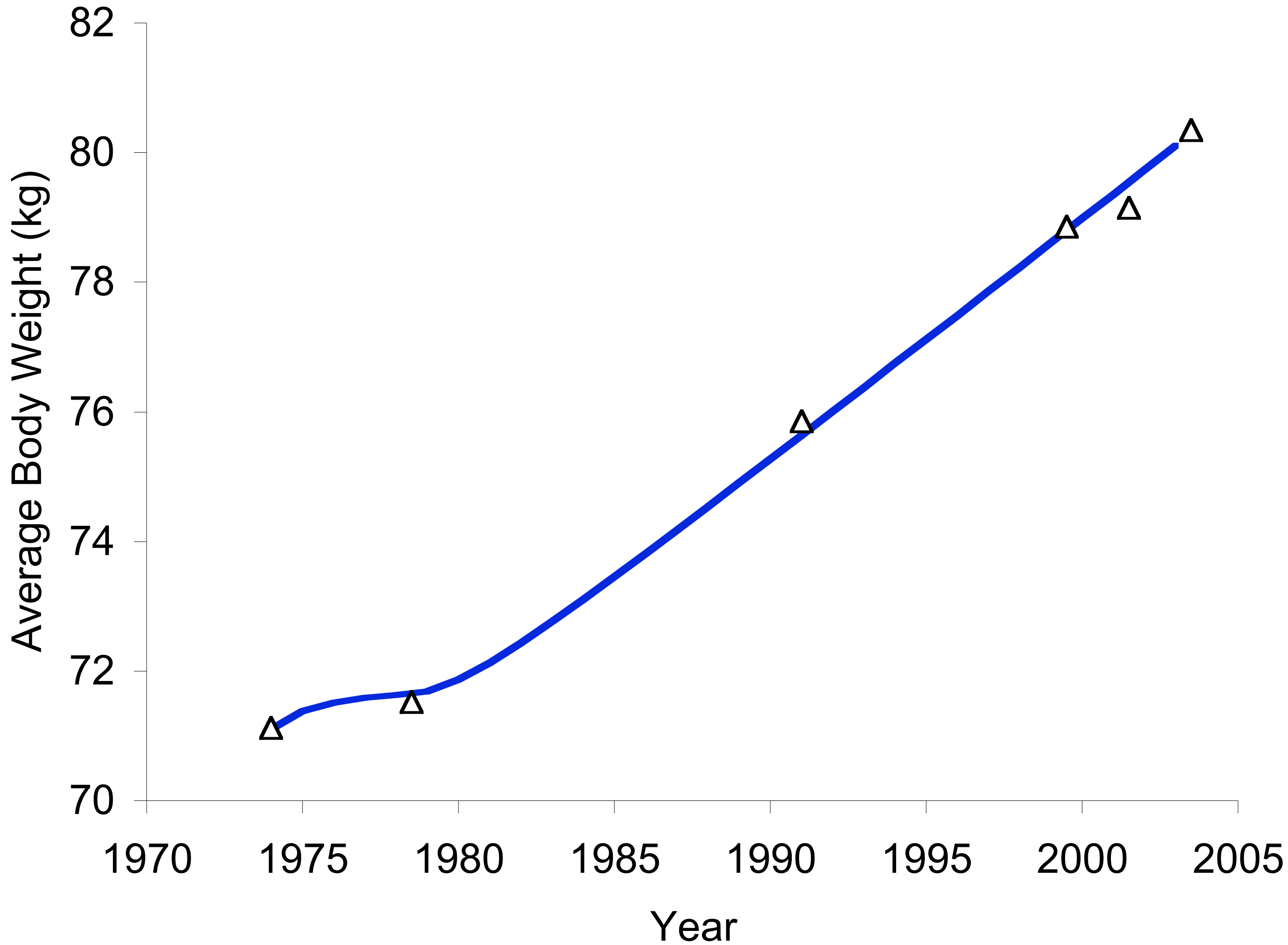
The physics of obesity



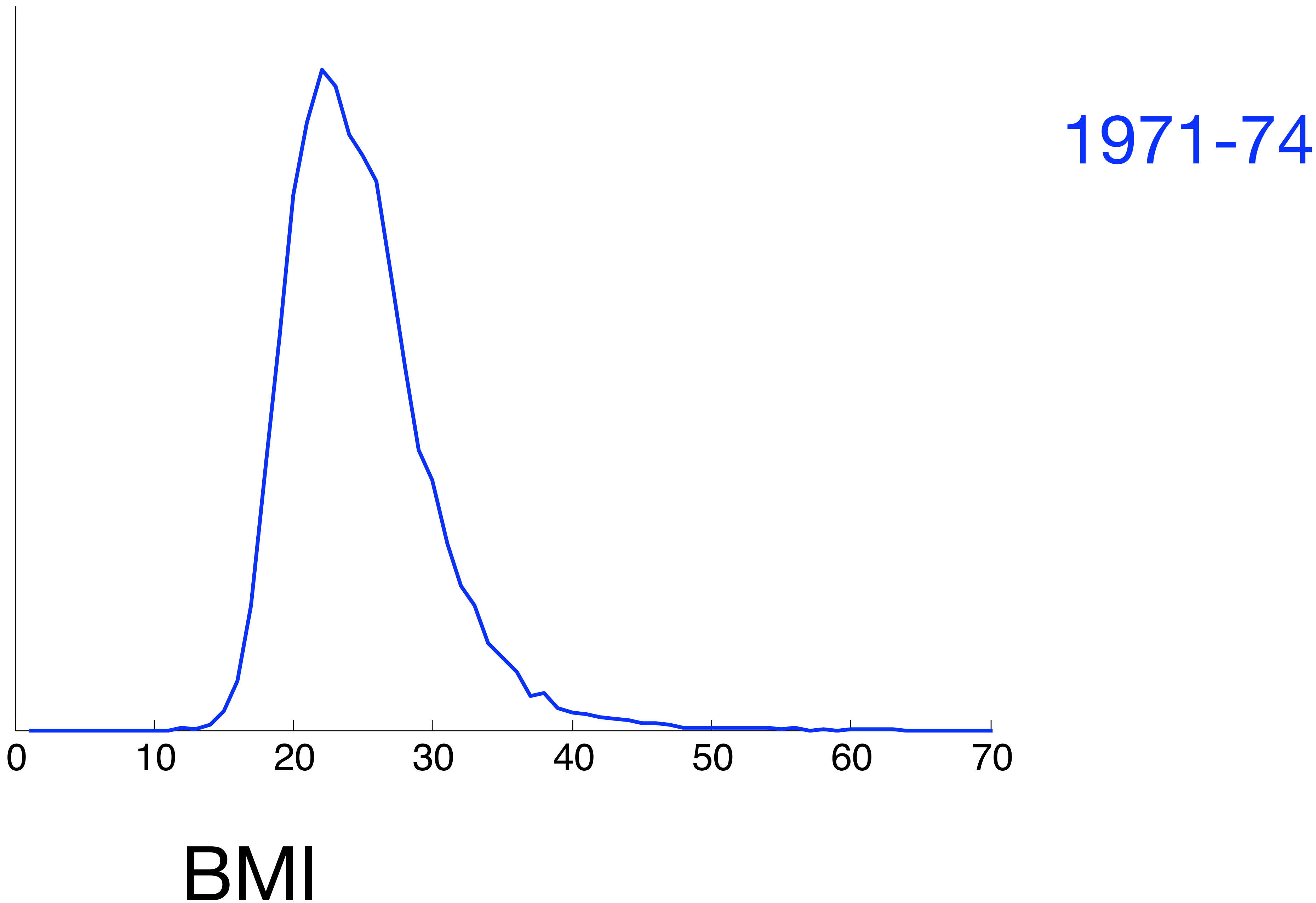
Carson Chow
Laboratory of Biological Modeling, NIDDK, NIH

US obesity epidemic



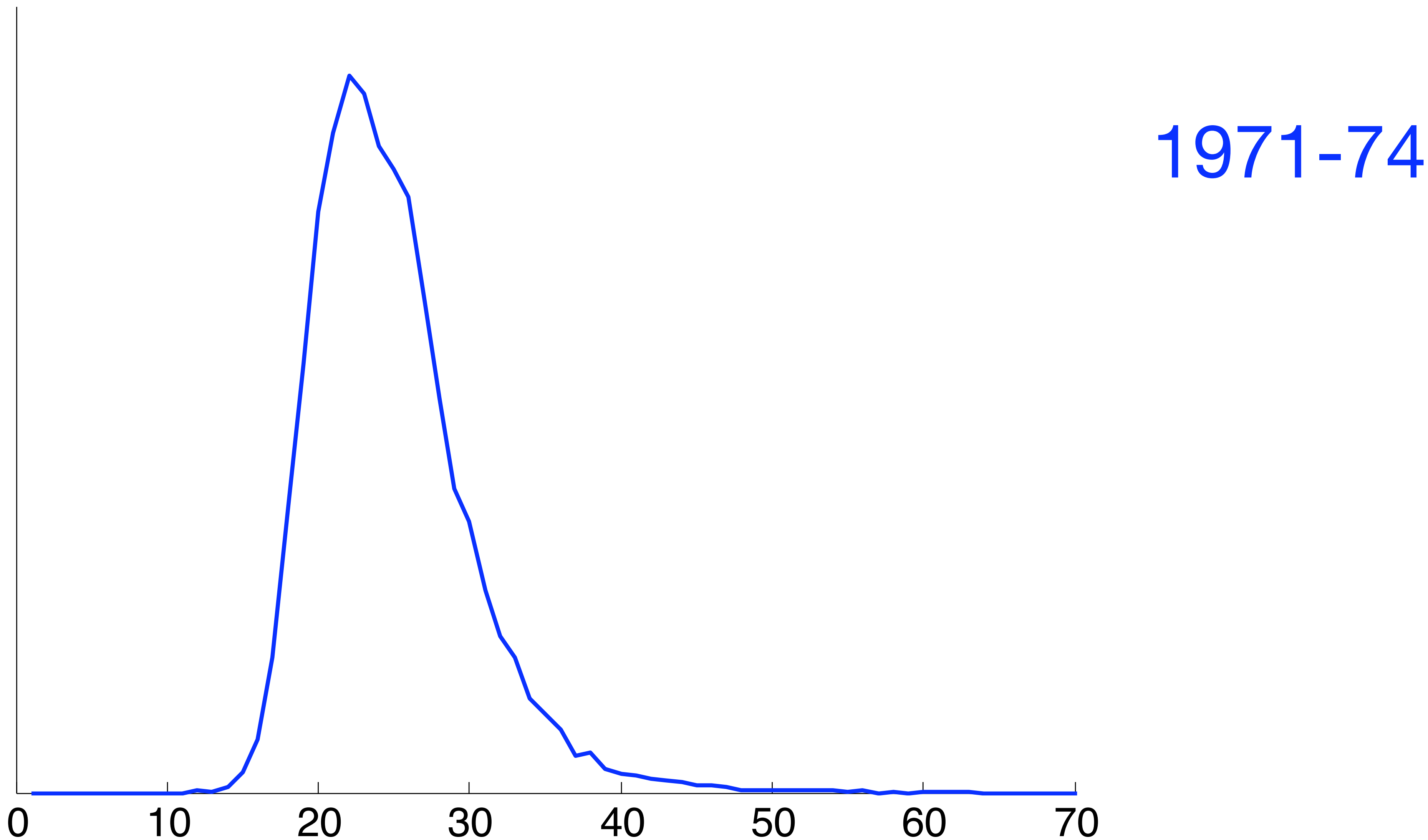
Data from National Health and Nutrition Examination Survey (NHANES)

US obesity epidemic



NHANES data

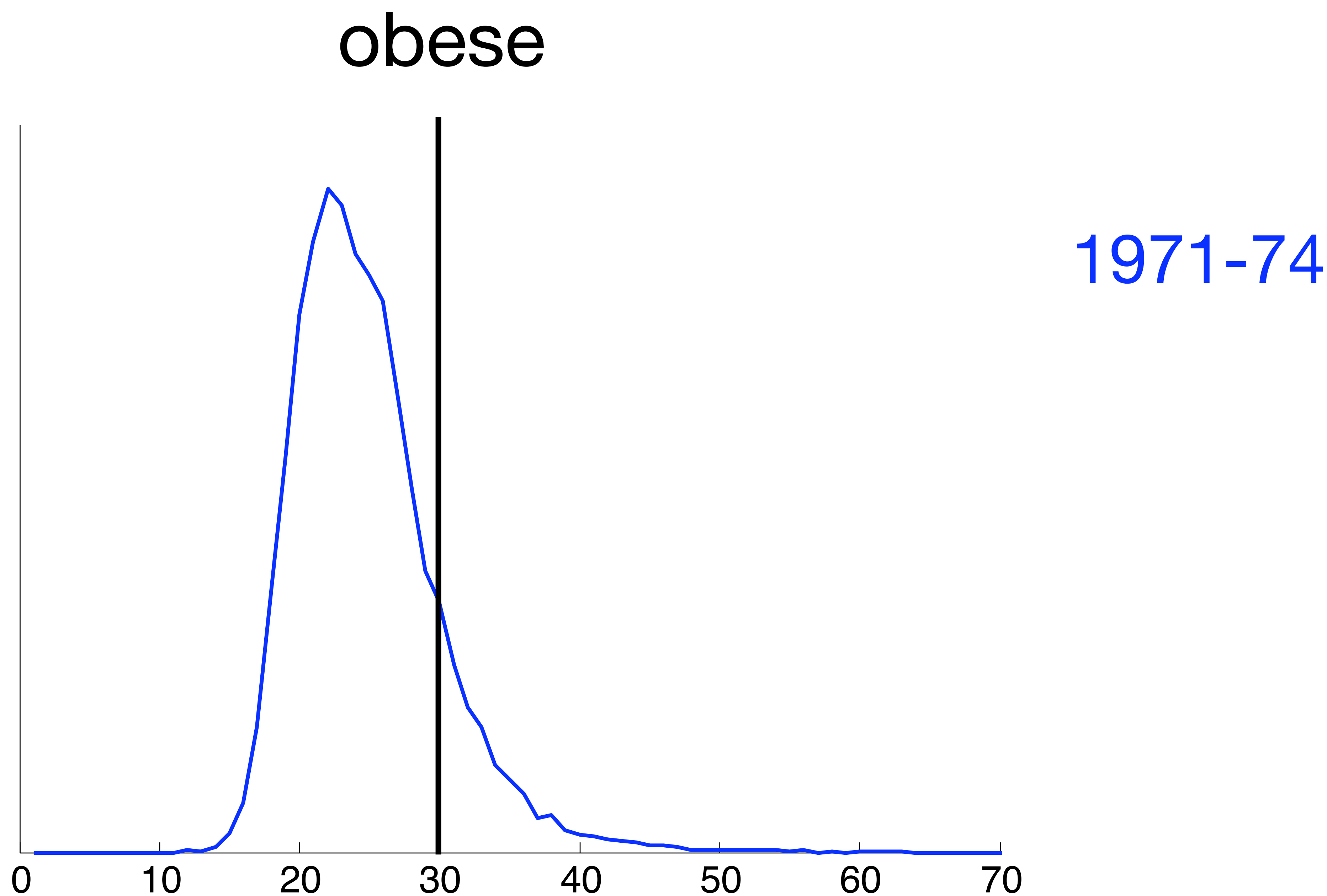
US obesity epidemic



BMI = weight / height^2

NHANES data

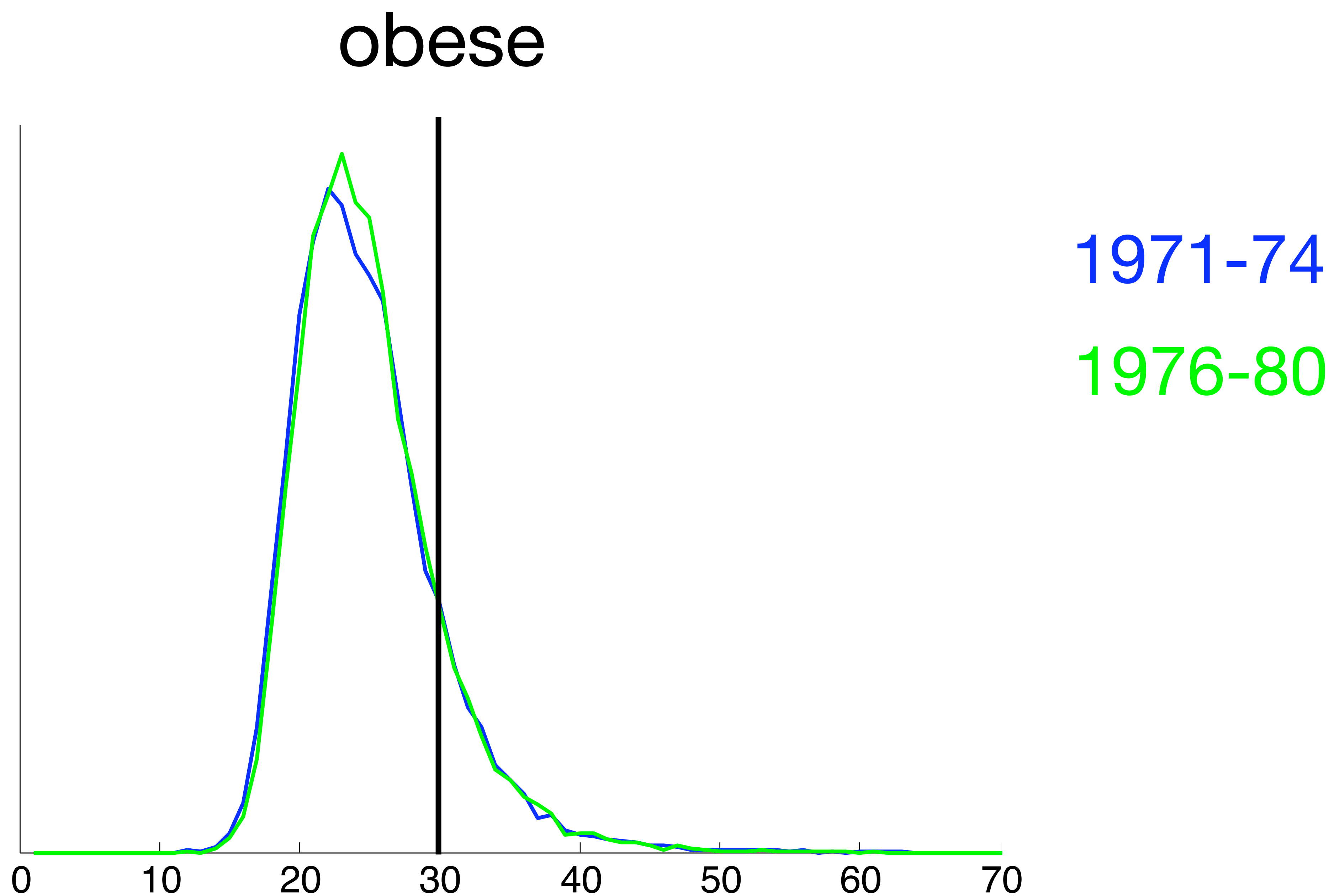
US obesity epidemic



BMI = weight / height^2

NHANES data

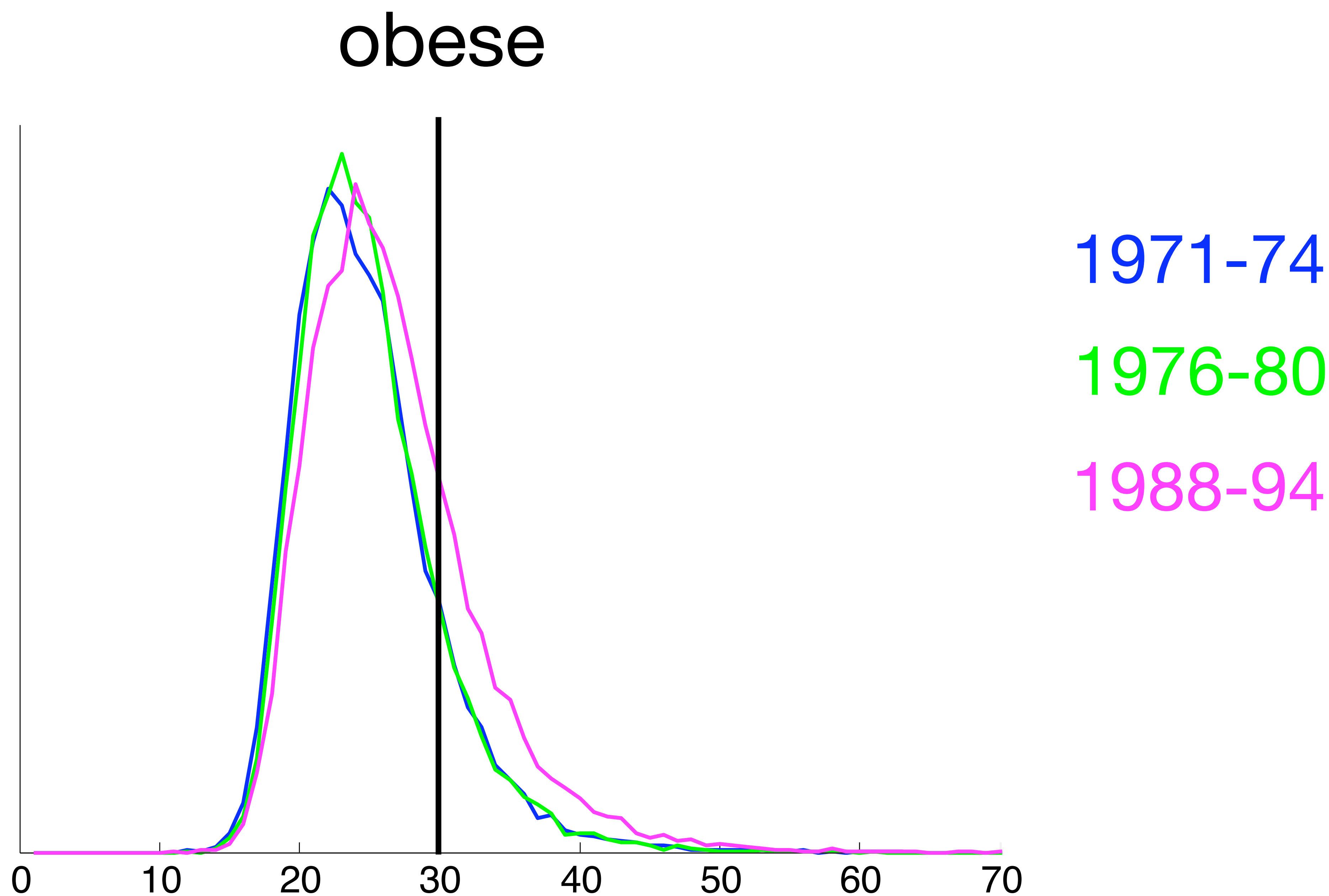
US obesity epidemic



BMI = weight / height^2

NHANES data

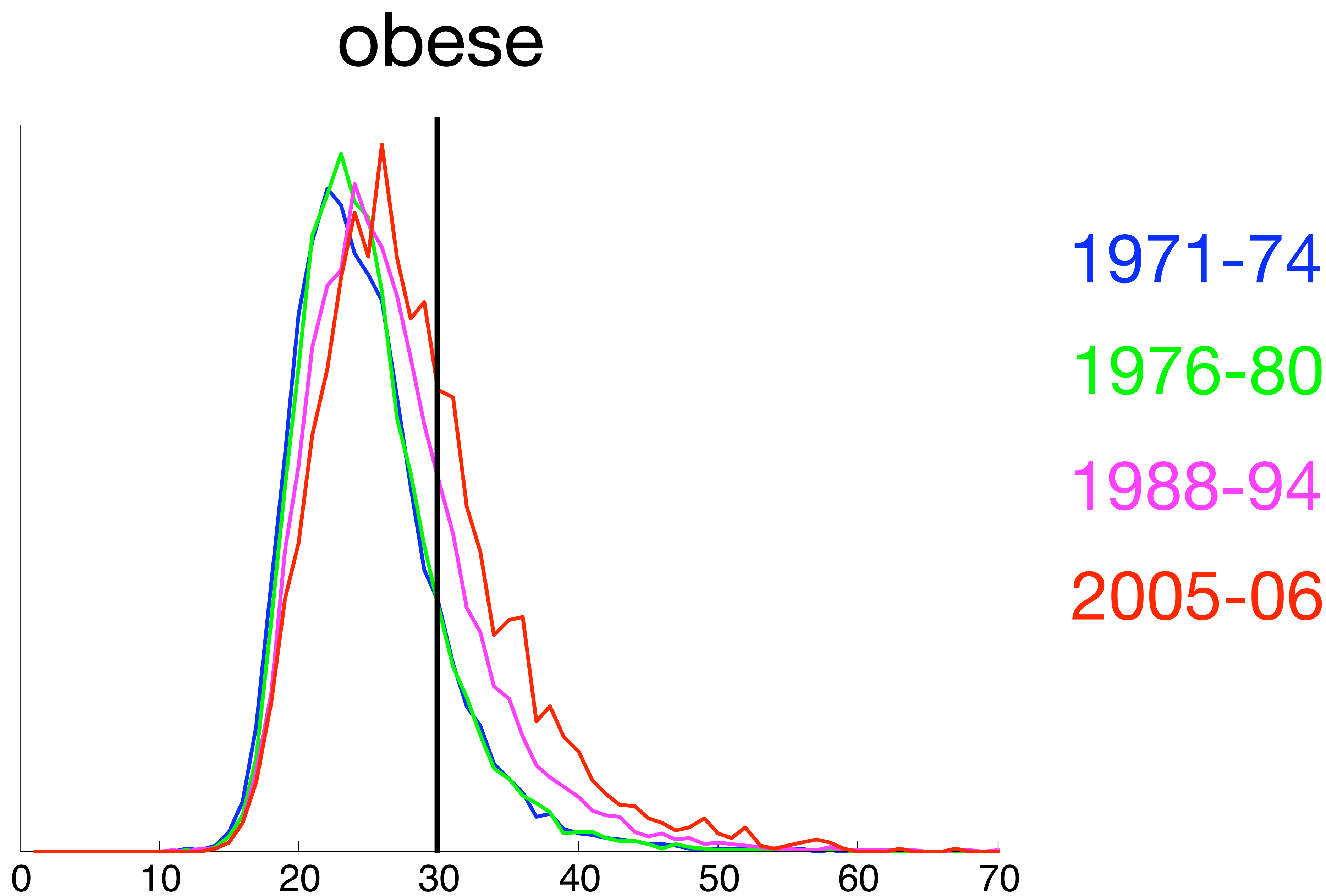
US obesity epidemic



BMI = weight / height^2

NHANES data

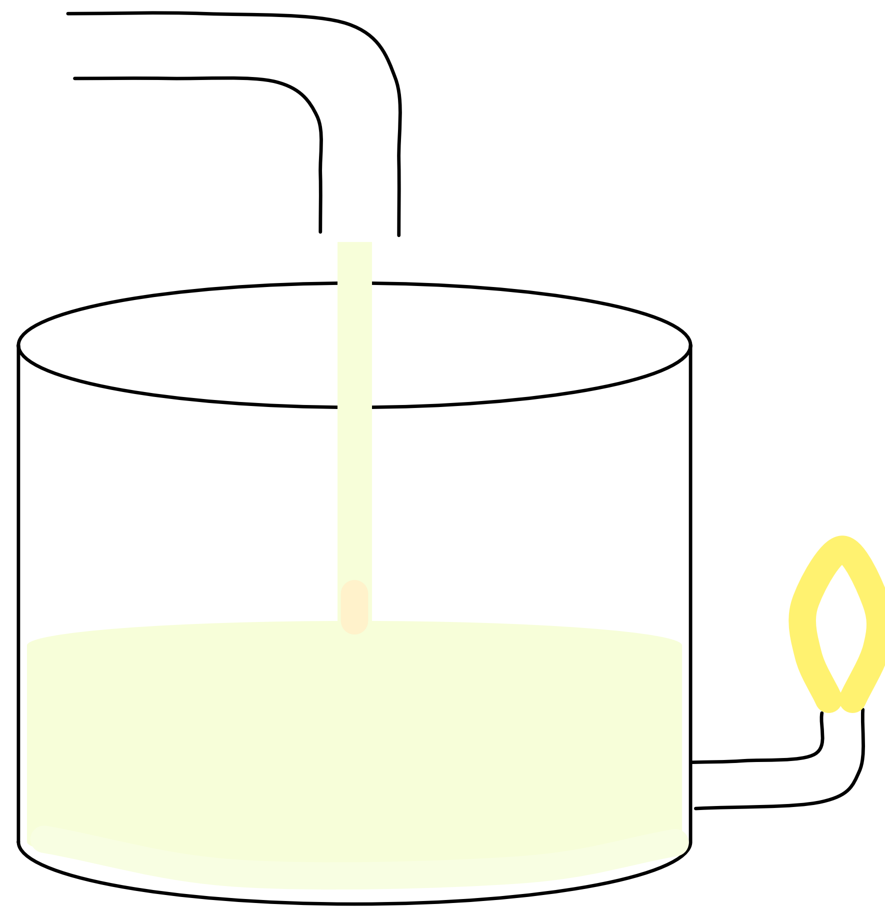
US obesity epidemic



BMI = weight / height^2

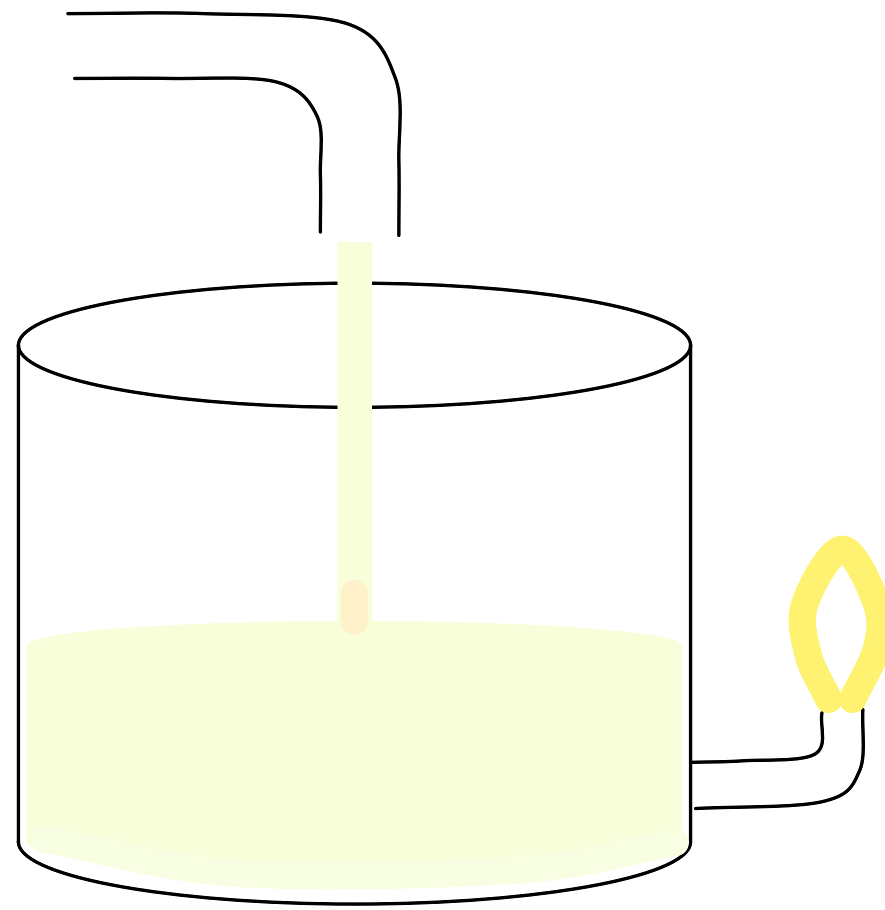
NHANES data

Conservation of energy

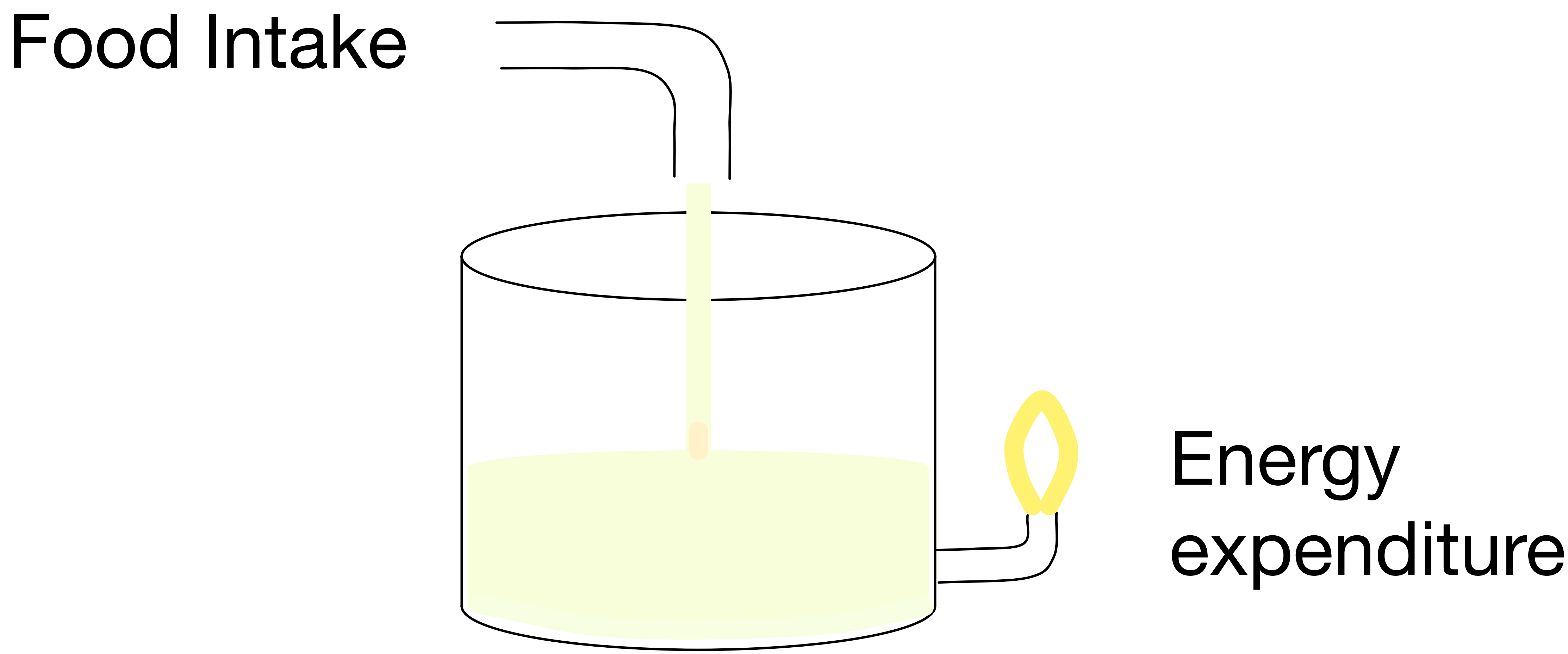


Conservation of energy

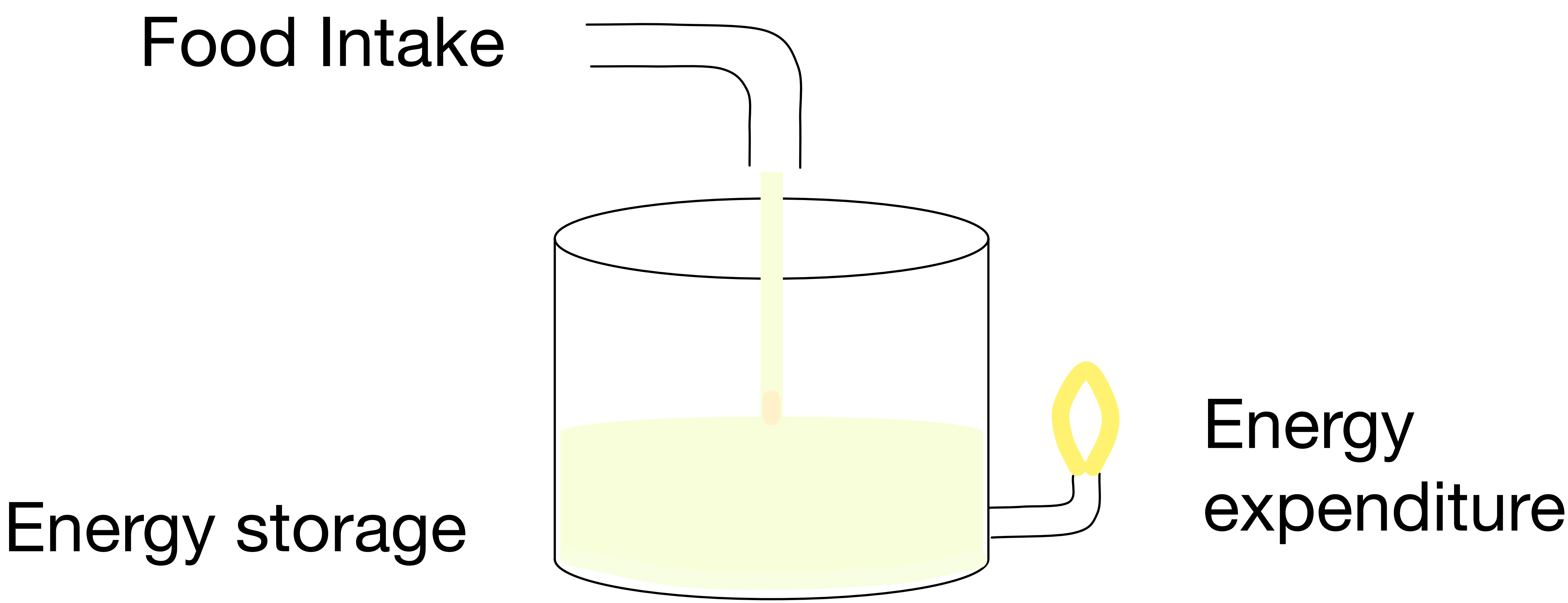
Food Intake



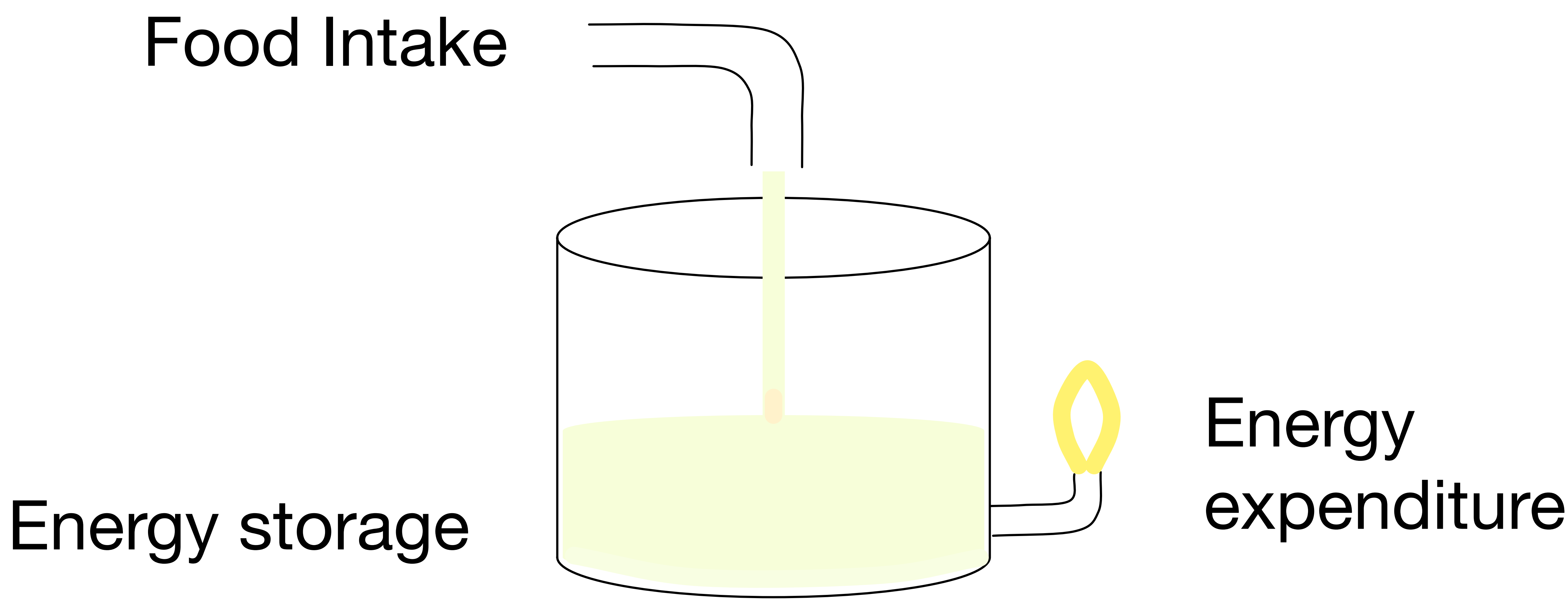
Conservation of energy



Conservation of energy



Conservation of energy



$$\Delta \text{Storage} = \text{Intake} - \text{Expenditure}$$

Energy flux

Rate of storage = intake rate - expenditure rate

$$\frac{d(\rho_M M)}{dt} = I - E$$

M = body mass

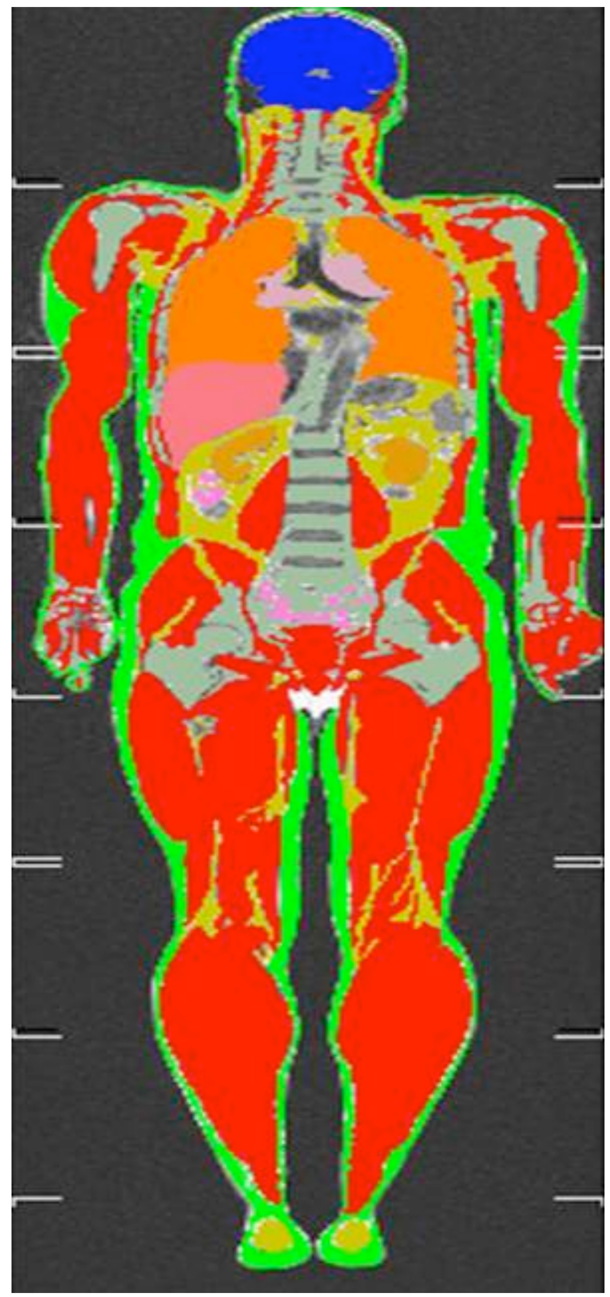
Energy density ρ_M converts energy to mass

Energy density

Fat
37.7 kJ/g

Carbs
(glycogen)
16.8 kJ/g

Protein
16.8 kJ/g

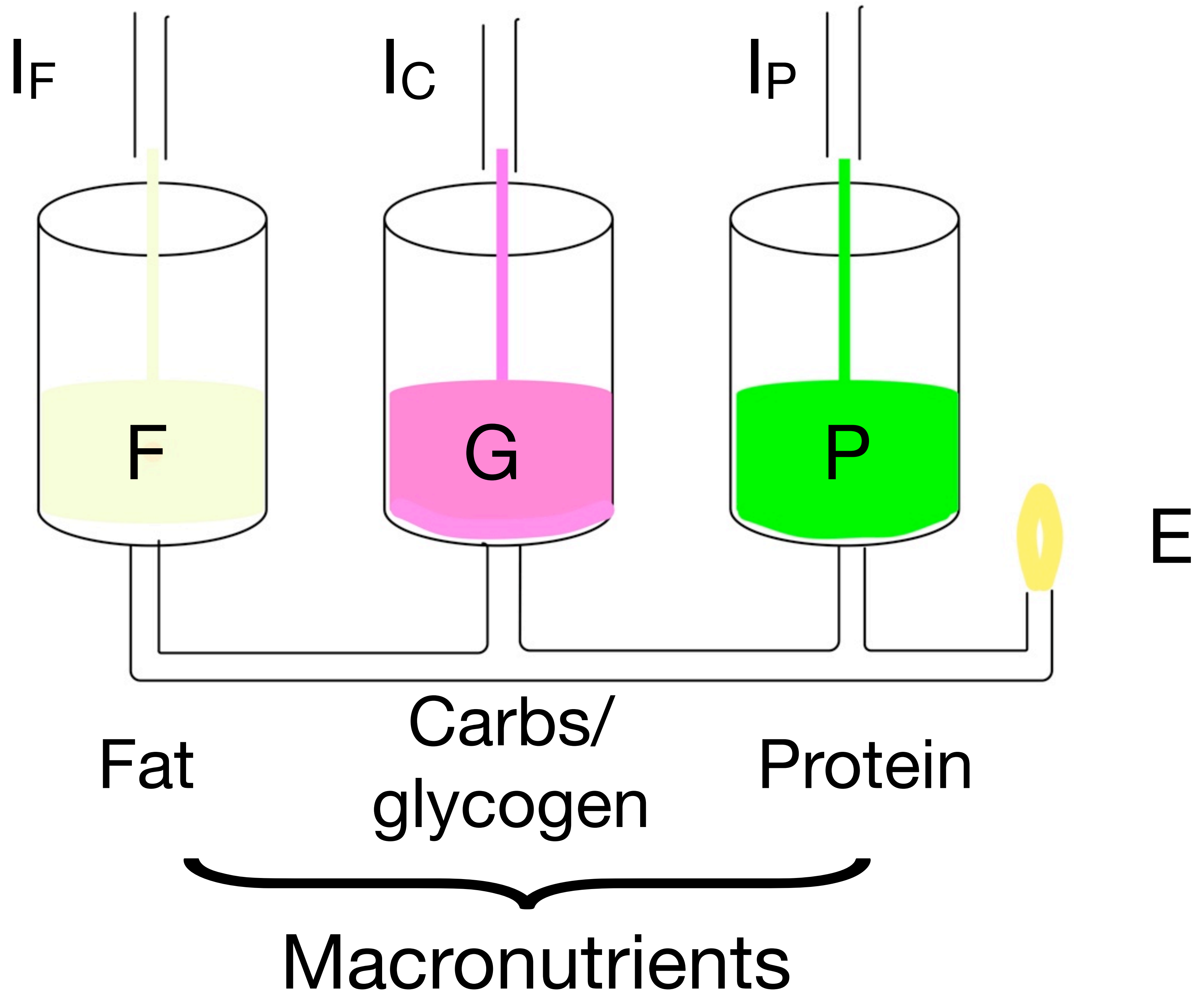


Water

Bone

Minerals

Multiple fuel sources



Macronutrient flux

$$\frac{d(\rho_M M)}{dt} = I - E$$

Macronutrient flux

$$\rho_F \frac{dF}{dt} + \rho_P \frac{dP}{dt} + \rho_G \frac{dG}{dt} = I - E$$

Macronutrient flux

$$\rho_F \frac{dF}{dt} + \rho_P \frac{dP}{dt} + \rho_G \frac{dG}{dt} = I_F + I_C + I_P - E$$

Macronutrient flux

$$\rho_F \frac{dF}{dt}$$

$$\rho_G \frac{dG}{dt} \qquad = I_F + I_C + I_P - E$$

$$\rho_P \frac{dP}{dt}$$

Macronutrient flux

$$\rho_F \frac{dF}{dt} = I_F$$

$$\rho_G \frac{dG}{dt} = I_C \qquad - E$$

$$\rho_P \frac{dP}{dt} = I_P$$

Macronutrient flux

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_G \frac{dG}{dt} = I_C - f_C E$$

$$\rho_P \frac{dP}{dt} = I_P - (1 - f_F - f_C) E$$

Macronutrient flux

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_G \frac{dG}{dt} = I_C - f_C E$$

$$\rho_P \frac{dP}{dt} = I_P - (1 - f_F - f_C) E$$

f_F = fraction of fat utilized

f_C = fraction of carbs utilized

Macronutrient flux

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_G \frac{dG}{dt} = I_C - f_C E$$

$$\rho_P \frac{dP}{dt} = I_P - (1 - f_F - f_C) E$$

f_F = fraction of fat utilized

f_C = fraction of carbs utilized

Reduction to $2D$

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_G \frac{dG}{dt} = I_C - f_C E$$

$$\rho_P \frac{dP}{dt} = I_P - (1 - f_F - f_C) E$$

f_F = fraction of fat utilized

f_C = fraction of carbs utilized

Reduction to *2D*

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_G \frac{dG}{dt} = I_C - f_C E$$

$$\rho_P \frac{dP}{dt} = I_P - (1 - f_F - f_C) E$$

Glycogen supply small, ~ fixed on long time scales

Reduction to $2D$

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_G \frac{dG}{dt} = I_C - f_C E = 0$$

$$\rho_P \frac{dP}{dt} = I_P - (1 - f_F - f_C) E$$

Glycogen supply small, ~ fixed on long time scales

Reduction to $2D$

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$f_C = \frac{I_C}{E}$$

$$\rho_P \frac{dP}{dt} = I_P - (1 - f_F - f_C) E$$

Glycogen supply small, ~ fixed on long time scales

Reduction to *2D*

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_P \frac{dP}{dt} = I_P - (1 - f_F - \frac{I_C}{E}) E$$

Glycogen supply small, ~ fixed on long time scales

Reduction to $2D$

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_P \frac{dP}{dt} = I_P - (1 - f_F) E + I_C$$

Glycogen supply small, ~ fixed on long time scales

Reduction to $2D$

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_P \frac{dP}{dt} = I_P + I_C - (1 - f_F) E$$

Glycogen supply small, ~ fixed on long time scales

Reduction to $2D$

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_P \frac{dP}{dt} = I_P + I_C - (1 - f_F) E$$

Divide mass into lean and fat $\quad M = L + F$

Reduction to $2D$

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\rho_P \frac{dP}{dt} = I_P + I_C - (1 - f_F) E$$

Divide mass into lean and fat $\quad M = L + F$

Change in L due to change in P and water $\quad \frac{dP}{dt} = \frac{1}{1 + h_P} \frac{dL}{dt}$

Reduction to *2D*

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\frac{\rho_P}{1+h_P}\frac{dL}{dt} = I_P + I_C - (1-f_F)E$$

Divide mass into lean and fat $M = L + F$

Reduction to $2D$

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\frac{\rho_P}{1 + h_P} \frac{dL}{dt} = I_P + I_C - (1 - f_F) E$$

Divide mass into lean and fat $M = L + F$

Lean intake = carbs + protein $I_P + I_C = I_L$

Reduction to *2D*

$$\rho_F \frac{dF}{dt} = I_F - f_F E$$

$$\frac{\rho_P}{1+h_P} \frac{dL}{dt} = \quad I_L \quad -(1-f_F)E$$

Divide mass into lean and fat $\quad M = L + F$

Body composition model

$$\rho_F \frac{dF}{dt} = I_F - fE$$

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E$$

Body composition model

$$\begin{aligned} \rho_F \frac{dF}{dt} &= I_F - fE \\ \rho_L \frac{dL}{dt} &= I_L - (1-f)E \end{aligned}$$

$$\rho_L = \rho_P/(1+h_P)$$

h_p protein hydration coefficient

Body composition model

$$\rho_F \frac{dF}{dt} = I_F - fE$$
$$\rho_L \frac{dL}{dt} = I_L - (1-f)E$$

$$\rho_L = \rho_P/(1+h_P)$$

h_p protein hydration coefficient

f is fraction of energy use that is fat

Body composition model

$$
\begin{aligned}
\rho_F \frac{dF}{dt} &= I_F - fE \\
\rho_L \frac{dL}{dt} &= I_L - (1-f)E
\end{aligned}
$$

$$\rho_L = \rho_P/(1+h_P)$$

h_p protein hydration coefficient

f is fraction of energy use that is fat

E and f are functions of F and L

Dynamical systems

Infer global dynamics from local information

Dynamical systems

$$\frac{dx}{dt} = x(1 - x)$$

Dynamical systems

$$\frac{dx}{dt} = x(1-x) = 0$$

Fixed points

Dynamical systems

$$\frac{dx}{dt} = x(1-x) = 0$$

Fixed points

$$x = 0, 1$$

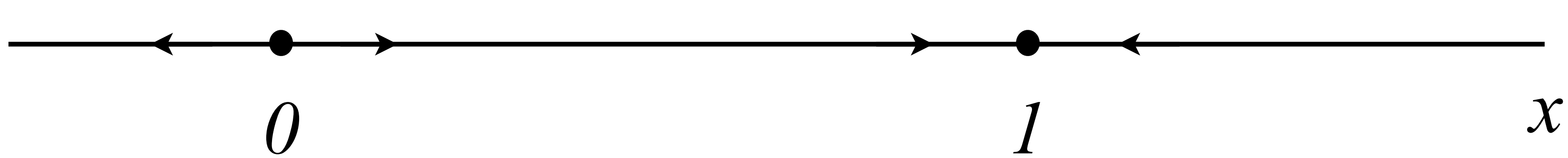
Dynamical systems

$$\frac{dx}{dt} = x(1-x) = 0$$

Fixed points

$$x = 0, 1$$

Represent geometrically in phase plane



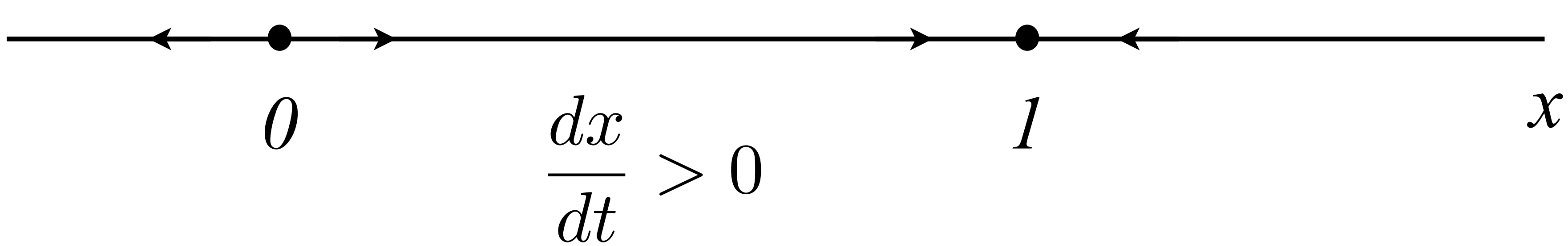
Dynamical systems

$$\frac{dx}{dt} = x(1-x) = 0$$

Fixed points

$$x = 0, 1$$

Represent geometrically in phase plane



Dynamical systems

$$\frac{dx}{dt} = x(1-x) = 0$$

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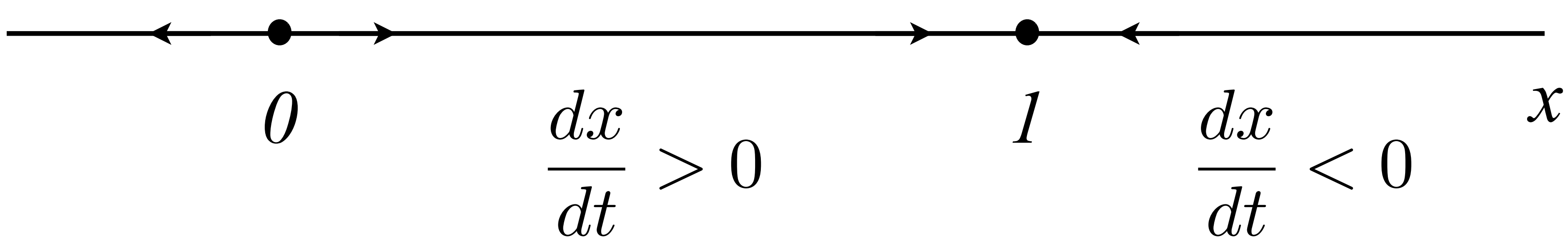

Dynamical systems

$$\frac{dx}{dt} = x(1-x) = 0$$

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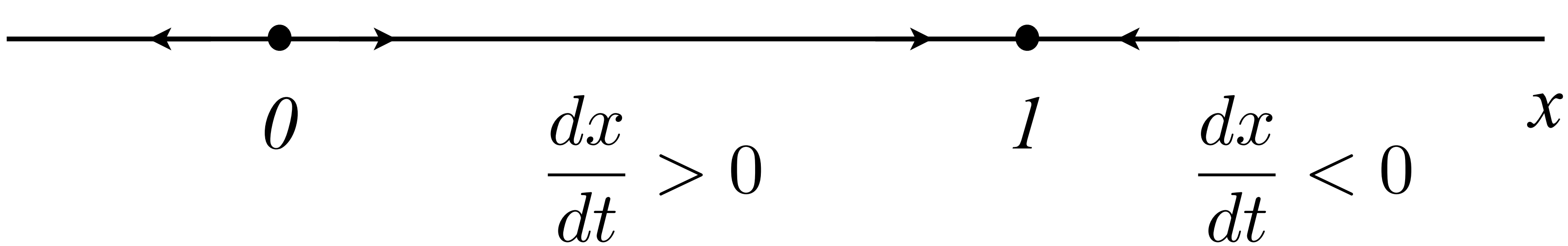
Represent geometrically in phase plane



vector field

Fixed points

$$\rho_F \frac{dF}{dt} = I_F - fE$$

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E$$

Fixed points

vector field

$$\begin{cases} \rho_F \dfrac{dF}{dt} = I_F - fE \\ \rho_L \dfrac{dL}{dt} = I_L - (1-f)E \end{cases}$$

Fixed points

Nullclines

vector field

$$\begin{cases} \rho_F \dfrac{dF}{dt} = I_F - fE = 0 & \text{L - Nullcline} \\ \rho_L \dfrac{dL}{dt} = I_L - (1-f)E = 0 & \text{F - Nullcline} \end{cases}$$

Fixed points

Nullclines

vector field

$$\rho_F \frac{dF}{dt} = I_F - fE = 0$$ L - Nullcline

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E = 0$$ F - Nullcline

$$E(F, L) = I$$

Fixed points

Nullclines

vector field

$$\rho_F \frac{dF}{dt} = I_F - fE = 0$$ L - Nullcline

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E = 0$$ F - Nullcline

$$E(F, L) = I$$ energy balance

Fixed points

Nullclines

vector field

$$\rho_F \frac{dF}{dt} = I_F - fE = 0$$

L - Nullcline

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E = 0$$

F - Nullcline

$$E(F,L) = I$$

energy balance

$$f(F,L) = \frac{I_F}{I}$$

Fixed points

Nullclines

vector field

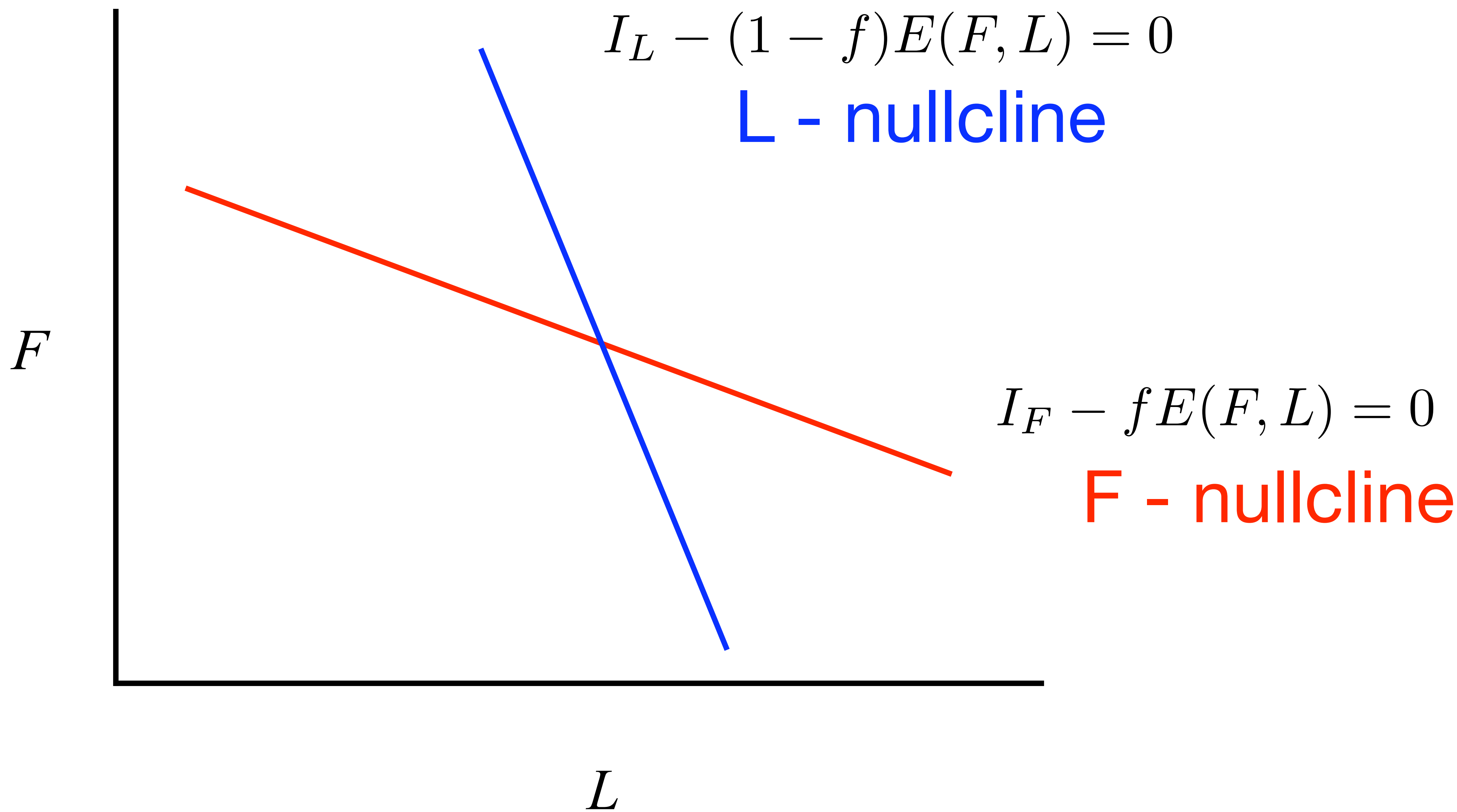
$$\rho_F \frac{dF}{dt} = I_F - fE = 0$$ L - Nullcline

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E = 0$$ F - Nullcline

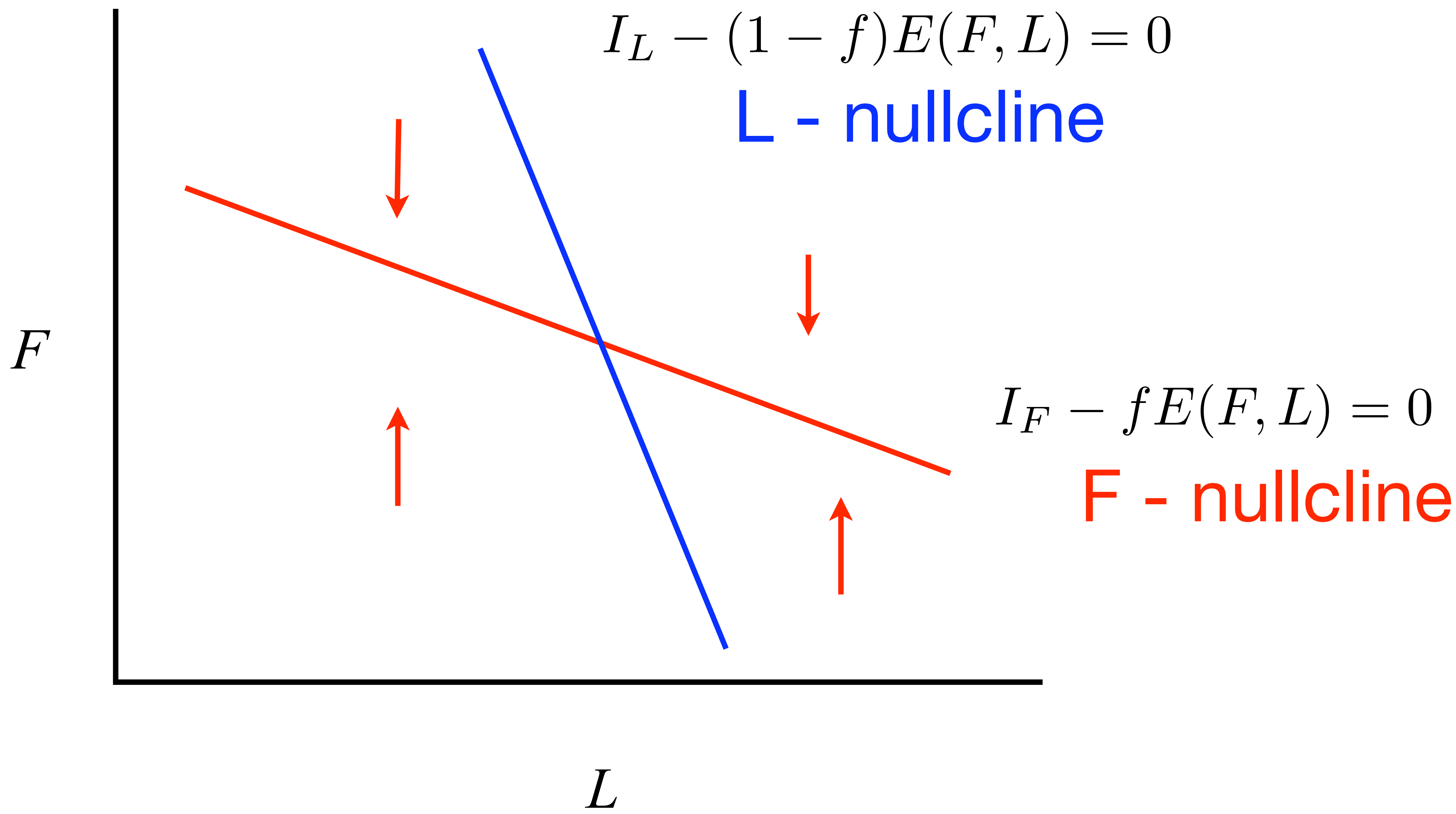
$$E(F,L) = I$$ energy balance

$$f(F,L) = \frac{I_F}{I}$$ fat balance

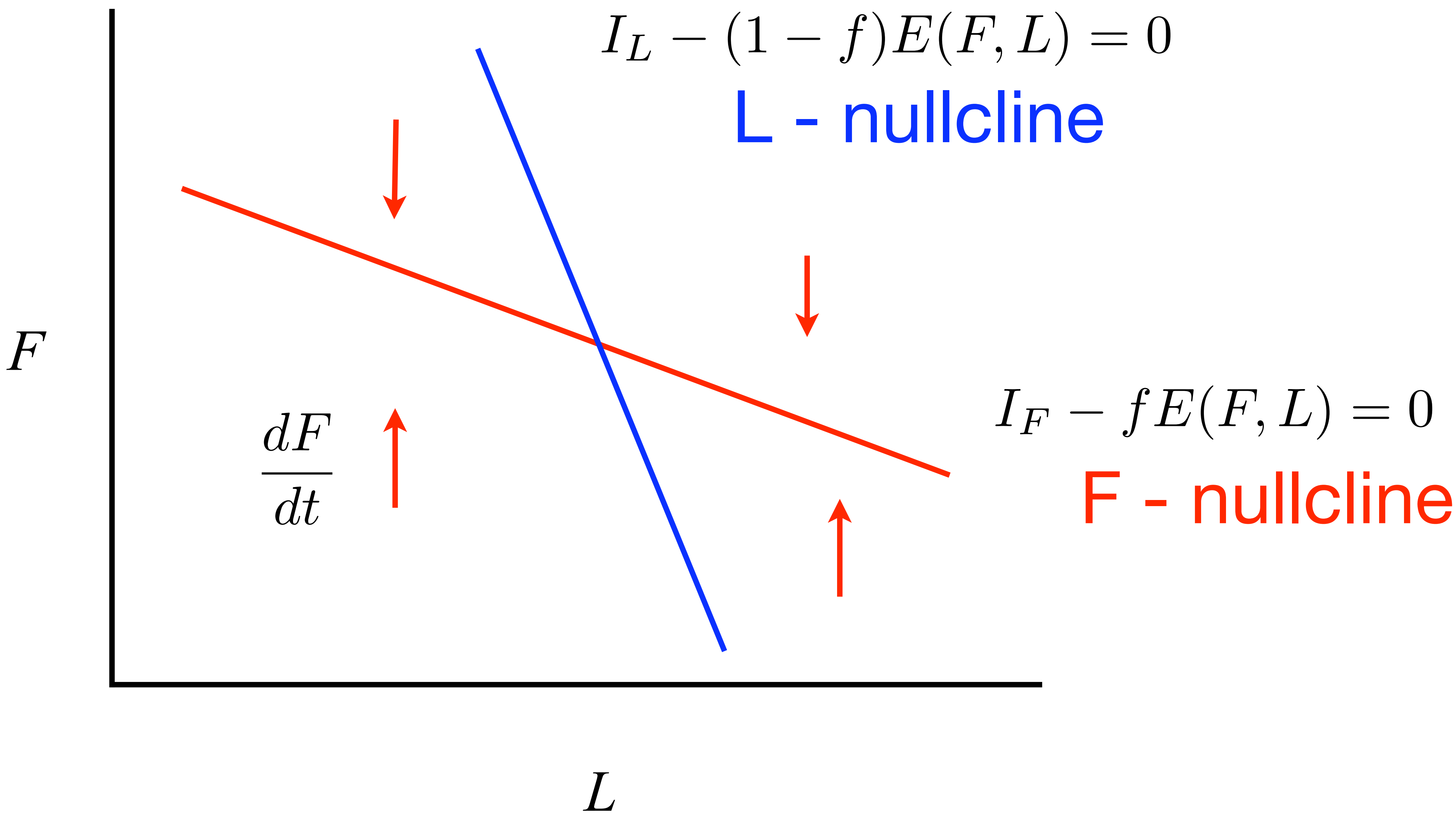
Phase plane



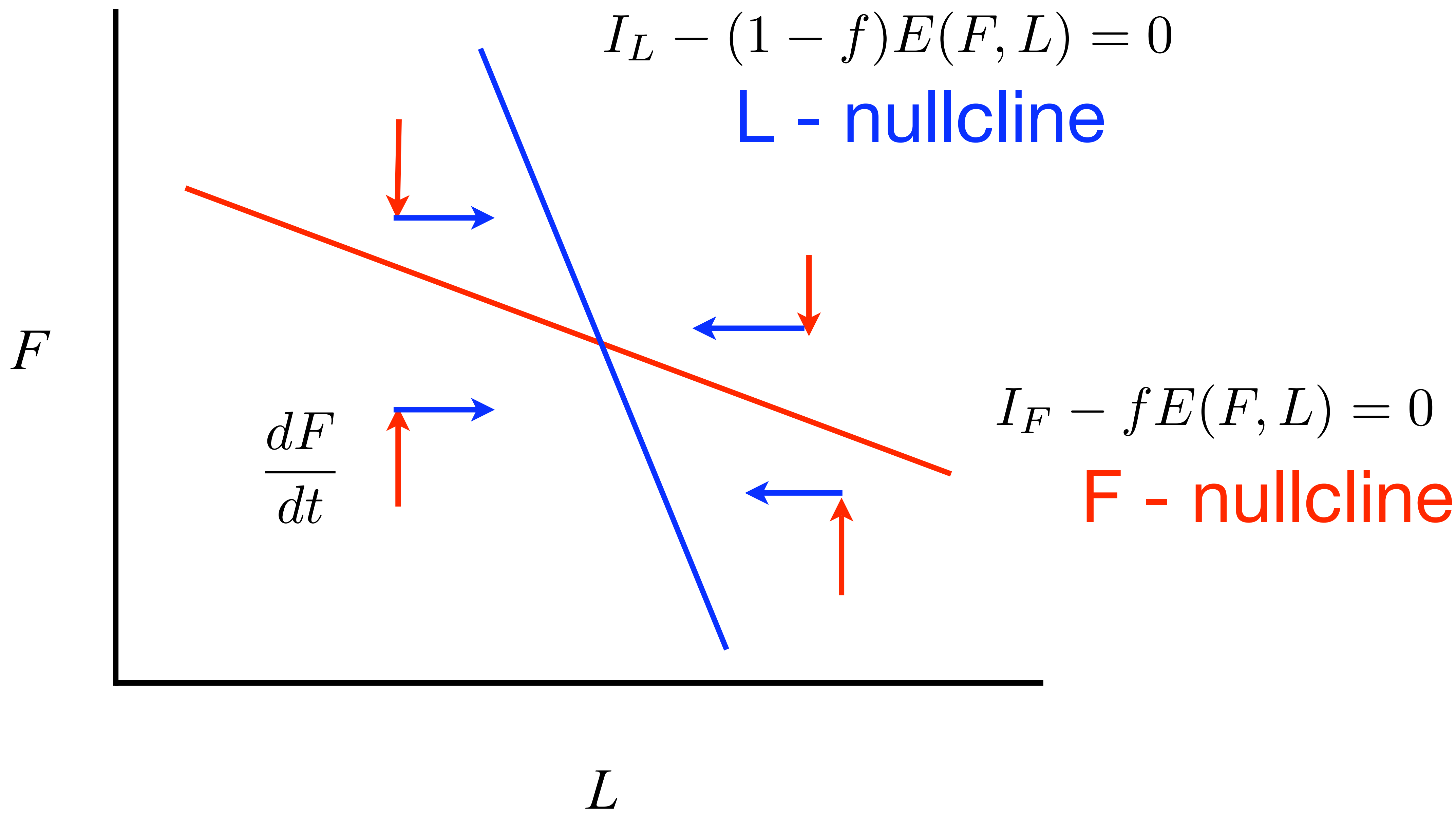
Phase plane



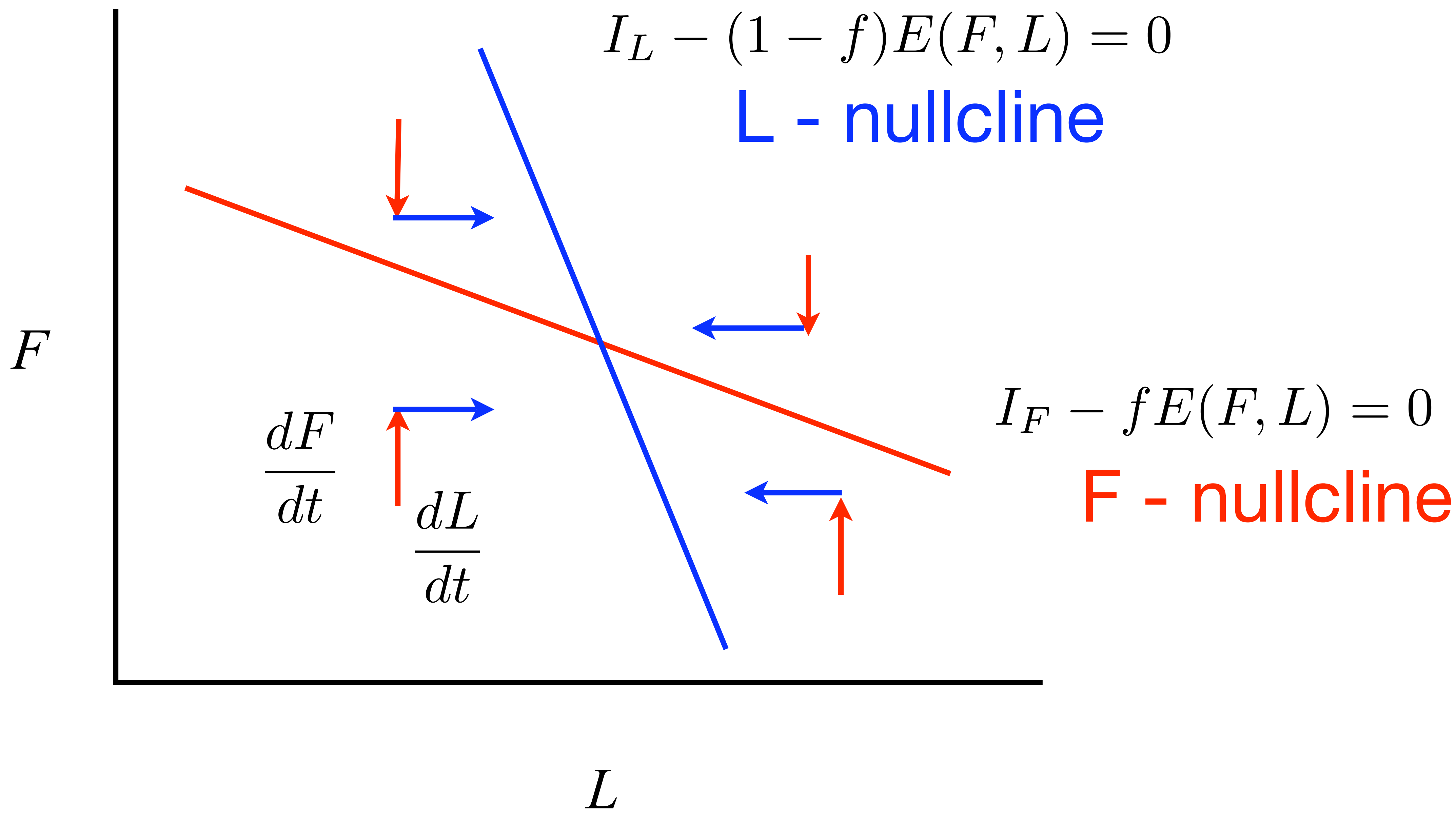
Phase plane



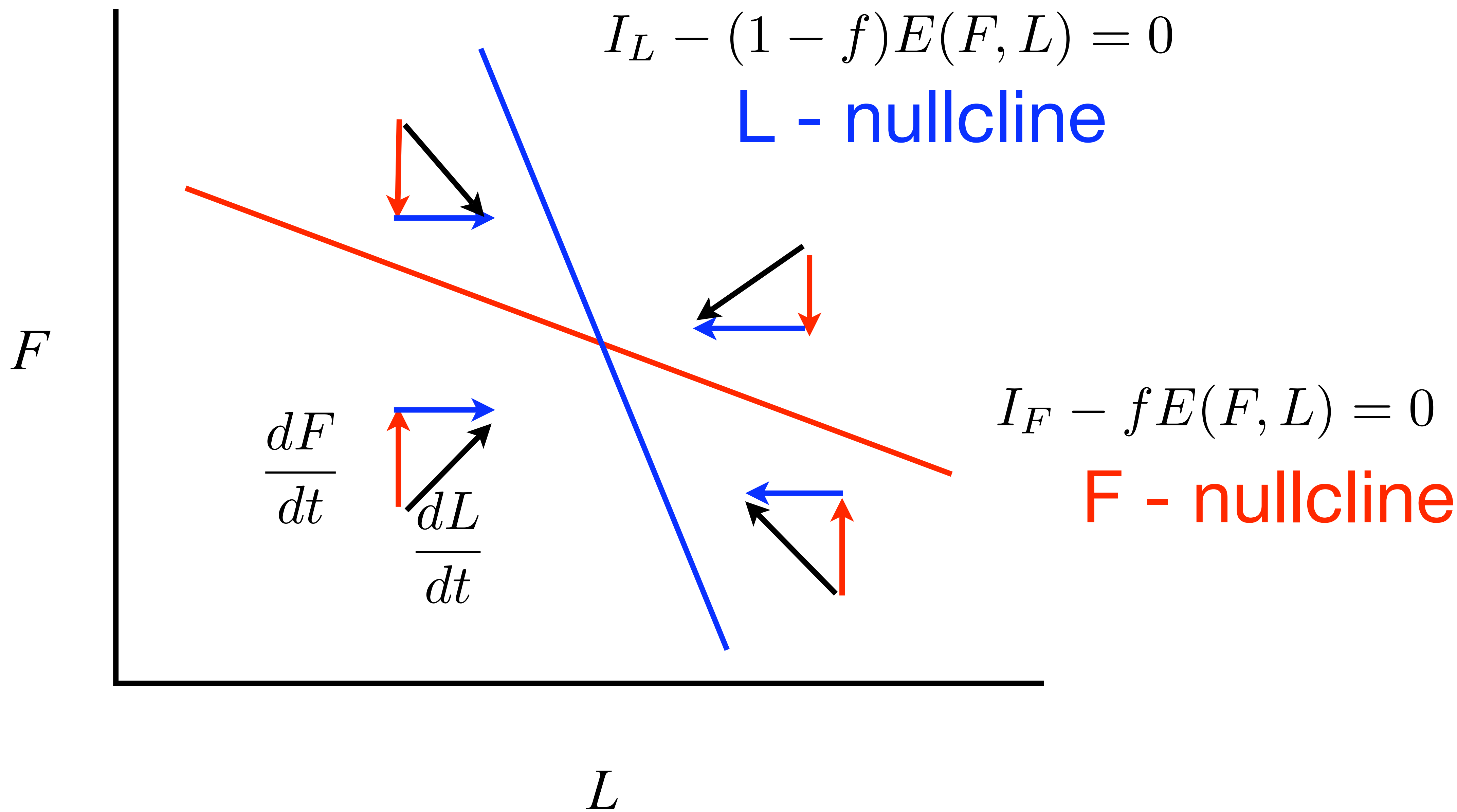
Phase plane



Phase plane


$I_L - (1-f)E(F,L) = 0$
L - nullcline
$I_F - fE(F,L) = 0$
F - nullcline
F
$\frac{dF}{dt}$
$\frac{dL}{dt}$
L

Phase plane



$I_L - (1-f)E(F,L) = 0$
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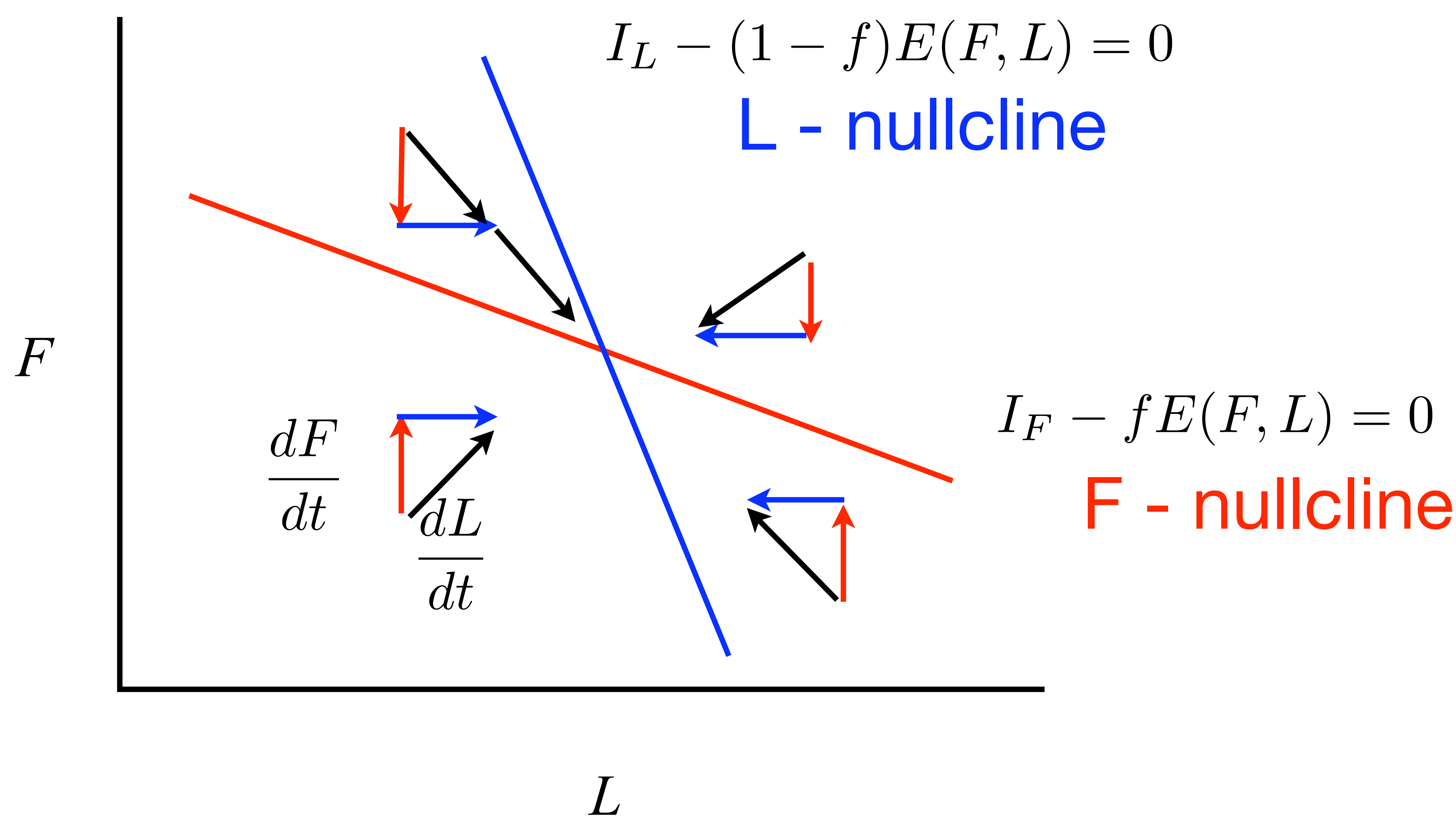
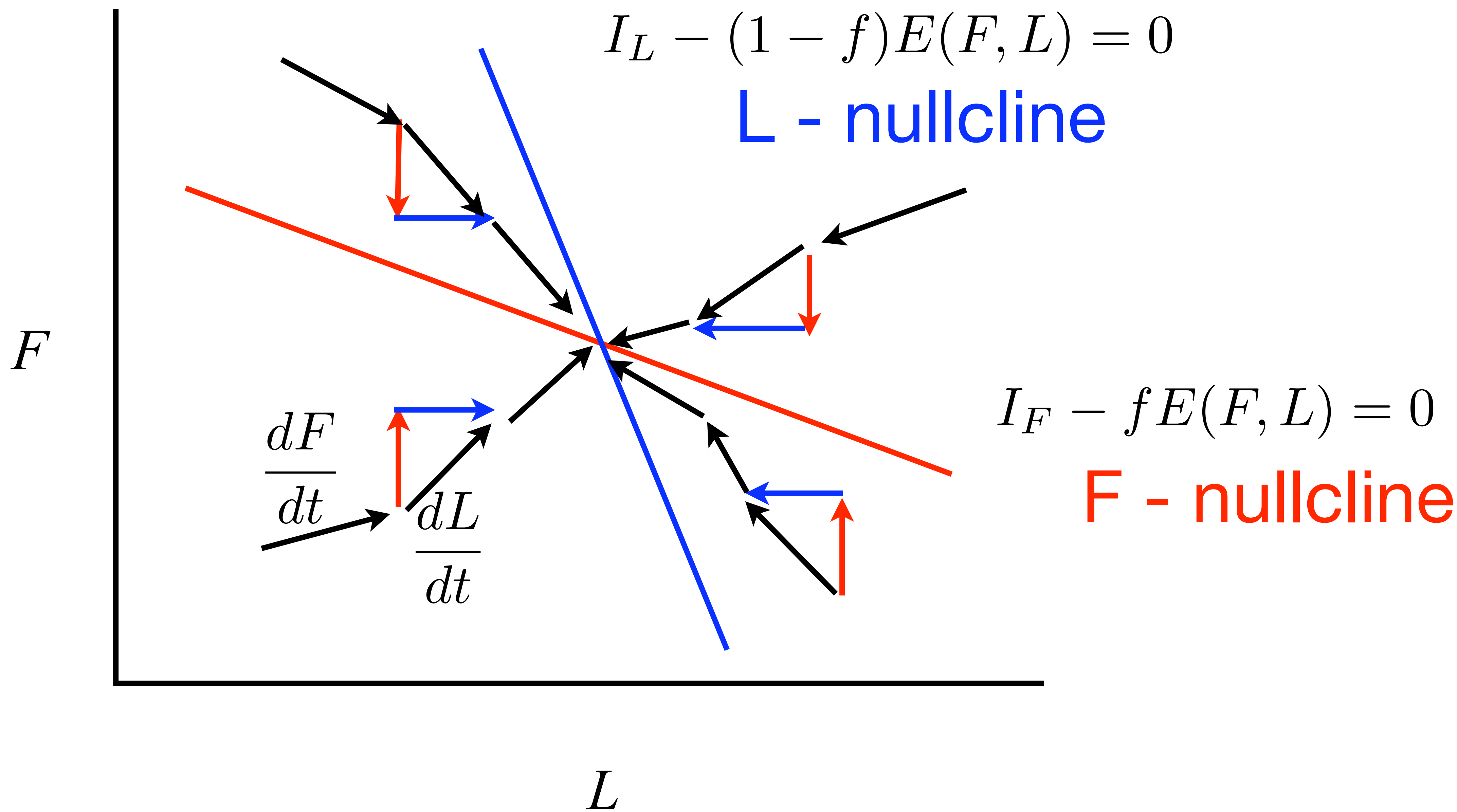
$I_F - fE(F,L) = 0$
F - nullcline
F
$\frac{dF}{dt}$
$\frac{dL}{dt}$
L

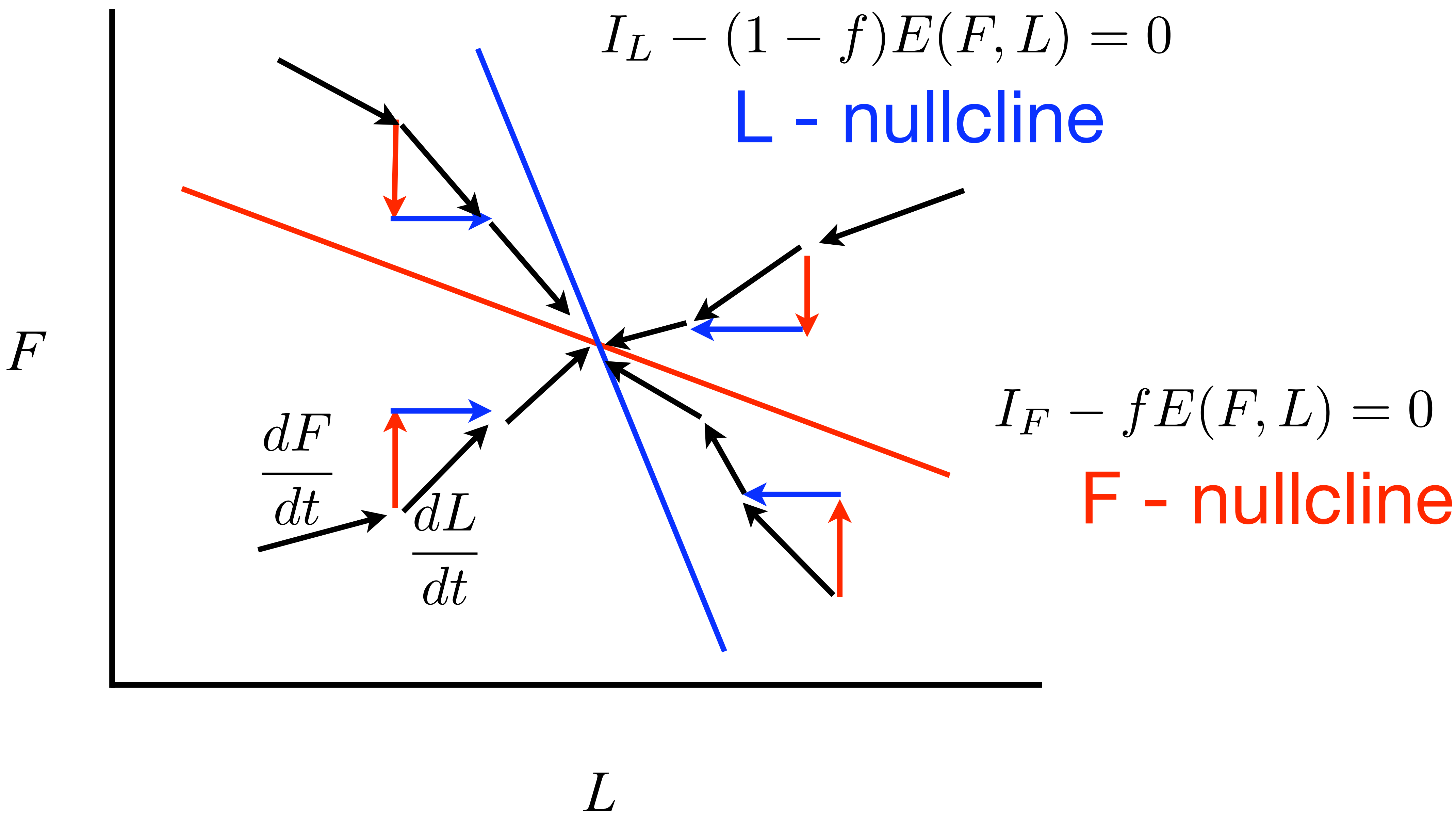
Phase plane


$I_L - (1-f)E(F,L) = 0$
L - nullcline
$I_F - fE(F,L) = 0$
F - nullcline
F
L
$\frac{dF}{dt}$
$\frac{dL}{dt}$

Phase plane

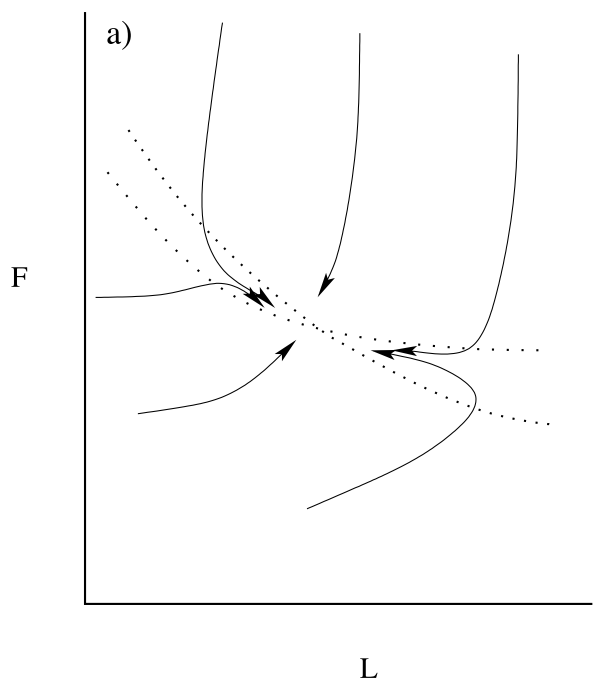

$I_L - (1-f)E(F,L) = 0$
L - nullcline
$I_F - fE(F,L) = 0$
F - nullcline
F
$\frac{dF}{dt}$
$\frac{dL}{dt}$
L

Phase plane

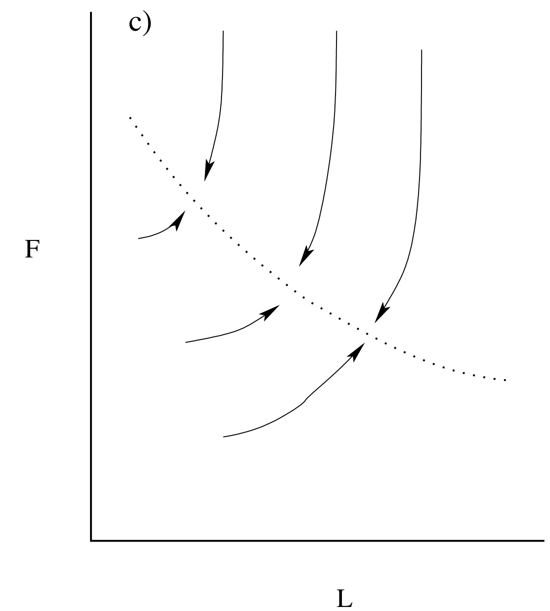


Fate of all initial conditions is known

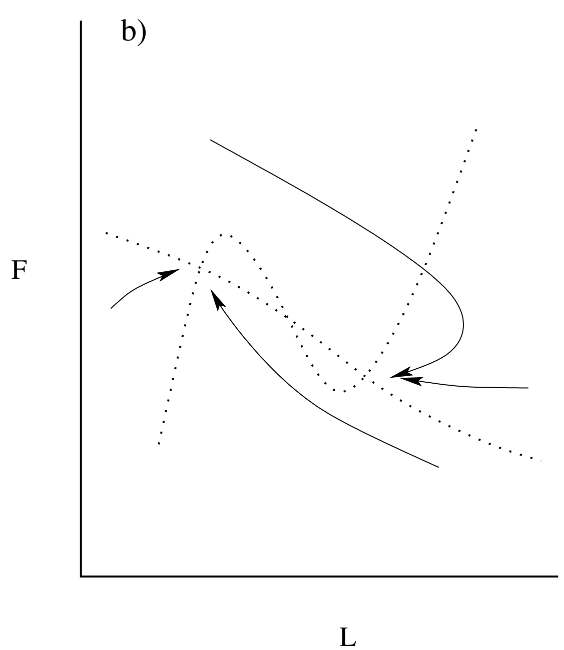
Possible phase plane dynamics



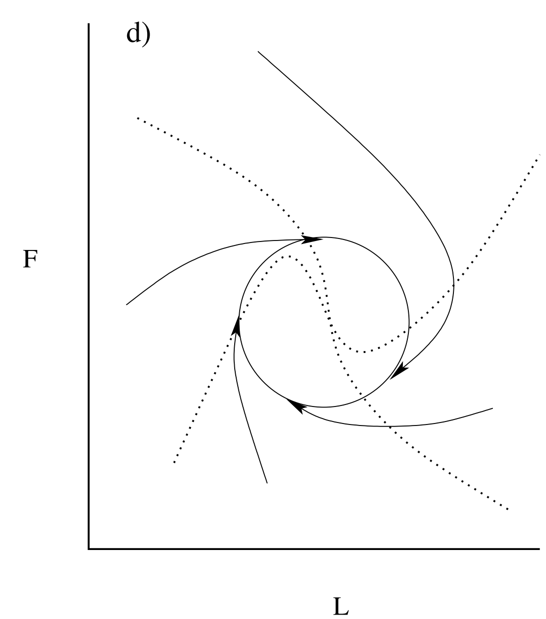
Fixed point



Line attractor



Multiple fixed points



Limit cycle

Chow and Hall, PLoS Comp Bio,4: e1000045, 2008

Indirect calorimetry

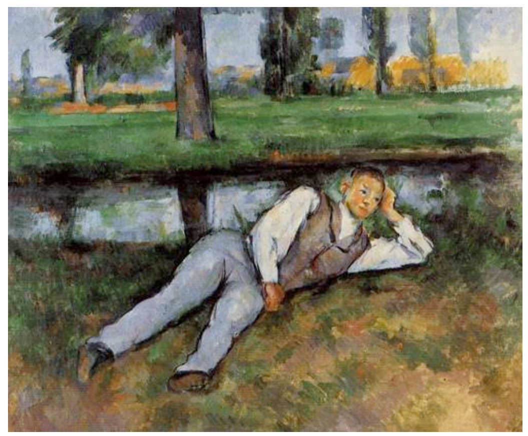
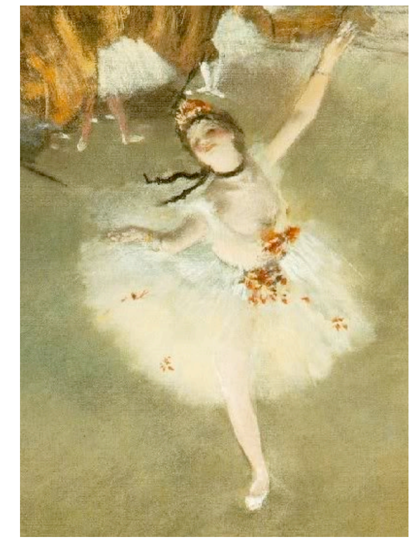
Carbohydrates: $C_6H_{12}O_6 + 6\ O_2 \rightarrow 6\ CO_2 + 6\ H_2O$

Fat: $C_{16}H_{32}O_2 + 23\ O_2 \rightarrow 16\ CO_2 + 16\ H_2O$

Protein: $C_{72}H_{112}N_2O_{22}S + 77\ O_2 \rightarrow 63\ CO_2 + 38\ H_2O + SO_3 + 9\ CO(NH_2)_2$

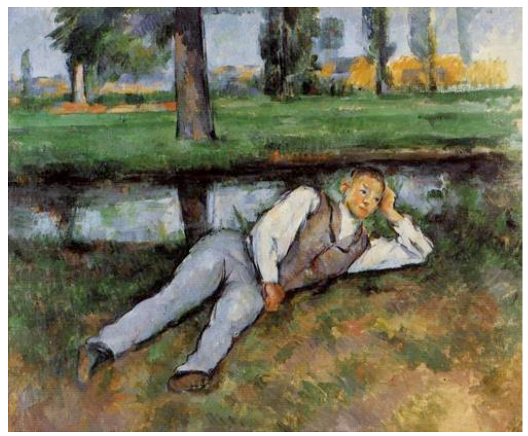
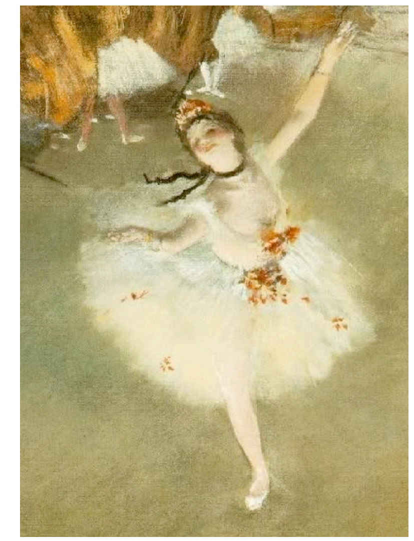
Flux of CO_2 and $O_2 \Rightarrow E$

Energy expenditure rate E

$E =$  $+$ 

Basal metabolic rate (BMR) Physical activity

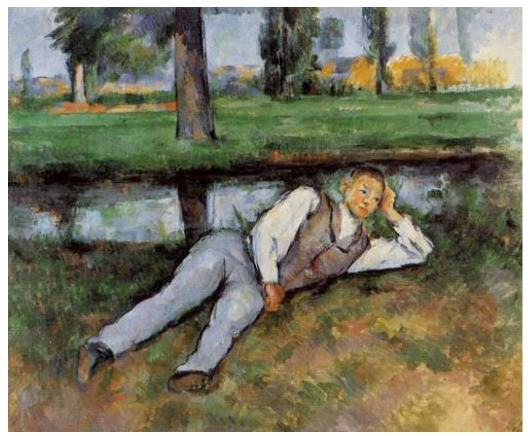
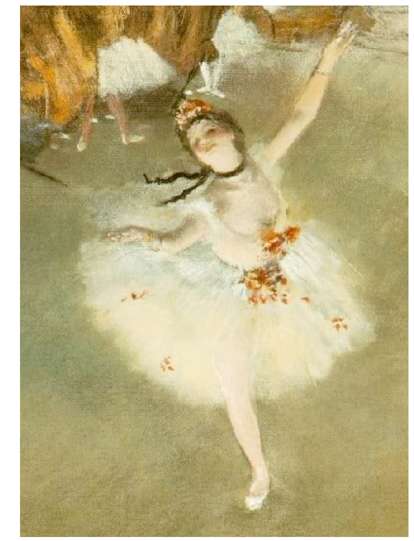
Energy expenditure rate E

$E =$  $+$ 

Basal metabolic rate (BMR) Physical activity

$E \sim$ 10 MJ/day

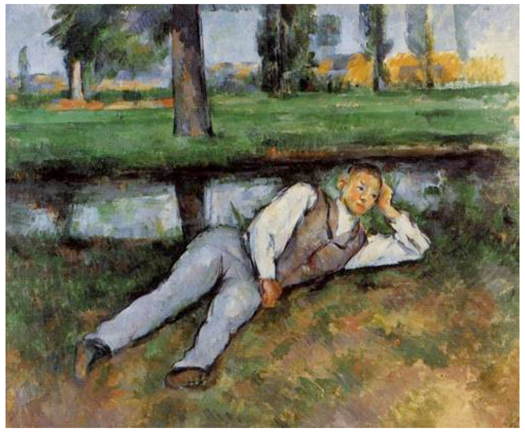
Energy expenditure rate E

$E =$  + 

Basal metabolic rate (BMR) Physical activity

E ~ 10 MJ/day ~115 W

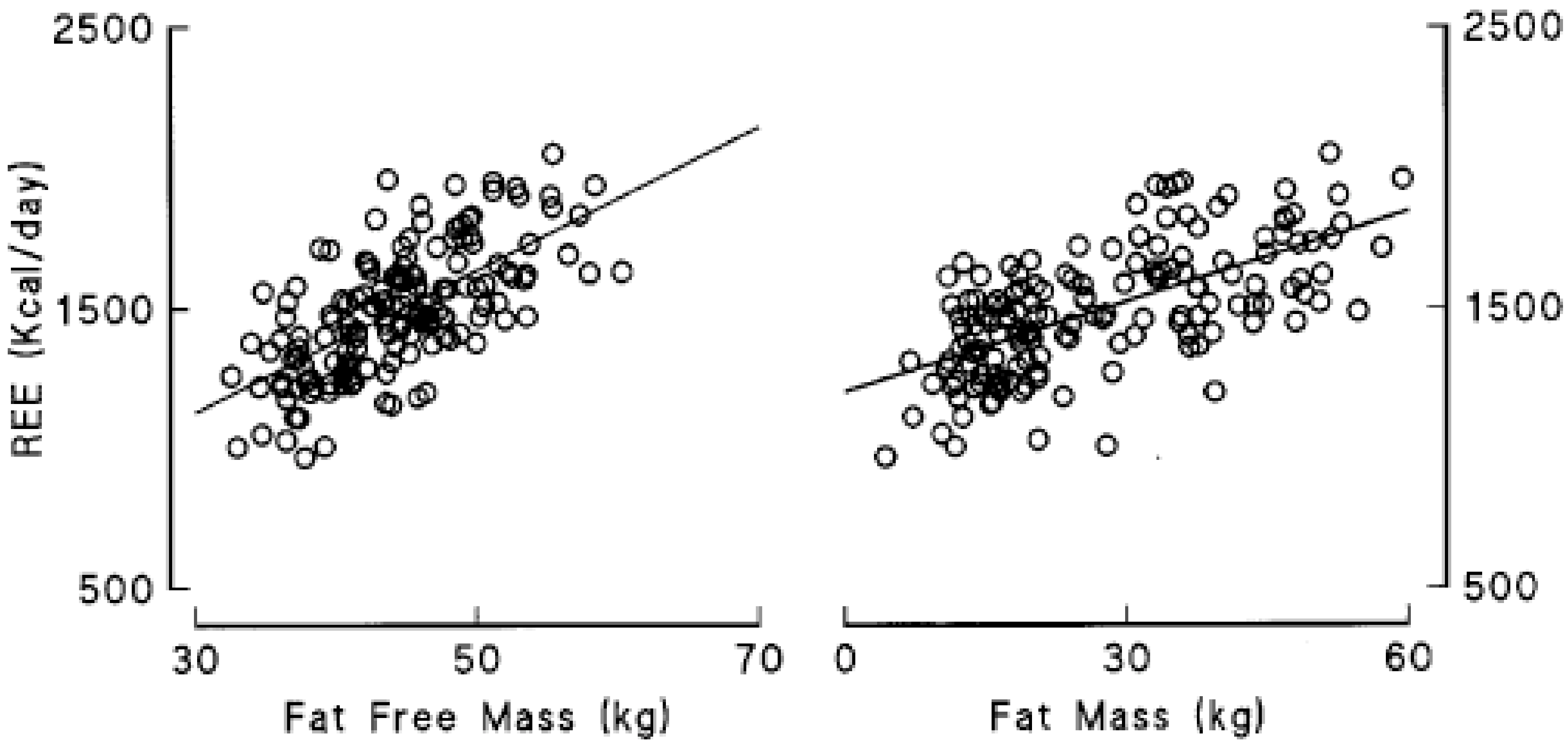
Energy expenditure rate E

$E =$  $+$ 

Basal metabolic rate (BMR) Physical activity

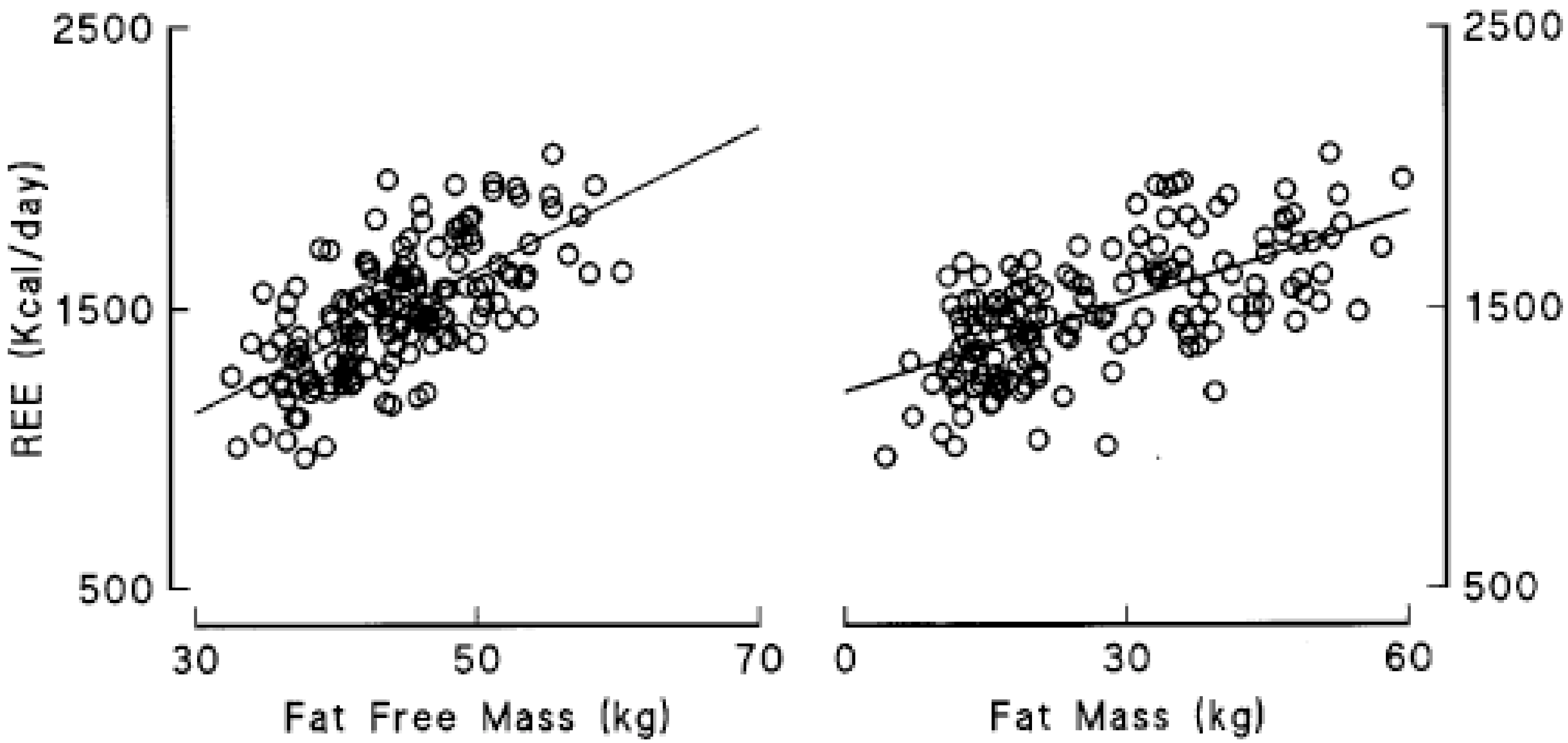
E ~ 10 MJ/day ~115 W ~ 3 KWH/day

Basal metabolic rate



Nielson, 2000

Basal metabolic rate



Nielson, 2000

e.g. $BMR\ (MJ/day) = 0.9\ L\ (kg) + 0.01\ F\ (kg) + 1.1$

Physical activity

Energy due to PA $\propto$ Mass

$$E_{PA} = aM = a(L + F)$$

a ranges from *0* to *0.1 MJ/kg/day*

Physical activity

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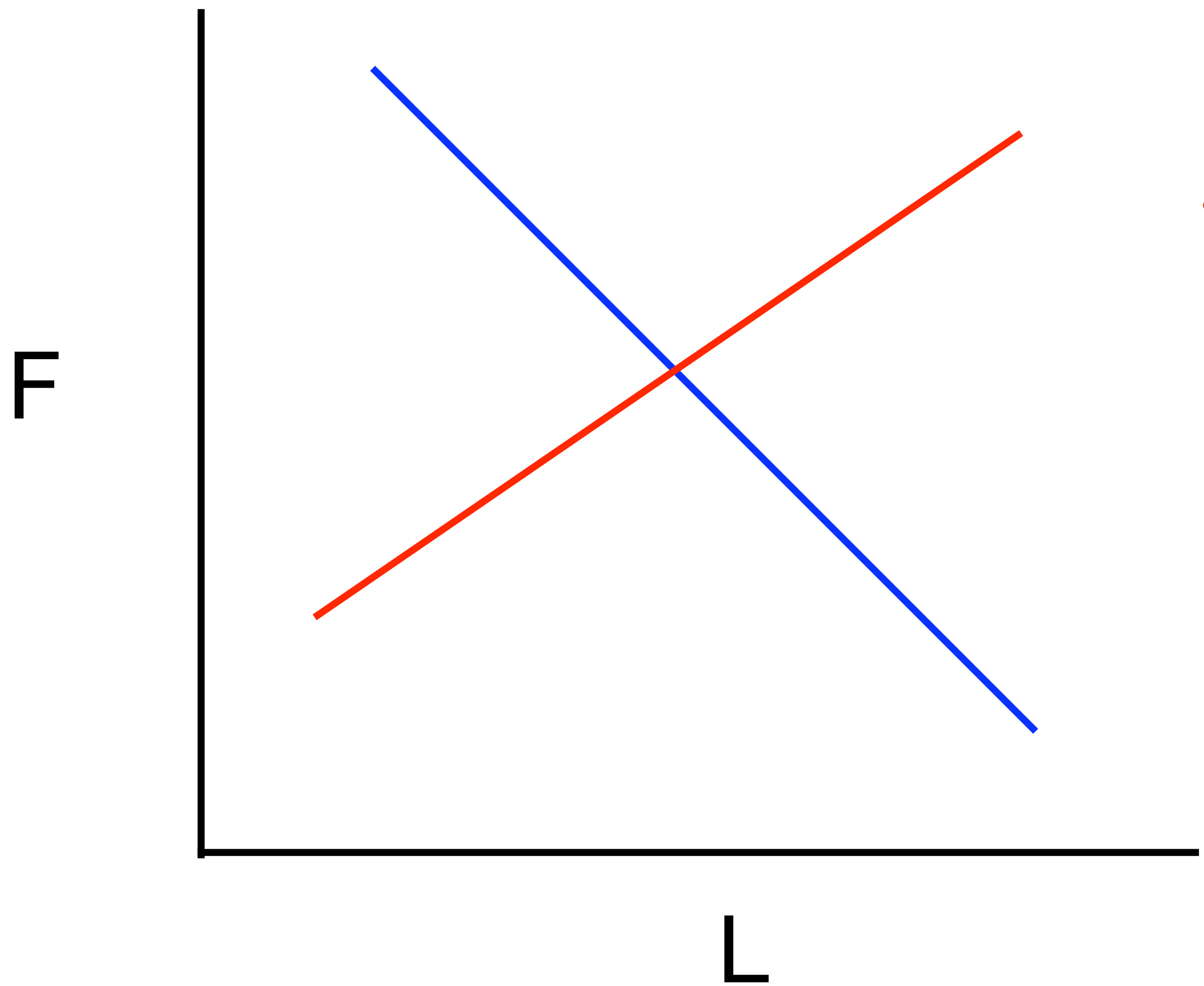
a ranges from *0* to *0.1 MJ/kg/day*

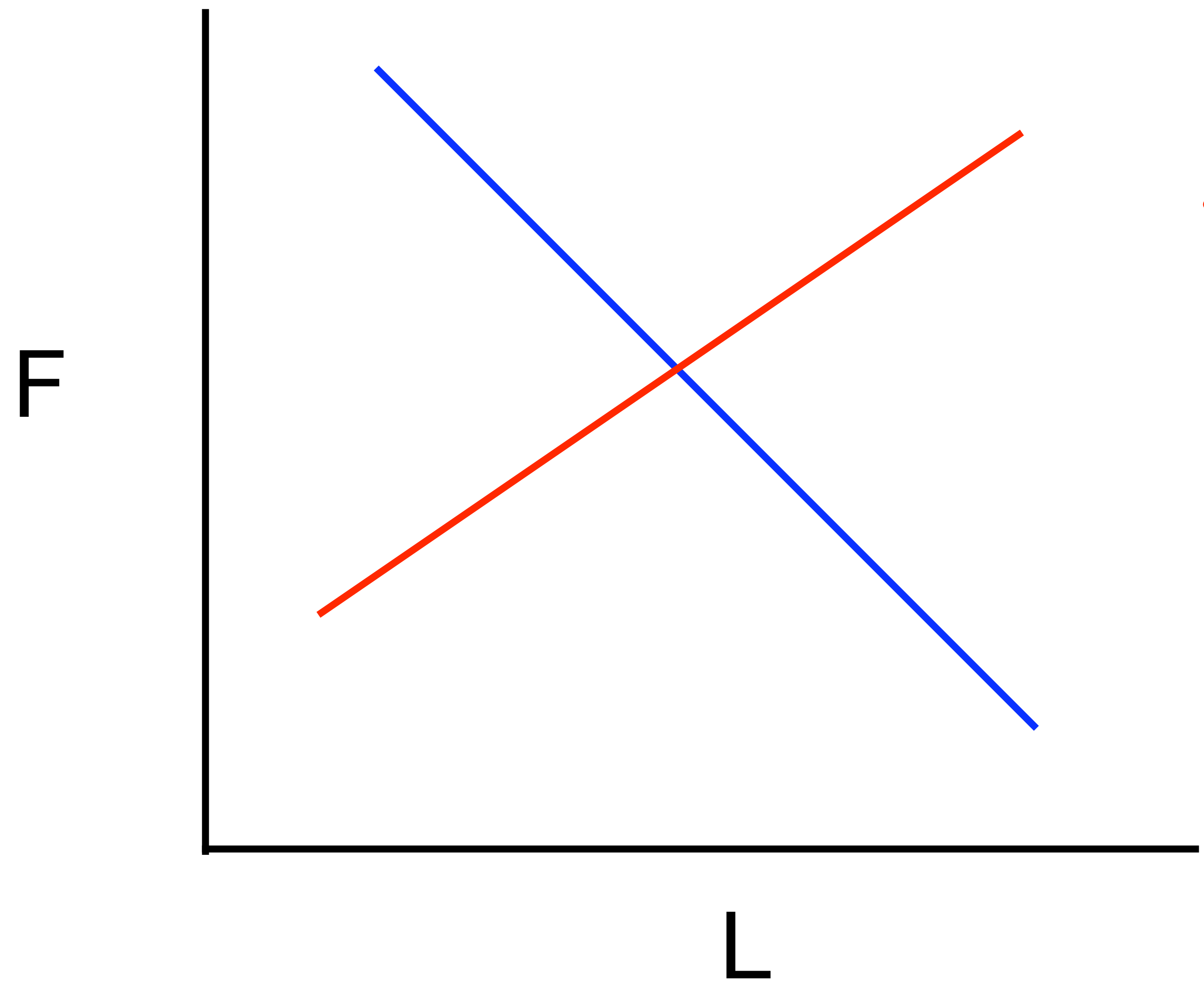
$\therefore$ E is linear in F and L

$$E(F,L) = bF + cL + d = I$$



$$f(F,L) = \frac{I_F}{I}$$

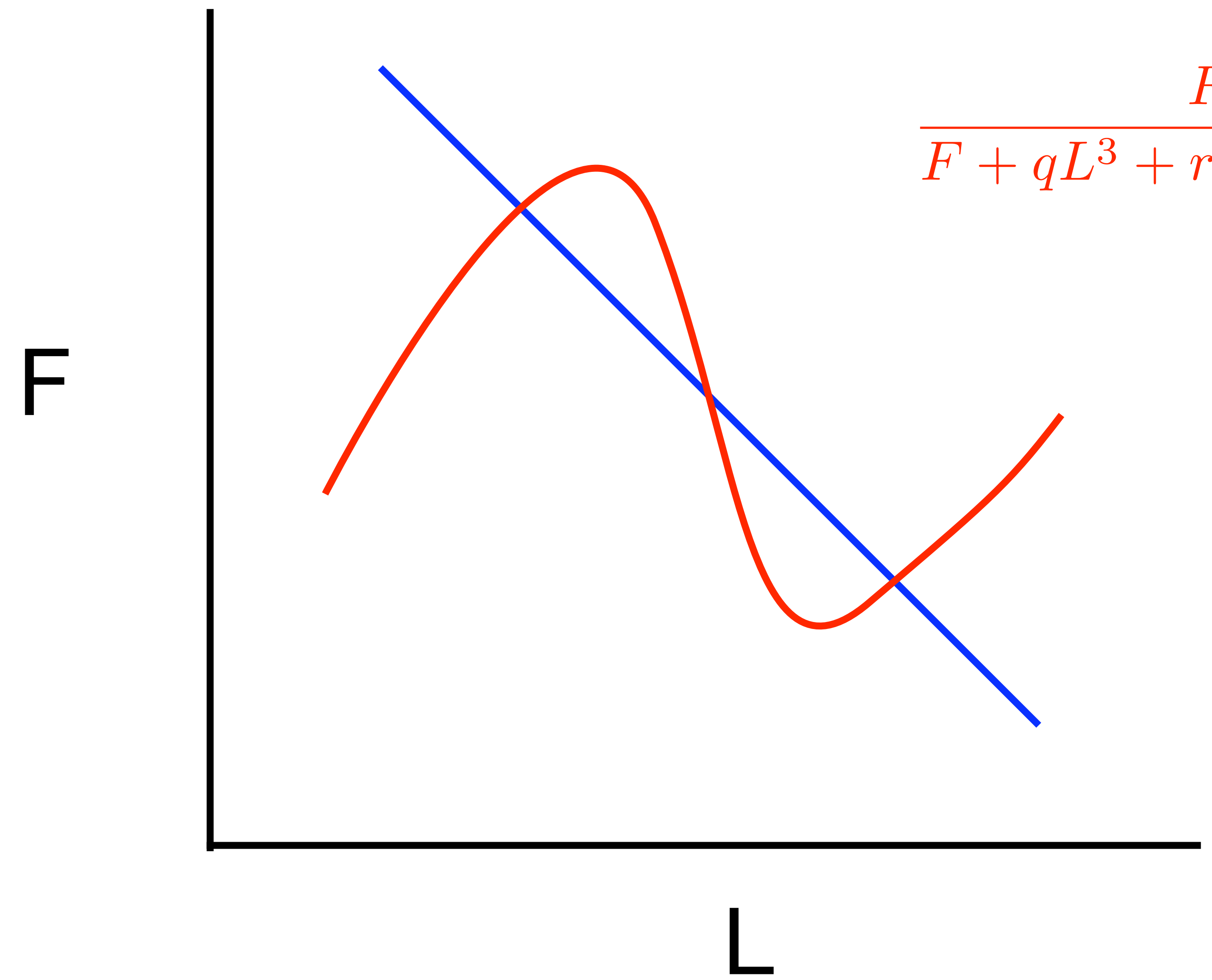
$$E(F, L) = bF + cL + d = I$$



$$f(F, L) = \frac{I_F}{I}$$

Single fixed point is generic

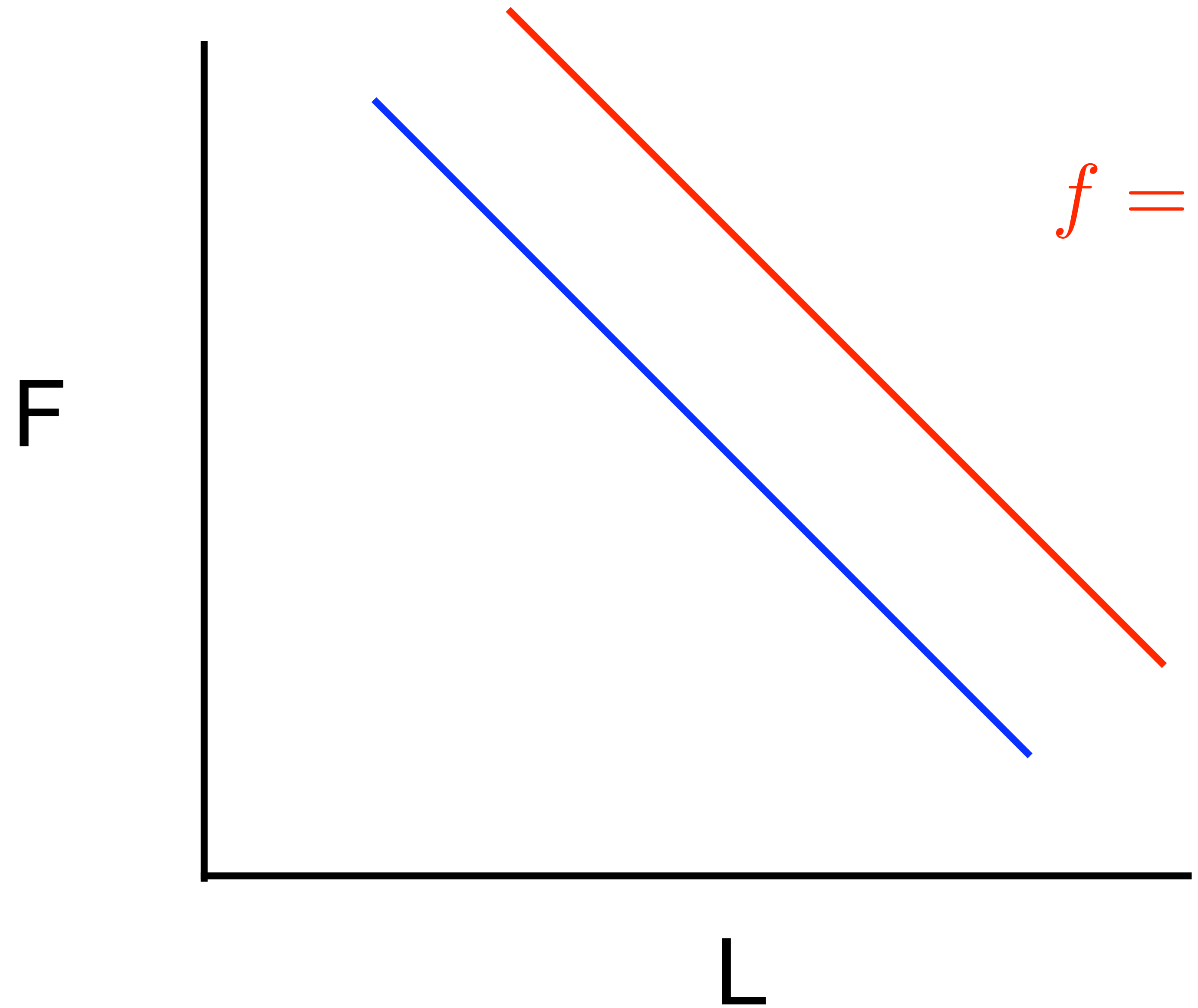
$$E(F, L) = bF + cL + d = I$$



$$\frac{F}{F + qL^3 + rL^2 + sL + t} = \frac{I_F}{E}$$

Multi-stability or limit cycle requires fine tuning

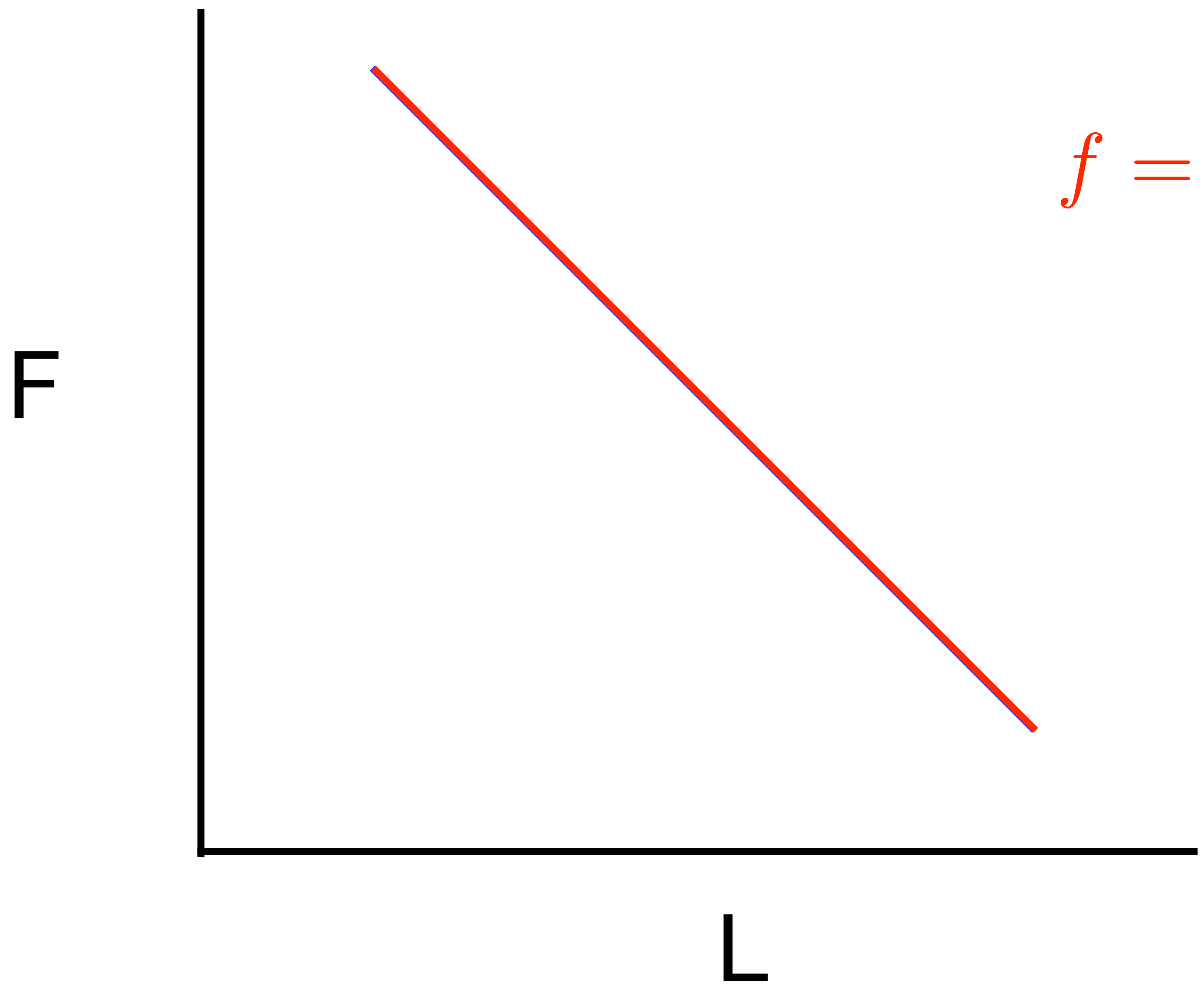
$$E(F, L) = bF + cL + d = I$$



$$f = \frac{I_F}{E} + \psi(I - E)$$

Line attractor requires special form

$$E(F, L) = bF + cL + d = I$$



$$f = \frac{I_F}{E} + \psi(I - E)$$

Line attractor requires special form

The problem with f

In energy balance, f reflects diet

The problem with f

In energy balance, f reflects diet $$f(F, L) = \frac{I_F}{I}$$

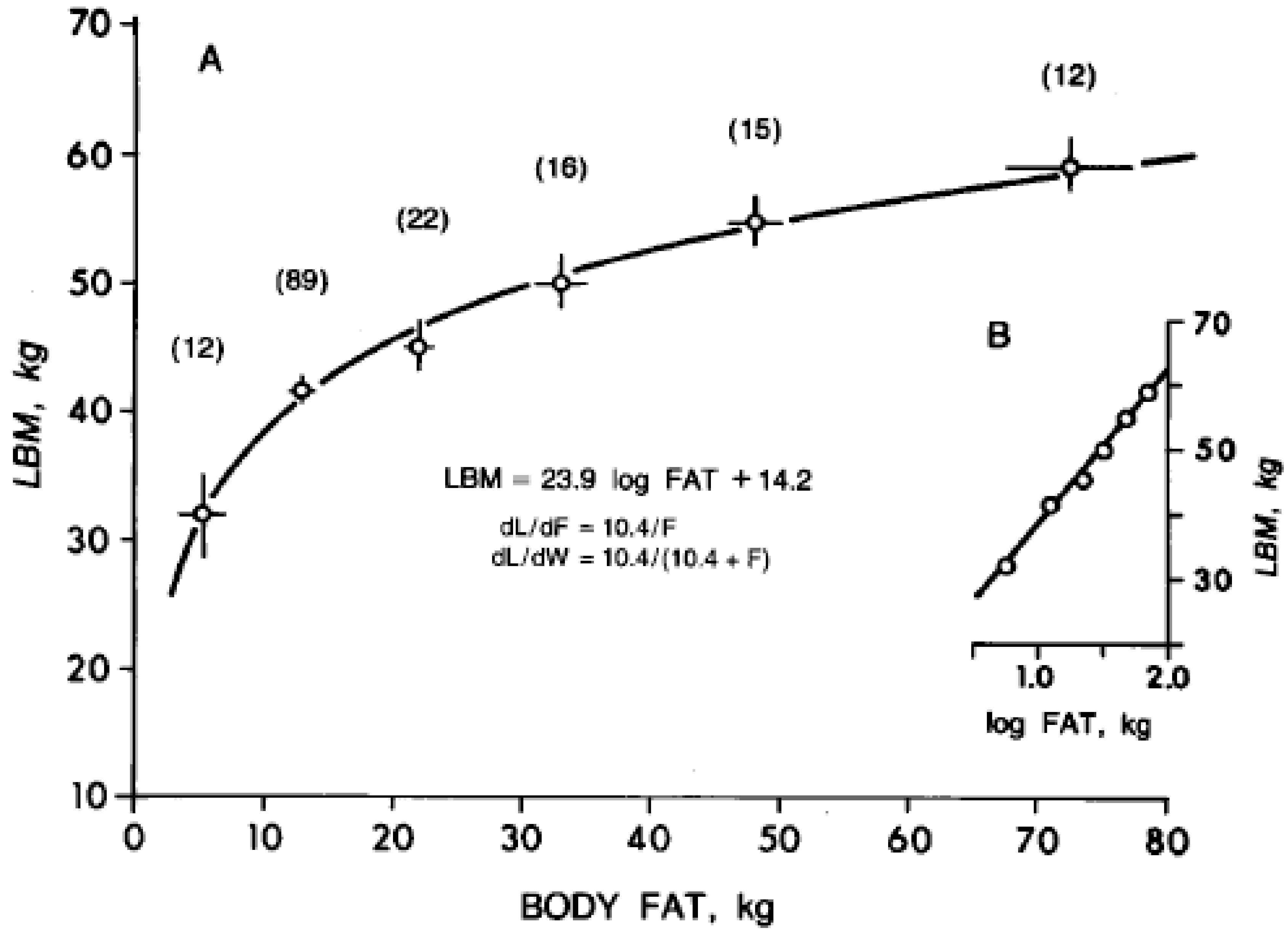
The problem with f

In energy balance, f reflects diet $\quad f(F,L) = \frac{I_F}{I}$

Must invert in dynamic situation

$$\begin{aligned}
\rho_F \frac{dF}{dt} &= I_F - fE \\
\rho_L \frac{dL}{dt} &= I_L - (1-f)E
\end{aligned}$$

Body composition



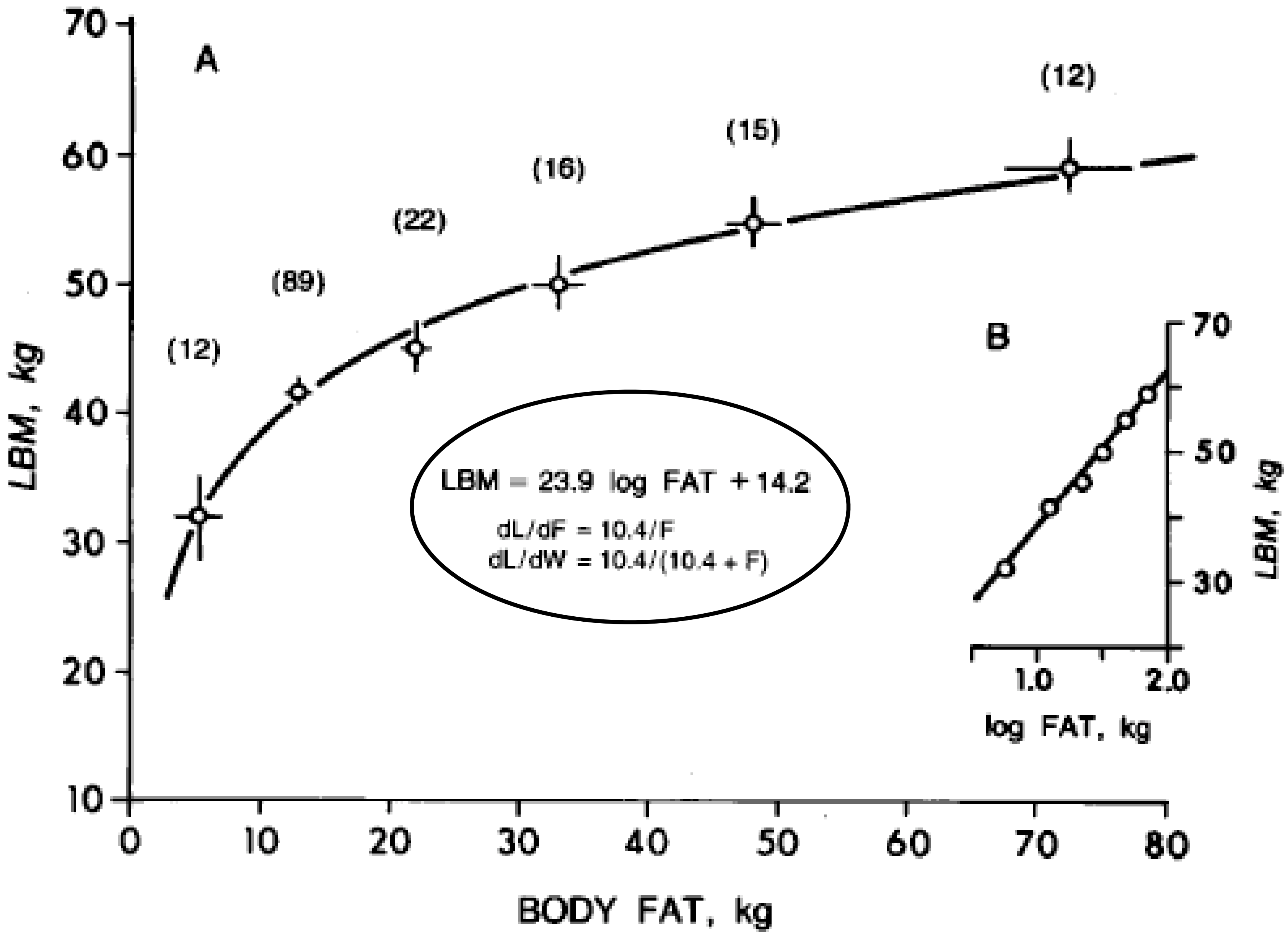
L

F

Forbes, 2000

Body composition

L



F

Forbes, 2000

Apply Forbes law to model

$$\frac{dF}{dL} = \frac{F}{10.4}$$

$$\rho_F \frac{dF}{dt} = (I_F - fE)$$

$$\rho_L \frac{dL}{dt} = (I_L - (1-f)E)$$

Apply Forbes law to model

$$\frac{dF}{dL} = \frac{F}{10.4}$$

$$dF = (I_F - fE)\,\frac{dt}{\rho_F}$$

$$\rho_L\,\frac{dL}{dt} = (I_L - (1-f)E)$$

Apply Forbes law to model

$$\frac{dF}{dL} = \frac{F}{10.4}$$

$$dF = (I_F - fE)\frac{dt}{\rho_F}$$

$$dL = (I_L - (1-f)E)\frac{dt}{\rho_L}$$

Apply Forbes law to model

$$\frac{dF}{dL} = \frac{F}{10.4}$$

$$\frac{dF}{dL} = \frac{(I_F - fE)\,\rho_L}{(I_L - (1-f)E)\rho_F}$$

Apply Forbes law to model

$$\frac{(I_F - fE)\,\rho_L}{(I_L - (1-f)E)\rho_F} = \frac{F}{10.4}$$

Apply Forbes law to model

$$\frac{(I_F - fE)\,\rho_L}{(I_L - (1-f)E)\rho_F} = \frac{F}{10.4}$$

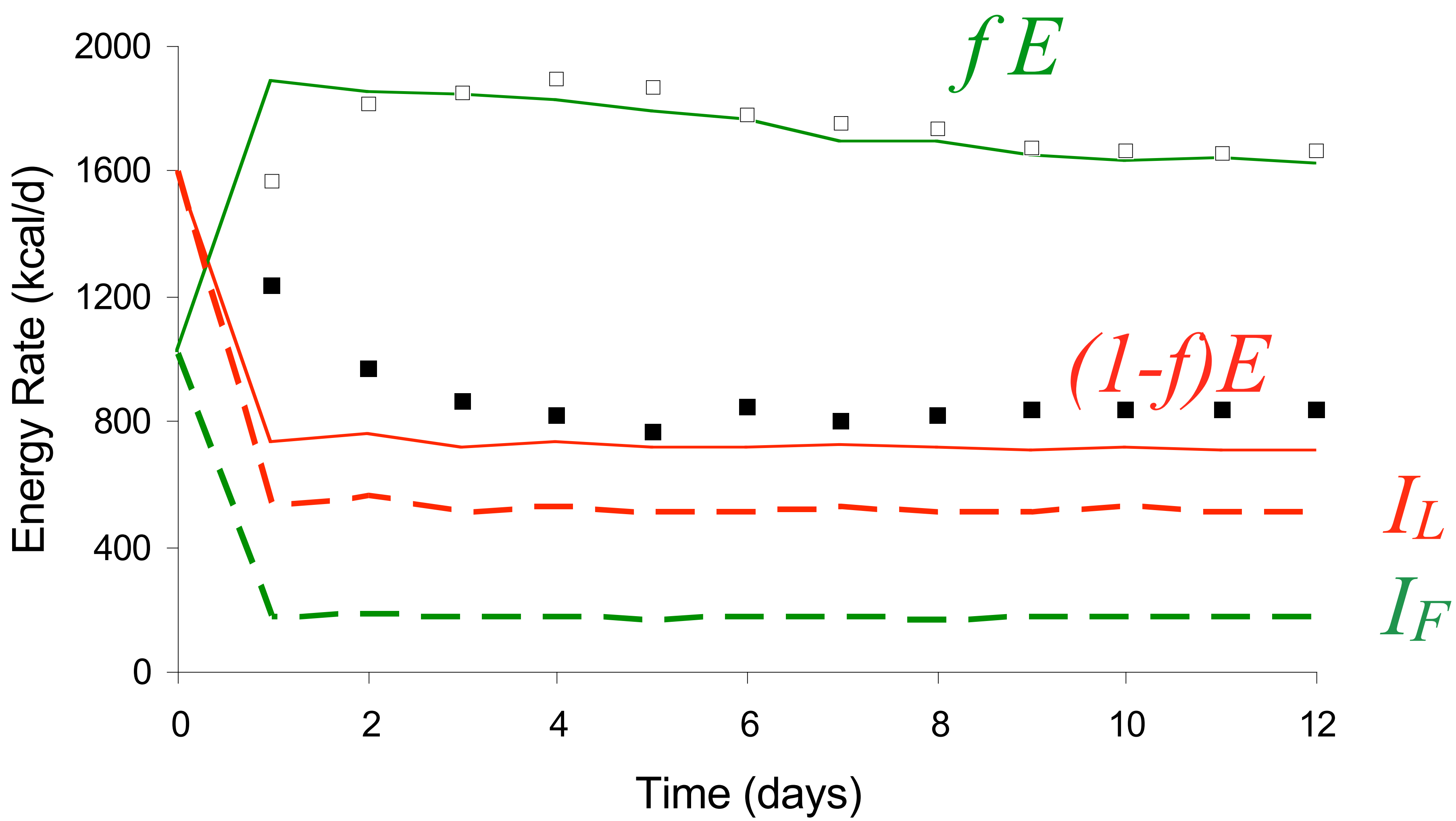
$$f = \frac{I_F - (1-p)(I-E)}{E} \qquad p = \frac{1}{1 + \frac{\rho_F}{\rho_L}\frac{F}{10.4}}$$

Apply Forbes law to model

$$\frac{(I_F - fE)\,\rho_L}{(I_L - (1-f)E)\rho_F} = \frac{F}{10.4}$$

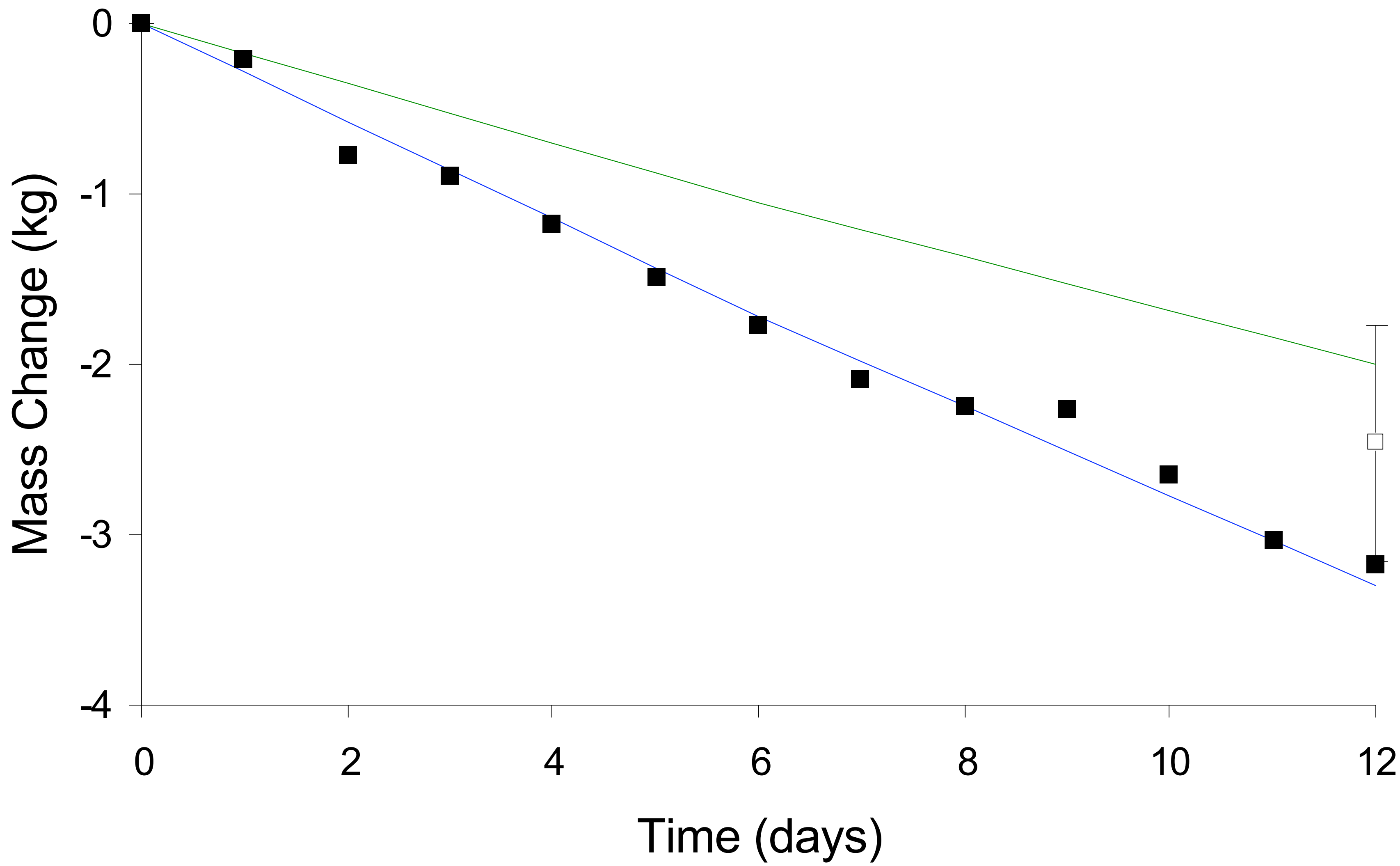
$$f = \frac{I_F - (1-p)(I-E)}{E} \qquad p = \frac{1}{1 + \frac{\rho_F}{\rho_L}\frac{F}{10.4}}$$

Matches data



Hall, Bain, and Chow, Int J. Obesity, (2007)

Weight and fat loss



Hall, Bain, and Chow, Int J. Obesity, (2007)

Energy partition model

$$\rho_F \frac{dF}{dt} = I_F - fE$$

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E$$

$$f = \frac{I_F - (1-p)(I-E)}{E}$$

Energy partition model

$$\rho_F \frac{dF}{dt} = I_F - \frac{I_F - (1-p)(I-E)}{E} E$$

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E$$

Energy partition model

$$\rho_F \frac{dF}{dt} = I_F - I_F + (1-p)(I-E)$$

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E$$

Energy partition model

$$\rho_F \frac{dF}{dt} = (1-p)(I-E)$$

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Energy partition model

$$\rho_F \frac{dF}{dt} = (1-p)(I-E)$$

$$\rho_L \frac{dL}{dt} = p(I-E)$$

Energy partition model

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Steady state is line attractor $E(F, L) = I$

Energy partition model

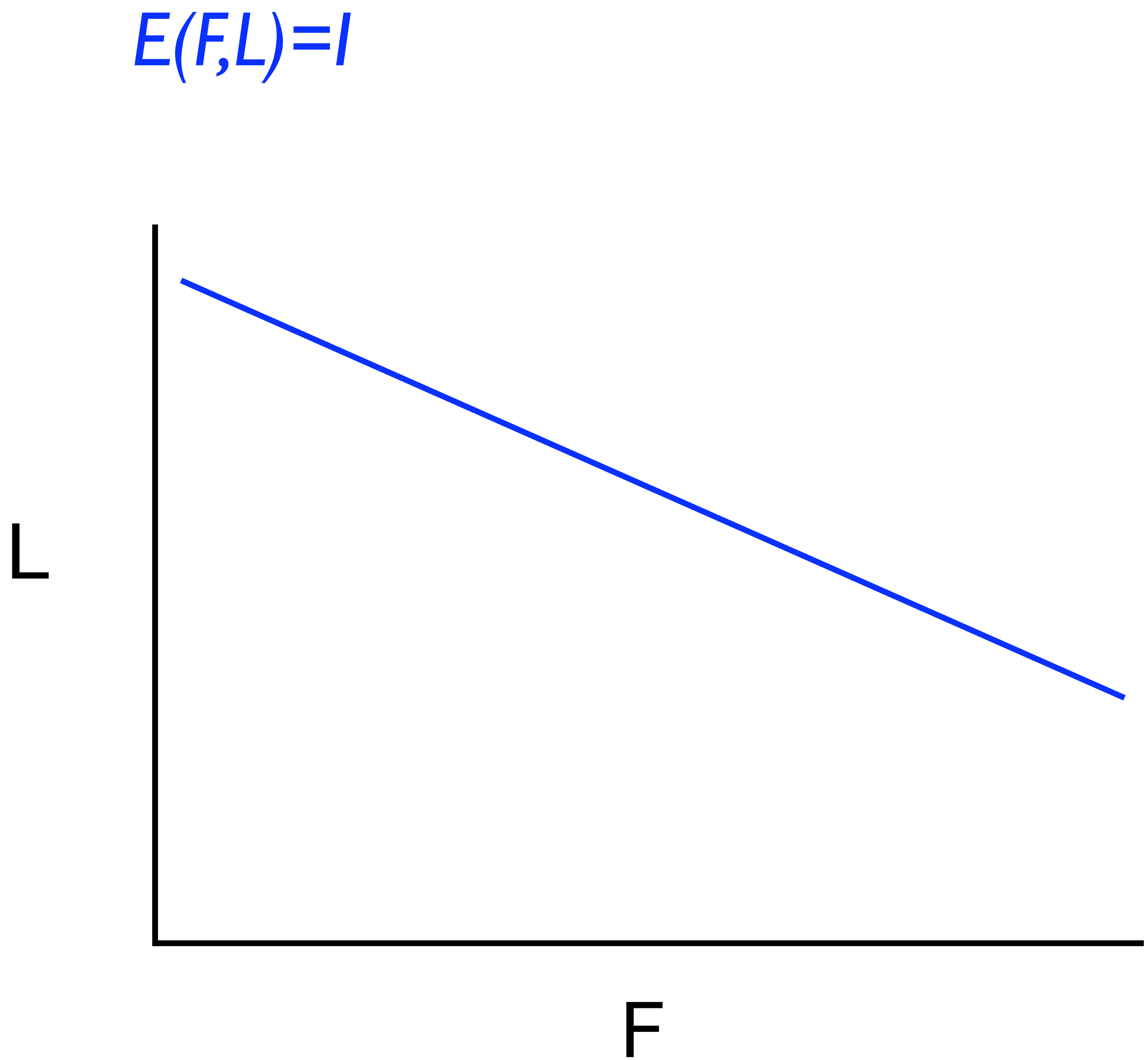
$$\rho_F \frac{dF}{dt} = (1-p)(I-E)$$

$$\rho_L \frac{dL}{dt} = p(I-E)$$

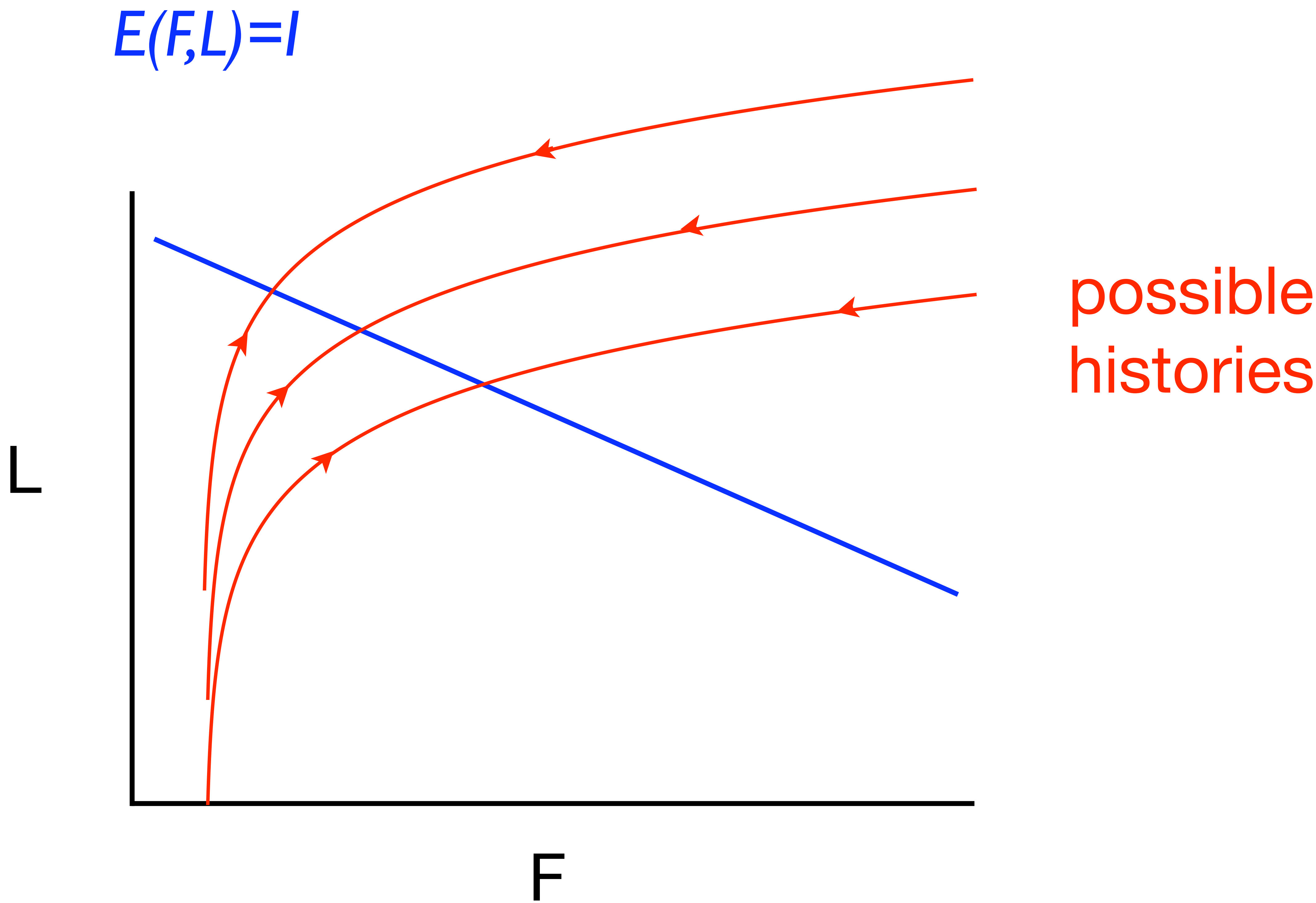
Steady state is line attractor $E(F, L) = I$

Almost all previous models use energy partition

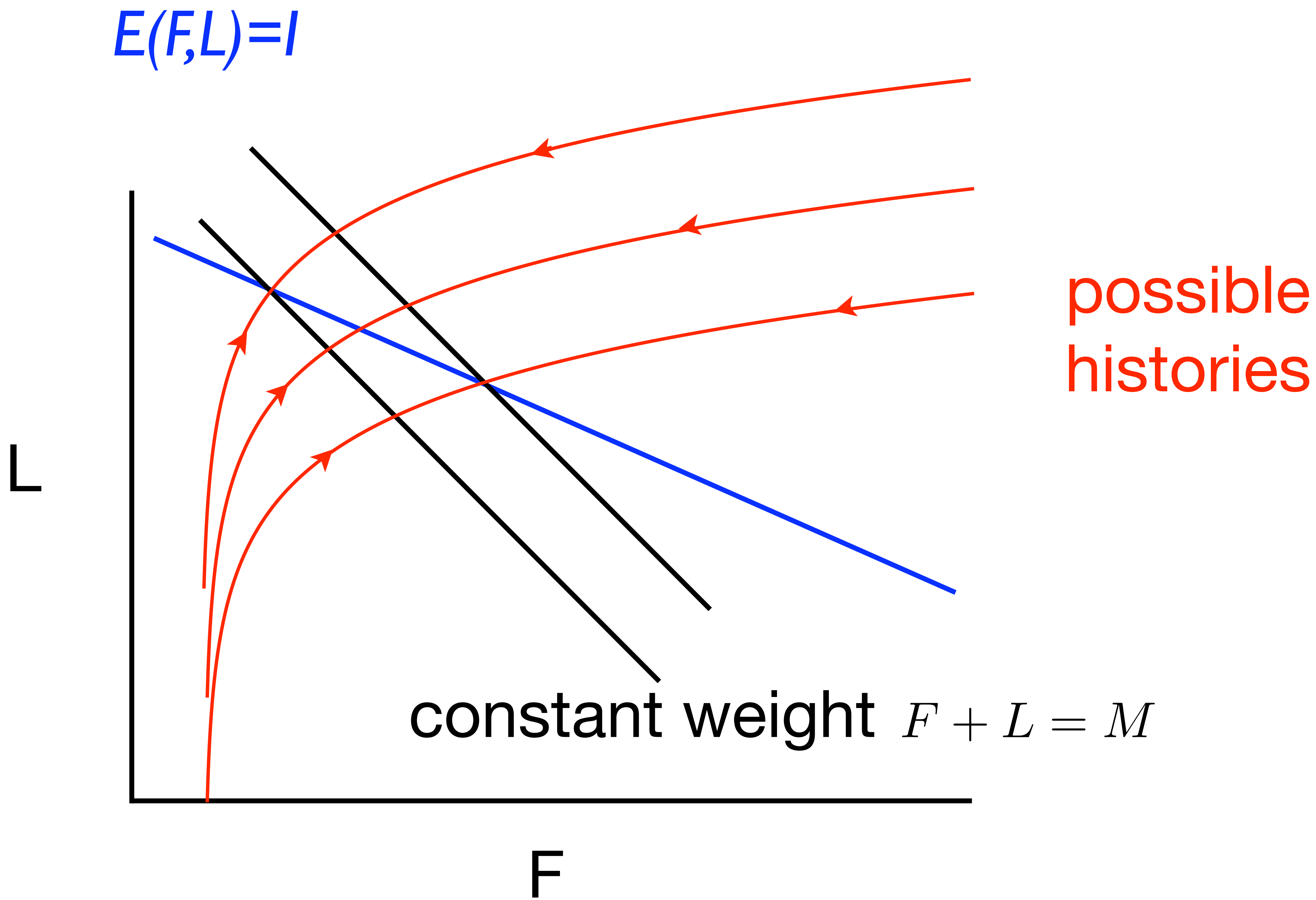
Consequences of line attractor



Consequences of line attractor



Consequences of line attractor



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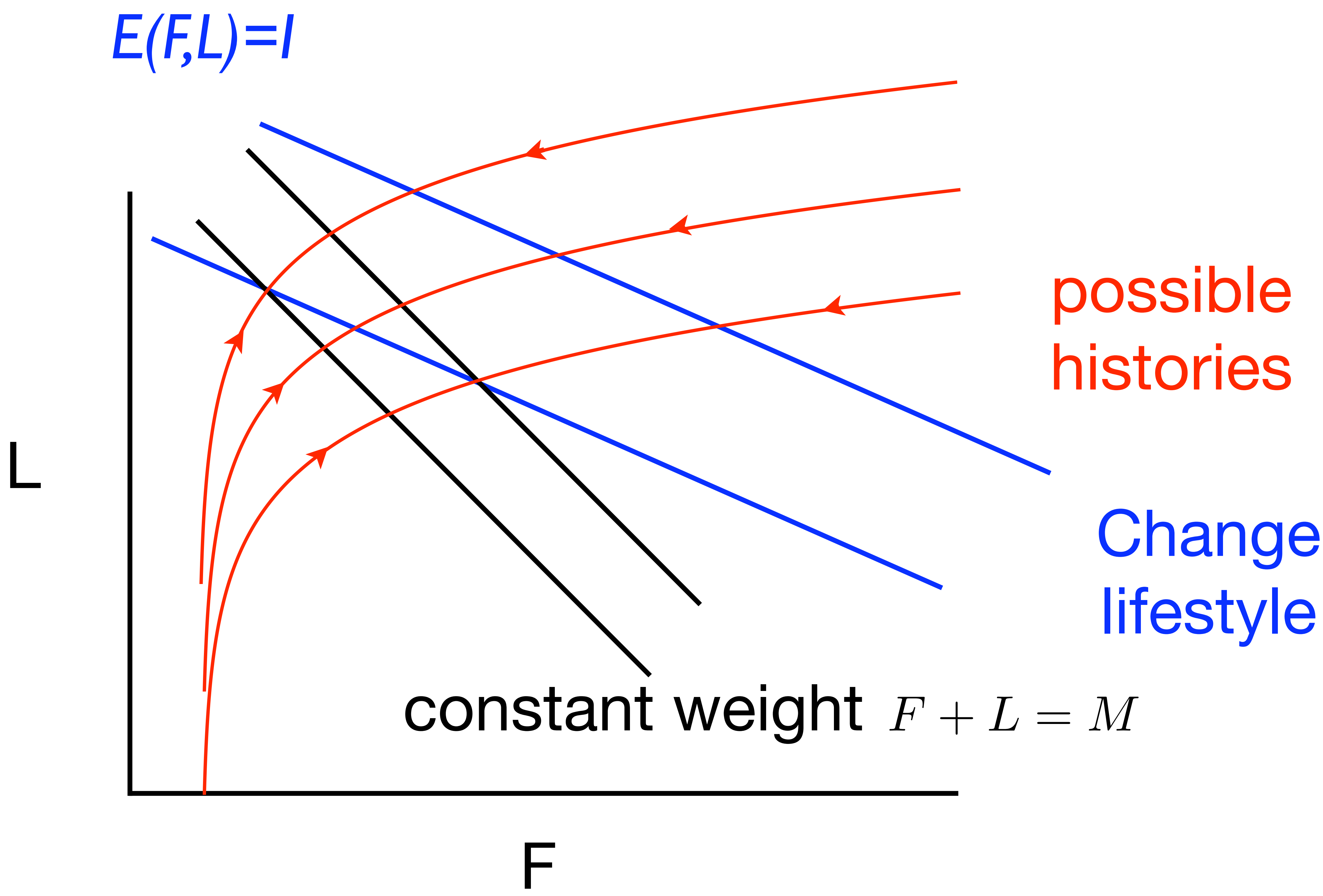
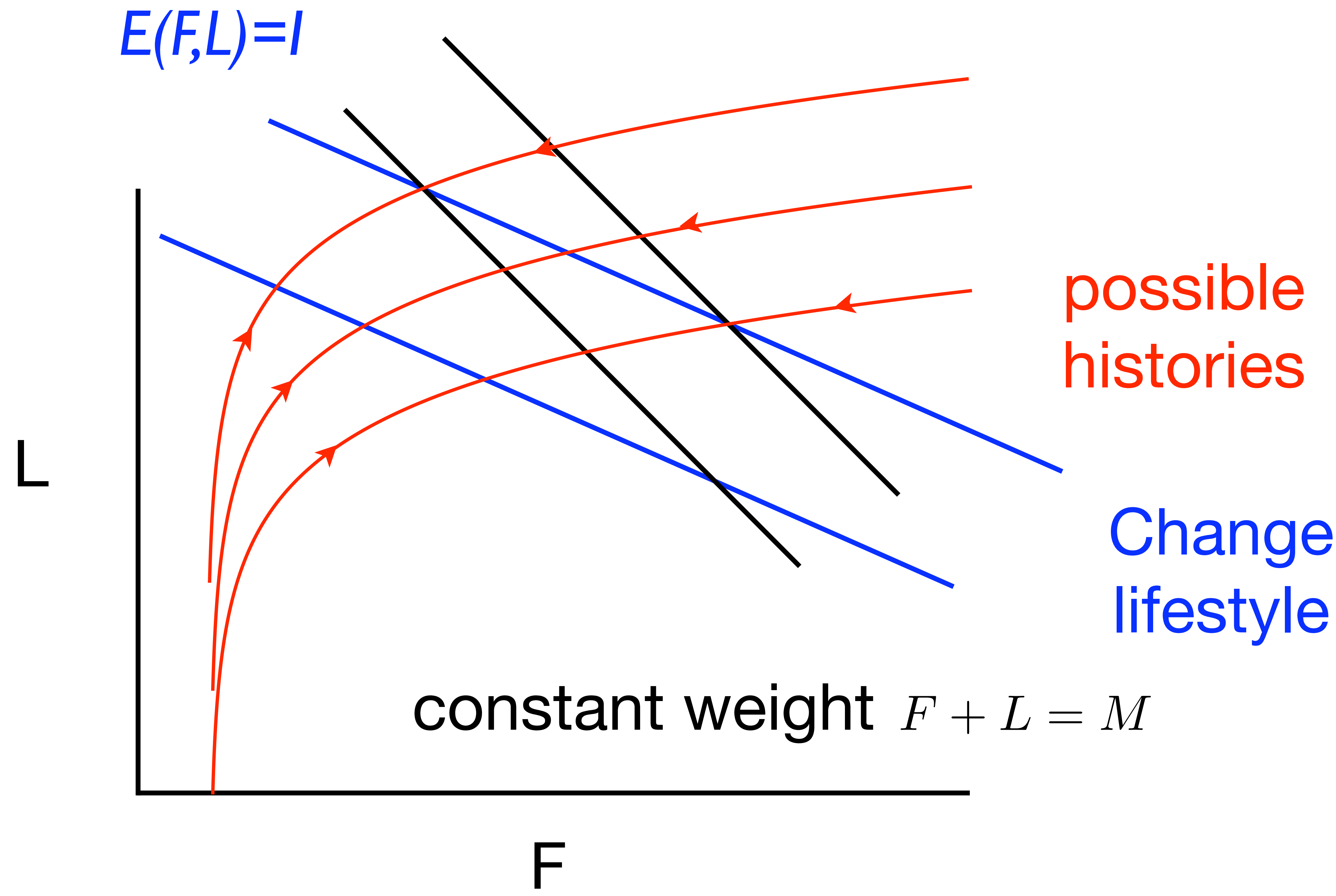
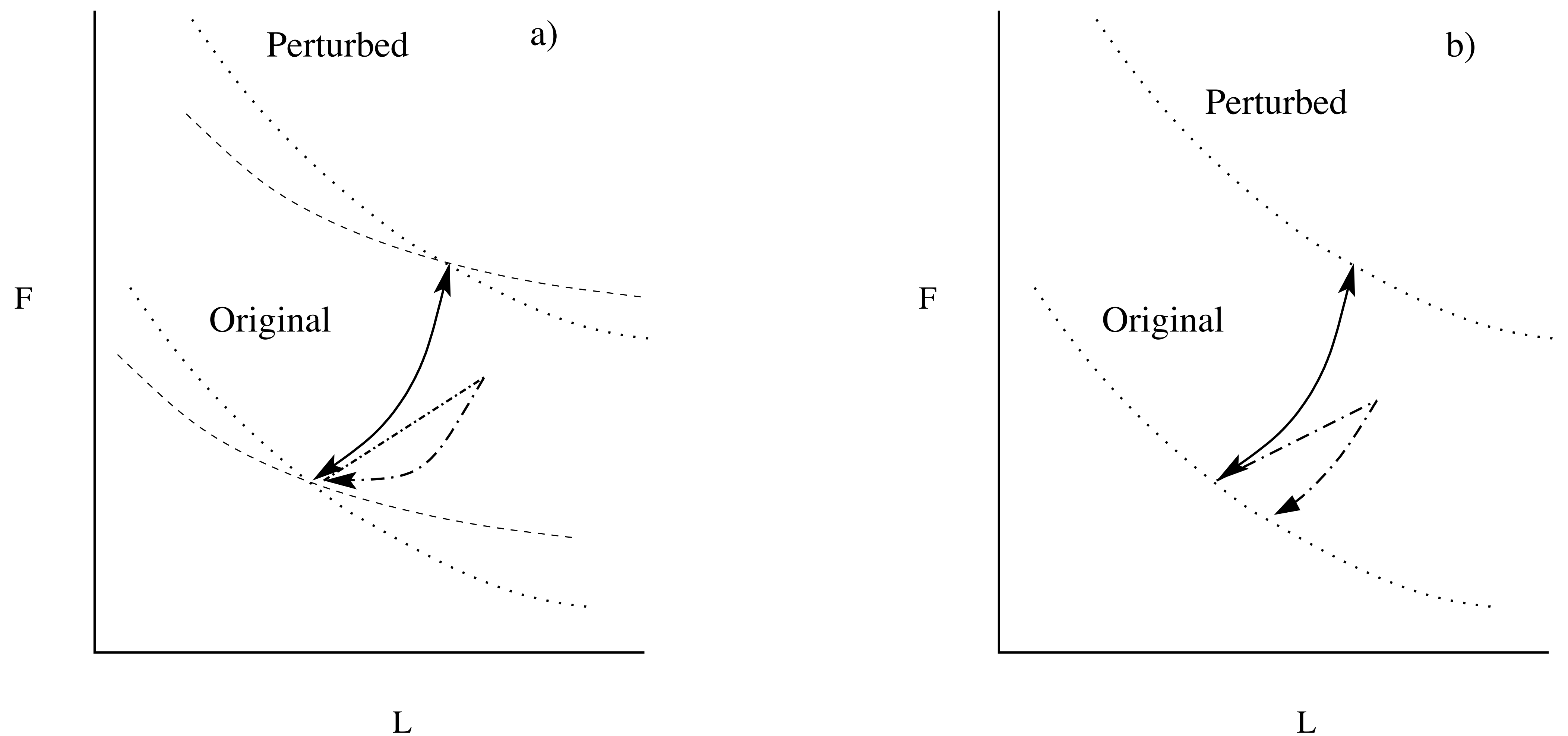


Consequences of line attractor



Effect of perturbations



fixed point line attractor

Numerical example

$$\rho_F \frac{dF}{dt} = I_F - fE$$

$$\rho_L \frac{dL}{dt} = I_L - (1-f)E$$

ρ_F = 37.7 *MJ/kg*

ρ_L = 7.6 *MJ/kg*

E (MJ/Day) =0.14 L (kg)+0.05 F (kg) +1.55

Numerical example

$$\rho_F \frac{dF}{dt} = I_F - fE$$

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$\rho_L = 7.6$ *MJ/kg*

E (MJ/Day) =0.14 L (kg)+0.05 F (kg) +1.55

$$f = \frac{I_F}{E} - (1-p)\frac{I-E}{E} - \frac{\psi}{E}$$

Numerical example

$$\rho_F \frac{dF}{dt} = (1-p)(I-E) + \psi$$

$$\rho_L \frac{dL}{dt} = p(I-E) - \psi$$

ρ_F = 37.7 *MJ/kg*

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E (MJ/Day) =0.14 L (kg)+0.05 F (kg) +1.55

Numerical example

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Forbes: $p = 2/(2+F)$

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$$\psi = \begin{cases} 0.05(F - 0.4\exp(L/10.4))/F \\ 0 \end{cases}$$

Numerical example

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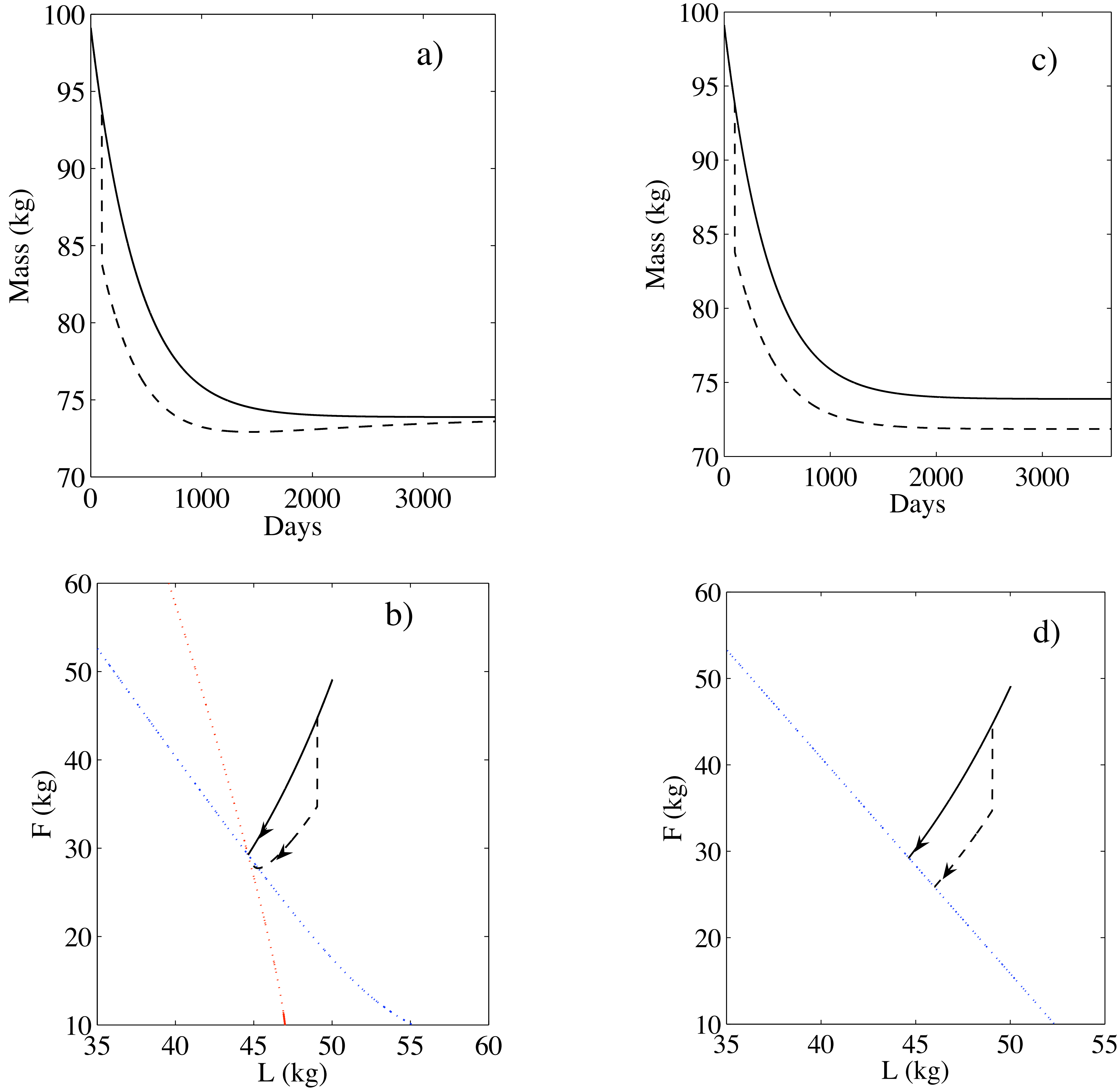
Forbes: $p = 2/(2+F)$

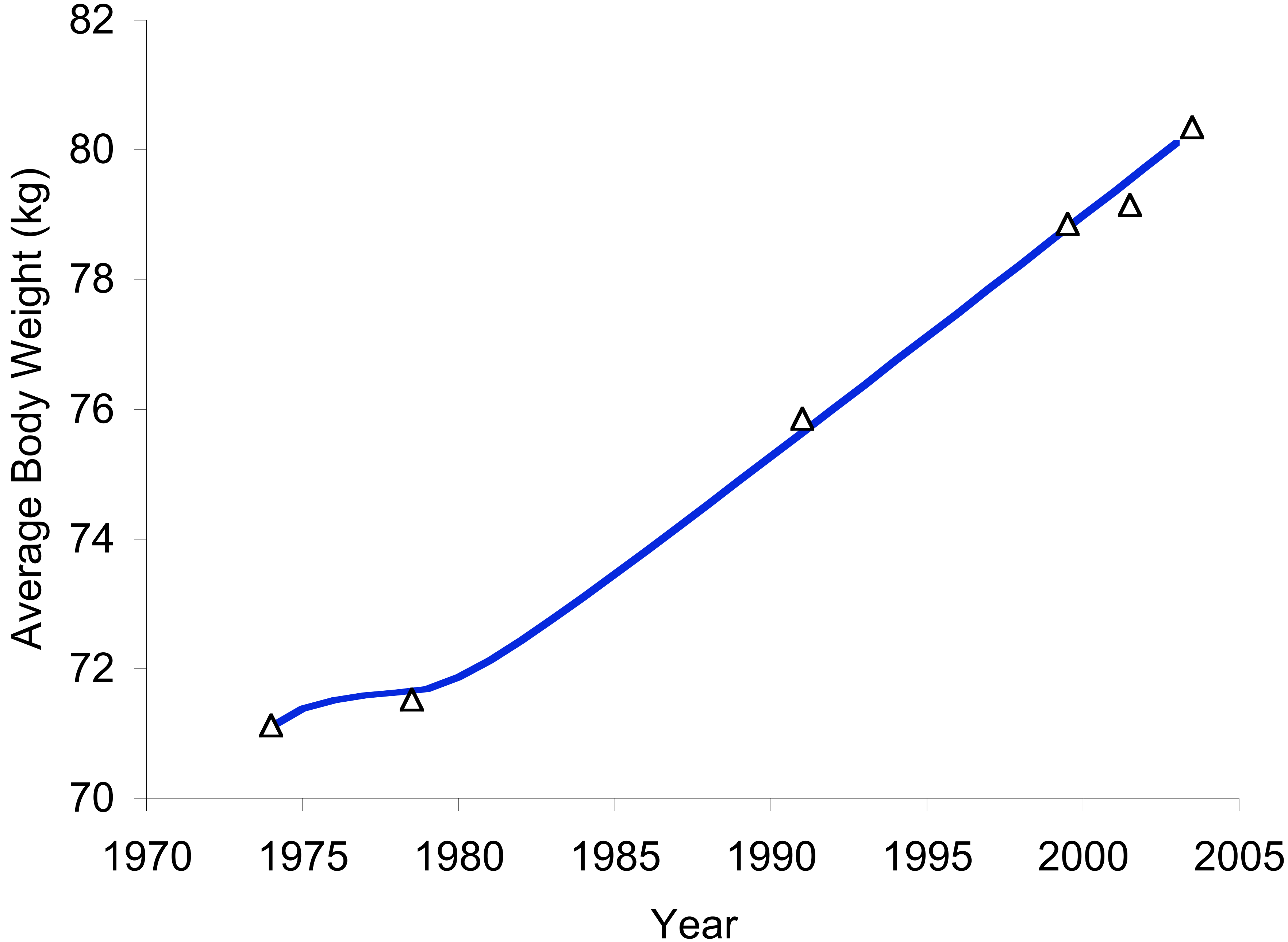
fixed point

$$\psi = \begin{cases} 0.05(F-0.4\exp(L/10.4))/F \\ 0 \end{cases}$$

invariant manifold

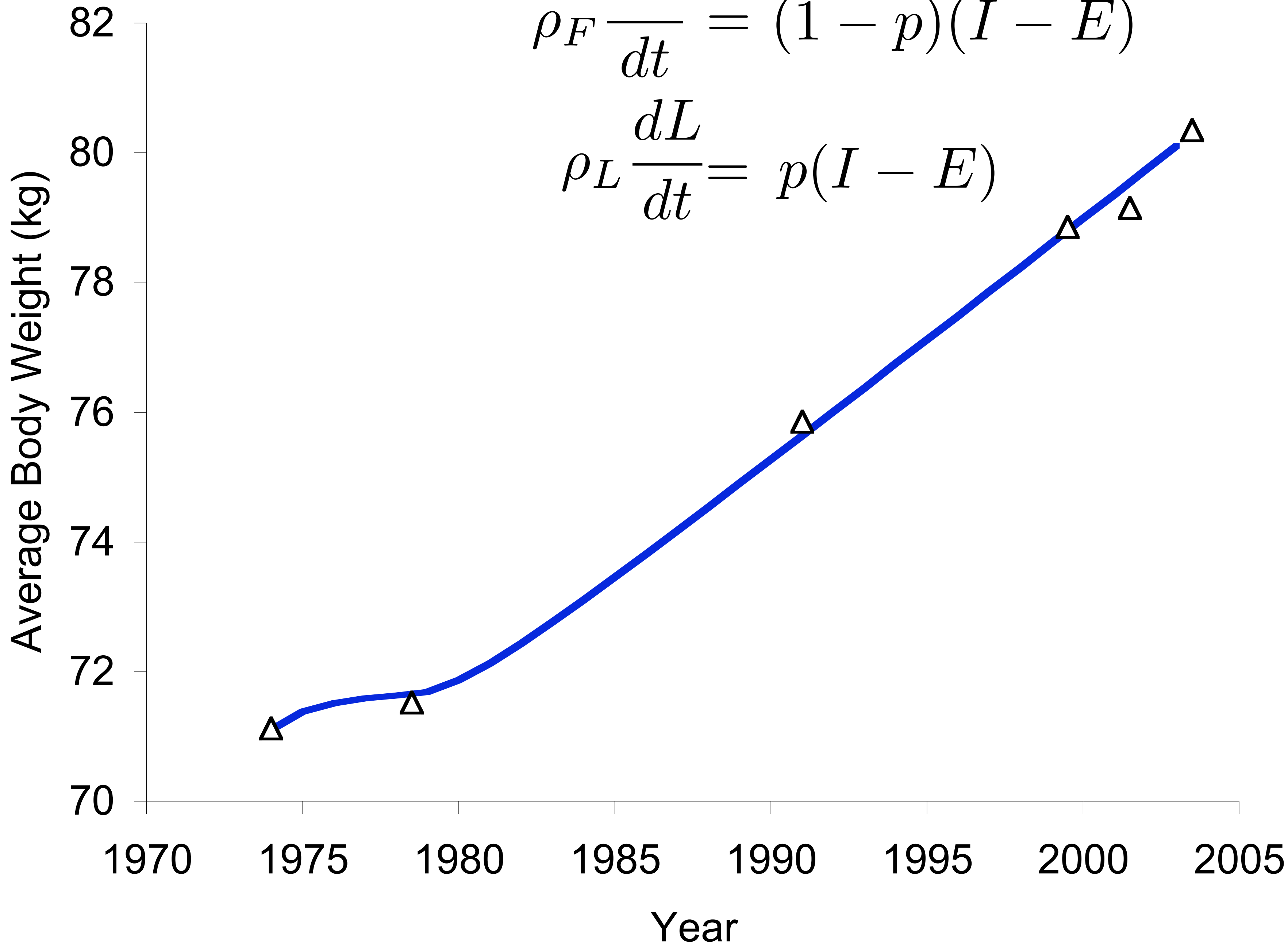

a)
Mass (kg)
Days
b)
F (kg)
L (kg)
c)
Mass (kg)
Days
d)
F (kg)
L (kg)

Mean US body weight



Data from National Health and Nutrition Examination Survey (NHANES)

Mean US body weight



$$\rho_F \frac{dF}{dt} = (1-p)(I-E)$$

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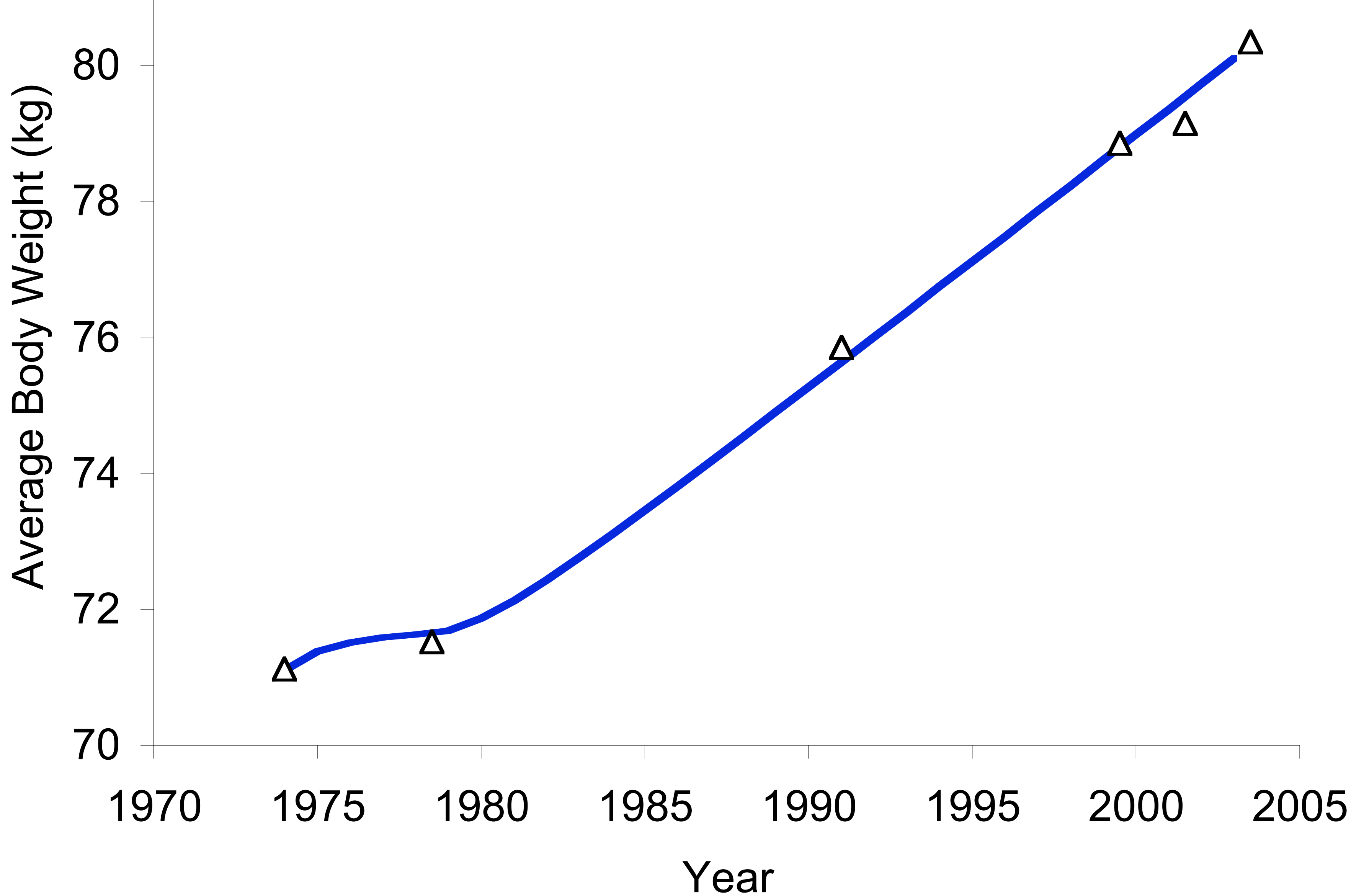
Data from National Health and Nutrition Examination Survey (NHANES)

Mean US body weight

$$\rho_L \frac{dL}{dt} + \rho_F \frac{dF}{dt} = I - E$$



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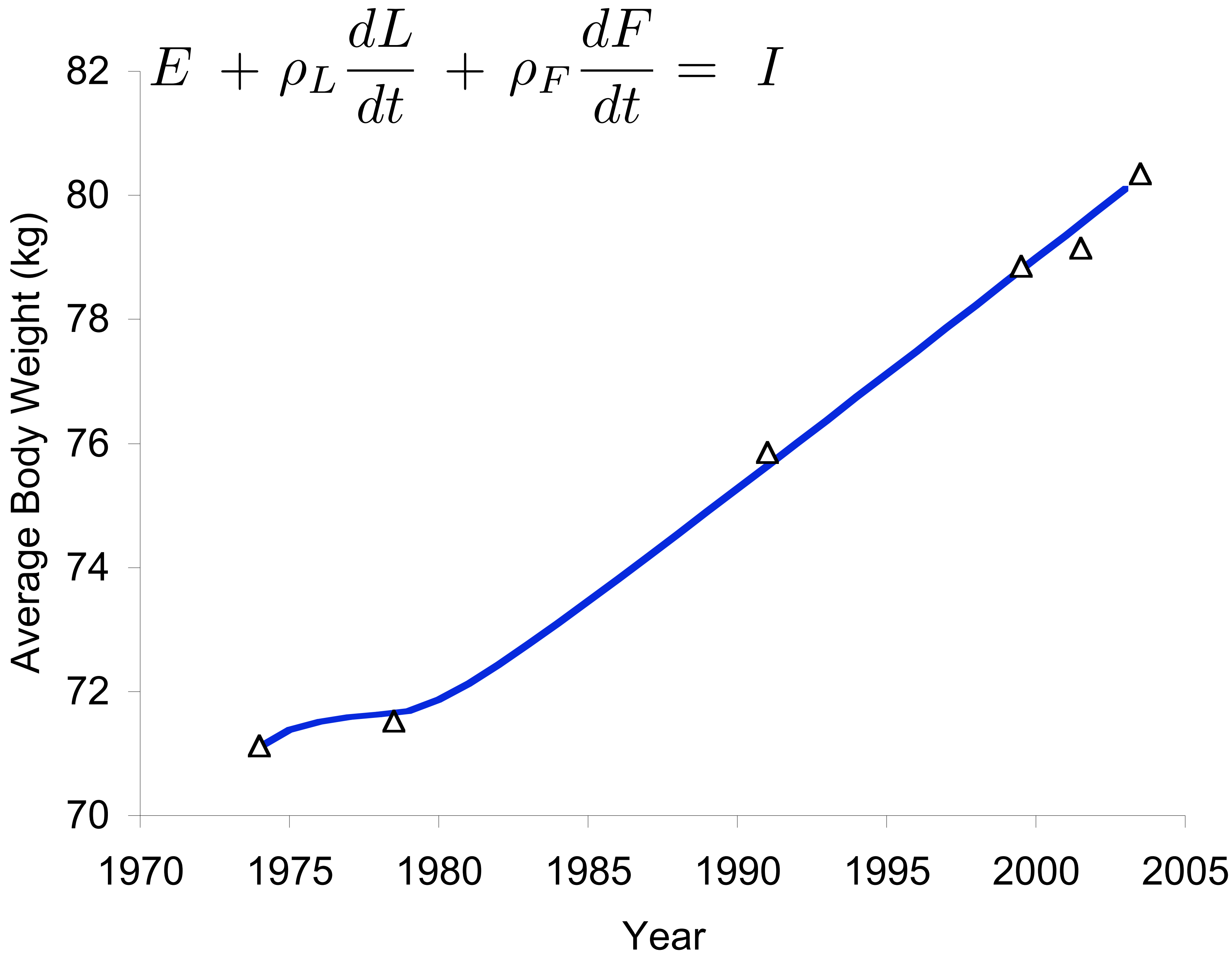
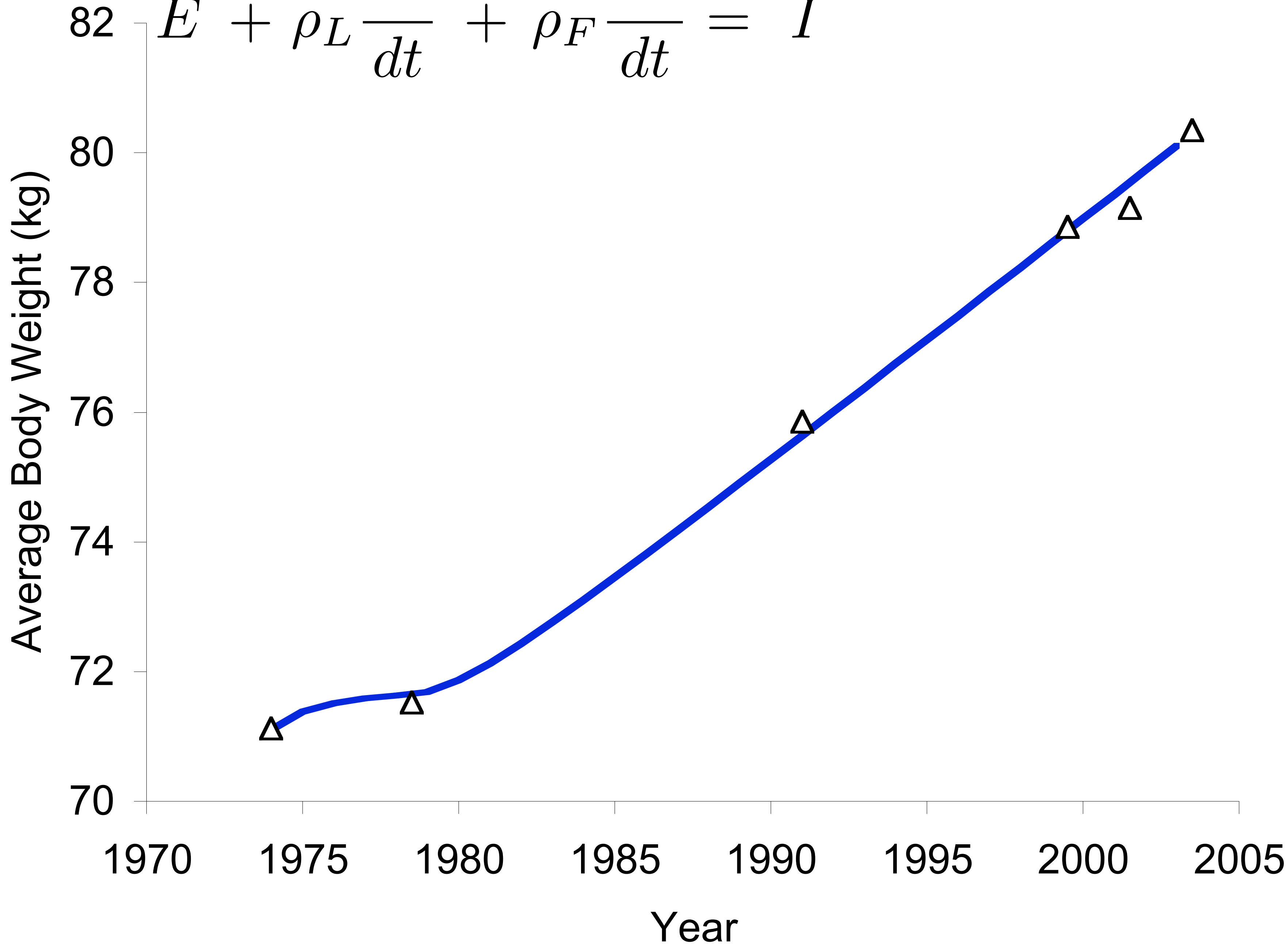


Data from National Health and Nutrition Examination Survey (NHANES)

Mean US body weight

$$\overline{E} + \rho_L \overline{\frac{dL}{dt}} + \rho_F \overline{\frac{dF}{dt}} = \overline{I}$$



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Mean US body weight

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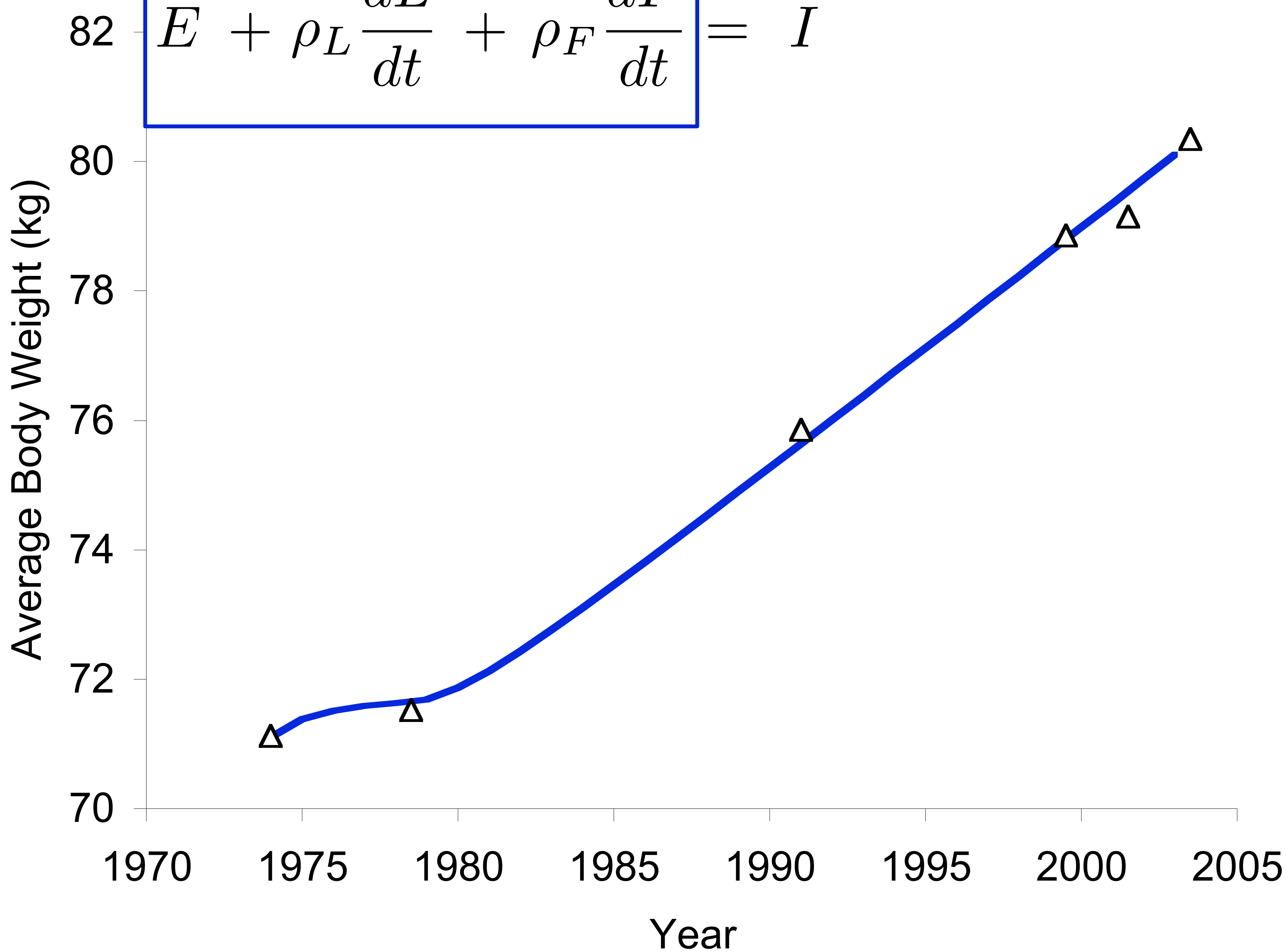
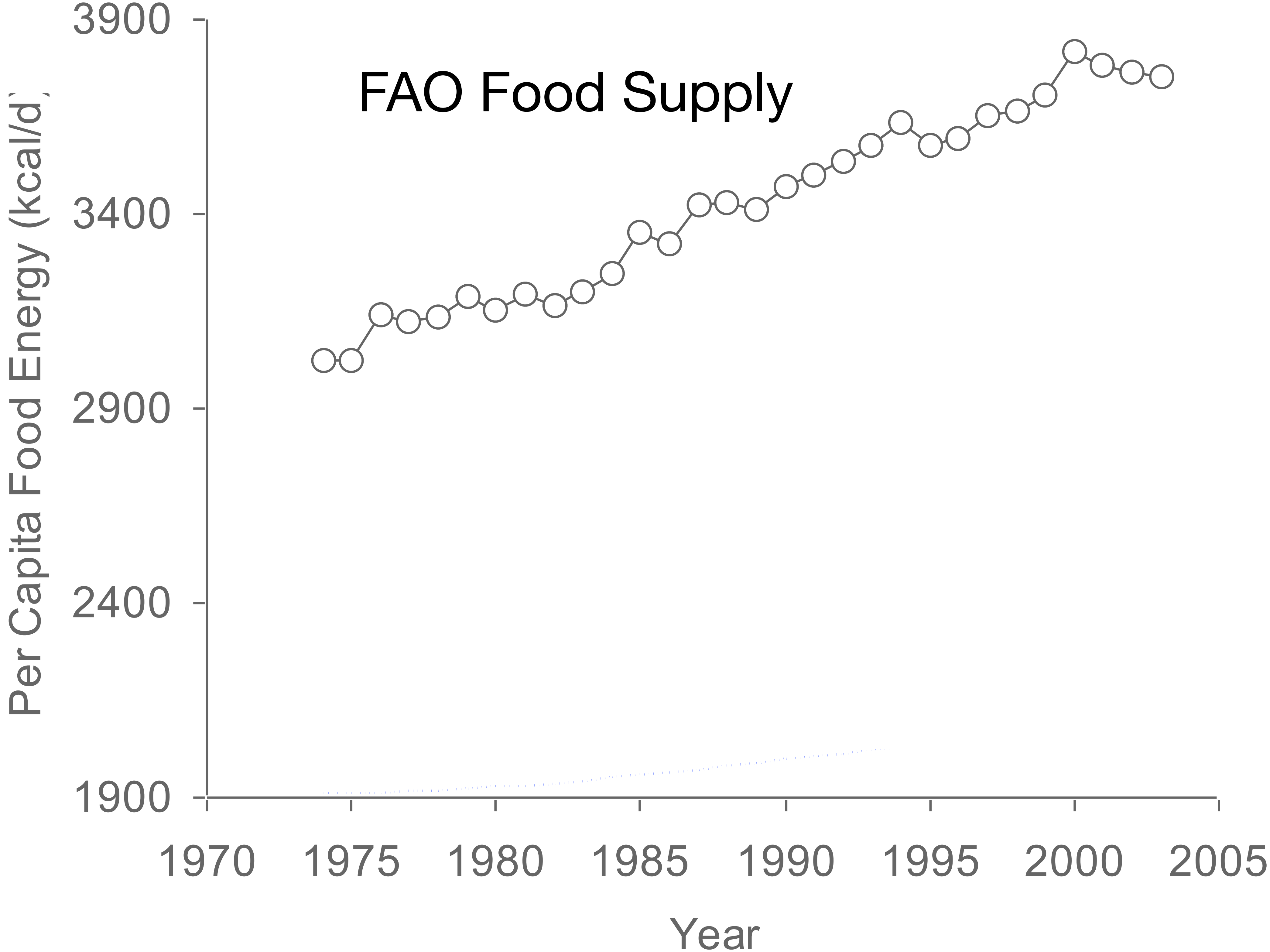


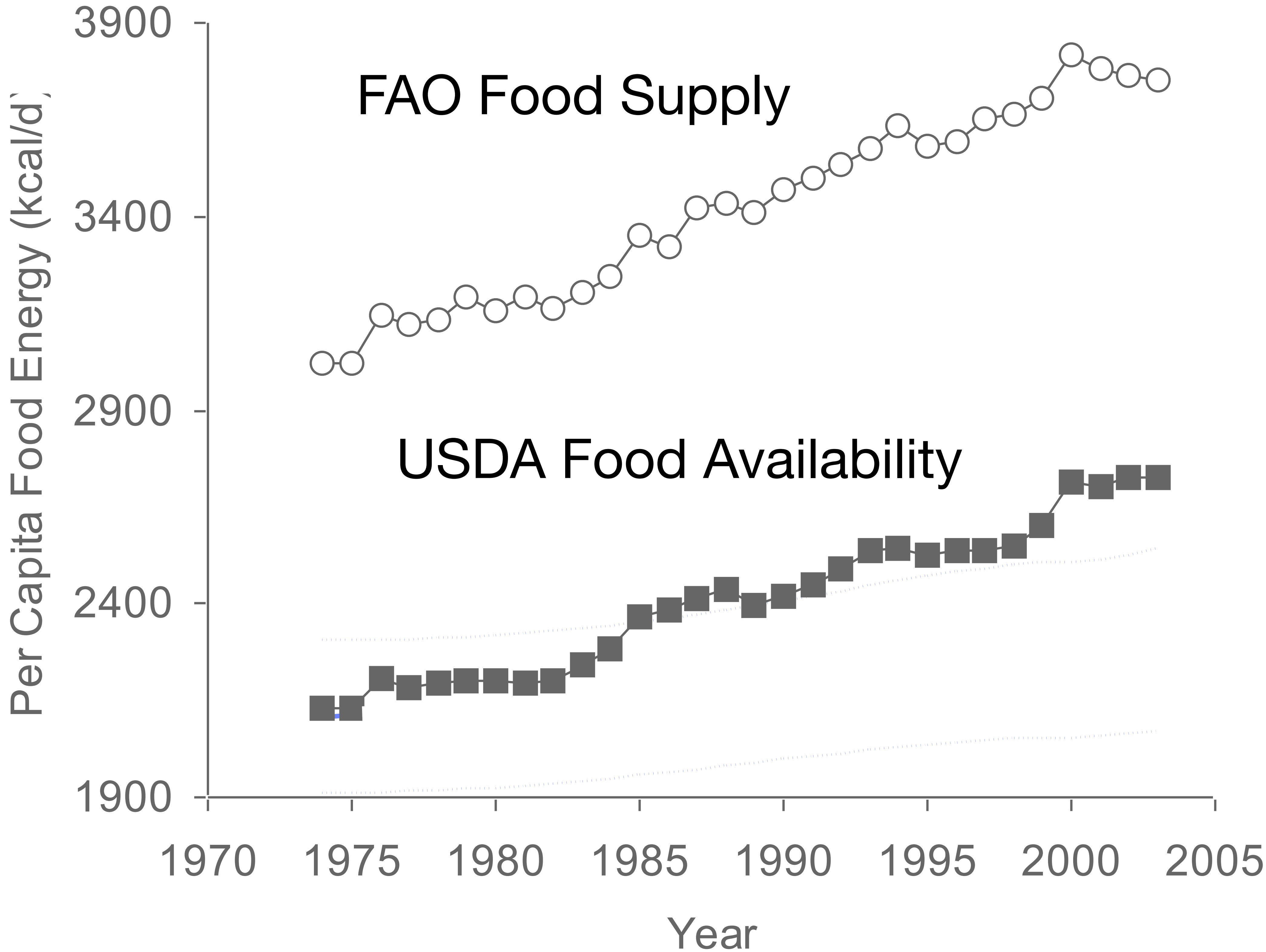
Data from National Health and Nutrition Examination Survey (NHANES)

U.S. Food Supply



Hall, Guo, Dore, Chow. *PLoS One* (2009)

U.S. Food Supply



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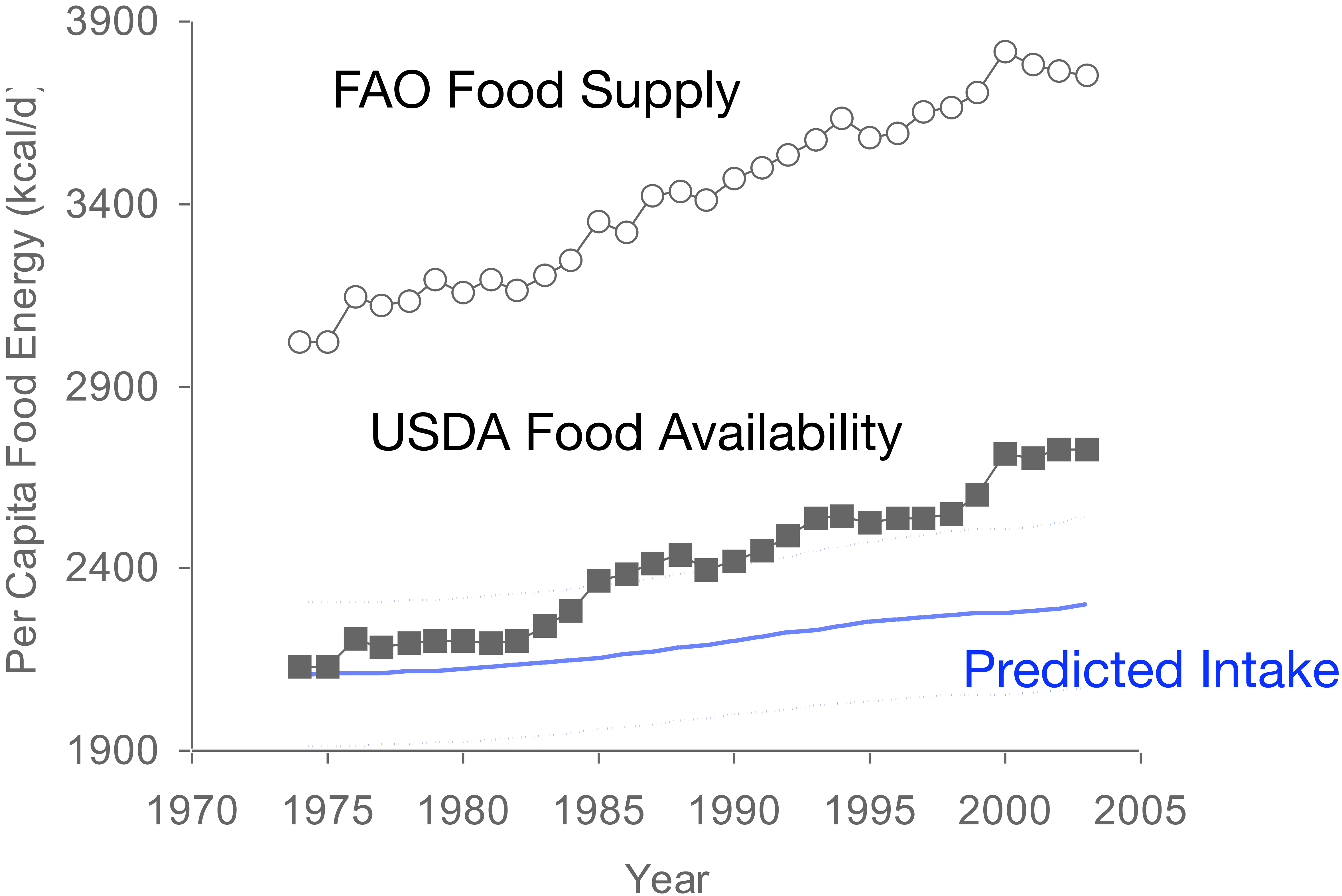

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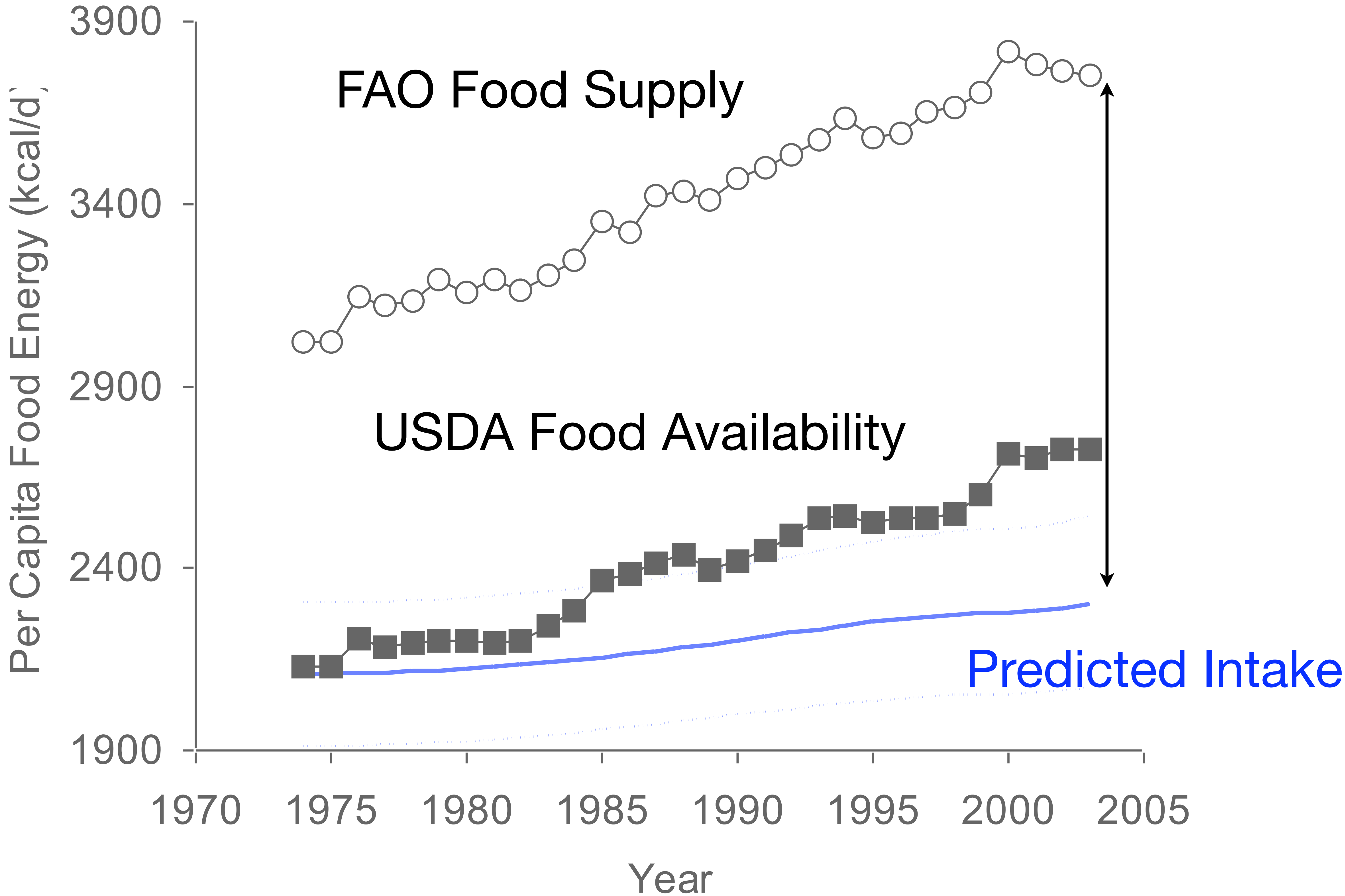

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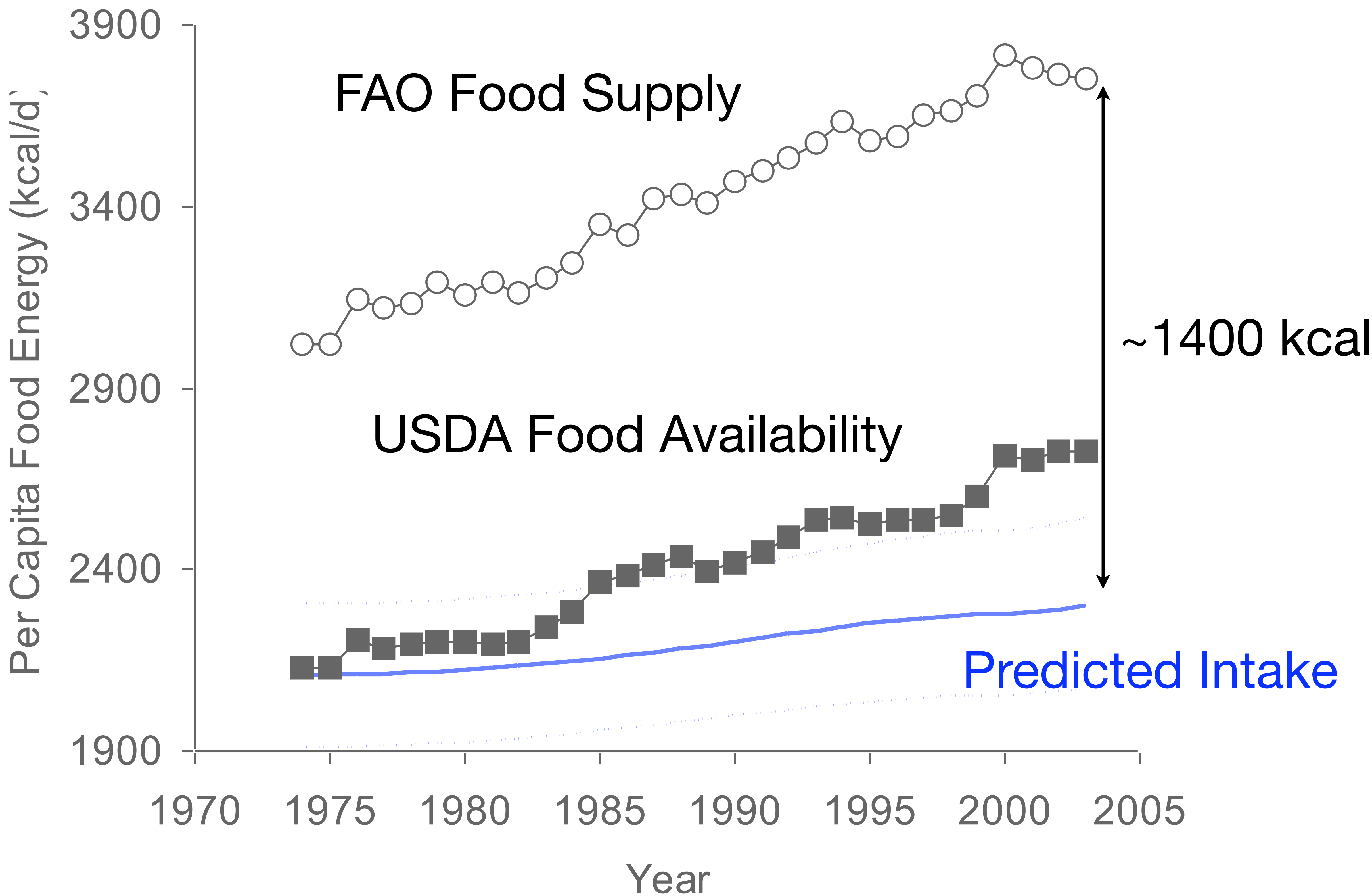
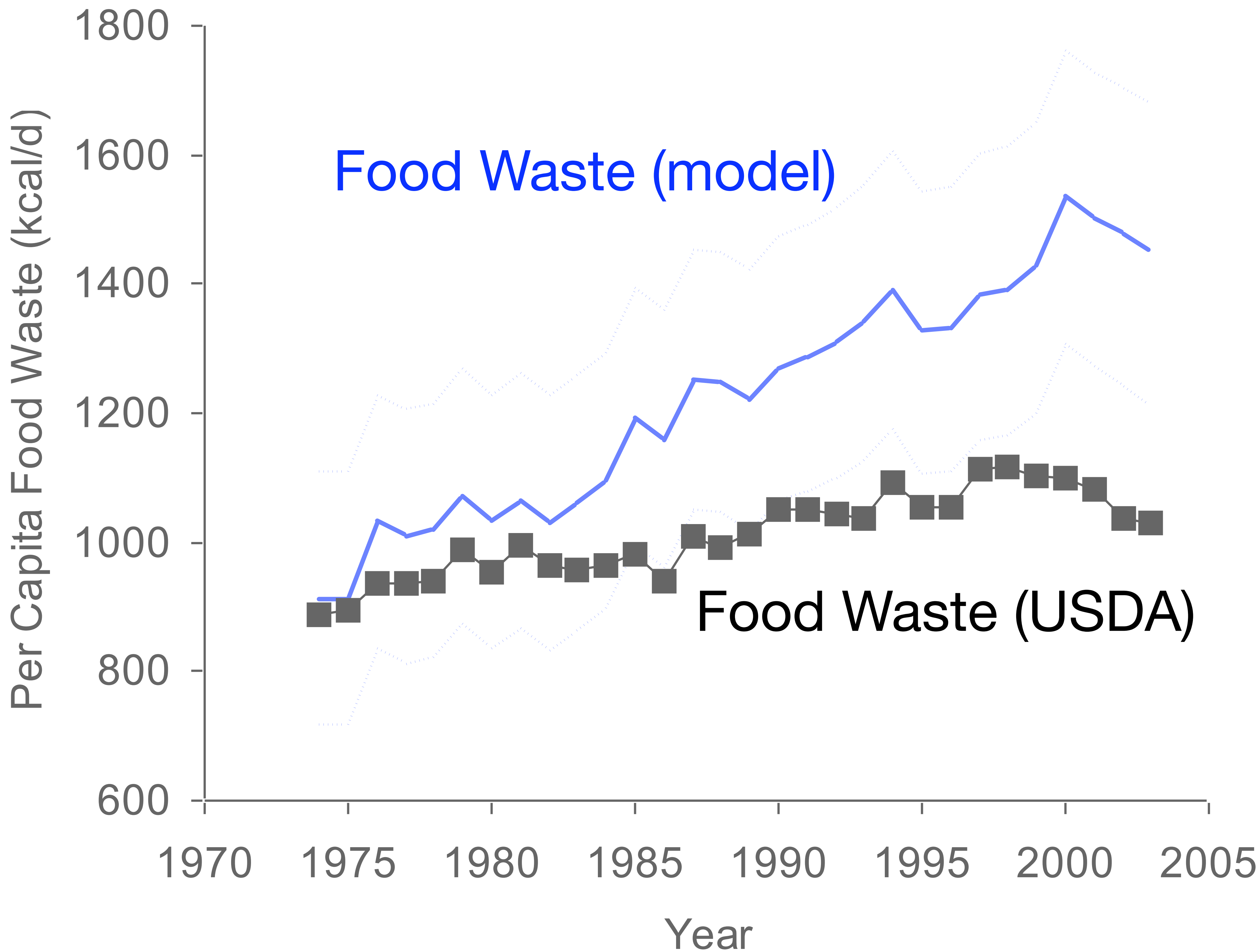
U.S. Food Supply



Hall, Guo, Dore, Chow. *PLoS One* (2009)

U.S. Food Waste



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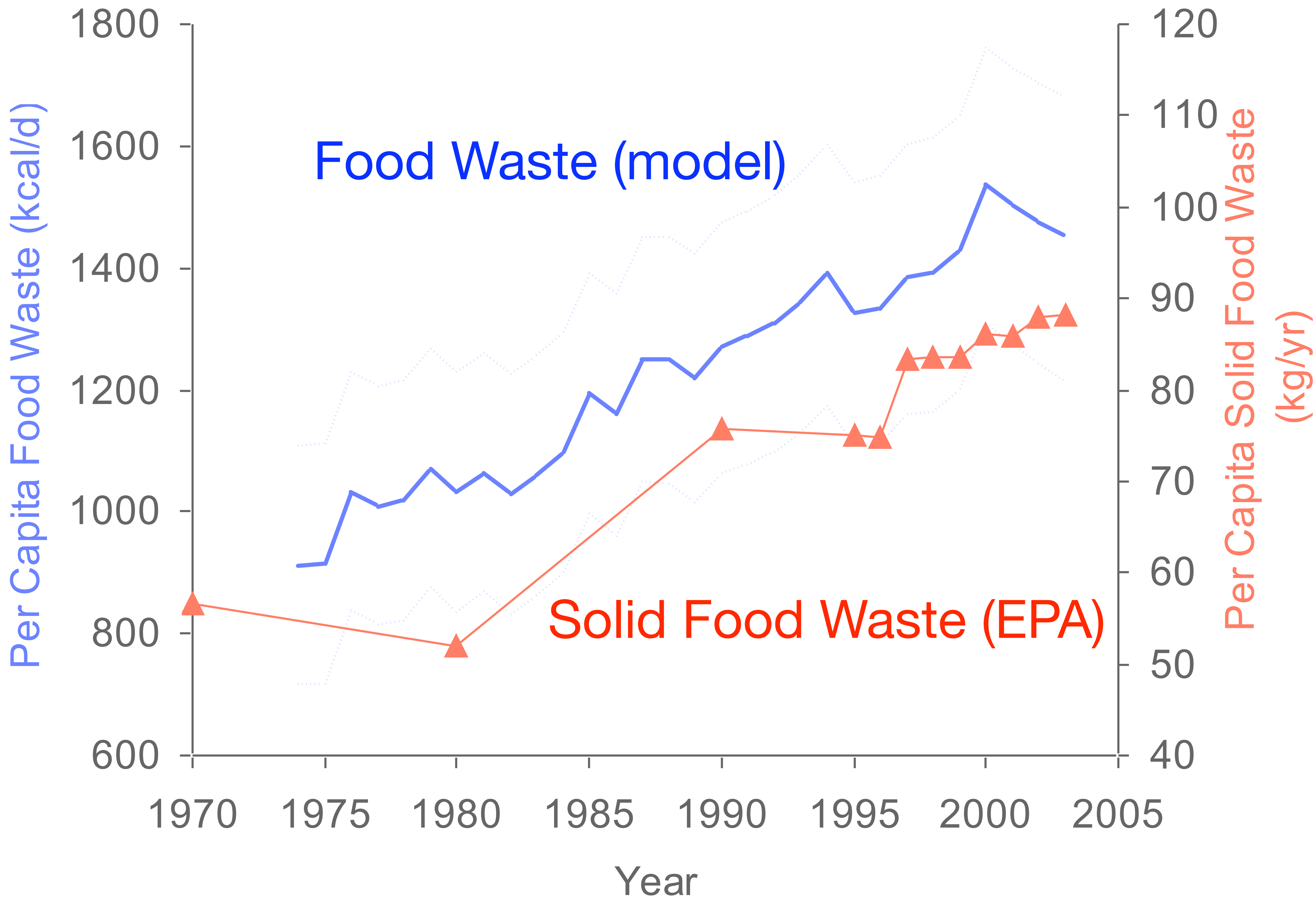
U.S. Food Waste



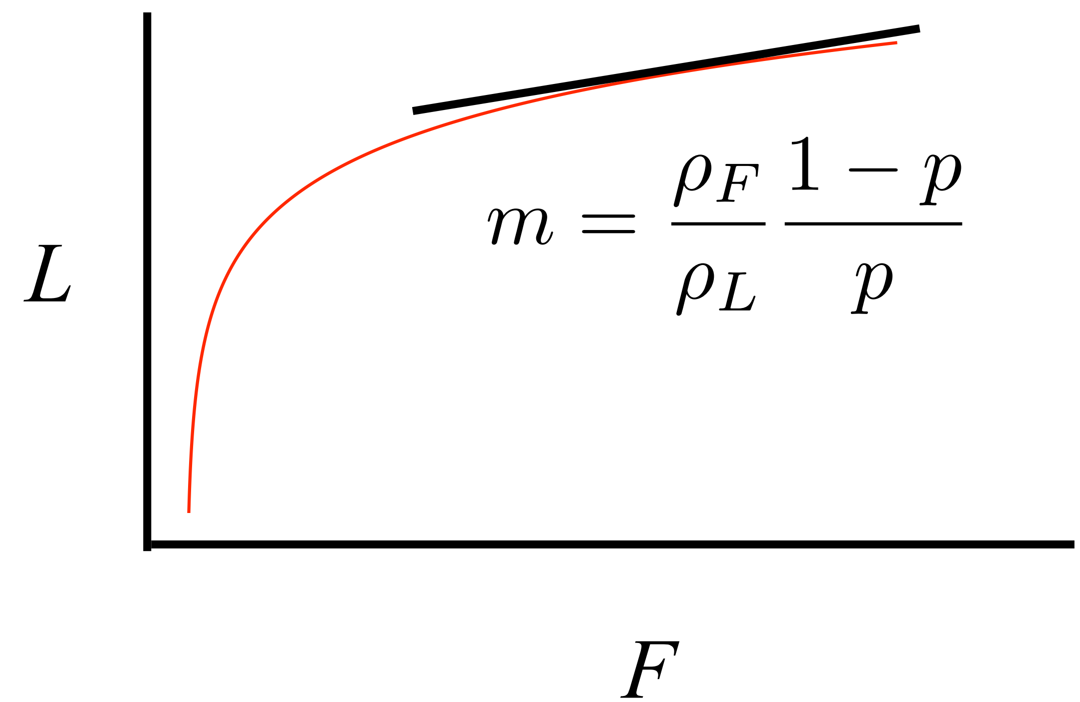
Hall, Guo, Dore, Chow. *PLoS One* (2009)

One dimensional models

Energy partition models are effectively *1D*

$$\rho_F \frac{dF}{dt} = (1-p)(I-E)$$

$$\rho_L \frac{dL}{dt} = p(I-E)$$

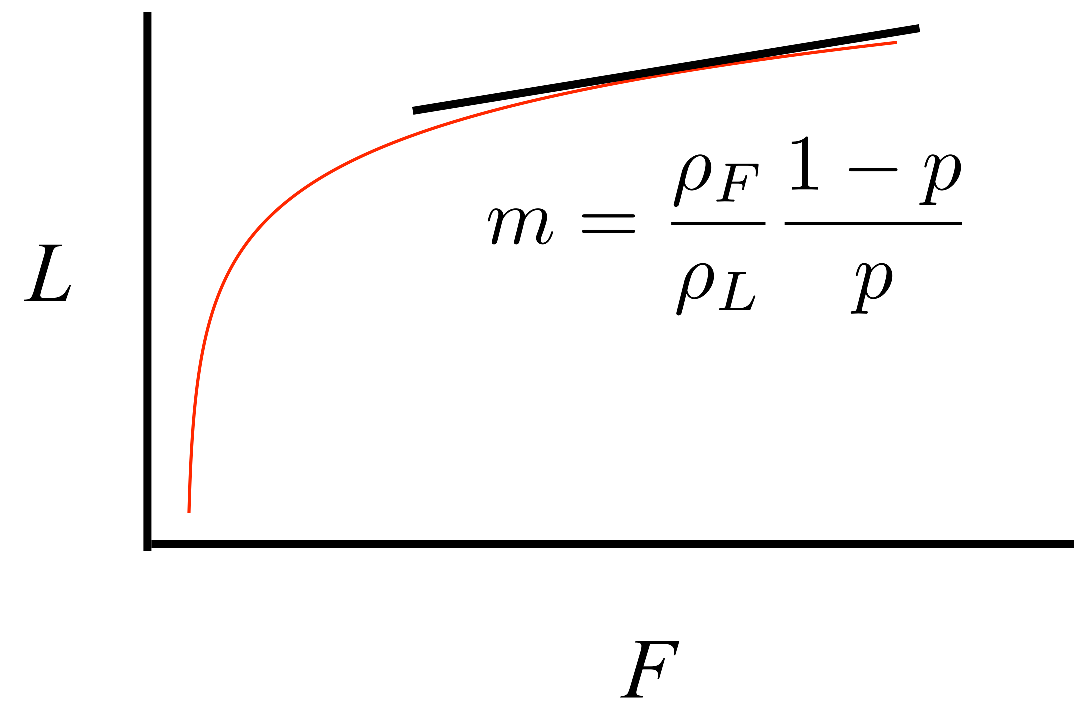


$$m = \frac{\rho_F}{\rho_L}\frac{1-p}{p}$$

One dimensional models

Energy partition models are effectively *1D*

$$\rho_L \frac{dL}{dt} + \rho_F \frac{dF}{dt} = I - E$$



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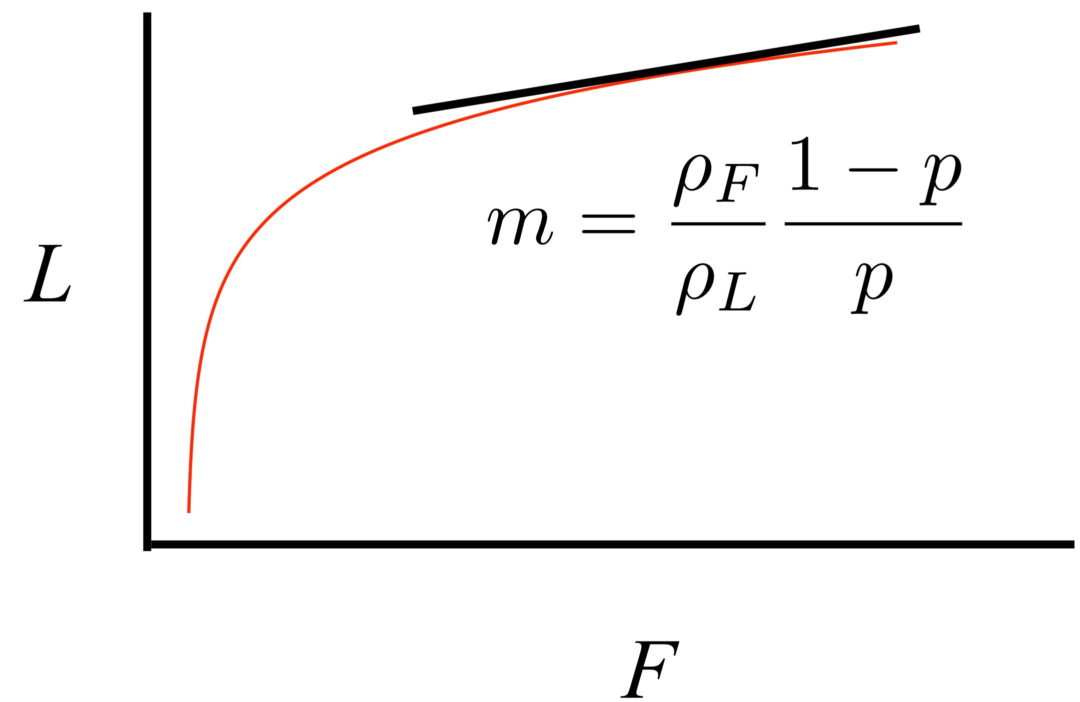
$$L \approx mF + b$$



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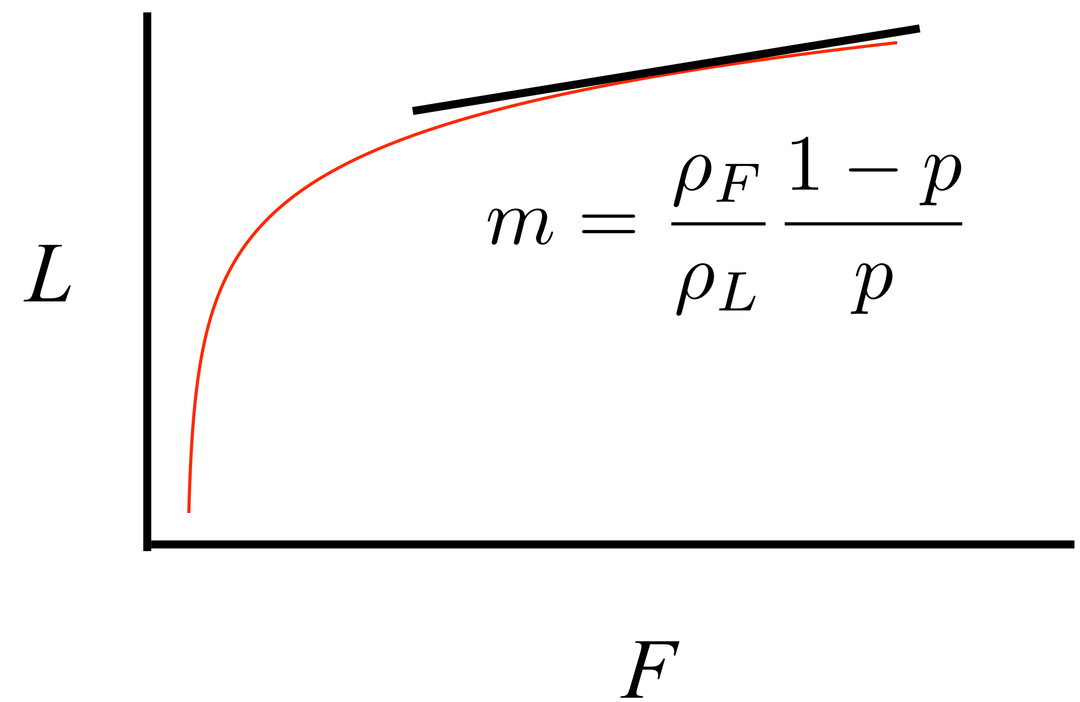
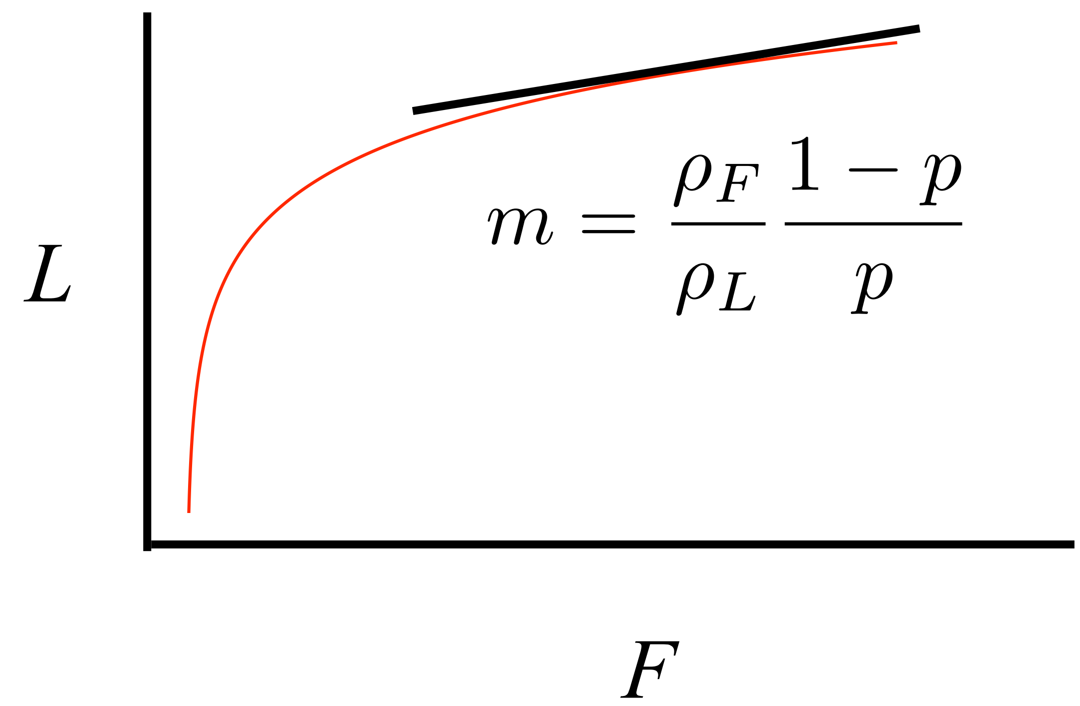


$$L \approx mF + b \qquad F = M - L$$

One dimensional models

Energy partition models are effectively *1D*

$$\rho_L \frac{dL}{dt} + \rho_F \frac{dF}{dt} = I - E$$



$$m = \frac{\rho_F}{\rho_L} \frac{1-p}{p}$$

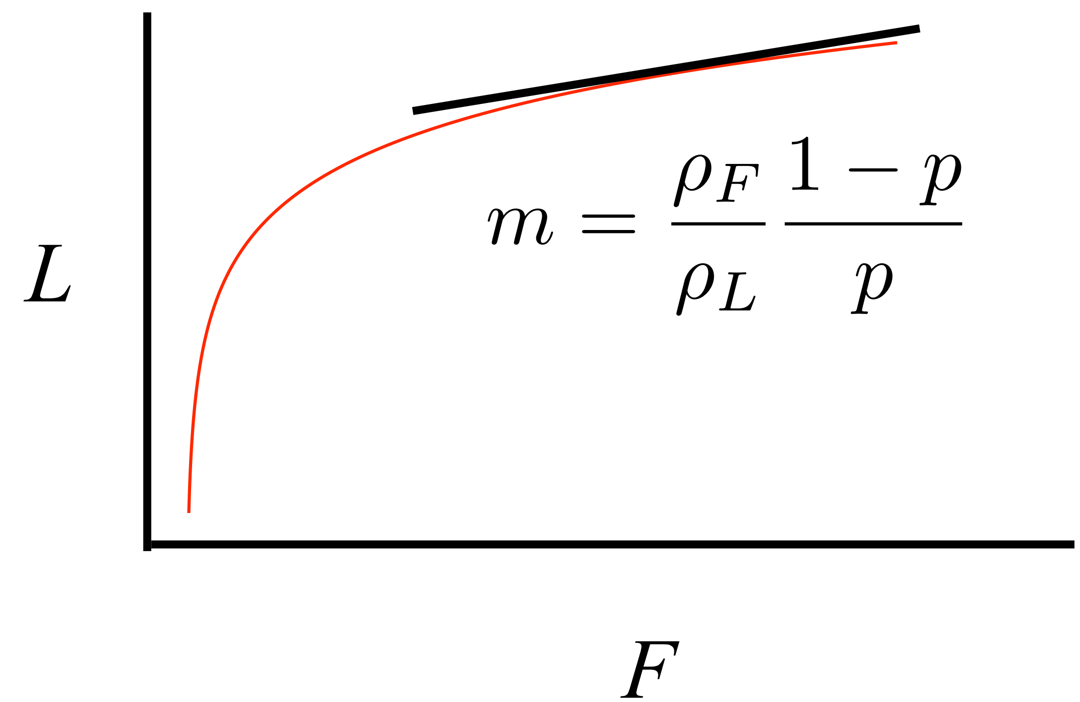
$$L \approx mF + b \qquad F = M - L$$

$$F = \frac{M - b}{1 + m}$$

One dimensional models

Energy partition models are effectively *1D*

$$\rho_L \frac{dL}{dt} + \rho_F \frac{dF}{dt} = I - E$$



$$L \approx mF + b \qquad F = M - L$$

$$L = \frac{mM + b}{1 + m} \qquad F = \frac{M - b}{1 + m}$$

One dimensional models

$$\rho_M \frac{dM}{dt} = I - \epsilon M - b$$

$$\rho_M = \frac{\rho_F \rho_L}{\rho_L + (\rho_F - \rho_L)p}$$

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Leaky integrator

One dimensional models

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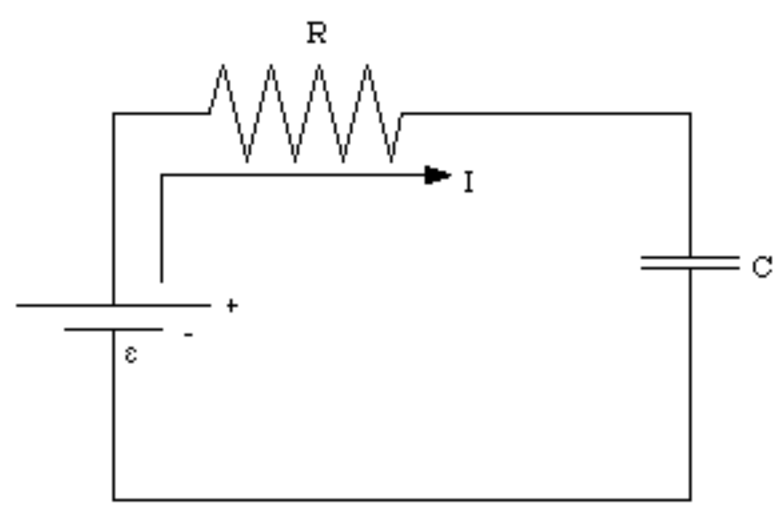
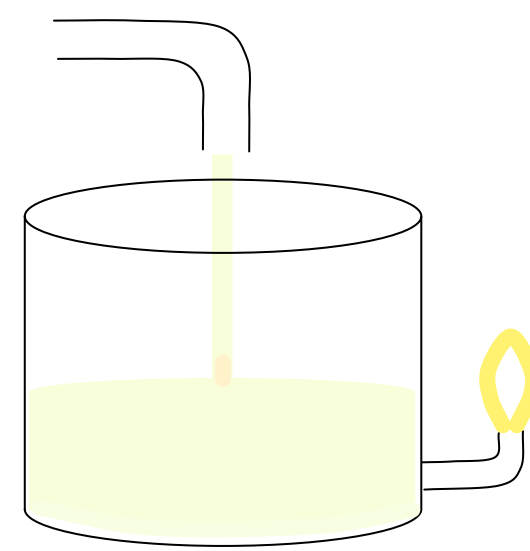
Leaky integrator





$$C\frac{dV}{dt} = I - \frac{1}{R}V$$

Steady state

$$\rho_M \frac{dM}{dt} = I - \epsilon M - b = 0$$

$$M = (I - b)/\epsilon \qquad \Delta M \sim \frac{1}{\epsilon} \Delta I$$

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$$\rho_M \frac{dM}{dt} = I - \epsilon M - b = 0$$

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ε decreases with weight and increases with activity

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ε ~ 0.1 MJ/kg/day or 23 kcal/kg/day

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$$M = (I - b)/\epsilon \qquad \Delta M \sim \frac{1}{\epsilon} \Delta I$$

ε decreases with weight and increases with activity

ε ~ 0.1 MJ/kg/day or 23 kcal/kg/day

Extra ~23 kcal/day is an extra kg

Time Constant

$$\rho_M \frac{dM}{dt} = I - \epsilon M - b$$

Time constant (half-life/.69) to reach steady state:

$$\tau = \rho_M / \epsilon$$

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$$\rho_M \frac{dM}{dt} = I - \epsilon M - b$$

Time constant (half-life/.69) to reach steady state:

$$\tau = \rho_M/\epsilon$$

ρ_M ~ 7700 kcal/kg, ε ~ 23 kcal/day, τ ~1 year

Time Constant

$$\rho_M \frac{dM}{dt} = I - \epsilon M - b$$

Time constant (half-life/.69) to reach steady state:

$$\tau = \rho_M / \epsilon$$

ρ_M ~ 7700 kcal/kg, ε ~ 23 kcal/day, τ ~1 year

τ increases with weight and decreases with activity

Intake precision

To maintain weight to 2 kg requires controlling intake to ~50 kcal/day, (out of ~2500 kcal/day)

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Paradox?

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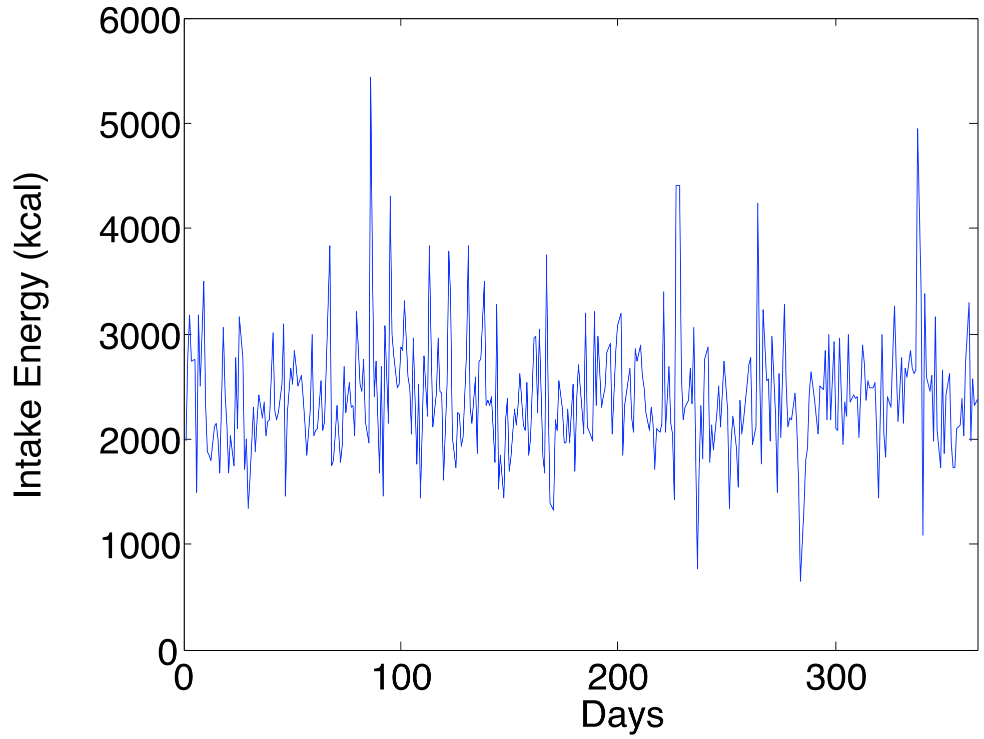
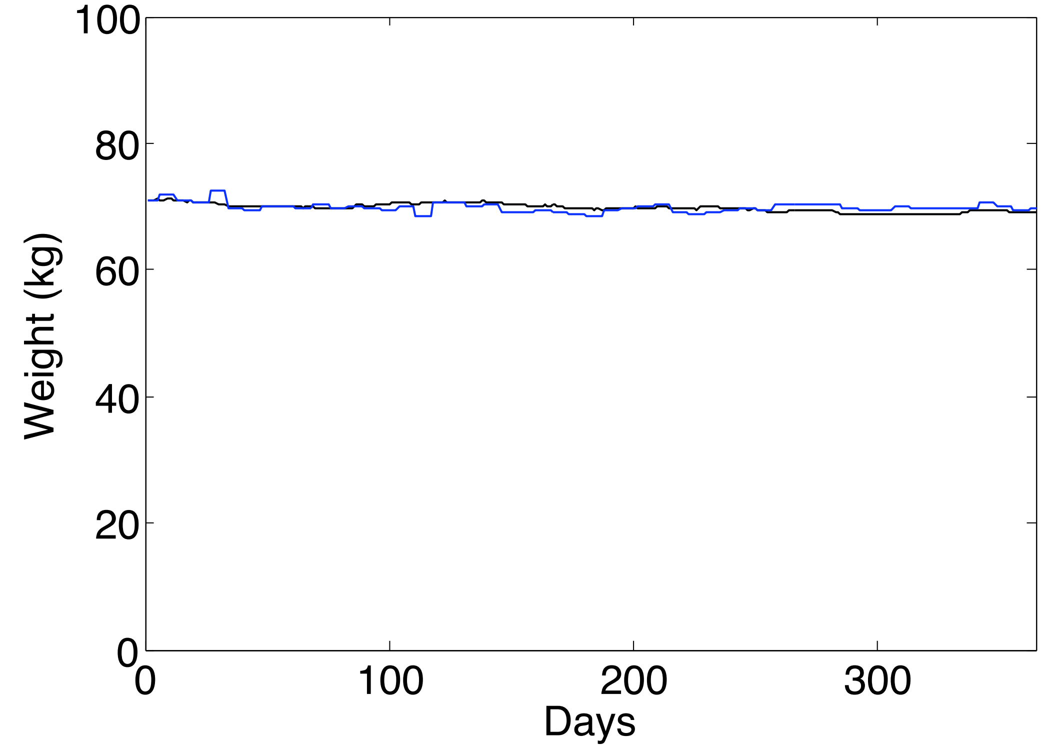
Cookie is ~150 kcal

Paradox? No, because of long time constant

Example daily intake energy

Beltsville one year intake study (courtesy of W. Rumpler)





$CV \sim 24\%$

$CV \sim 1\%$

Intake varations have little effect on weight

Time varying intake

$$I(t) = \bar{I} + \eta(t)$$

Noisy intake

$$\langle \eta(t)\eta(t') \rangle = \sigma^2 \delta(t - t')$$

White noise

$$\rho_M \frac{dM}{dt} = \bar{I} - b - \epsilon M + \eta(t)$$

Ornstein-Uhlenbeck process

CV(I) is σ/I , find *CV(M)*

$$\langle (M(t) - \langle M \rangle)^2 \rangle = \frac{\sigma^2}{2\rho_M \epsilon}$$

$$\langle M \rangle = \frac{I - b}{\epsilon}$$

$$\mathrm{CV}(M) = \frac{1}{\sqrt{2\tau}} \frac{\bar{I}}{I - b} \mathrm{CV}(\bar{I})$$

$$\tau = \rho_M / \epsilon$$

$$\langle (M(t) - \langle M \rangle)^2 \rangle = \frac{\sigma^2}{2\rho_M \epsilon}$$

$$\langle M \rangle = \frac{I - b}{\epsilon}$$

$$\boxed{\mathrm{CV}(M) = \frac{1}{\sqrt{2\tau}} \frac{\bar{I}}{I - b} \mathrm{CV}(\bar{I})}$$

$$\tau = \rho_M / \epsilon$$

$$\langle (M(t) - \langle M \rangle)^2 \rangle = \frac{\sigma^2}{2\rho_M \epsilon}$$

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$$\mathrm{CV}(M) = \frac{1}{\sqrt{2\tau}} \frac{\bar{I}}{I - b} \mathrm{CV}(\bar{I})$$

$$\tau = \rho_M / \epsilon$$

For $\sqrt{2\tau} \sim 30, \bar{I} \sim 2500, b \sim 600$

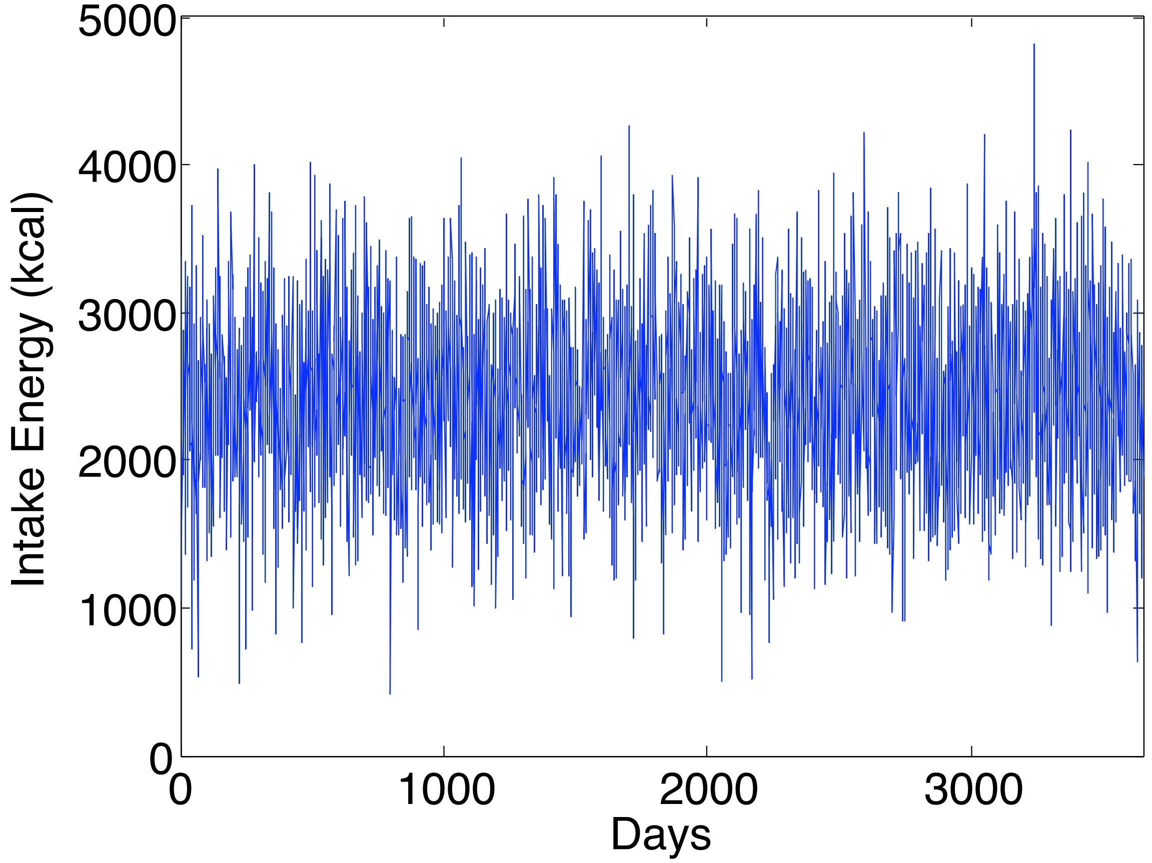
$$\langle (M(t) - \langle M \rangle)^2 \rangle = \frac{\sigma^2}{2\rho_M \epsilon} \qquad \langle M \rangle = \frac{I - b}{\epsilon}$$

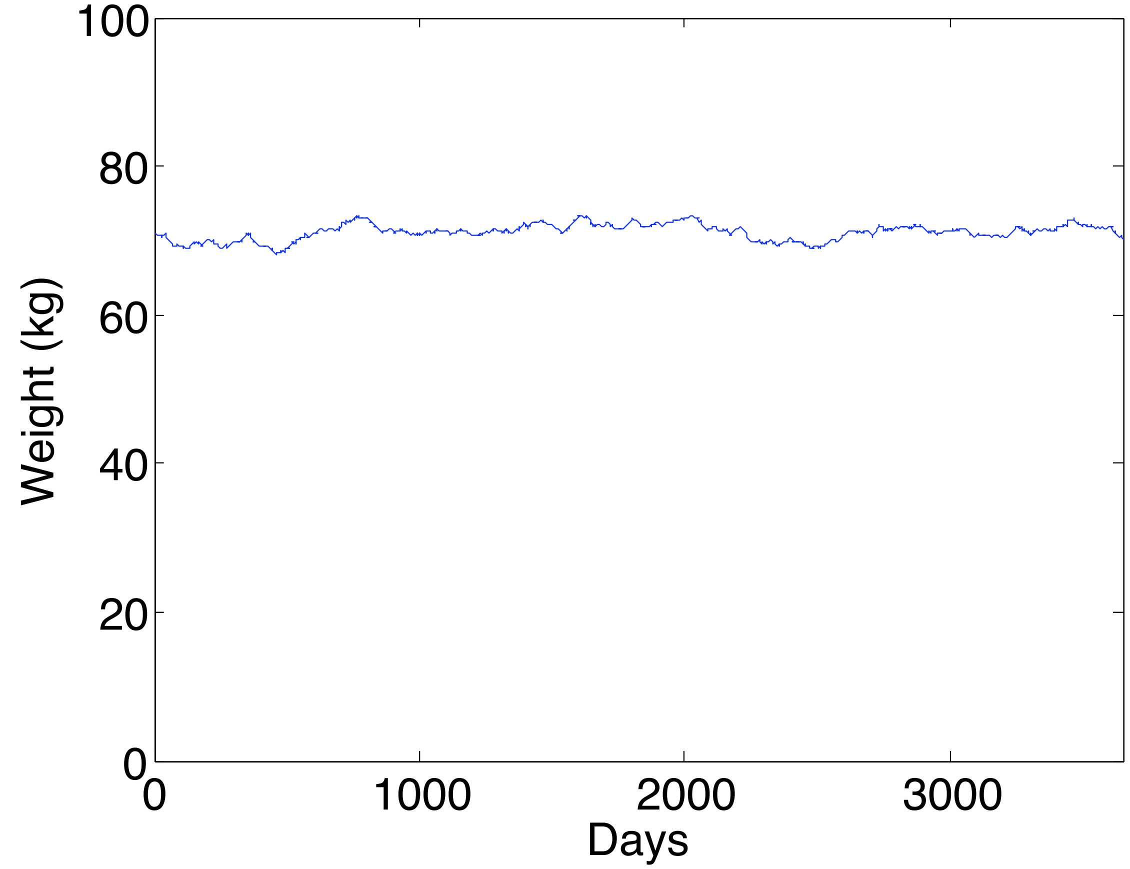
$$\mathrm{CV}(M) = \frac{1}{\sqrt{2\tau}} \frac{\bar{I}}{I - b} \mathrm{CV}(\bar{I}) \qquad \tau = \rho_M / \epsilon$$

For $\sqrt{2\tau} \sim 30, \bar{I} \sim 2500, b \sim 600$

CV (*M*) reduced by factor of *15-20* vs *CV* (*I*)

Simulated Beltsville data 10 years

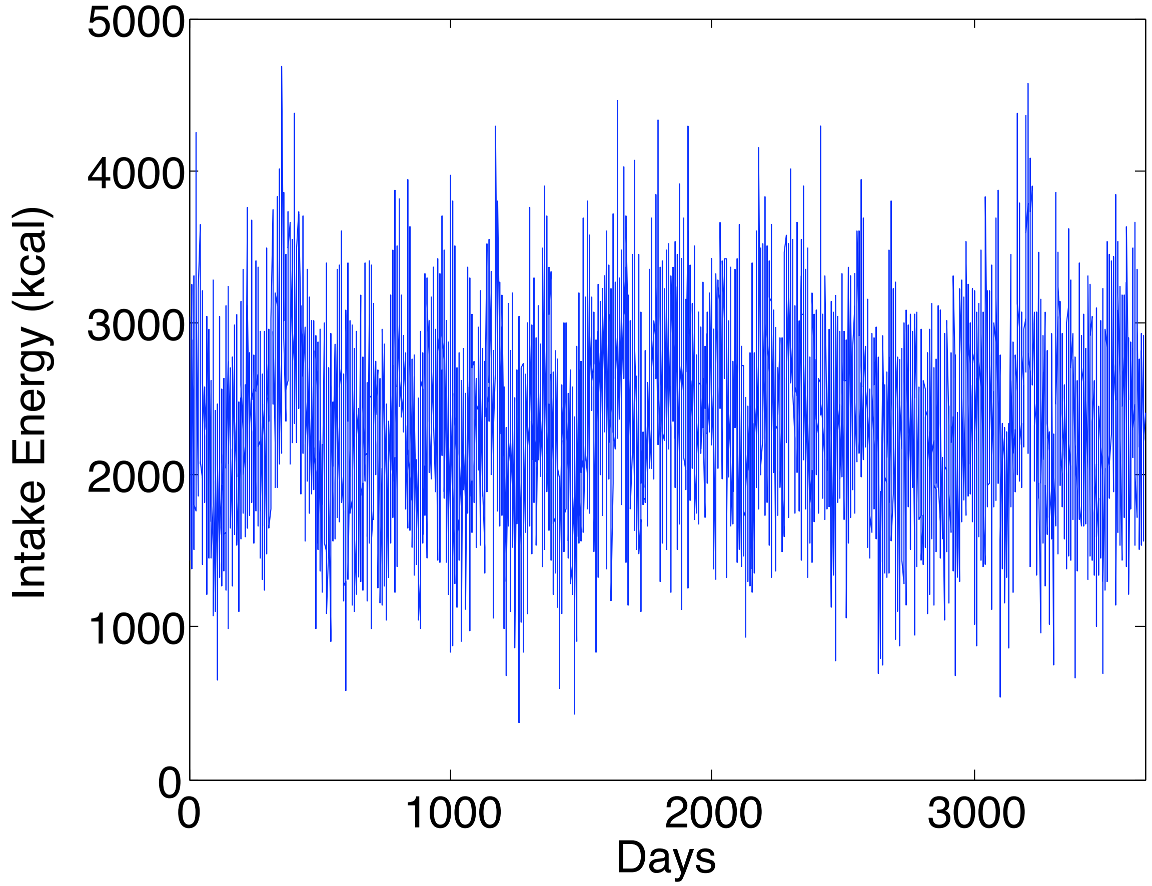


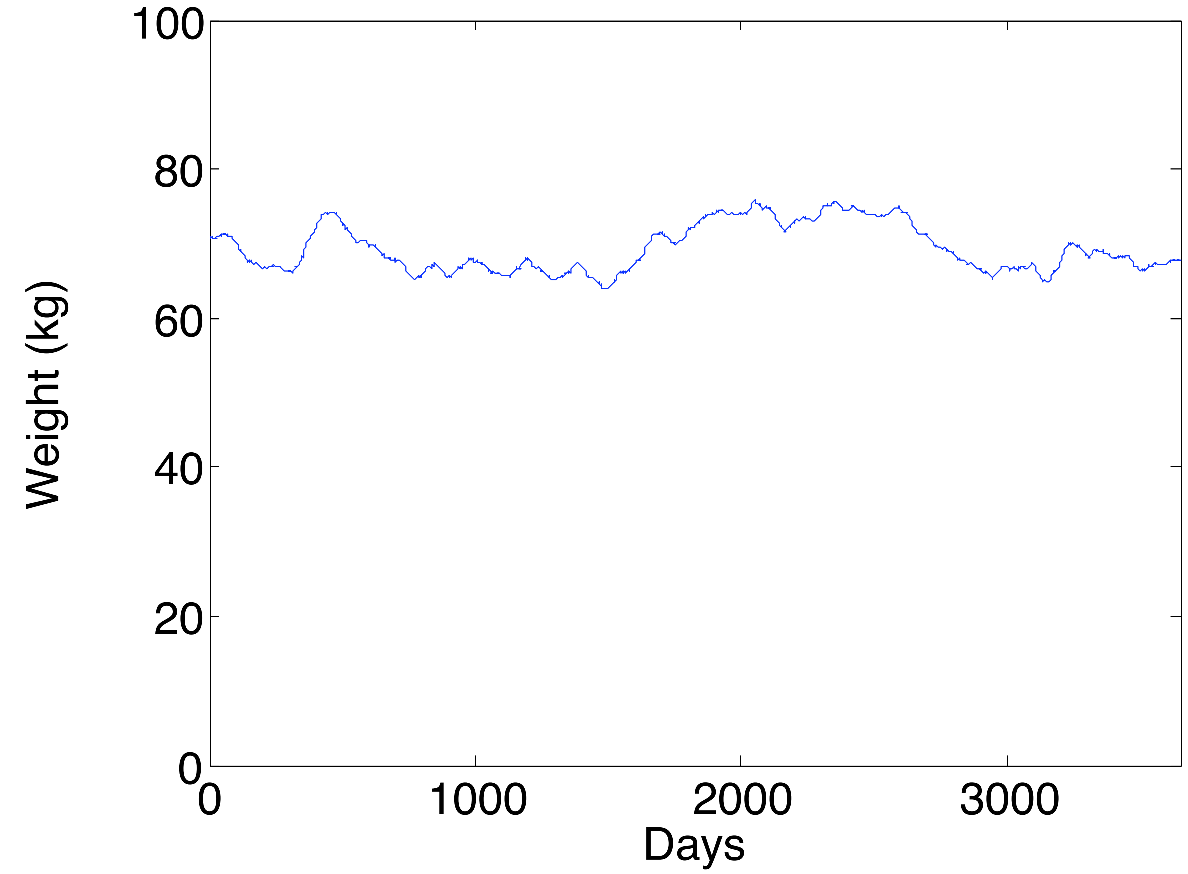


CV ~ *23%*

CV ~ *2%*

Correlations increase fluctuations

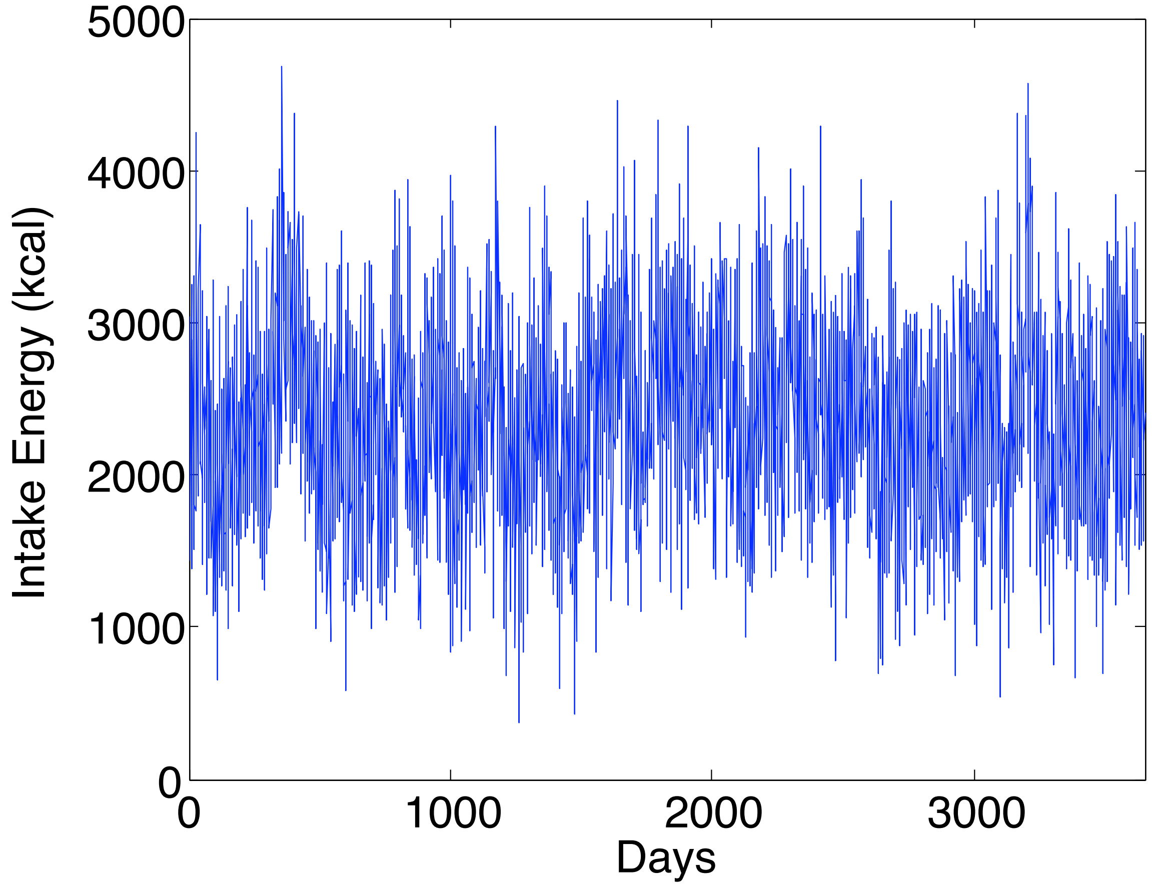


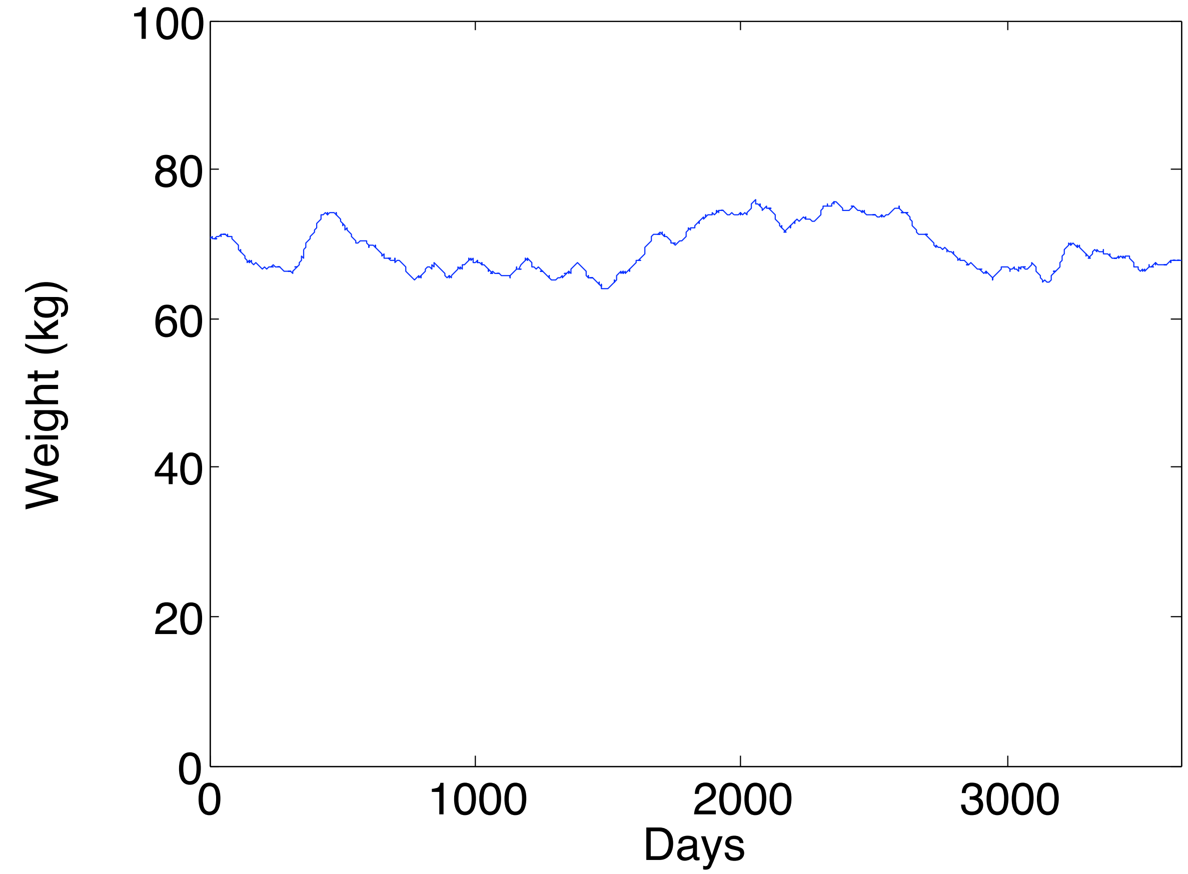


CV ~ *26%*

CV ~ *5%*

Correlations increase fluctuations





CV ~ *26%*

CV ~ *5%*

Longer correlations ⇒ higher BMI

Periwal and Chow, AJP:EM, 291:929-36 (2006)

Acknowledgments

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