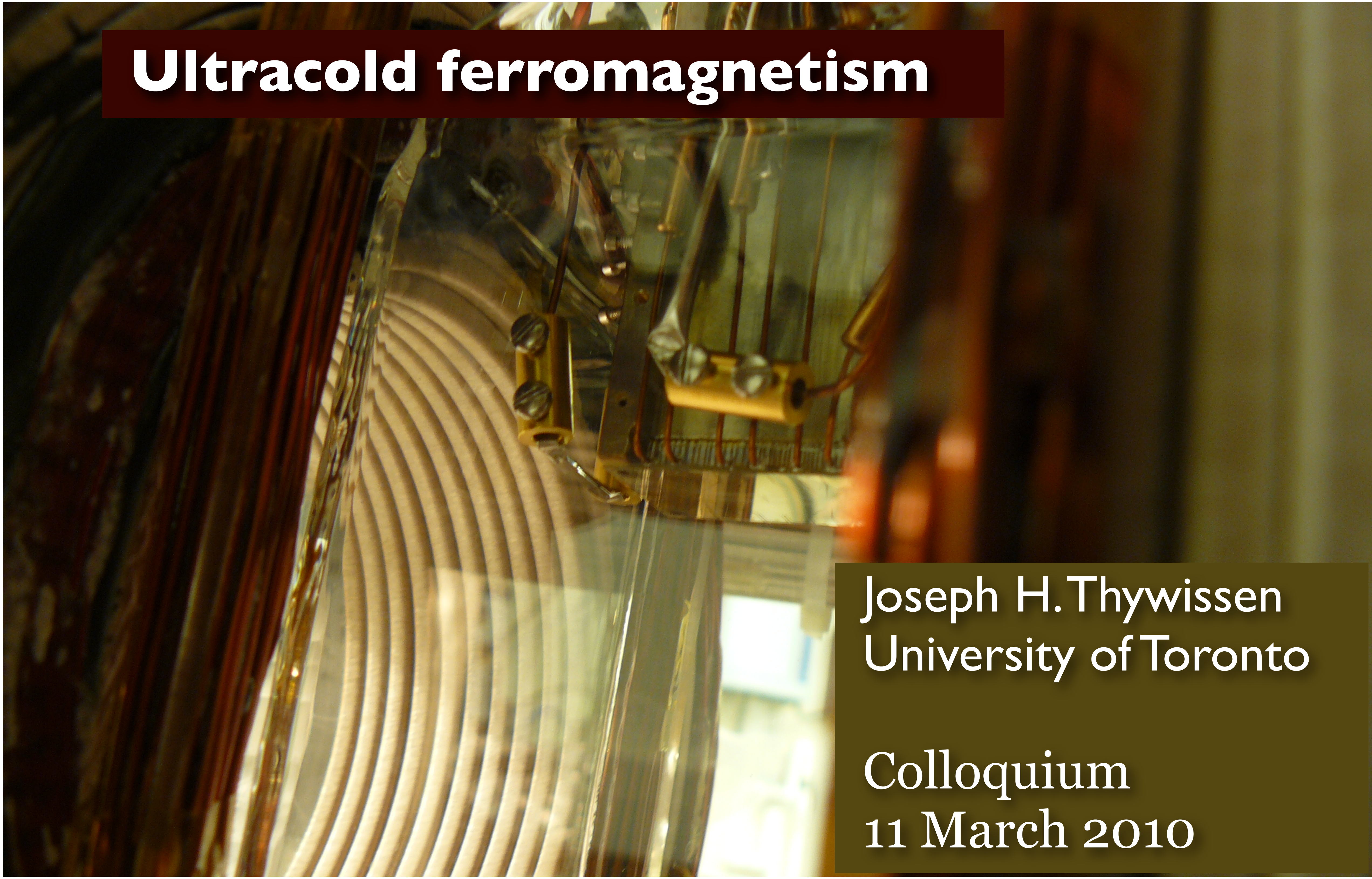
Ultracold ferromagnetism



Joseph H. Thywissen
University of Toronto

Colloquium
11 March 2010

Ontario
NSERC
CRSNG
CIFAR
CUA
VELUT ARBOR ÆVO

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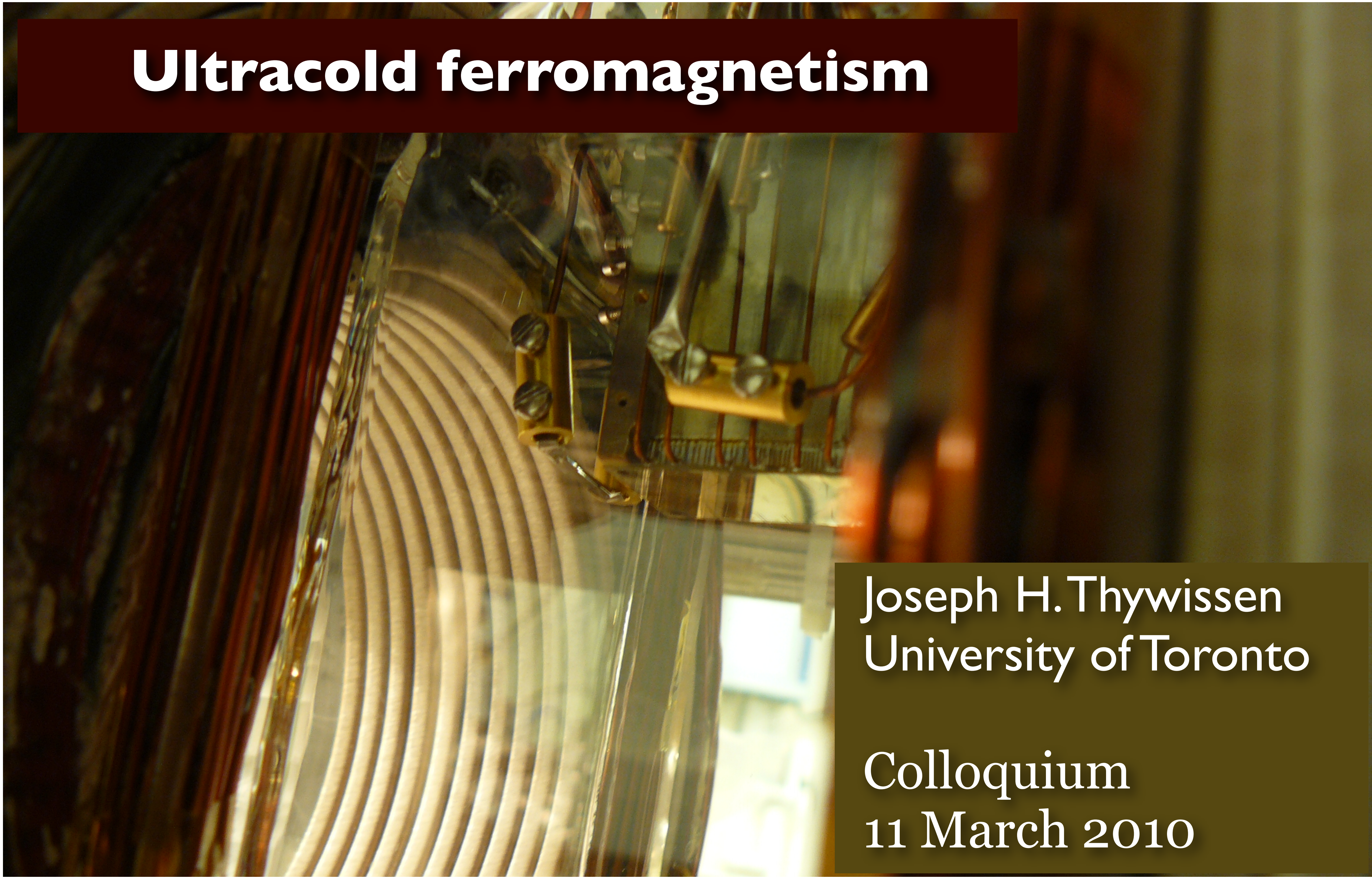

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Ontario NSERC CRSNG CIFAR CUA

Collaborators

MIT experimental group
Gyu-Boong Jo
Ye-Ryoung Lee
Jae-Hoon Choi
Caleb A. Christensen
Tony H. Kim
David Pritchard
Wolfgang Ketterle

Toronto Theory
Arun Paramekanti

Waterloo Theory
Anton Burkov

Toronto experimental group
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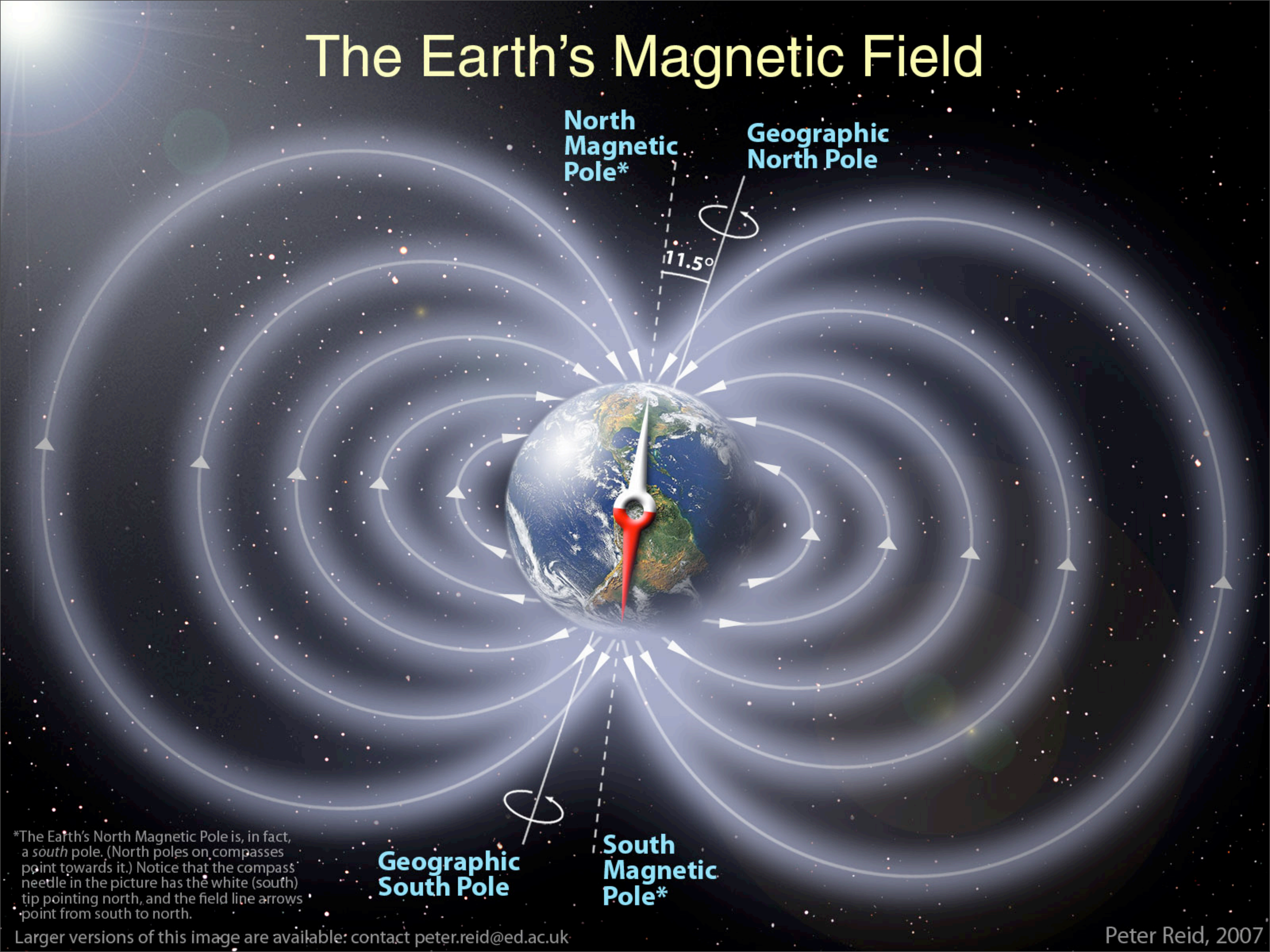
Dylan Jervis
Lindsay J. LeBlanc
David McKay
Dr. Jason McKeever
Dr. Karl Pilch
Matthias Scholl
Michael Sprague
Alan Stummer
JHT

The shepherd Magnes



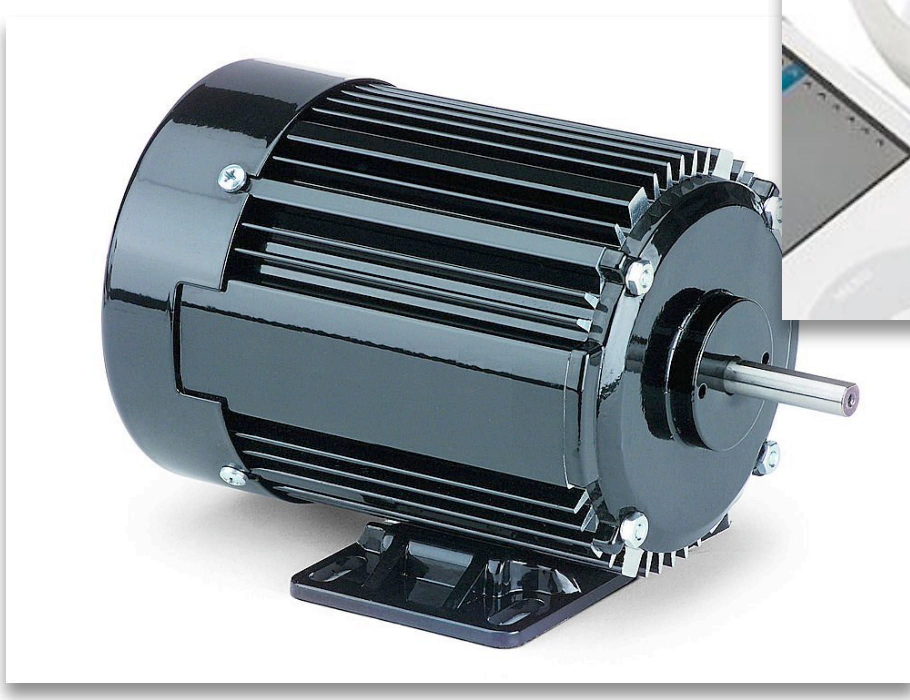
The Earth's Magnetic Field
North Magnetic Pole*
Geographic North Pole
11.5°
Geographic South Pole
South Magnetic Pole*
*The Earth's North Magnetic Pole is, in fact, a *south* pole. (North poles on compasses point towards it.) Notice that the compass needle in the picture has the white (south) tip pointing north, and the field line arrows point from south to north.
Larger versions of this image are available: contact peter.reid@ed.ac.uk
Peter Reid, 2007

Long history of magnetism

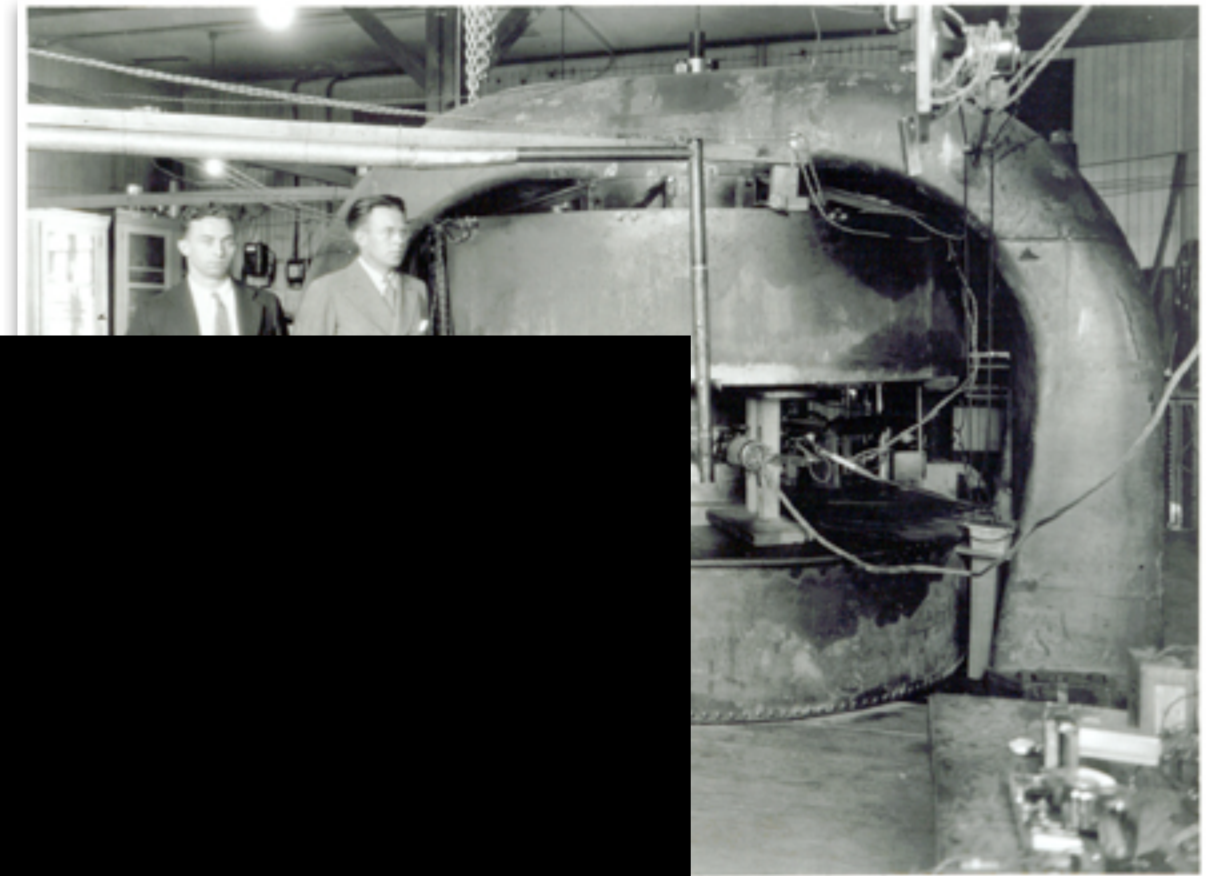
- Greeks (600 BC): lodestone attracts iron
- Gilbert (England, 16 c.): Earth is a weak magnet
- Gauss (Germany, 18 c.): theory…
- Coulomb (France, 18 c.): inverse square law
- Oersted (Denmark, 19c.): connection to electricity
- Ampere, Faraday (19 c.): how E-fields relate to B-fields
- Maxwell (Scotland, 19 c.): E&M unification
- Curie, Weiss (19 c.): effect of T on magnet
- Ising, Heisenberg, Bloch, Stoner (20 c.): quantum theory
- Weinberg, Salam (20 c.): electroweak unification

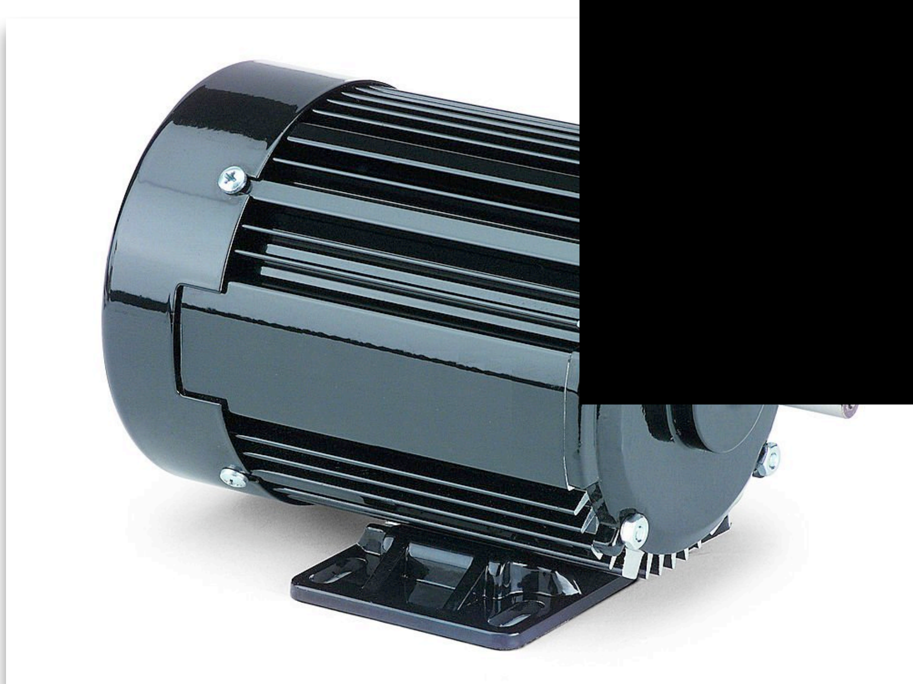
Applications

Applications



Applications





Applications

Magnetism in materials

Magnetism in materials

- for an isolated atom:
  -magnetic dipole moment => paramagnetic
  -at extremely high field, a diamagnetic term. Effectively only visible when all e^- dipole moments are paired.

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- Permanent magnetism: ferromagnetism
  -topic of discussion today
  -result of Fermi statistics and electron interactions

Magnetism in materials

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- Bulk material:
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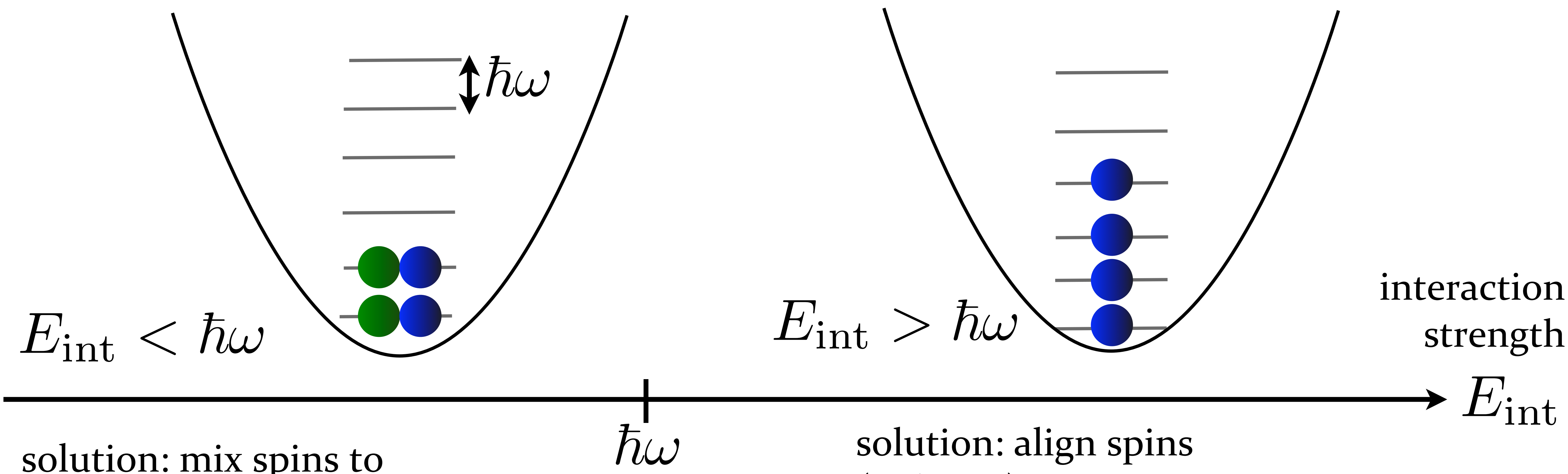
- Permanent magnetism: ferromagnetism
  -topic of discussion today
  -result of Fermi statistics and electron interactions
- Antiferromagnetism even more common, but not easily discernible (eg, not to Magnes the shepherd.)

Basic energetics of ferromagnetism

Total energy = single-particle energy + interaction energy

$$E_{\text{tot}} = \hbar\omega \sum n_i + E_{\text{int}} N_{\uparrow} N_{\downarrow}$$

For example, what configuration minimizes energy for 4 particles ?



solution: mix spins to minimize maximum occupied level.

solution: align spins (polarize) to minimize interaction energy.

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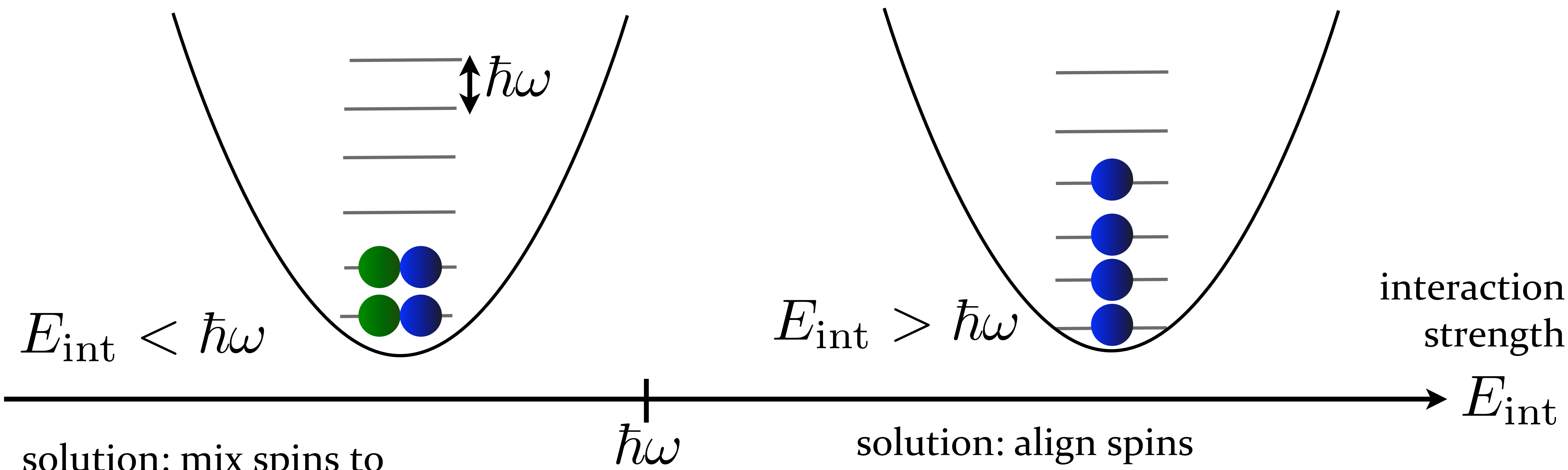


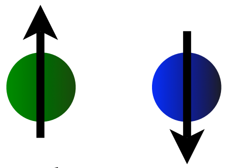
solution: mix spins to minimize maximum occupied level.

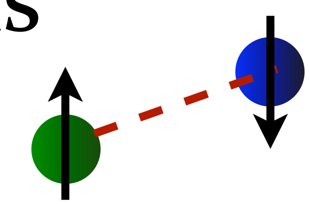
solution: align spins (polarize) to minimize interaction energy.

Ferromagnetic configuration is *strongly interacting*:
Interaction energy must be higher than single-particle energy.

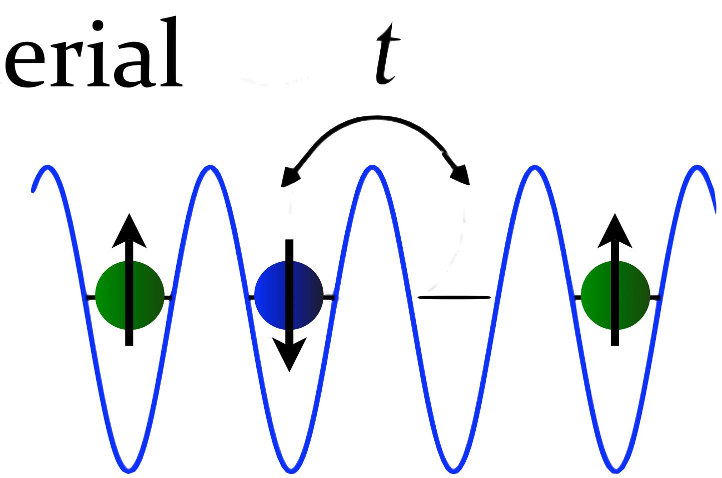
Ingredients

that we find in ferromagnetic materials

1. Fermions 
   -unpaired electrons

2. Repulsive interactions
   -Coulomb repulsion 

3. Lattice
   -structure of material



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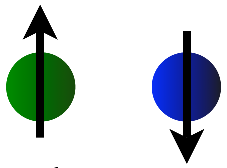
1. Fermions 
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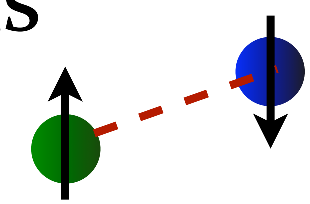
   -Coulomb repulsion 

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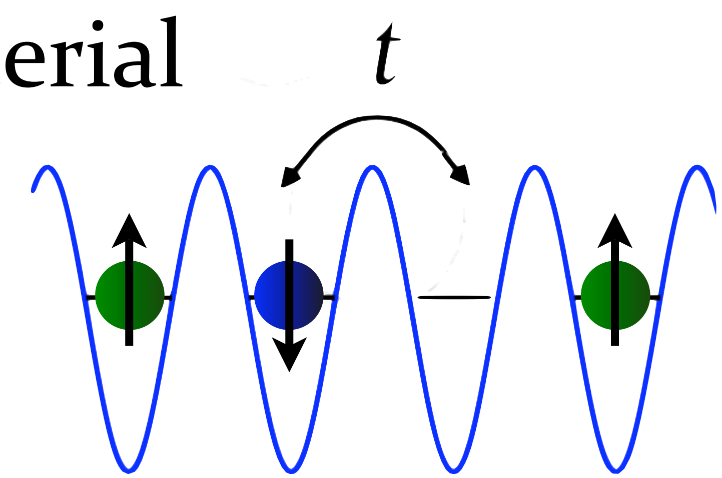

} Necessary to energetics.

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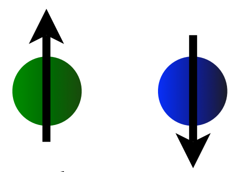
that we find in ferromagnetic materials

1. Fermions 
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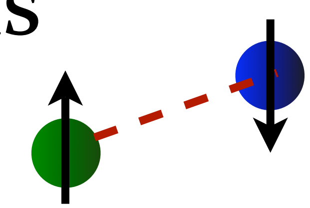
   -Coulomb repulsion 

Necessary to energetics.

3. Lattice
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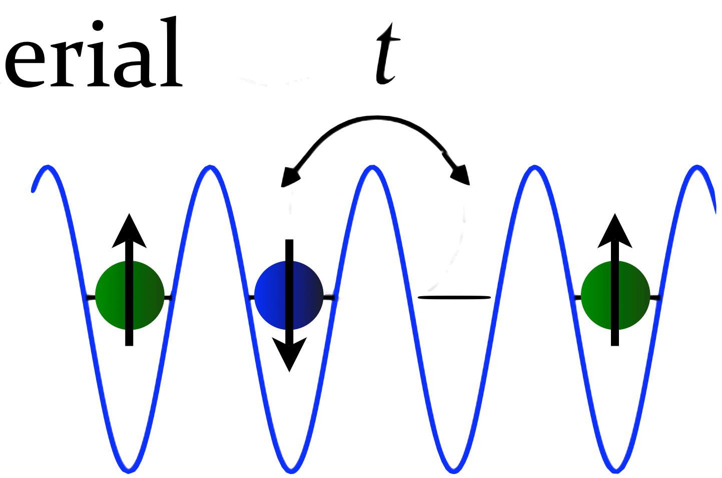


Necessary?
What about a gas?

What is the simplest condition in which permanent magnetism can occur?

Ultra-cold atoms

- **Neutral gases cooled to quantum degeneracy**
  …Bosons (integer spin): form Bose-Einstein condensate
  …Fermions (half-integer spin): degenerate Fermi sea
- Can choose the mixture of states
  …in this problem, use two states of fermions
- Either with or without a lattice environment
- **Interactions can be tuned** to infinitely strong + or -
  …using a Feshbach resonance

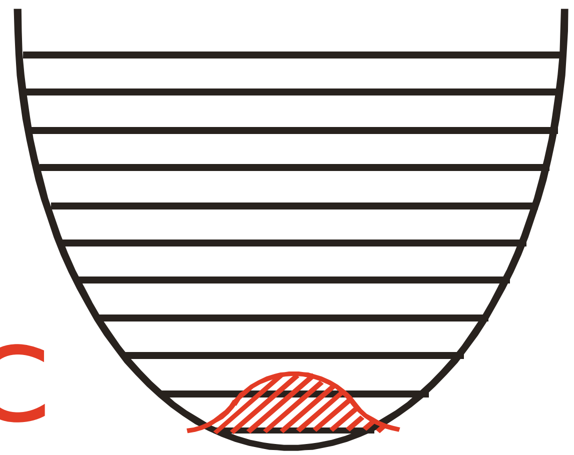
A fun “sand box” for physicists!

Bosons versus Fermions

"The social particle"

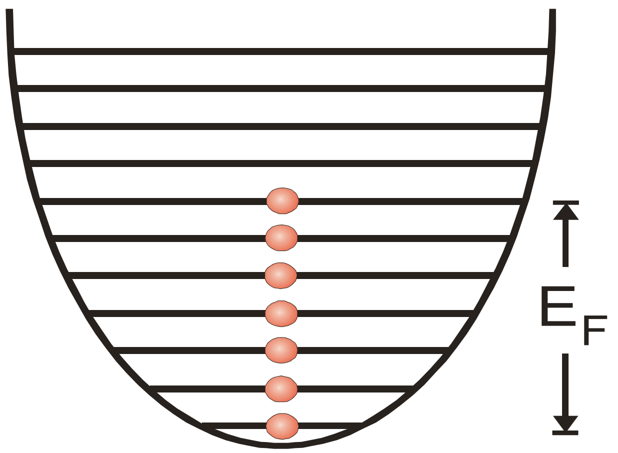
"Antisocial"

@low temperature:



BEC

(Bose-Einstein condensate)



DFG

(Degenerate Fermi gas)

examples:

photons (J=1)
Helium (J=0)
^{87}Rb (F=2)

examples:

electrons (J=1/2)
protons, neutrons
^{40}K (F=9/2)

Bosons versus Fermions

"The social particle" | "Antisocial"

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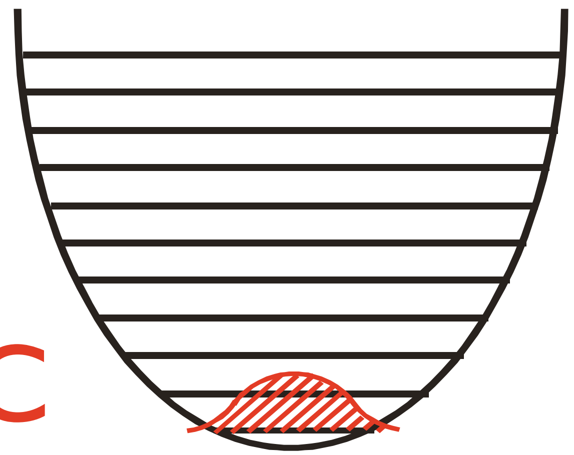

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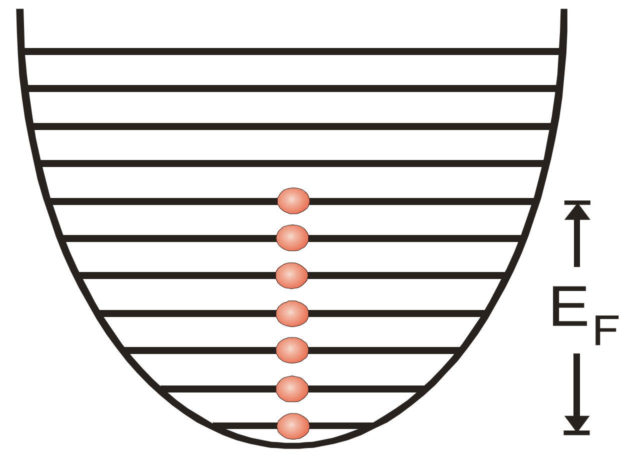


DFG

(Degenerate Fermi gas)

Bosons — examples:

photons (J=1) → lasers

Helium (J=0) → superfluid

^{87}Rb (F=2)

Fermions — examples:

electrons (J=1/2) → periodic table

protons, neutrons → stability of neutron stars

^{40}K (F=9/2)

^{87}Rb and ^{40}K → alkali gases

How cold do you have to be?

- Einstein's criterion: $n\lambda_{\mathrm{dB}}^3 \geq 1$
- LHS of inequality scales as $\sqrt{\frac{n^2}{T^3}}$

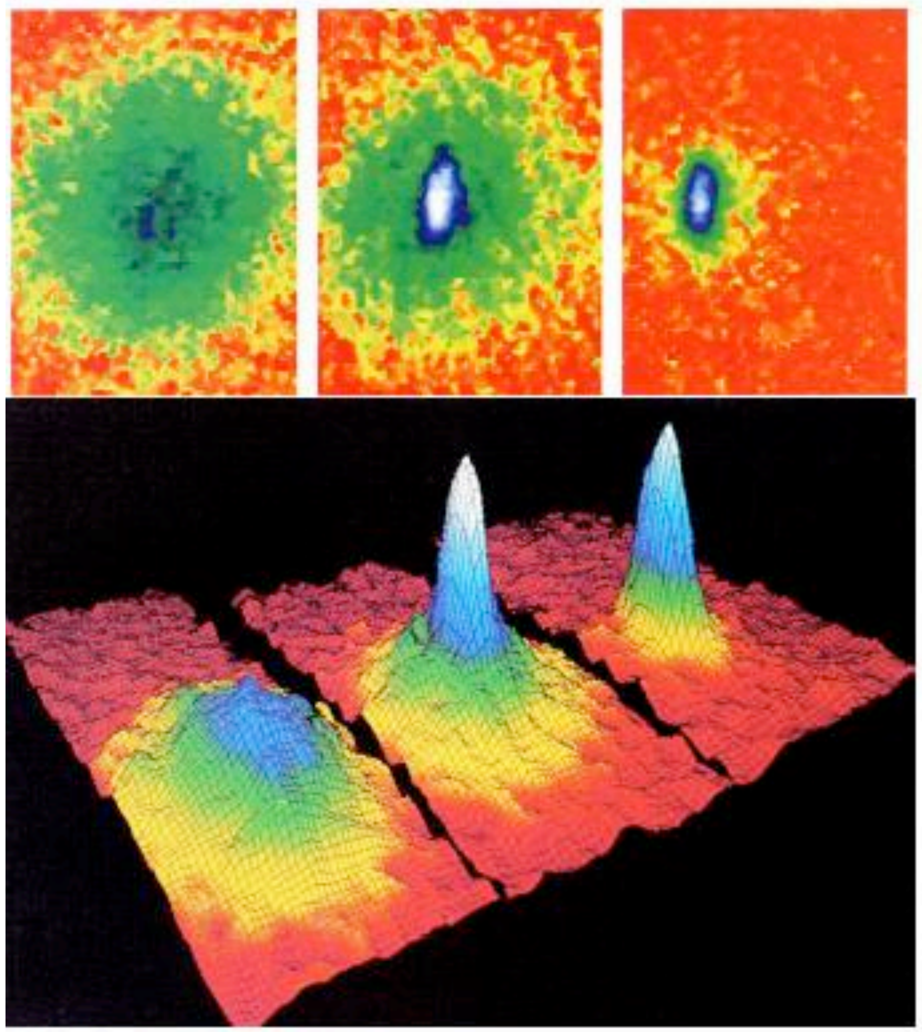
where:
λ_{dB} =deBroglie wavelength
n =density
T =temperature

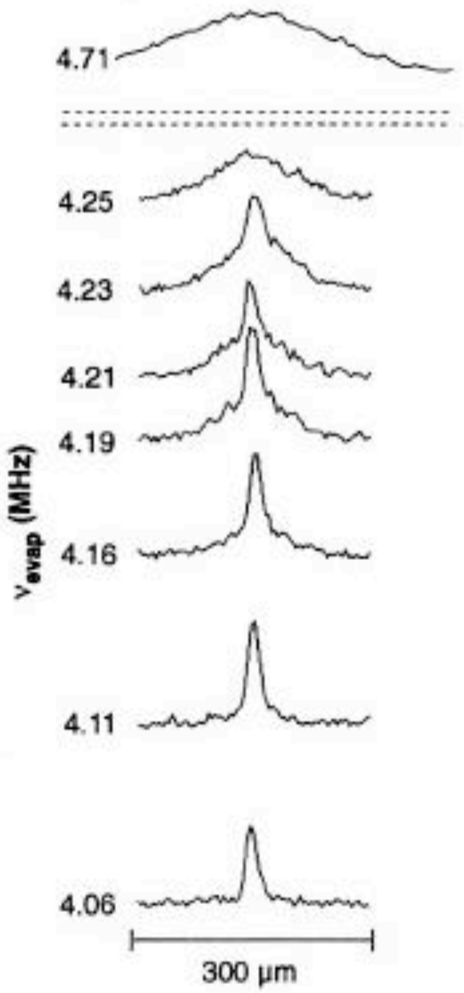
- Superfluid ^{4}He: 2 Kelvin, but 10^{22} cm^{-3}
gas density: 10^{13} cm^{-3} --> 10^9 less dense

We need to be a million times colder than superfluid liquid He.

- In real numbers: **about 500 nK!**

1995 BEC (Boulder)





M.H.Anderson, J.R. Ensher, M.R. Matthews, C.E Wieman and E.A. Cornell, *Science* ***269***, *198 (1995)*.

<u>measured:</u>

T=170 nK

n=2.5 x 10^{12} cm^{-3}

<u>Rb:</u>

a=5 nm

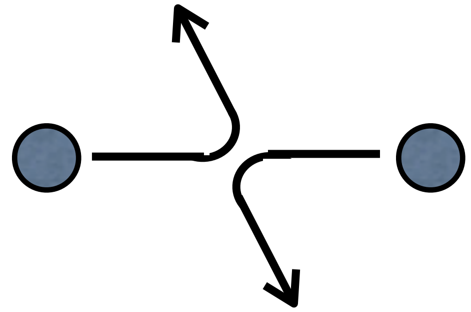
r_0~2 nm

Interactions:

1. Dilute?
2. Ultra-cold?
3. Weakly interacting?

Collisions when ultra-cold

- At low energy, only L=0 partial wave contributes to scattering: $\sigma = 4\pi a^2$



- Always true for dilute degenerate gas:

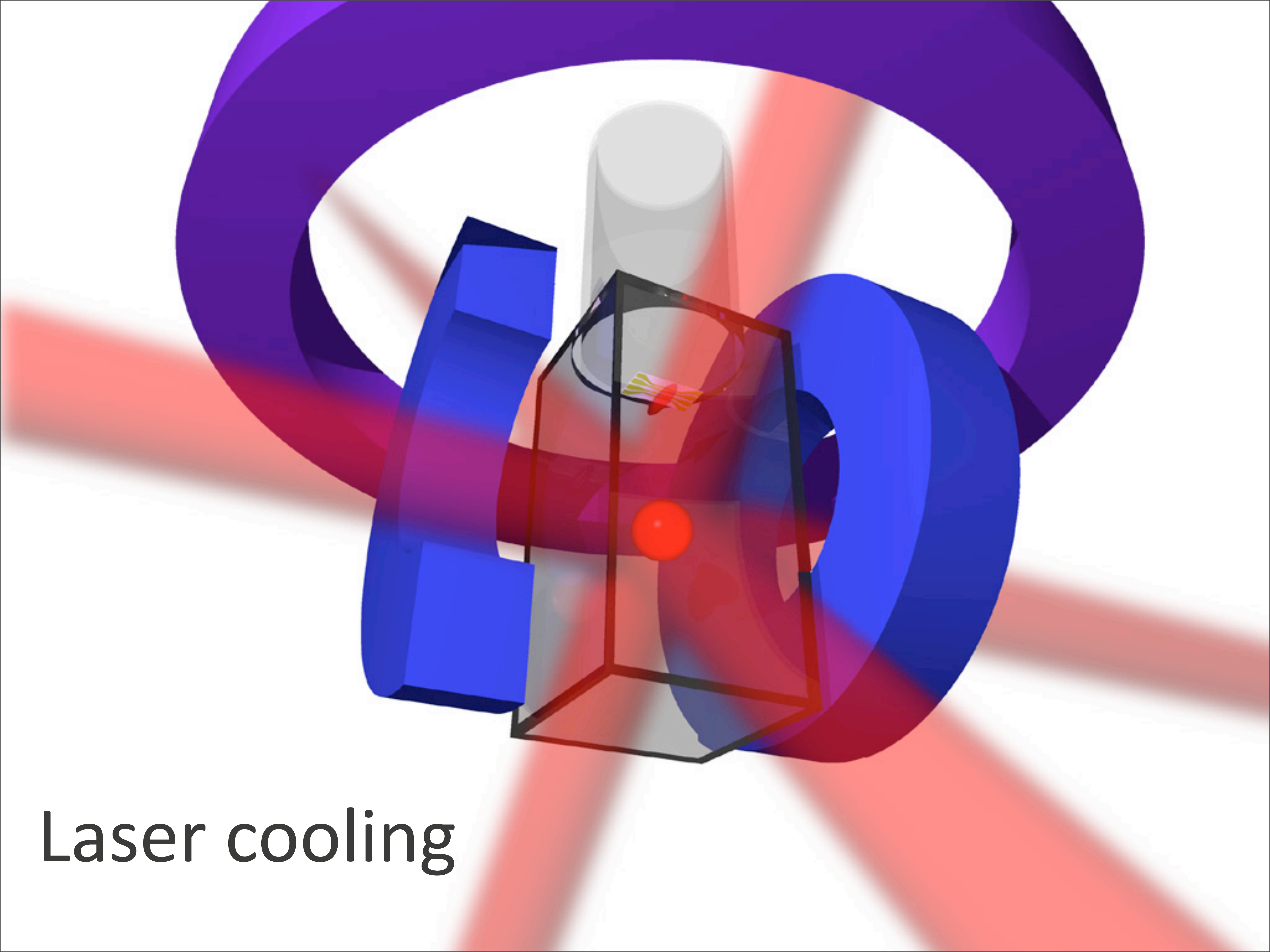
$$\frac{h}{p} \sim \lambda_{\rm dB} > d \gg r_0$$

...where p is a typical collision momentum.

- Can therefore use *any potential* with correct s-wave scattering length!

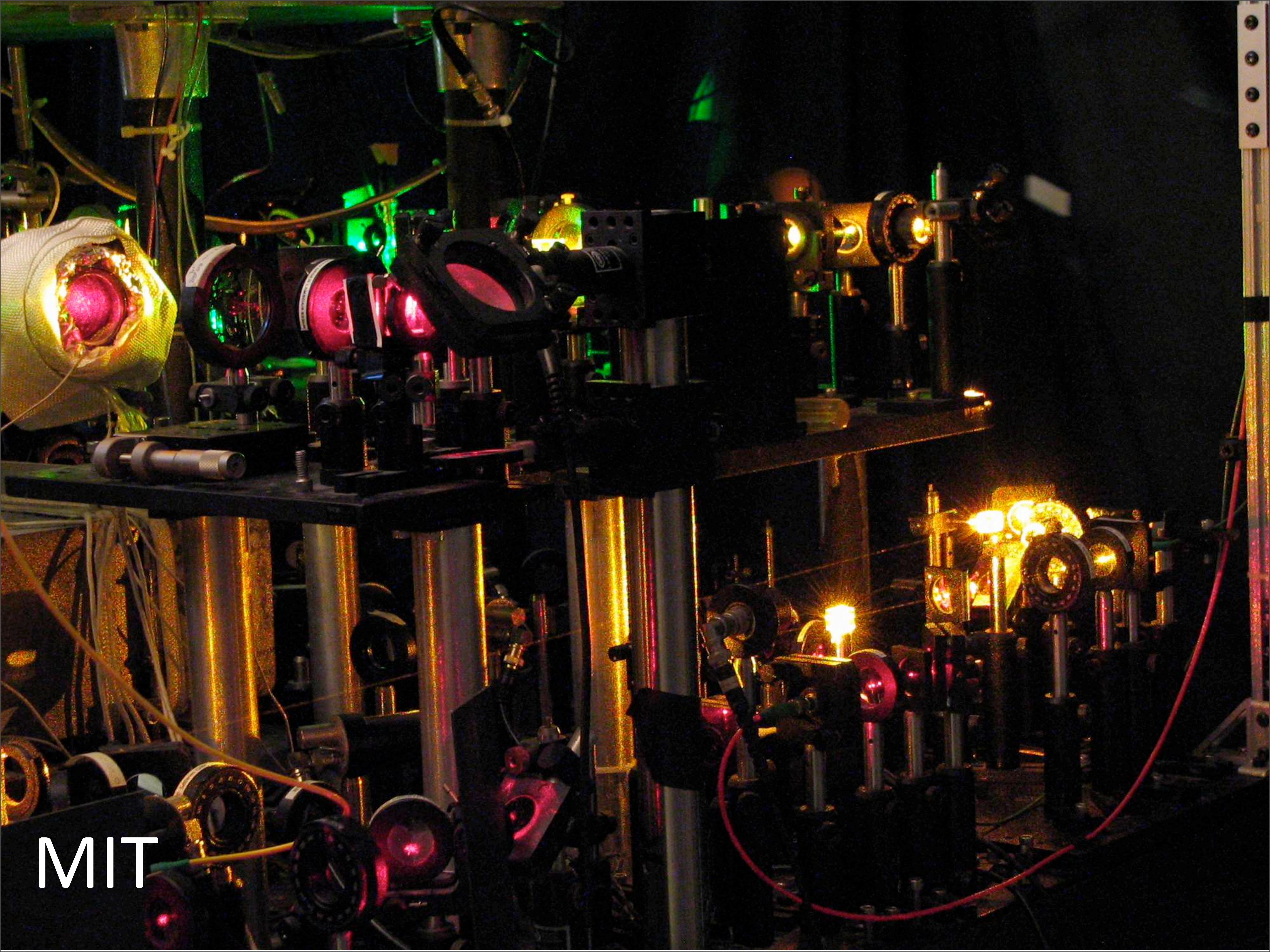
$$V(\vec{r'} - \vec{r}) \to \frac{4\pi\hbar^2 a}{m}\delta(\vec{r'} - \vec{r})$$

(let's choose an easy one)



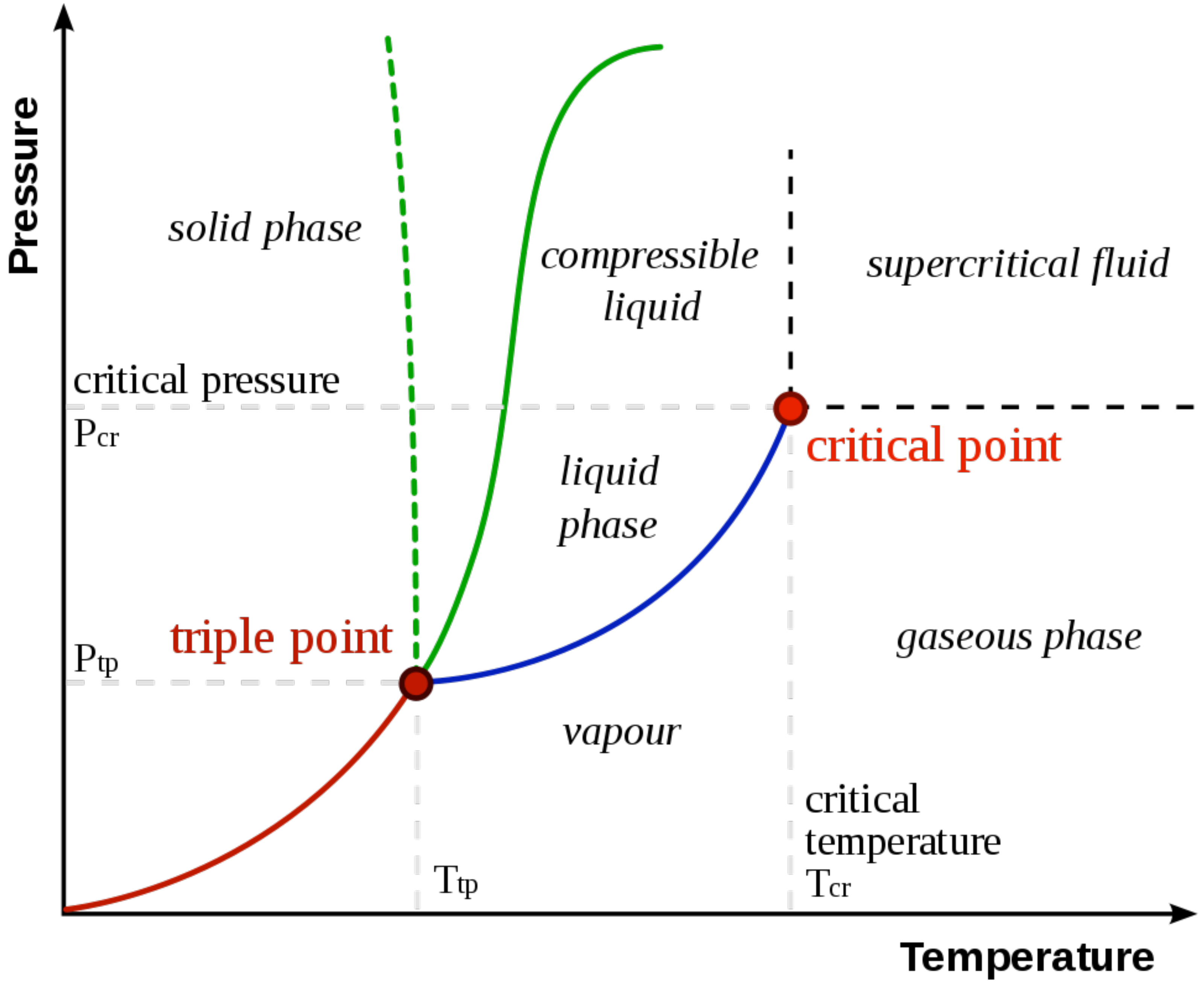
Laser cooling


Toronto


MIT

Won't it just freeze?

- At low enough T, everything but He a solid.
- Where is a BEC on the phase diagram?



"If it were really impossible, they wouldn't have bothered to forbid it."

--Eric Cornell, paraphrasing Joseph Heller

Metastability

- In fact, a BEC is not in equilibrium, but slowly dying, via *3-body recombination*

$$\Gamma = Ln^2$$

L=3-body rate

- Must also preserve thermal equilibrium, via *2-body* collisions: $\gamma = n\sigma\bar{v}$

$\left\{ \begin{array}{l} \sigma = \text{cross-section} \\ \bar{v} = \text{collision velocity} \end{array} \right.$

- Combine $\Gamma/\gamma < 10^{-2}$ with degeneracy:

Using numbers for ^{87}Rb:

$$k_B T < \frac{4\pi\hbar^3\sigma}{10^2 L m^2} \approx \underline{15\mu\text{K}}$$

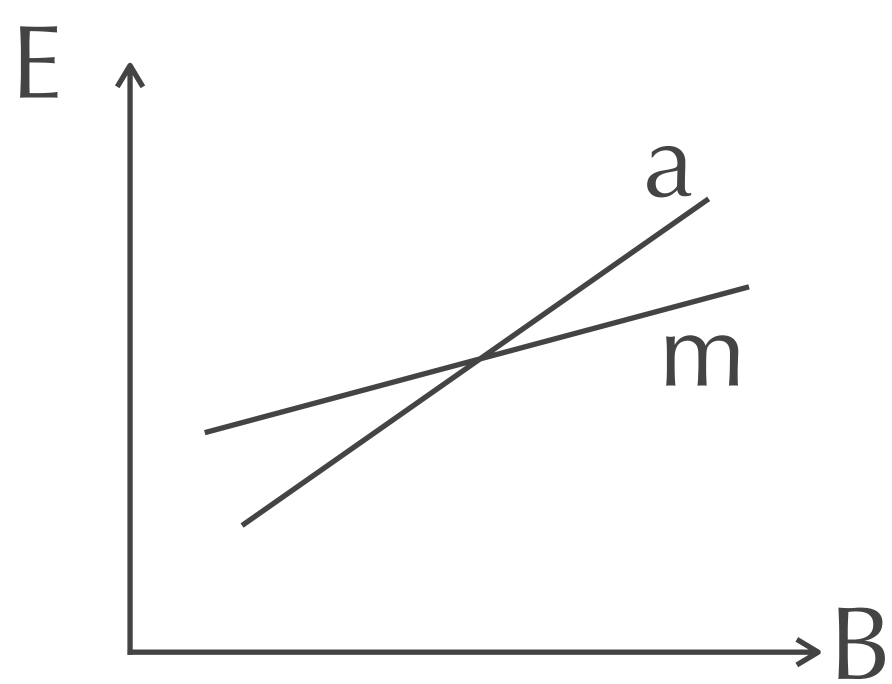
Experiments: typically 100nK to 5μK

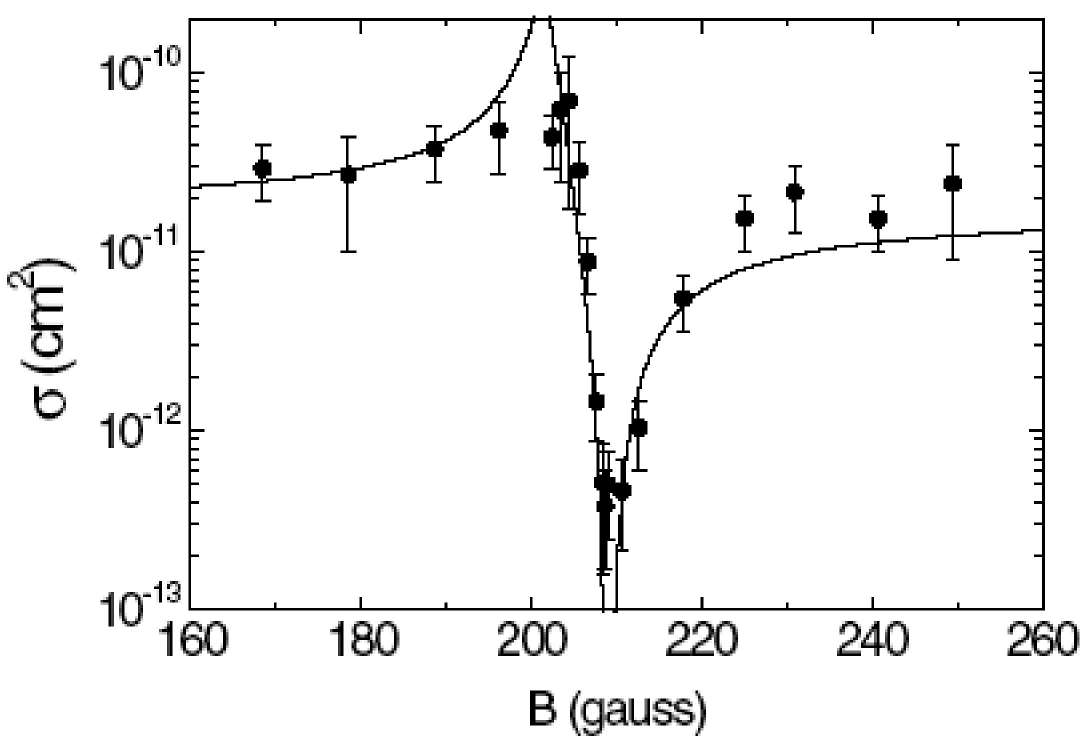
Degenerate gas can be metastable if it's ultra-cold.

Weak or Strong interactions

- Weakly interacting when $n|a|^3 \ll 1$ {Boulder: 10^{-7}}
- However scattering length tuned by magnetic field:

Verhaar and co-workers: Phys. Rev. A **46**, R1167 (1992); Phys. Rev. A **47**, 4114 (1993).

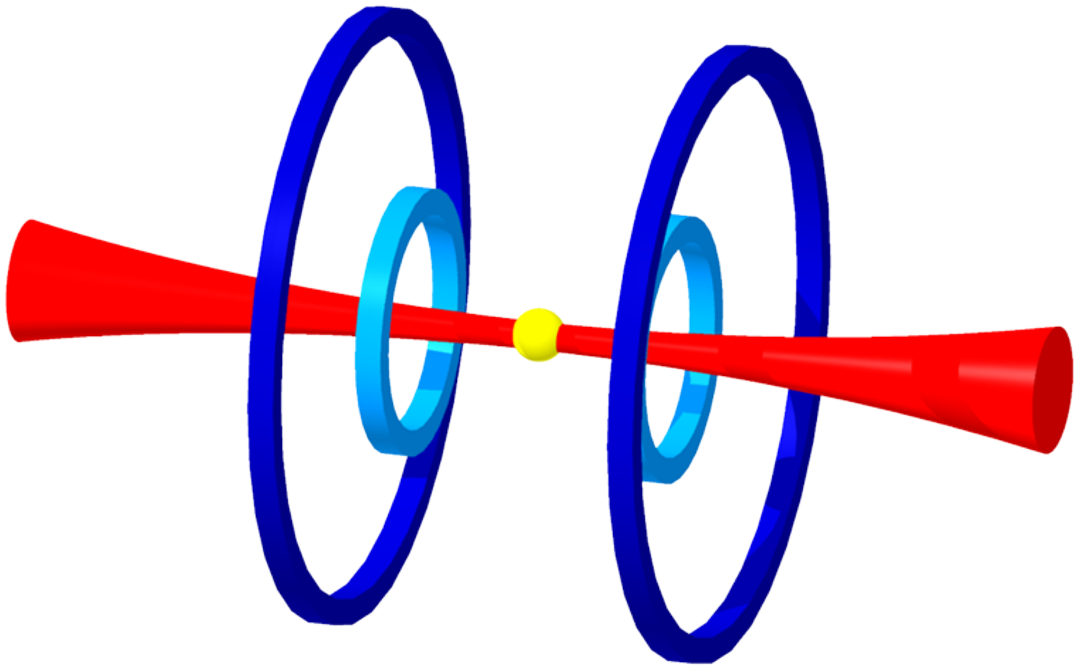


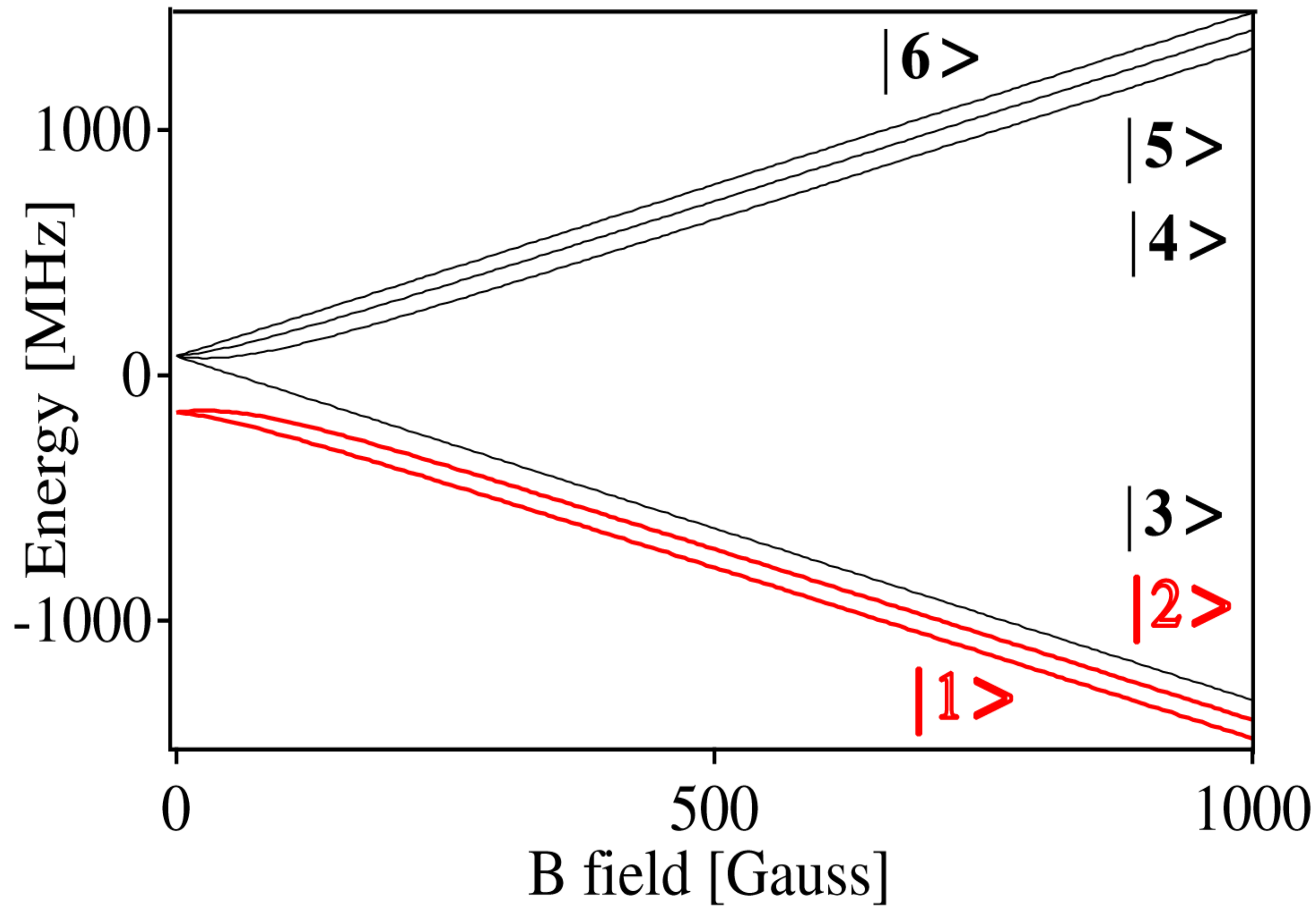


Loftus...Jin, Phys. Rev. Lett. **88**, 173201 (2002).

- In strongly interacting regime, *still dilute and s-wave*, but more than pairwise interaction.

Preparation of an repulsively interacting Fermi system in Li-6

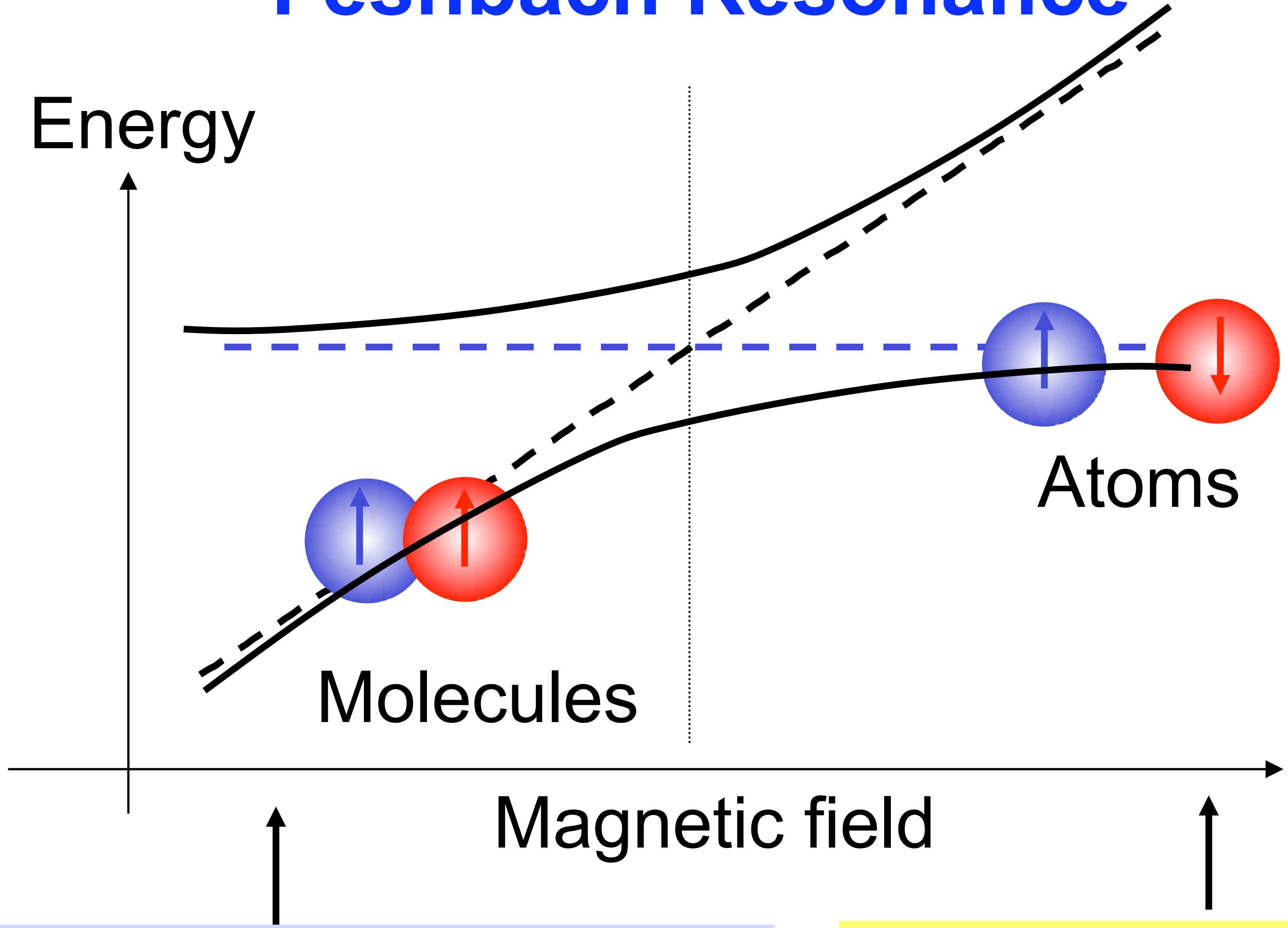




One million Li-6 atoms at 150 nK trapped in the focus of an infrared laser beam

States |1> and |2> correspond to |↑> and |↓>

Feshbach Resonance



Atoms form stable molecules

Atoms repel each other
a>0

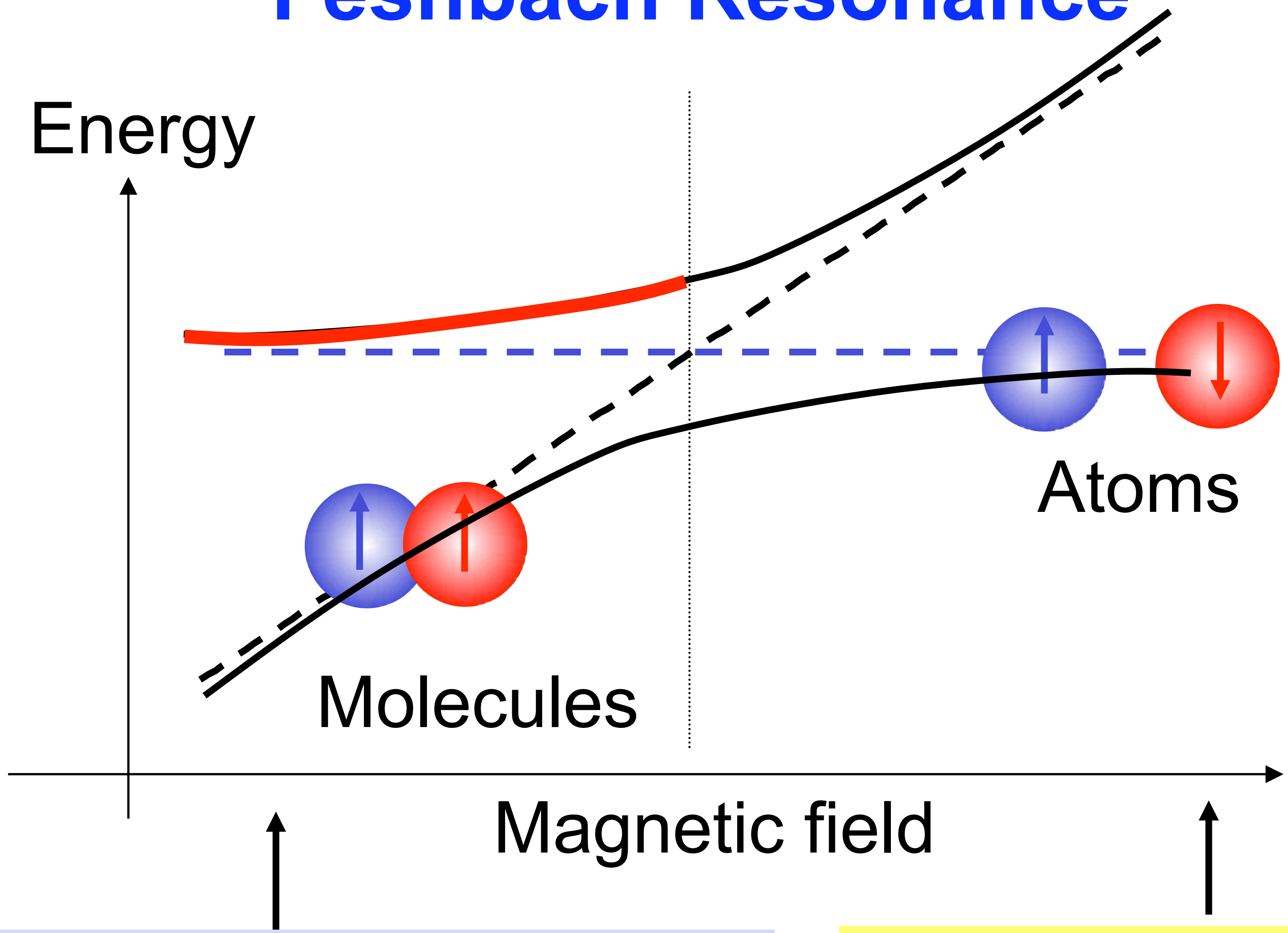
BEC of Molecules:
Condensation of
tightly bound fermion pairs

Molecules are unstable

Atoms attract each other
a<0

BCS-limit:
Condensation of
long-range Cooper pairs

Feshbach Resonance



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Atoms attract each other
a<0

Itinerant Ferromagnetism
Stoner instability
in a free gas

BCS-limit:
Condensation of
long-range Cooper pairs

CM-AMO line-up

Electron

charged:
(screened) Coulomb interaction

Crystal environment

Curie point: >300K

Found naturally
+$$$

Atom (composite fermion)

neutral:
contact interaction
tuneable with Feshbach resonance

Gas (trapped)
could apply a lattice though…

<300 nK

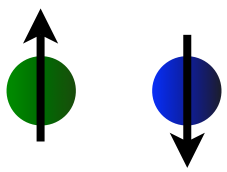
Coldest spot in the universe
-$$$

Stoner ferromagnetism

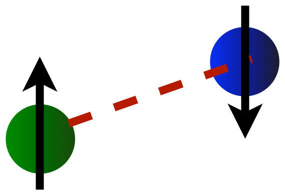
(a simple model)

$$\hat{H} = \int d^3x \left[\sum_{\sigma} a_{\sigma}^{+} \left(\frac{p^2}{2m}\right) a_{\sigma} + g a_{\uparrow}^{+} a_{\downarrow}^{+} a_{\downarrow} a_{\uparrow} \right]$$

1. Fermions
   - cold atoms
   - two states

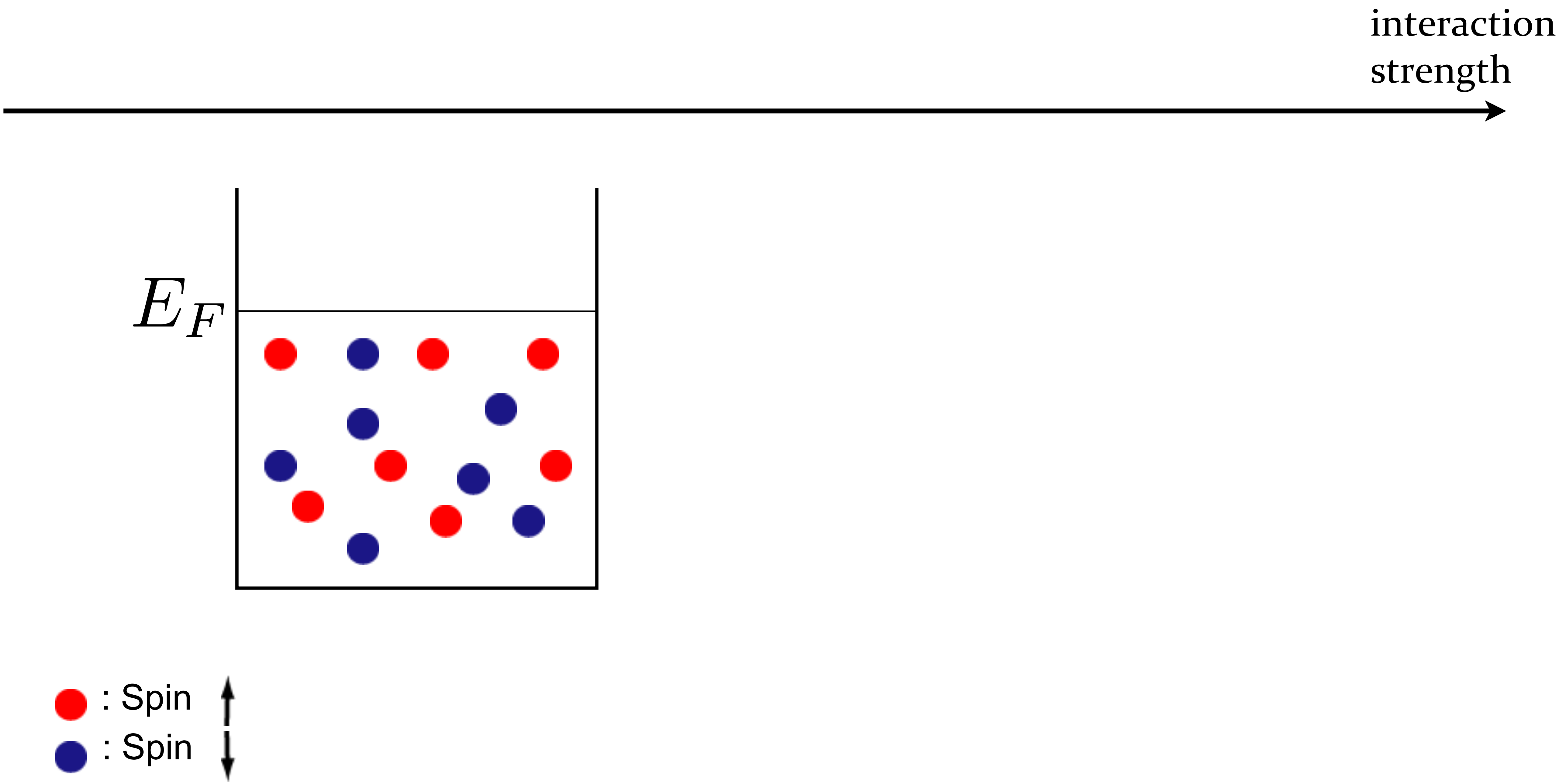


2. Repulsive interactions
   - contact potential
   - strength adjustable



3. No lattice (… but will consider a trapping potential)

Polarization in a 3D uniform Fermi gas



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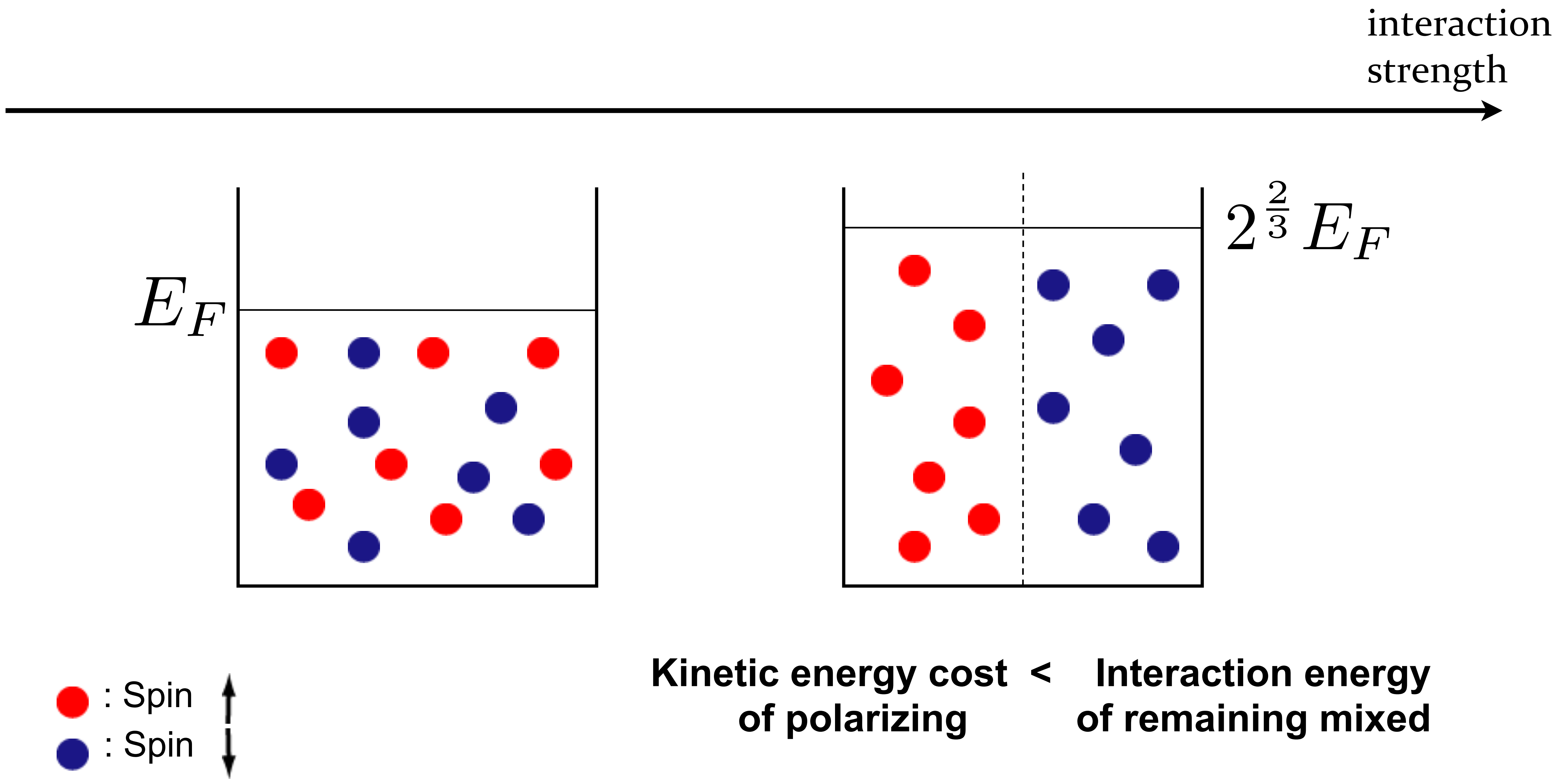


Kinetic energy cost of polarizing < Interaction energy of remaining mixed

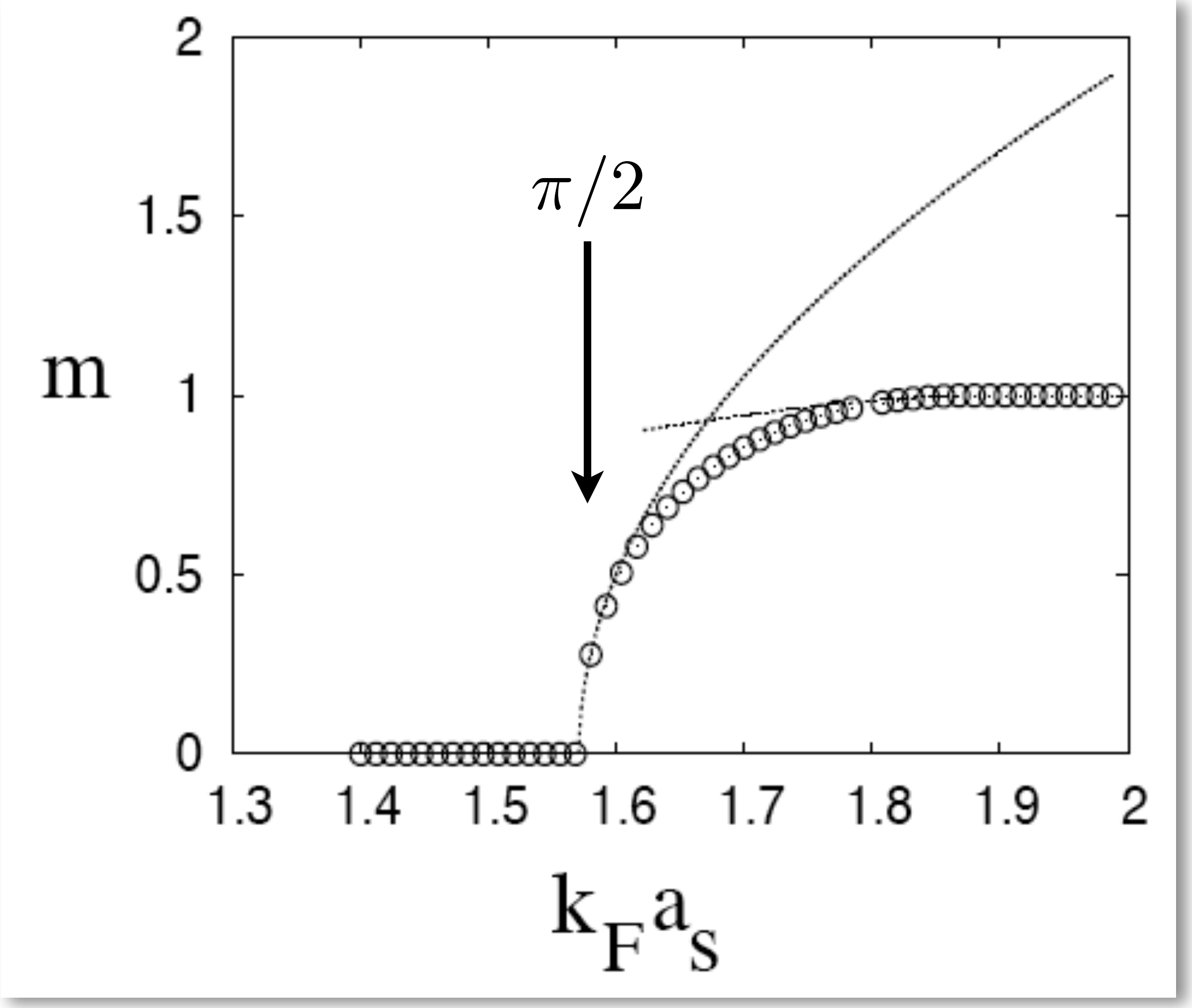
domain formation!

Mean field treatment of Stoner

Uniform 3D system:

$$E = \frac{3}{10} E_{\mathrm{F}}[(1+m)^{5/3} + (1-m)^{5/3} + \frac{20}{9\pi} k_{\mathrm{F}} a_{\mathrm{S}} (1-m^2)]$$

- spontaneous magnetization $m>0$ occurs at $k_F a_S = \pi/2$
- (note immediately that this is beyond the expected validity of mean field…)



a<0 and a>0 mean field considered in:
Stoof *et al.*, PRA **56**, 4864 (1997); PRL **76**, 10 (1996).

Paramekanti, notes 2008

Spin textures in a trapped Fermi gas

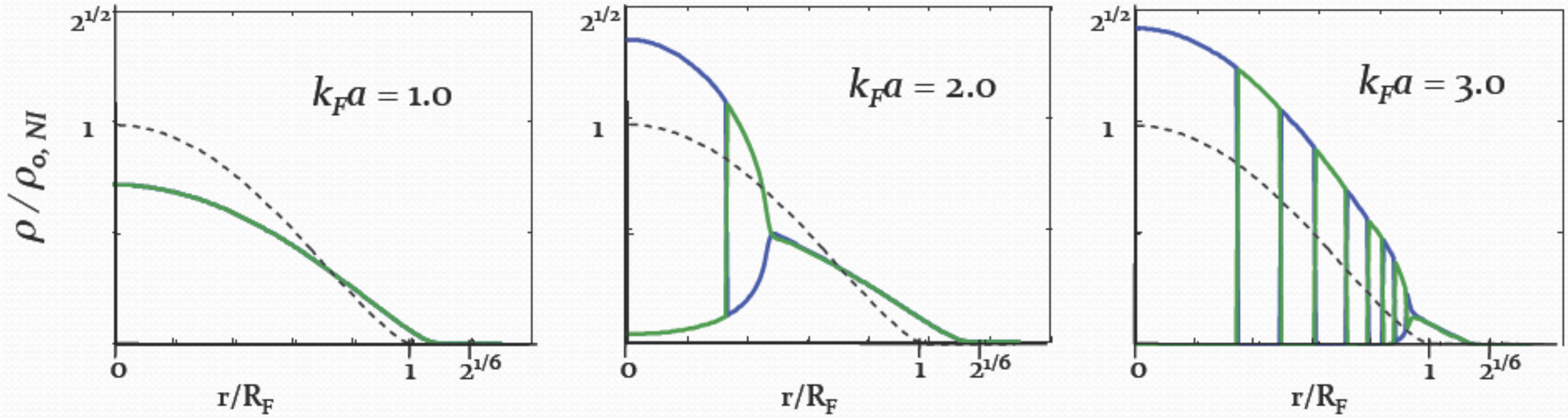
LeBlanc, Burkov, Thywissen, & Paramekanti PRA **80**, 013607 (2009).

Ground state energy functional:

$$E[\{\rho_\sigma(\mathbf{r})\}] = \int d^3\mathbf{r} \left[\underbrace{\frac{3}{5}\sum_\sigma \frac{\hbar^2(6\pi^2\rho_\sigma)^{2/3}}{2m}\rho_\sigma(\mathbf{r})}_{\text{kinetic energy, like } \frac{\hbar^2 k_F^2(\mathbf{r})}{2m}} + \underbrace{V(\mathbf{r})\sum_\sigma \rho_\sigma(\mathbf{r})}_{\text{potential energy}} + \underbrace{g\rho_\uparrow(\mathbf{r})\rho_\downarrow(\mathbf{r})}_{\text{interaction energy}} \right]$$

interaction energy $g = \frac{4\pi a\hbar^2}{m}$

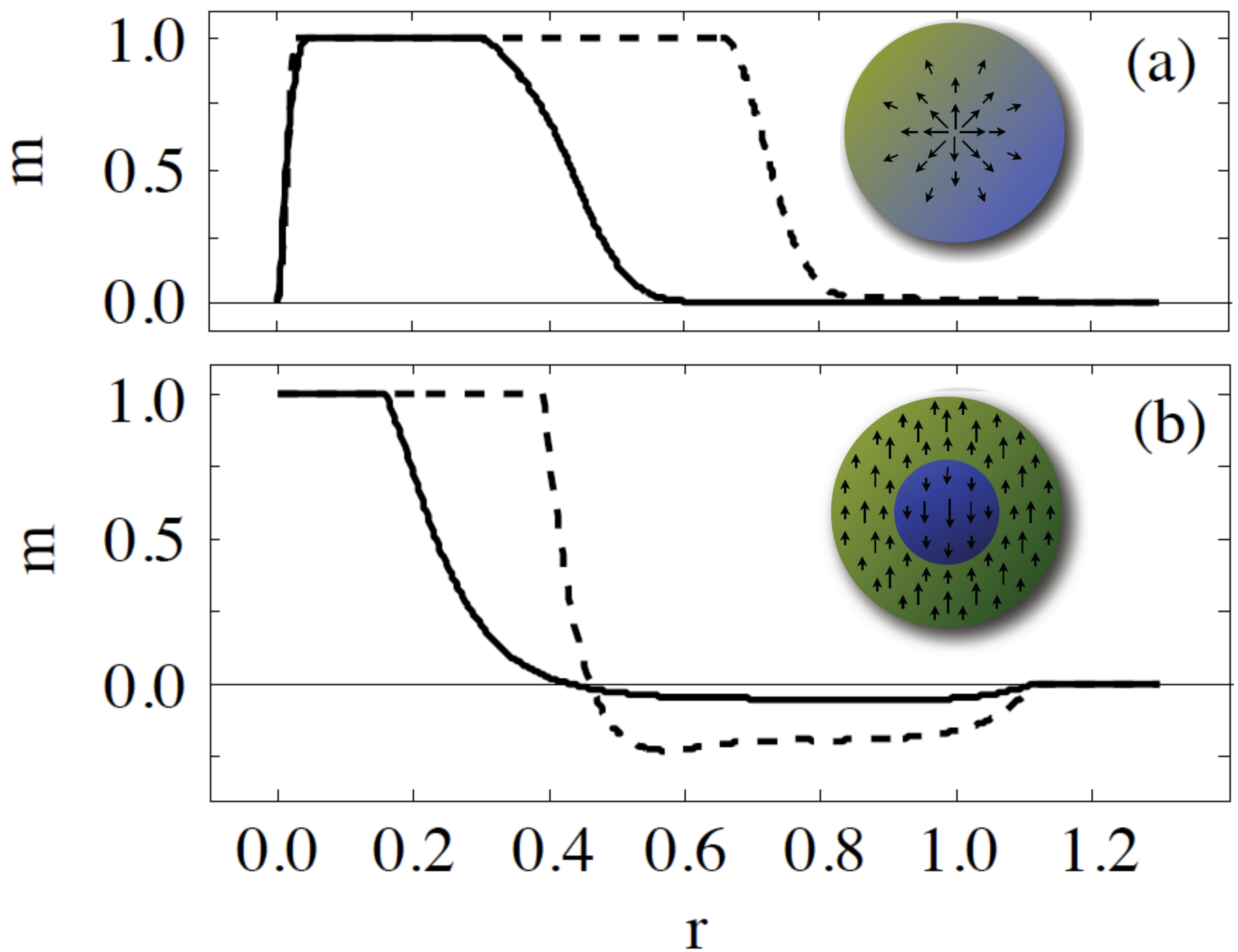
numerical minimization



LeBlanc, Burkov, Thywissen, & Paramekanti PRA **80**, 013607 (2009).

What is the ground state?

- What is the lowest energy state of the system?
  - -Important to remember: spin conserved.

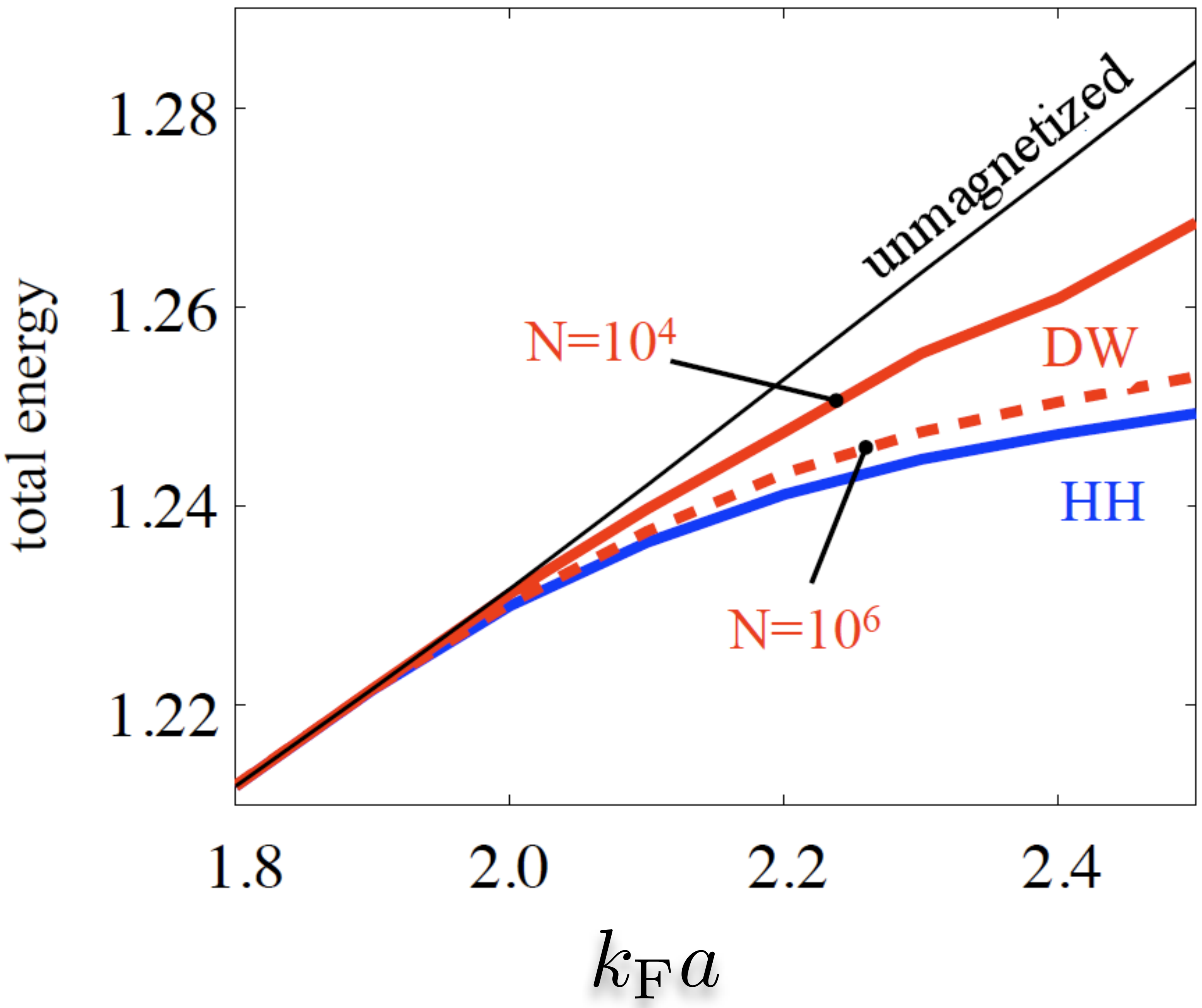


"Hedgehog"

"Domain wall"

What is the ground state? (II)

Total energy of various spin textures:



LeBlanc, Burkov, Thywissen, & Paramekanti PRA **80**, 013607 (2009).

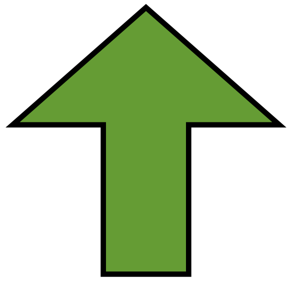
Energetics of the FM transition

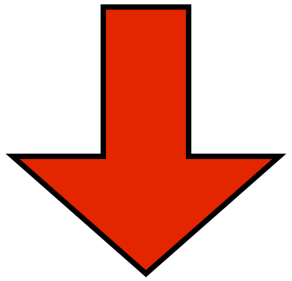
$$E[\{\rho_\sigma(\mathbf{r})\}] = \int d^3\mathbf{r} \left[\underbrace{\frac{3}{5} \sum_\sigma \frac{\hbar^2 (6\pi^2 \rho_\sigma)^{2/3}}{2m} \rho_\sigma(\mathbf{r})} + \underbrace{V(\mathbf{r}) \sum_\sigma \rho_\sigma(\mathbf{r})} + \underbrace{g \rho_\uparrow(\mathbf{r}) \rho_\downarrow(\mathbf{r})} \right]$$

kinetic energy, like $\frac{\hbar^2 k_F^2(\mathbf{r})}{2m}$

potential energy

interaction energy $g = \frac{4\pi a \hbar^2}{m}$





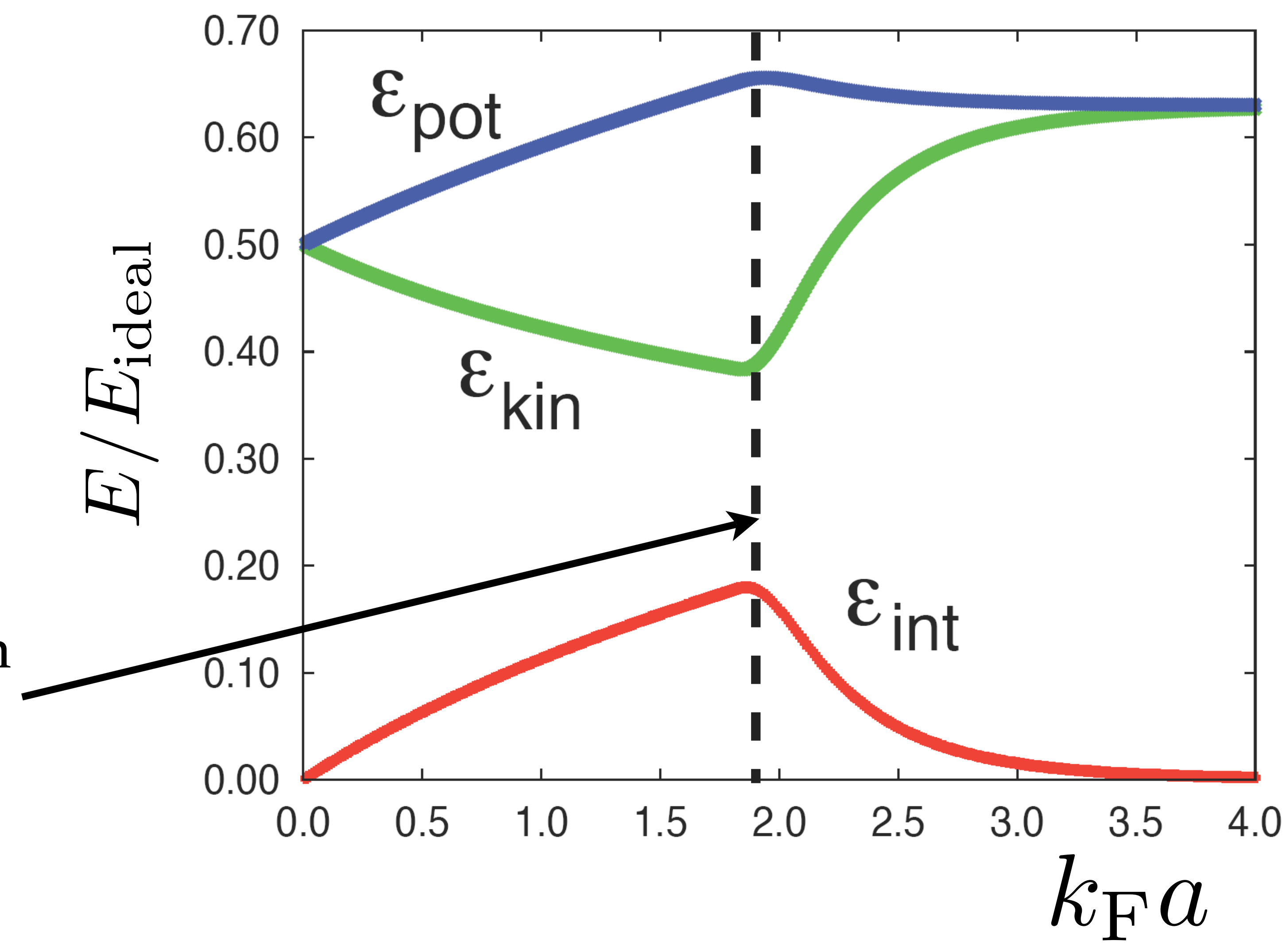
LeBlanc, Burkov, Thywissen, & Paramekanti PRA **80**, 013607 (2009).

Energetic signatures

- using calculated density profiles, find kinetic, potential, and interaction energies.

- compare expansion energy with and without tuning to a = 0 regime before release.

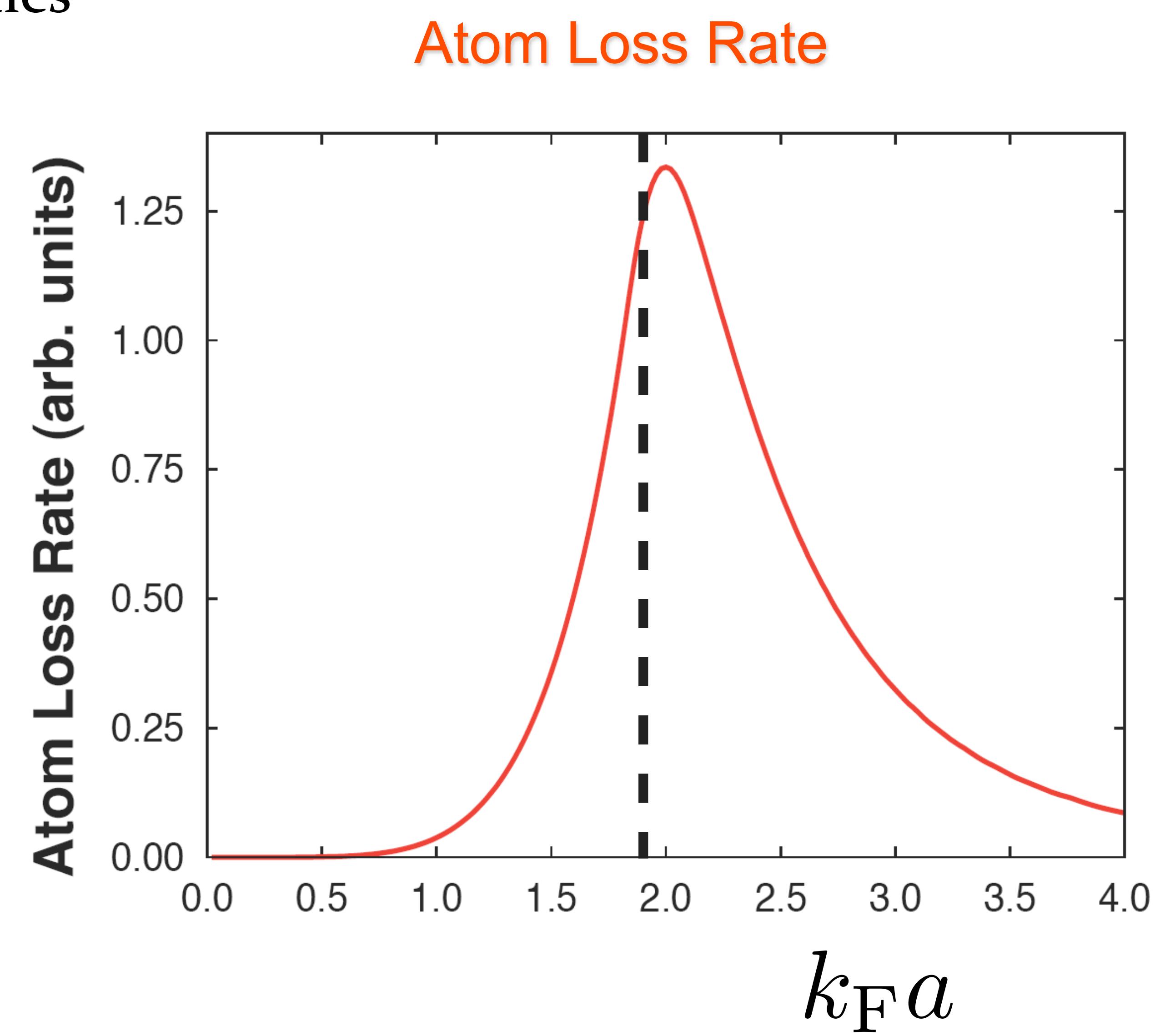
- “kink” in energy vs. interaction strength indicates a crossover to ferromagnetic regime



LeBlanc, Burkov, Thywissen, & Paramekanti PRA **80**, 013607 (2009).

Loss signature

- a polarized gas cannot recombine into molecules



$$\Gamma = \Gamma_0 \lambda^6 \int d^3\mathbf{r}\ n_\uparrow(\mathbf{r}) n_\downarrow(\mathbf{r}) (n_\uparrow(\mathbf{r}) + n_\downarrow(\mathbf{r}))$$

For every problem, there is a simple, elegant solution....

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For every problem, there is a simple, elegant solution....

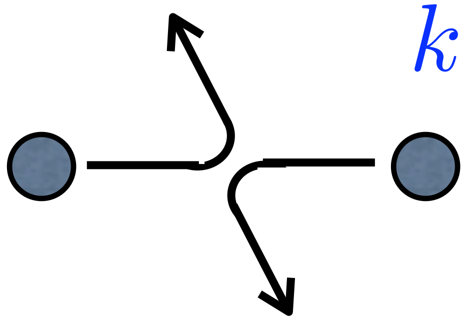
...which is wrong.

- Nowhere is this more true than in CM physics!
- Stoner model does *not* lead to FM in one dimension (1D) -Lieb (1962)
- No proof to date about 2D or 3D

How could the Stoner model fail?

- At some point, shouldn't interactions be strong enough to make spin alignment energetically favourable?
- Unfortunately interactions can only be so strong.

Recall scattering theory:



$$k = 2\pi/\lambda_{\mathrm{dB}}$$

Cross-section

$$\sigma_0 = \frac{4\pi}{k^2}\sin^2\delta_0(k)$$

Contact potential:

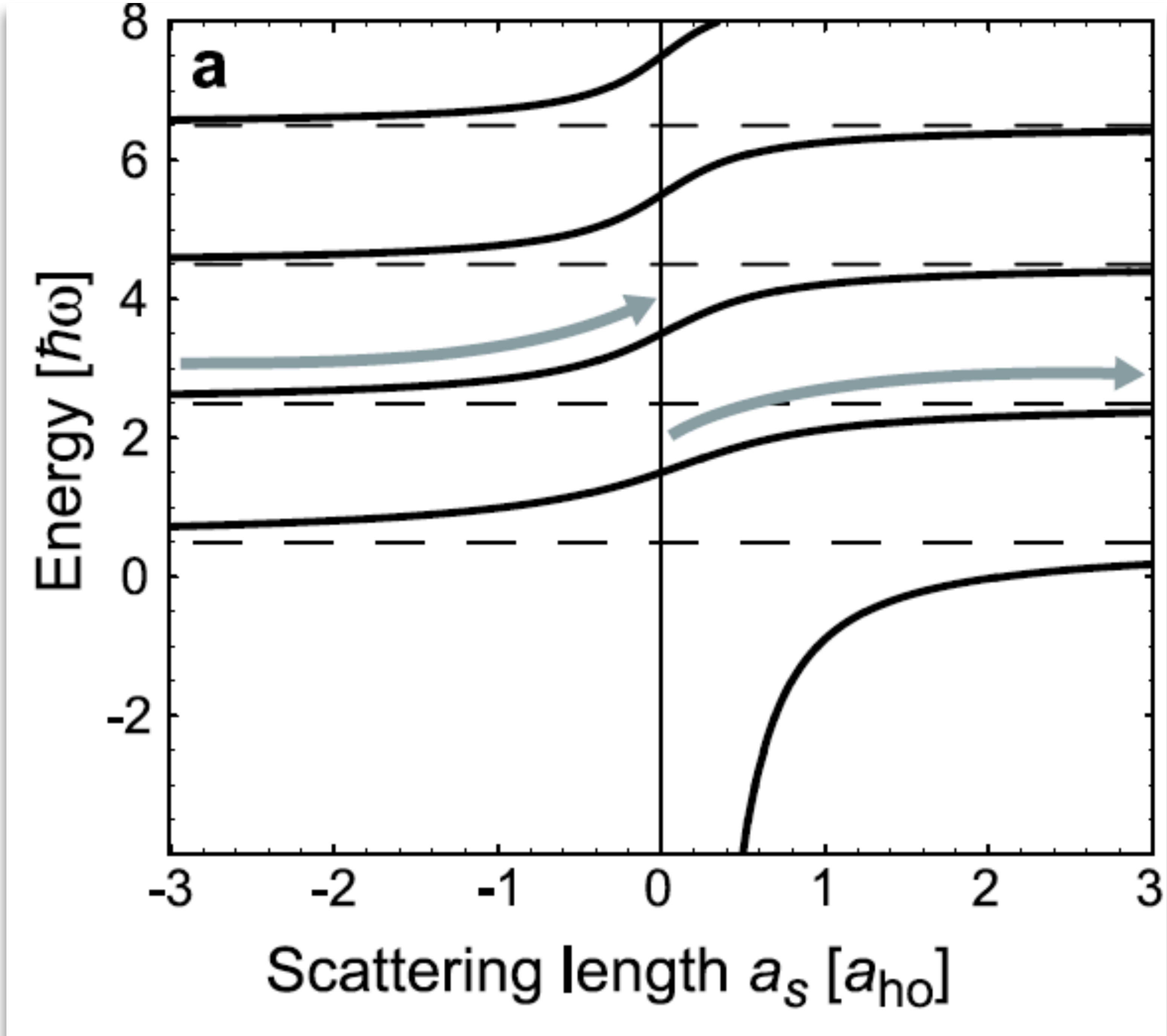
$$f_{\vec{k}} = -[1/a + ik]^{-1}$$

$$\sigma_0 = \frac{4\pi a^2}{1 + a^2k^2} \to \frac{4\pi}{k^2}$$

"Unitarity limit:" Can't do more than reflect back. In fact, resonant scattering of a wave always has a cross-section of lambda squared!

2 particles in a single well

- The ground state is a singlet (and thus not locally polarized/FM)



(Esslinger 2006)

Experiment

Experiment

“Itinerant Ferromagnetism in a Fermi Gas of Ultracold Atoms”
G.-B. Jo, Y. R. Lee, J.-H. Choi, C. A. Christensen, H. Kim,
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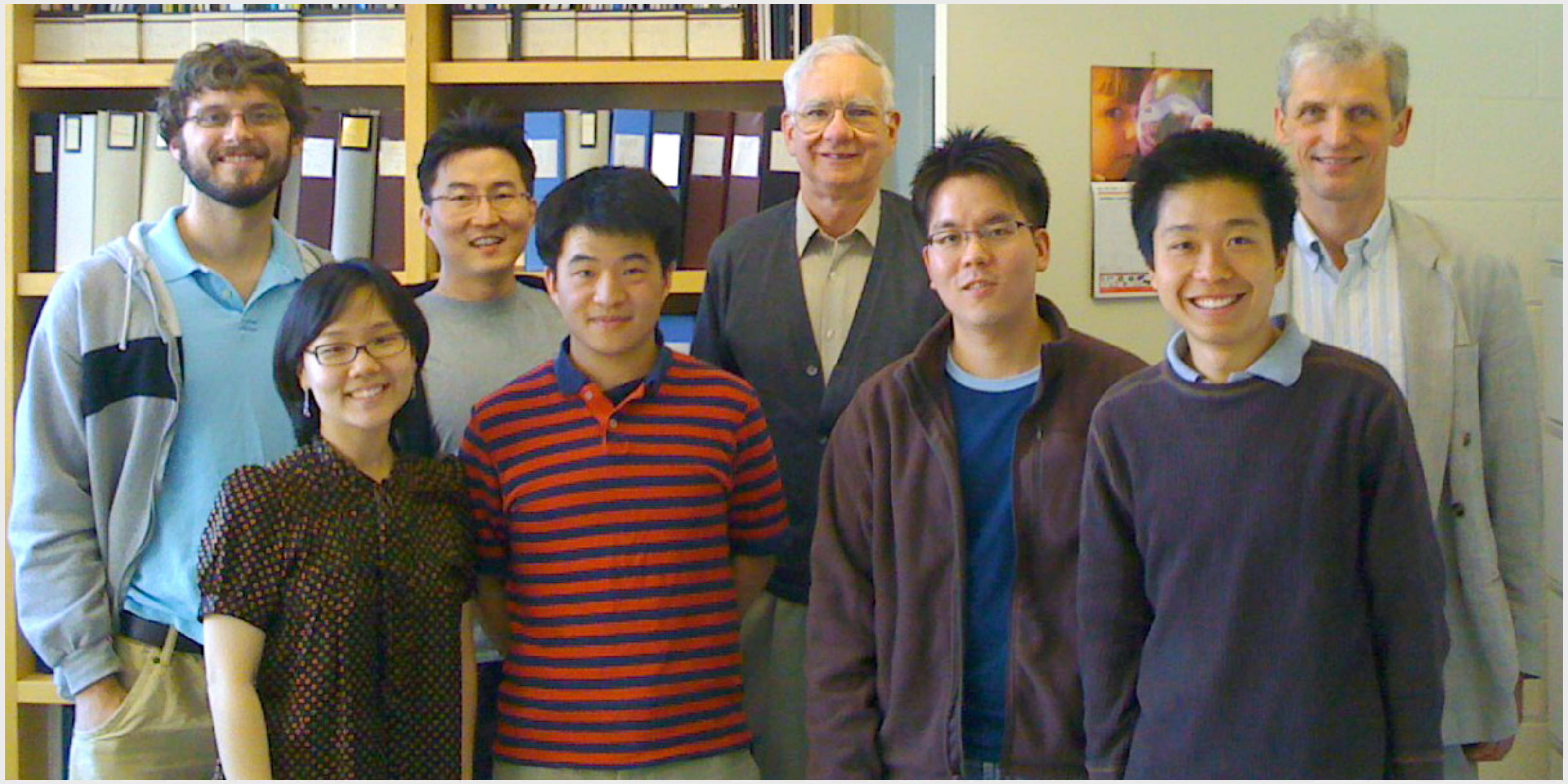
Science 325, **1521 (2009)**



MIT team

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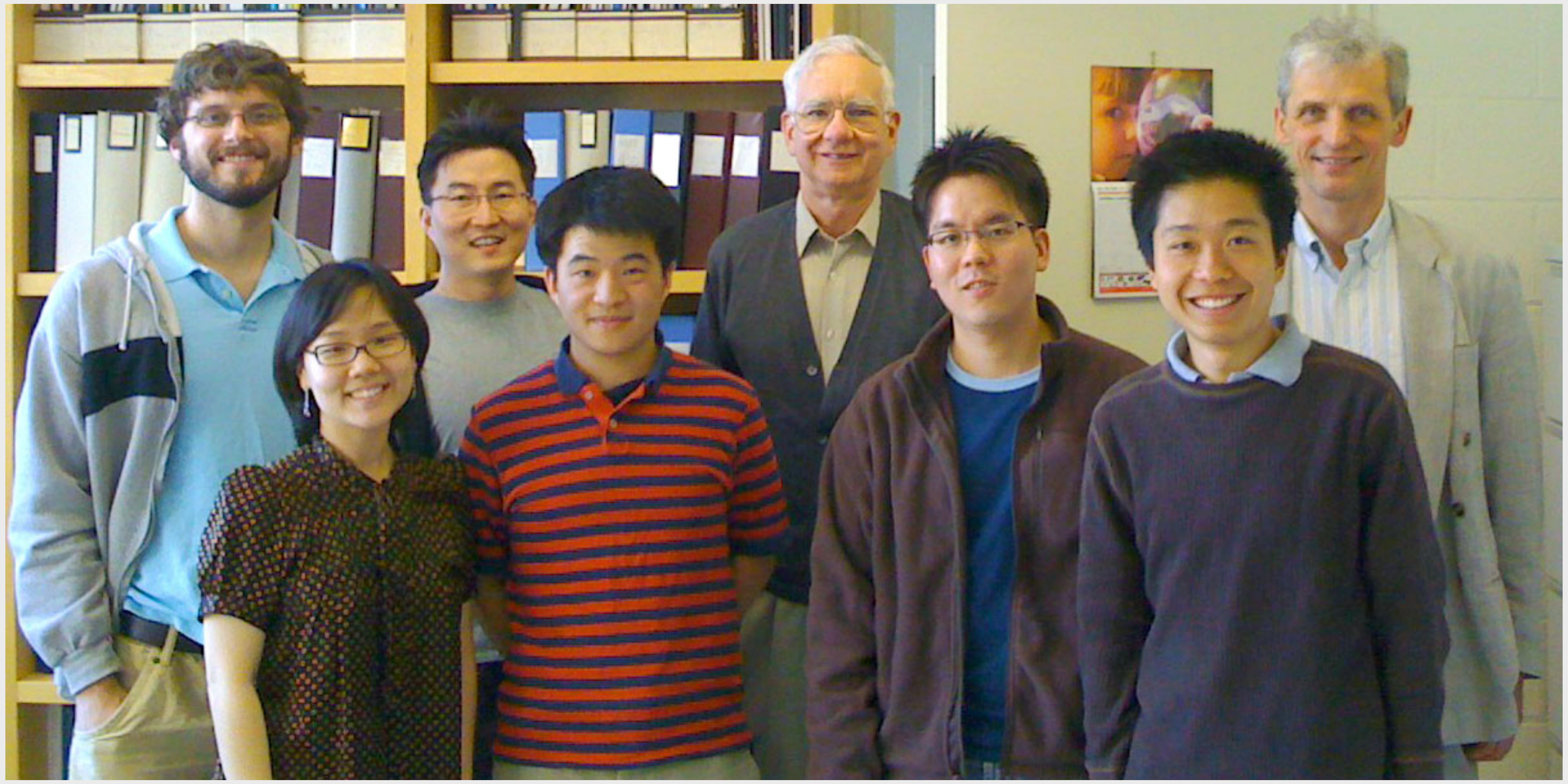
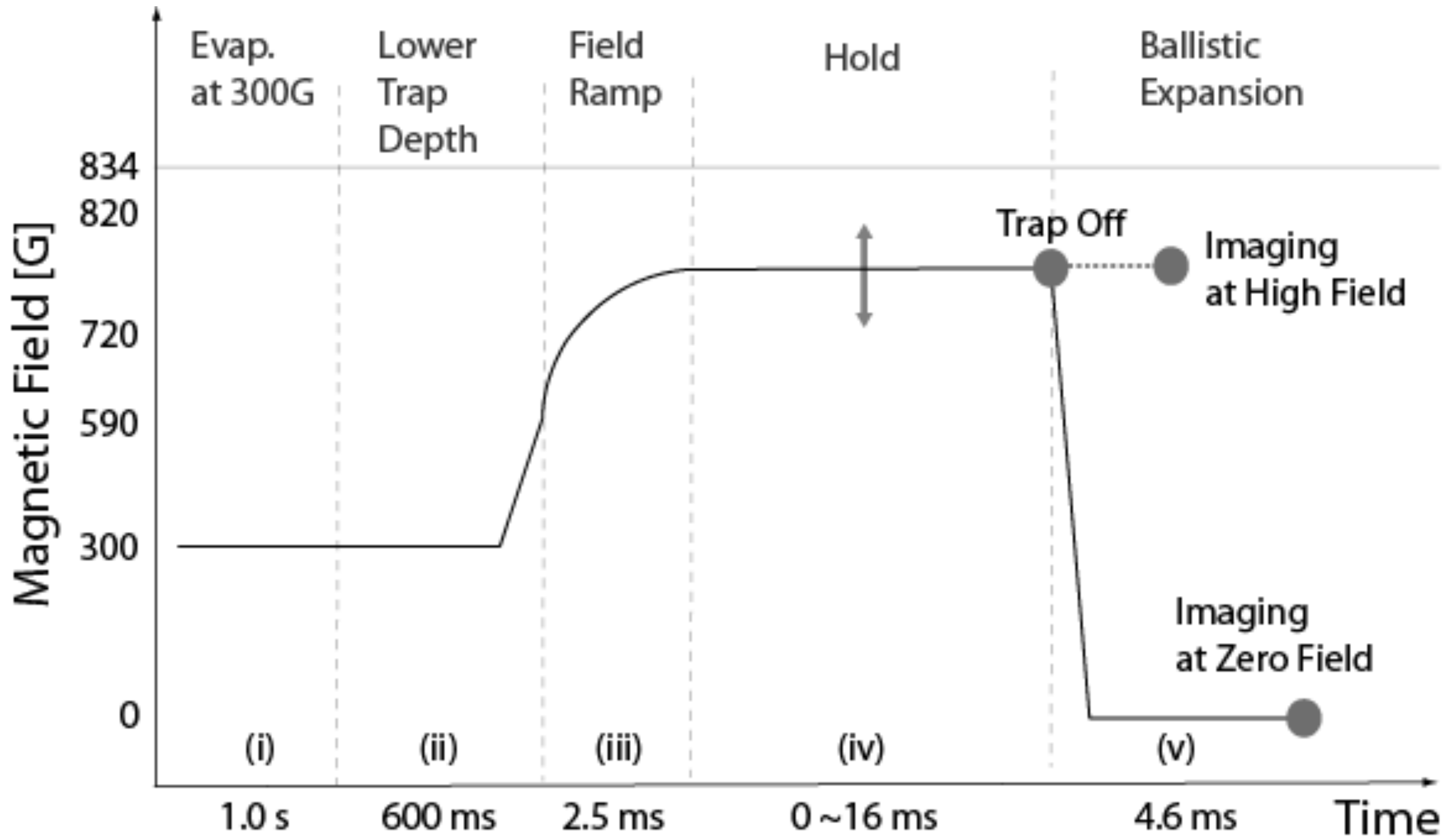
2009 sabbatical visitor



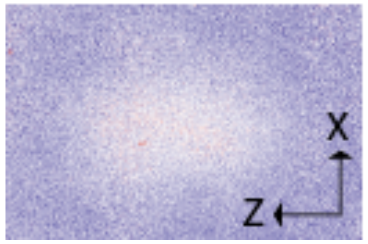
MIT team

Time sequence

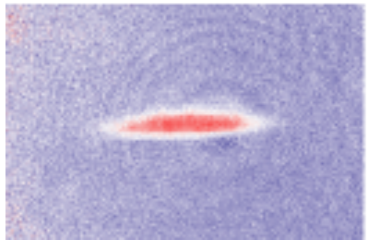
- Prepared a two-component Fermi gas(~ 0.65 million per each spin state)
- Vary repulsive interactions near the Feshbach resonance located at 834 G



4.6ms ToF at 0G



In-trap image at 812G



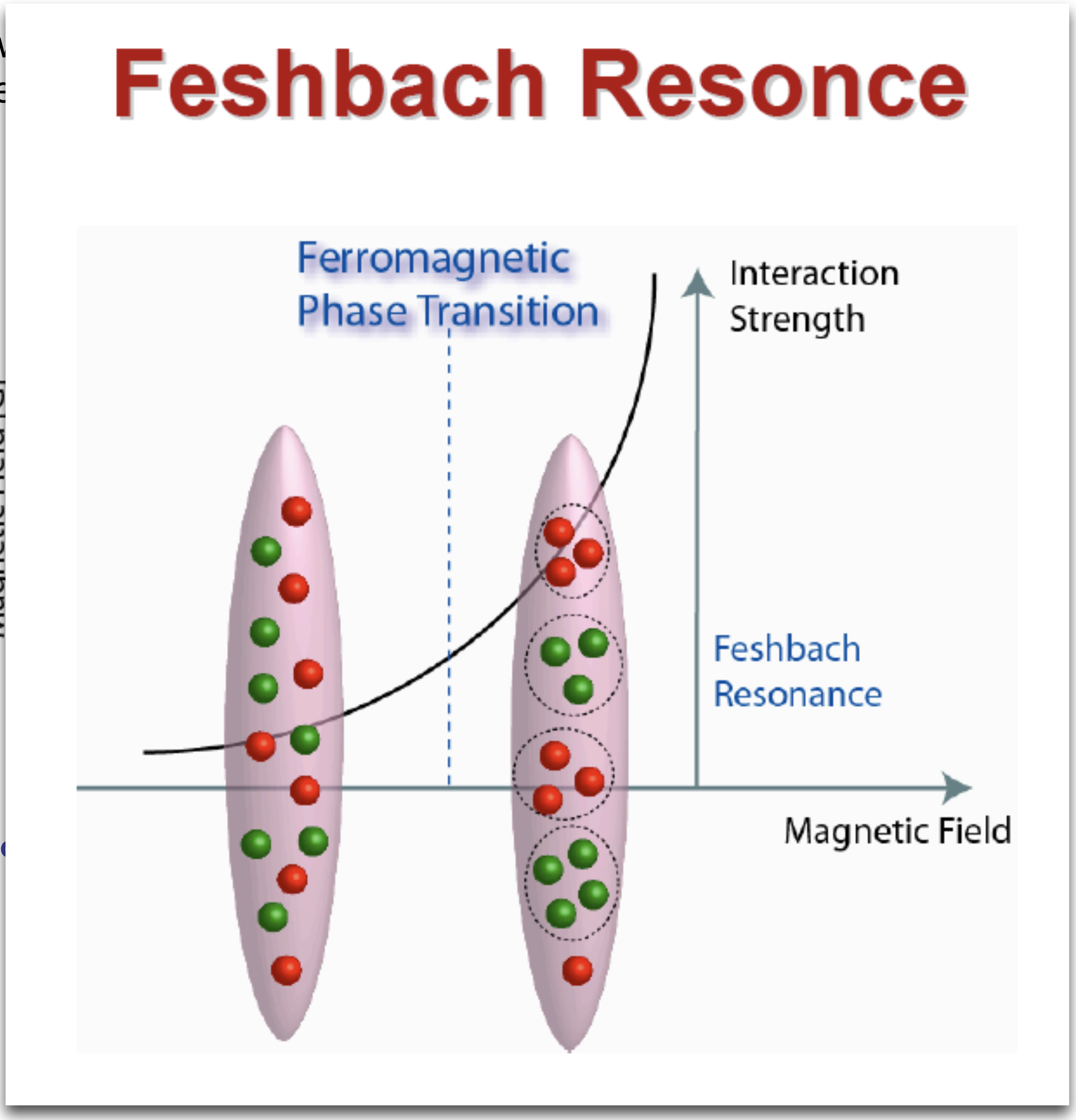
Control knobs

1. Magnetic Field → Interaction parameter $k_F a$
2. Temperature
3. Wait time

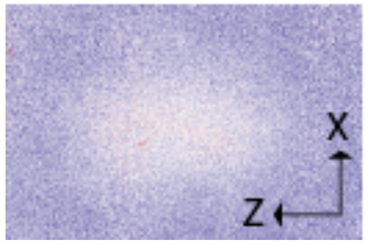
Time sequence

- Prepared a tw
- Vary repulsive

Magnetic Field [G]



4.6ms ToF at 0G



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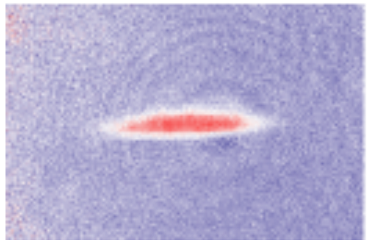
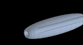
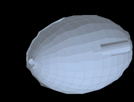


Contr

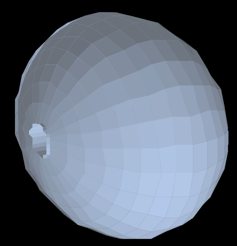
Free expansion



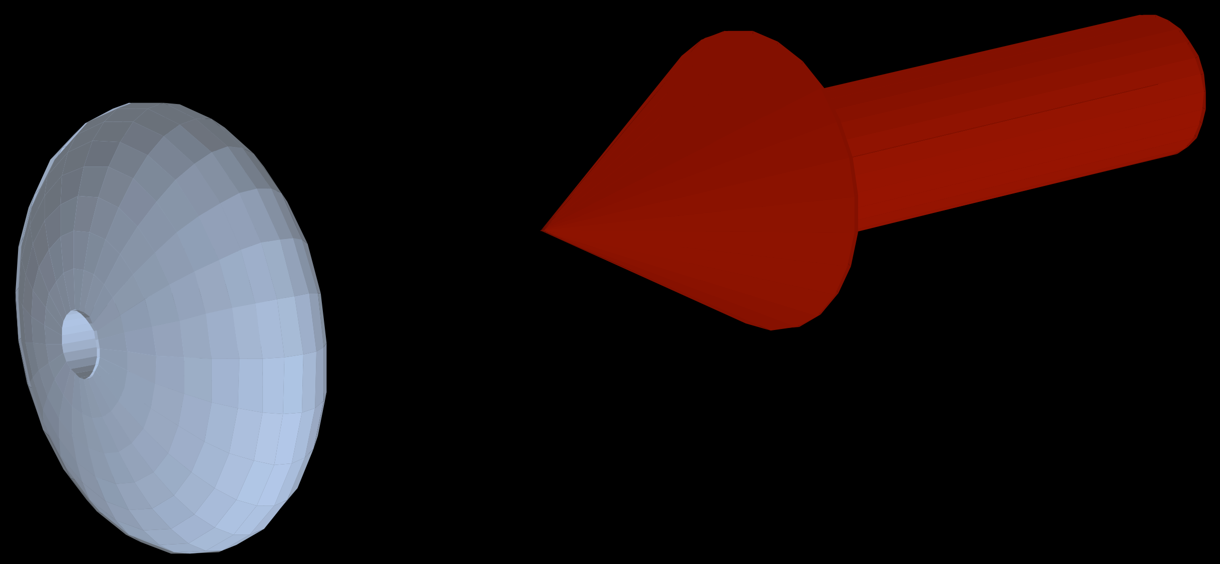
Free expansion



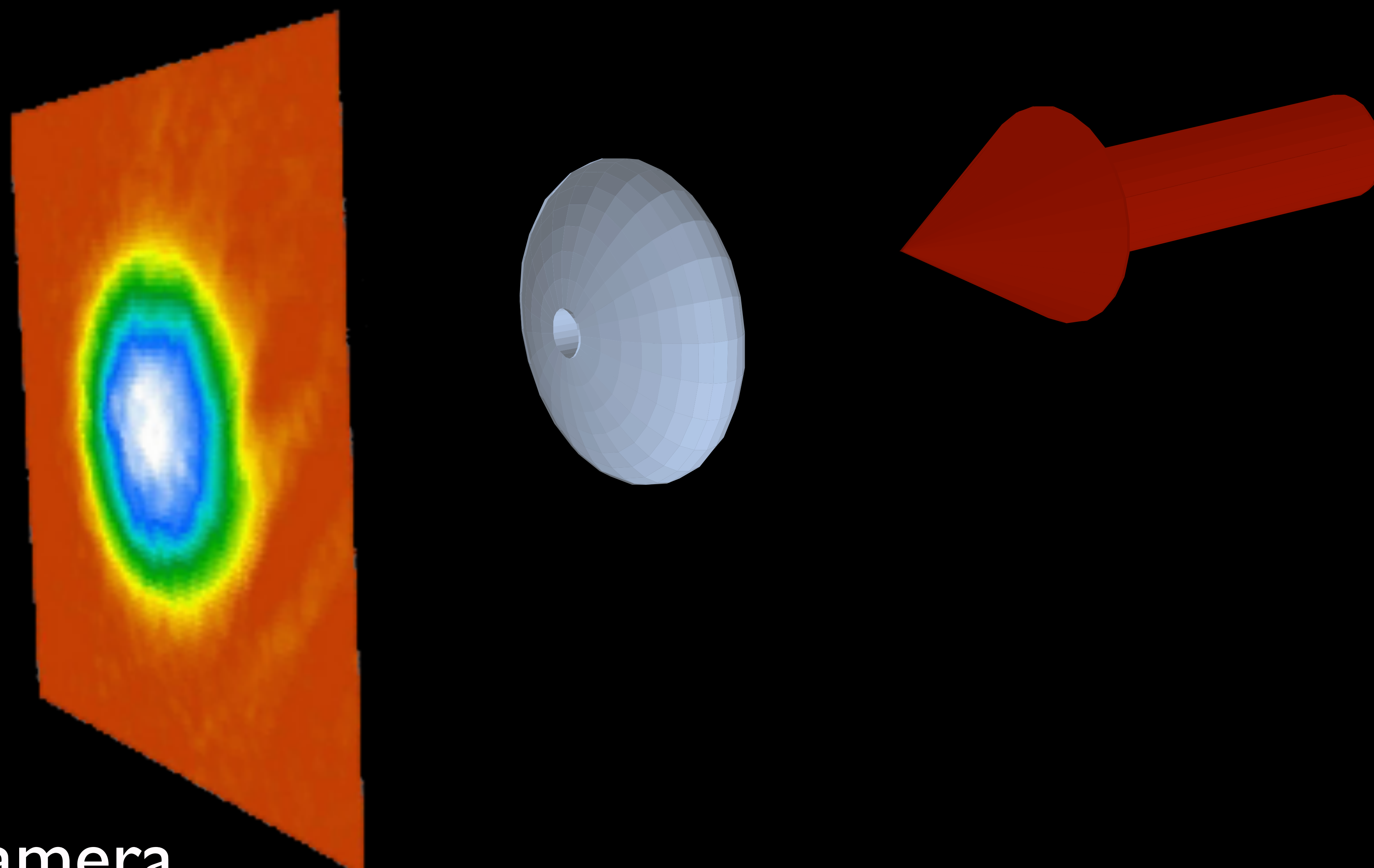
Free expansion



Free expansion



Free expansion

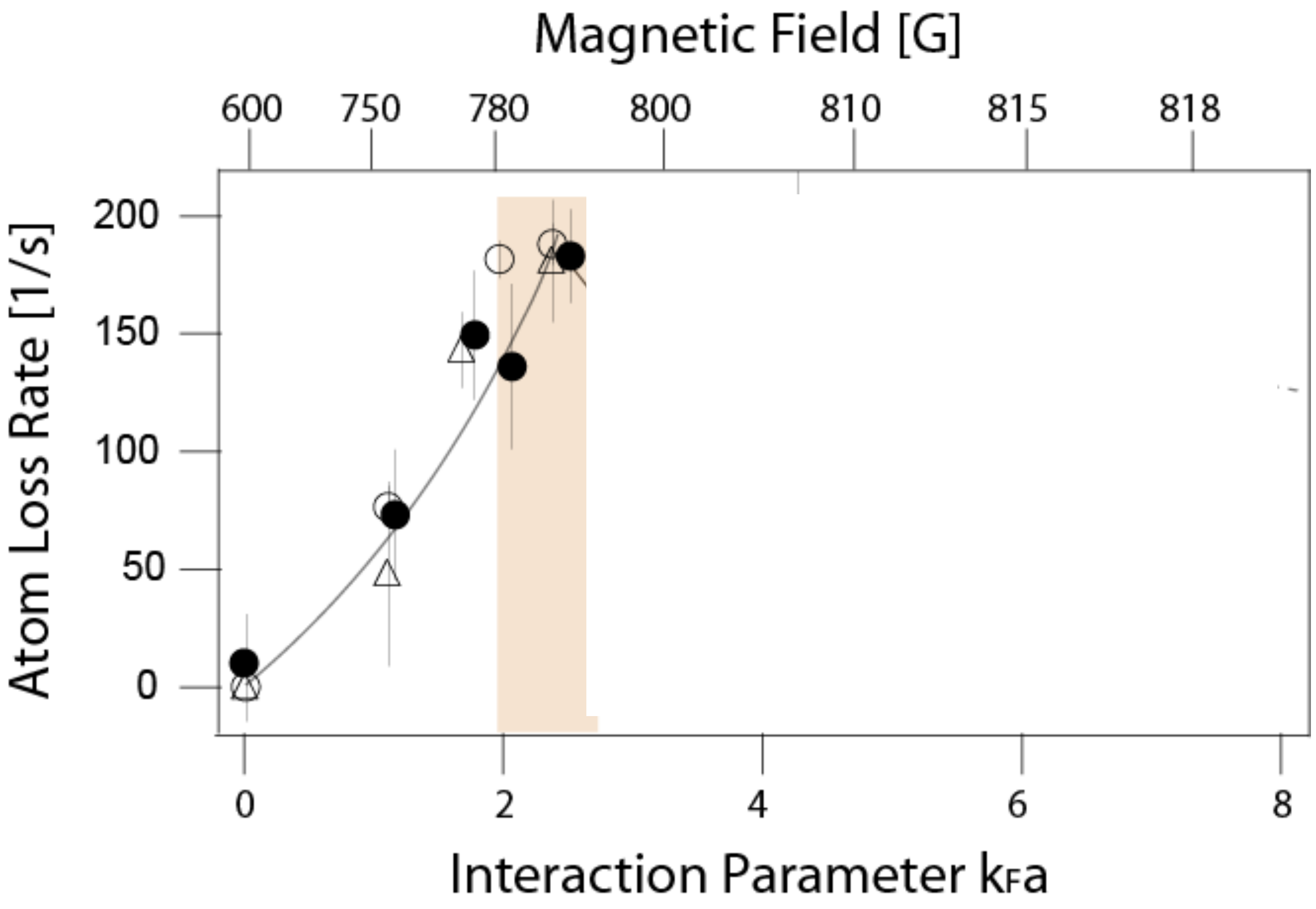


shadow
imaged
onto a
CCD camera

[Credit for these slides: Madison]

Local Probe for Magnetization

Three-body recombination rate $\propto a^6 n_{total}^2$



Here, k_F: Fermi wave vector for non-interacting gas

Local Probe for Magnetization

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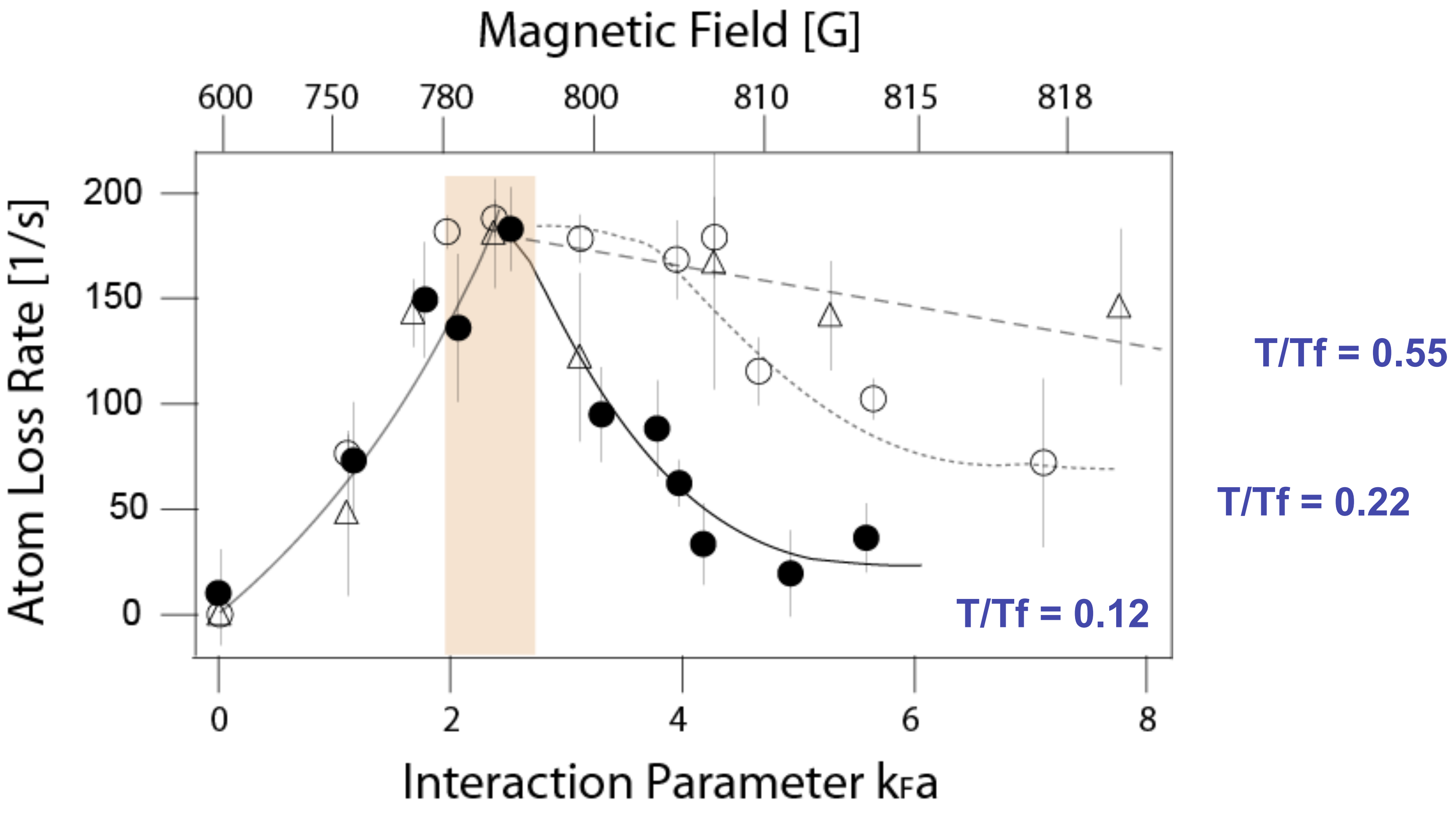
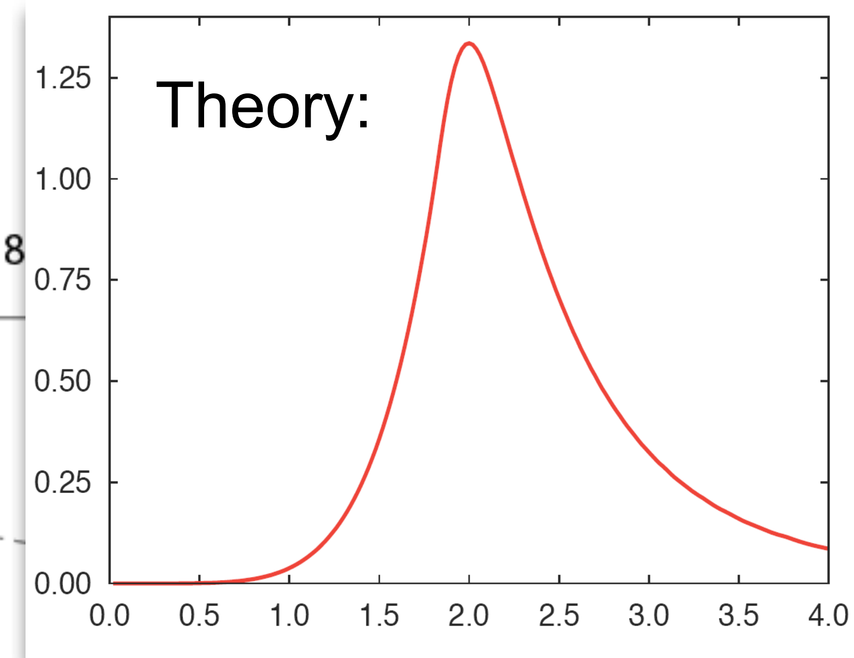


Here, k_F: Fermi wave vector for non-interacting gas

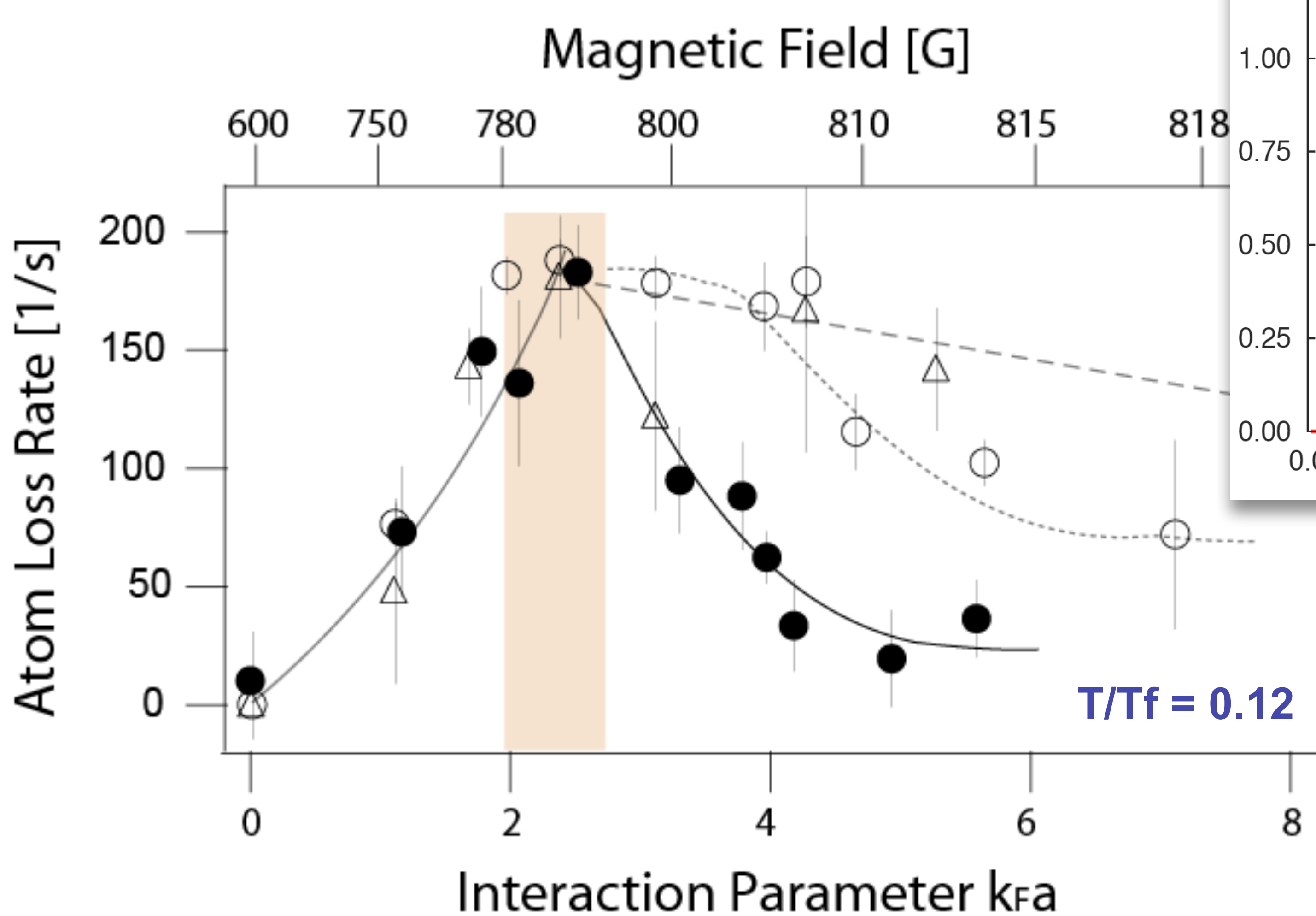
Local Probe for Magnetization

Three-body recombination rate $\propto a^6 n_{total}^{\ 2}(1 - m^2)$

m=1 Fully polarized
m=0 50/50 mixture



T/Tf = 0.22



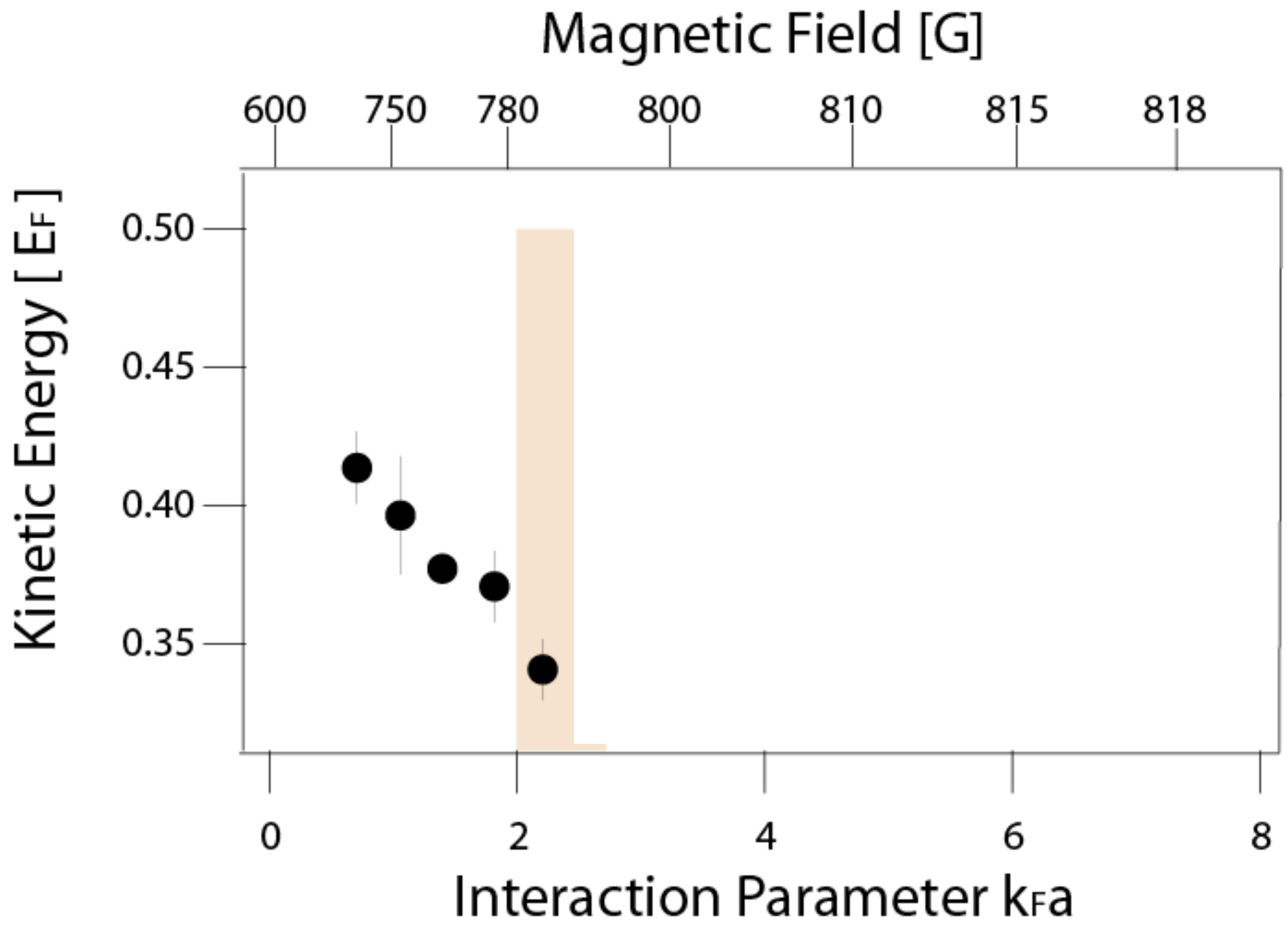
T/Tf = 0.12

Here, k_F: Fermi wave vector for non-interacting gas

Highly suppressed atom-atom collisions

Kinetic Energy of the gas

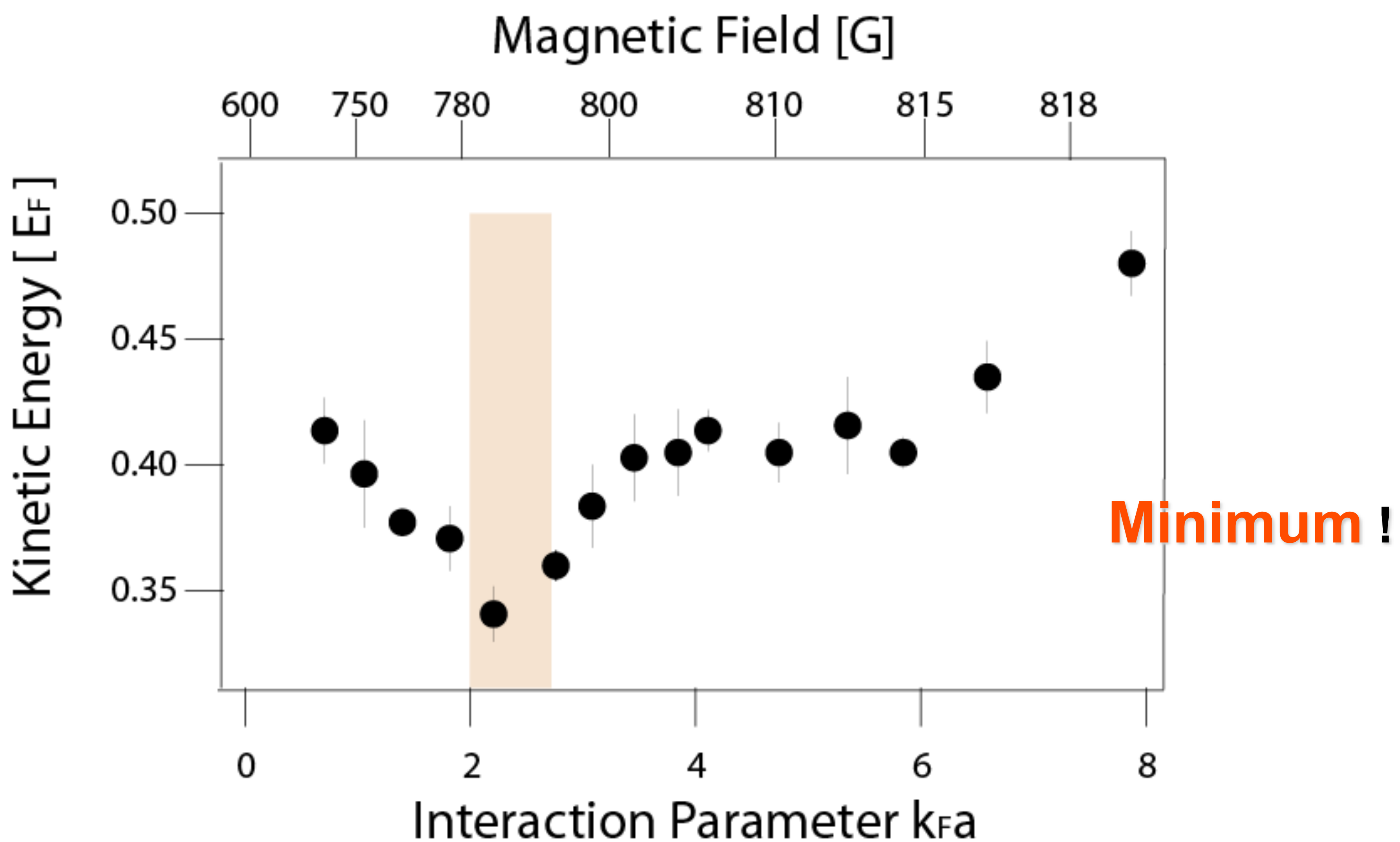
At T/Tf = 0.12



Note : The atom loss rate peaks at the minimum in the kinetic energy !

Kinetic Energy of the gas

At T/Tf = 0.12



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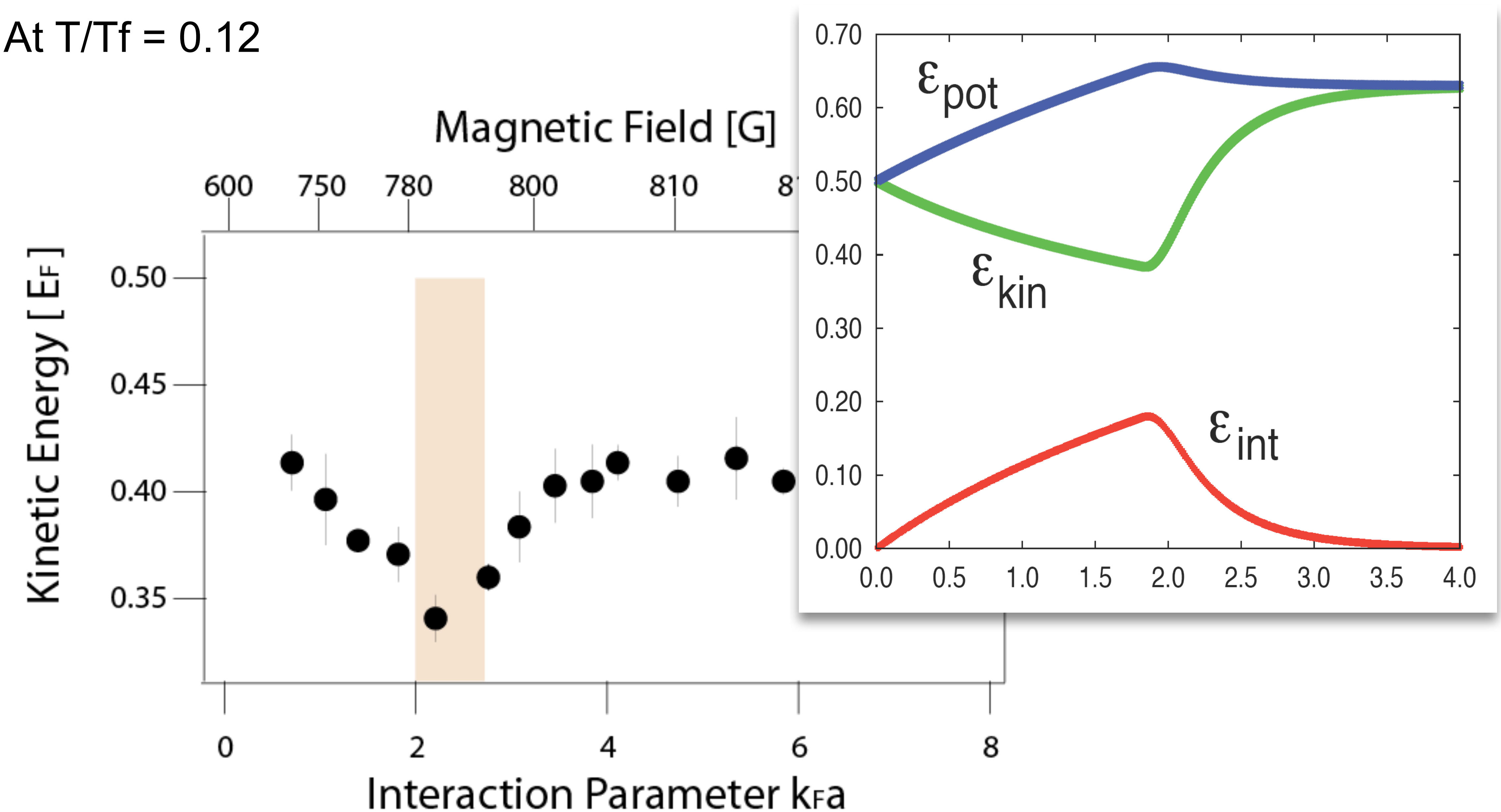
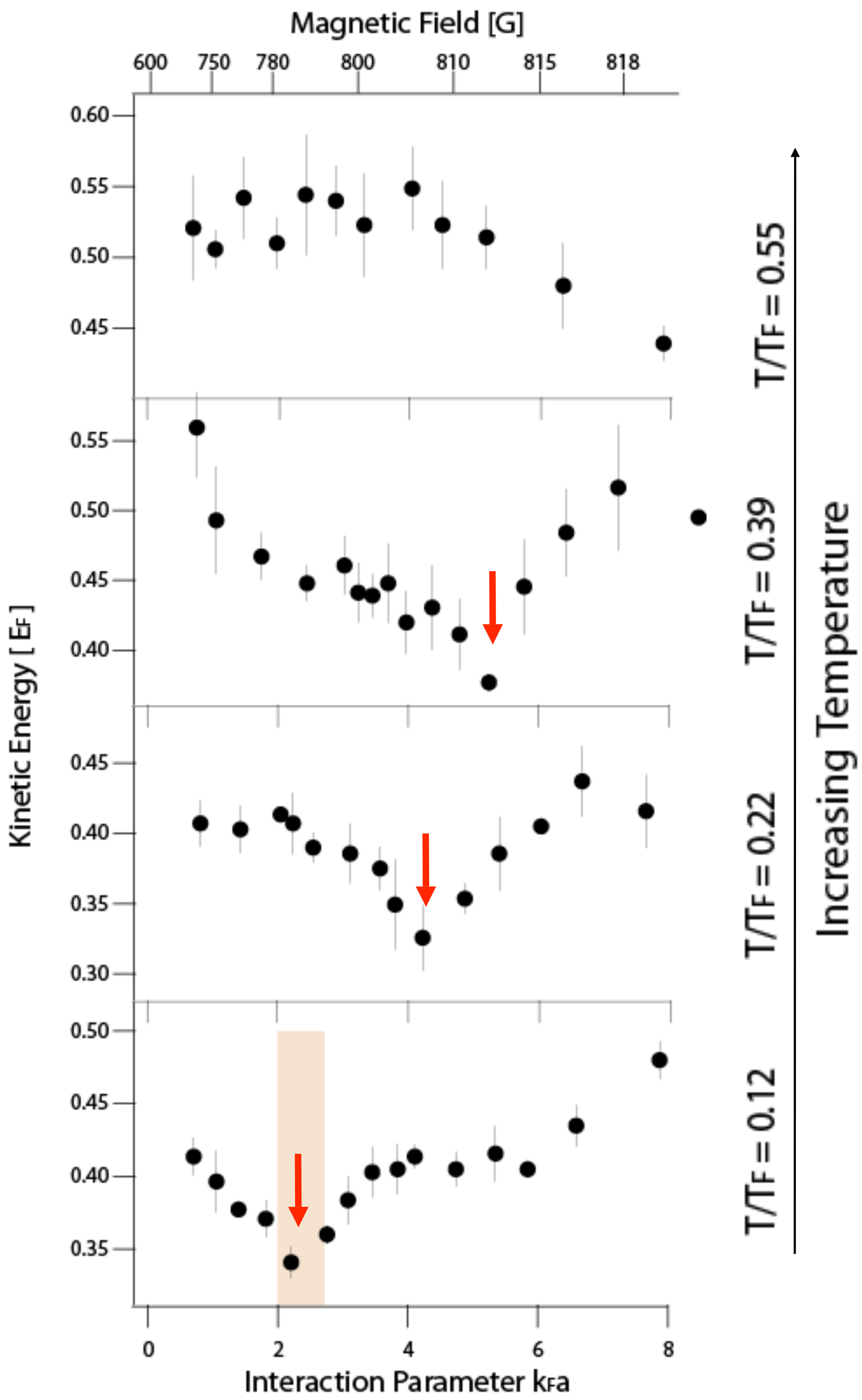
Kinetic Energy of the gas

At T/Tf = 0.12

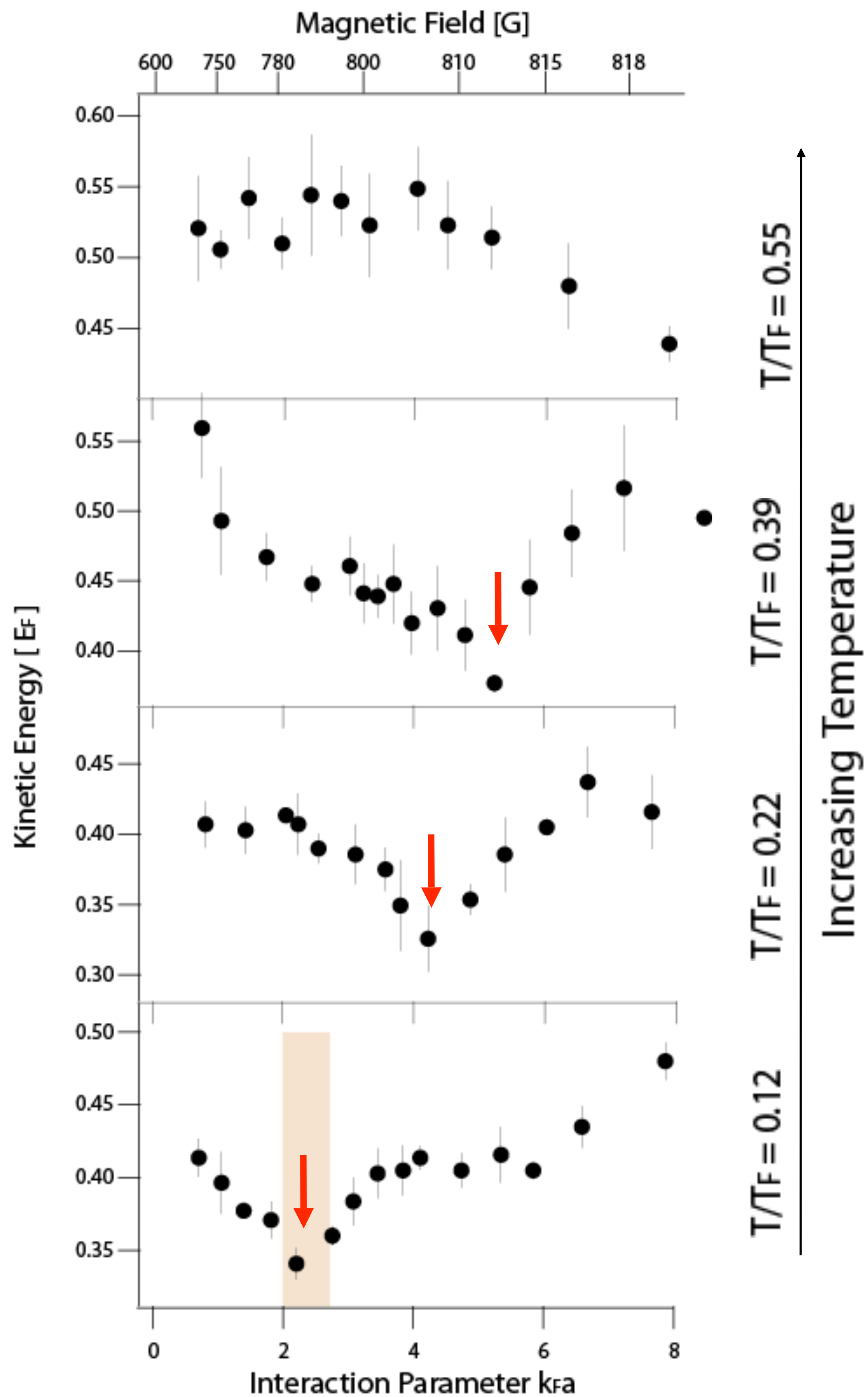


Note : The atom loss rate peaks at the minimum in the kinetic energy !

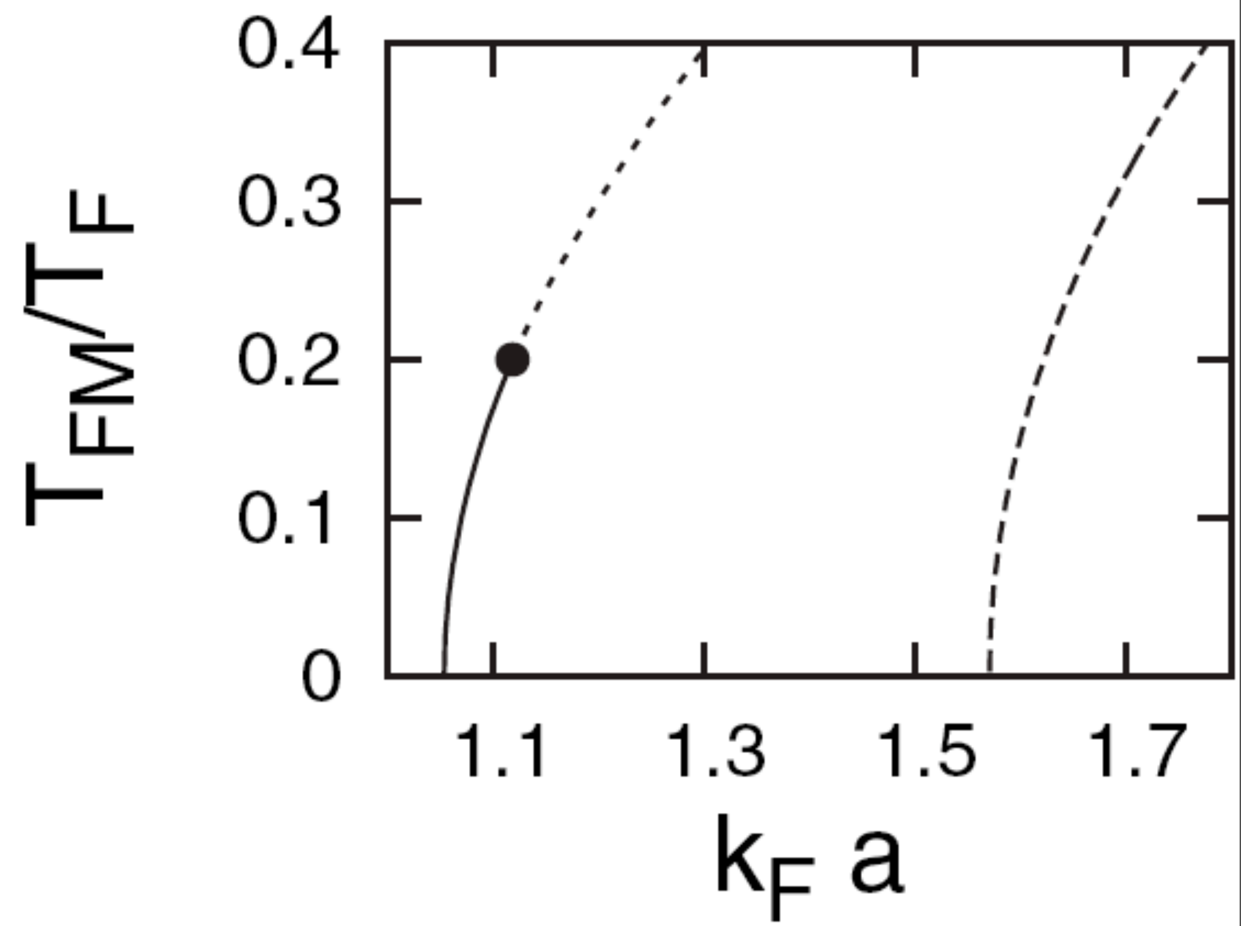
Kinetic Energy of the gas : Temperature dependence



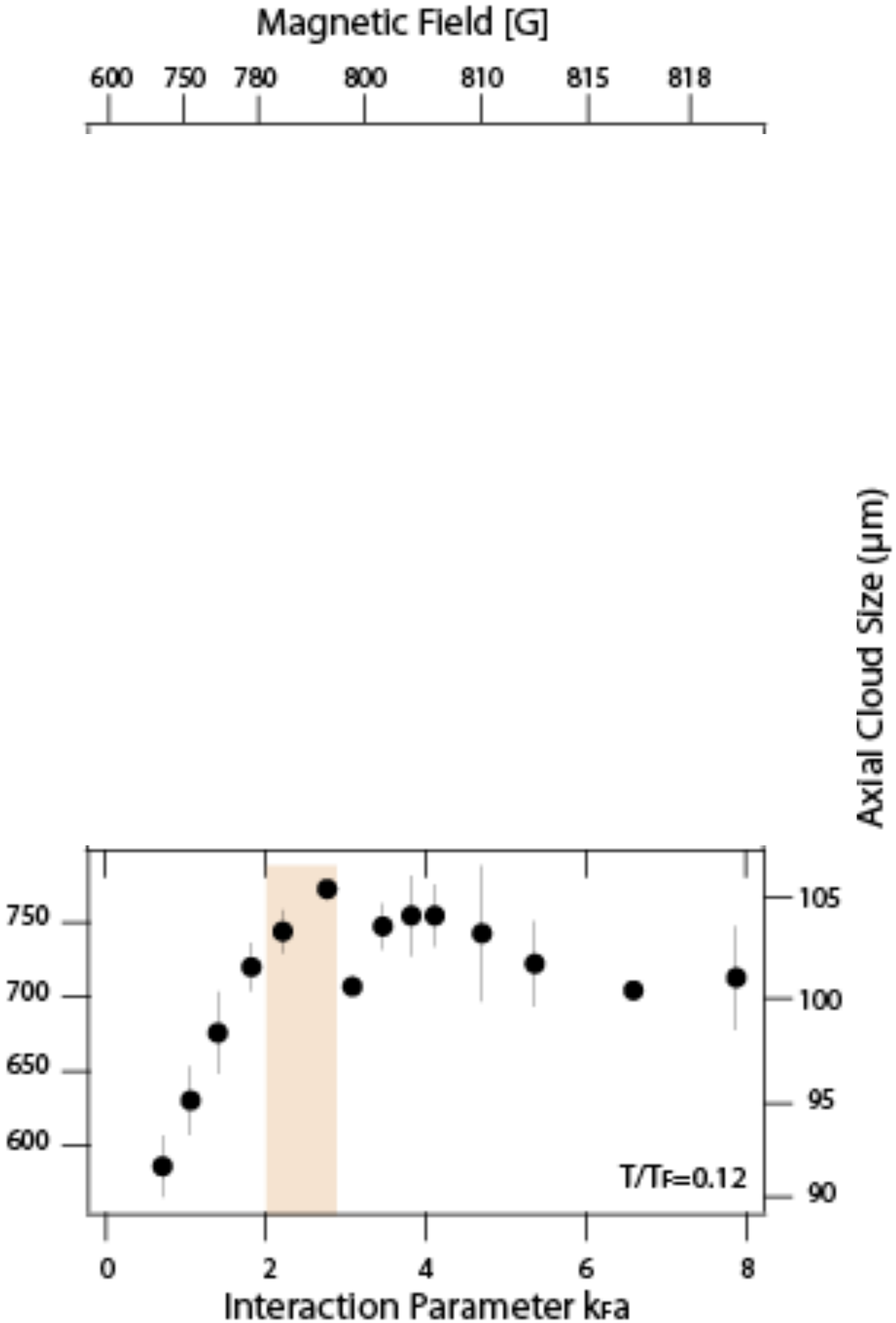
Kinetic Energy of the gas : Temperature dependence



Duine & MacDonald, PRL **95**, 230403 (2005).



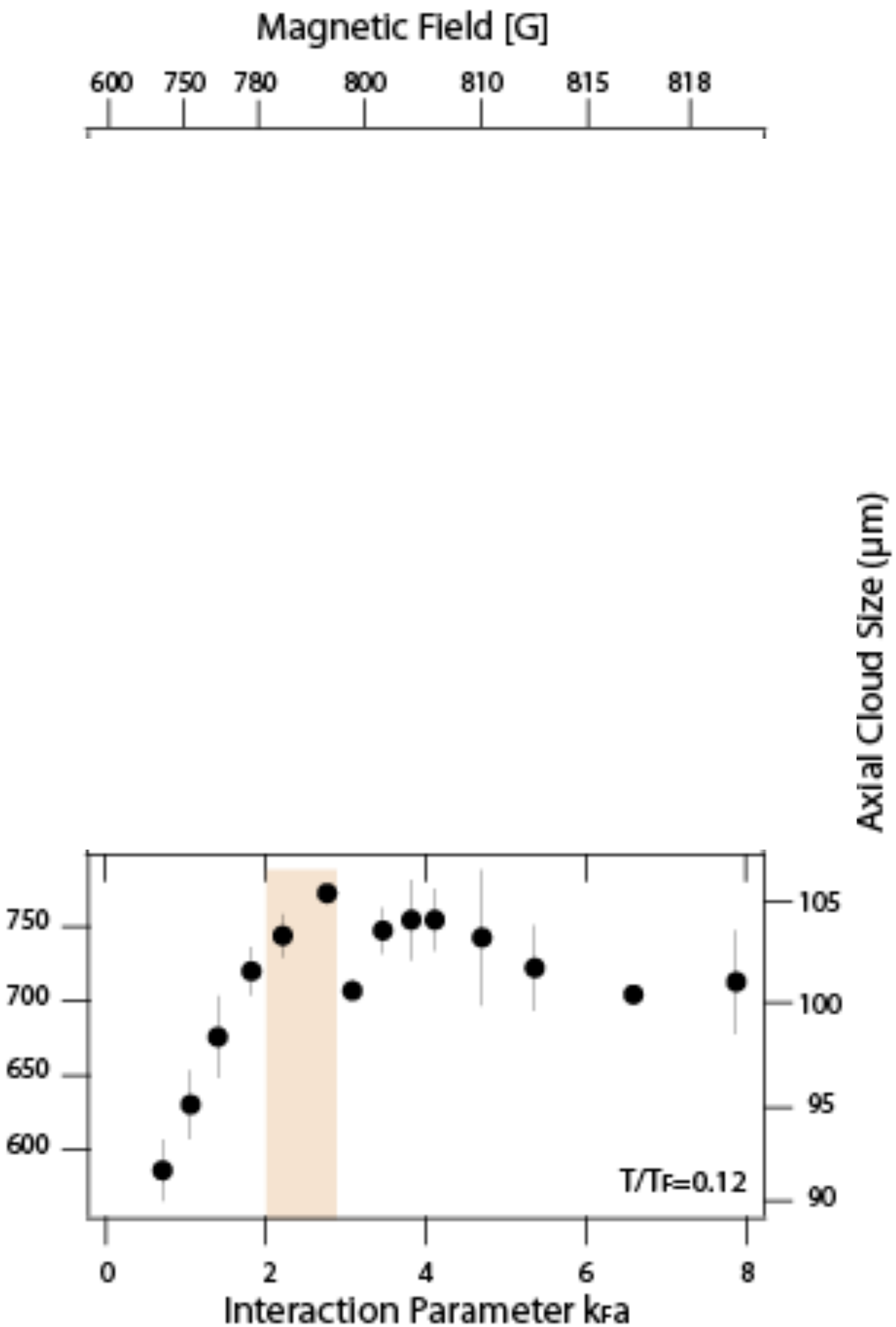
Chemical Potential



Macroscopic size of the gas

→ **Maximum !**

Chemical Potential



Macroscopic size of the gas

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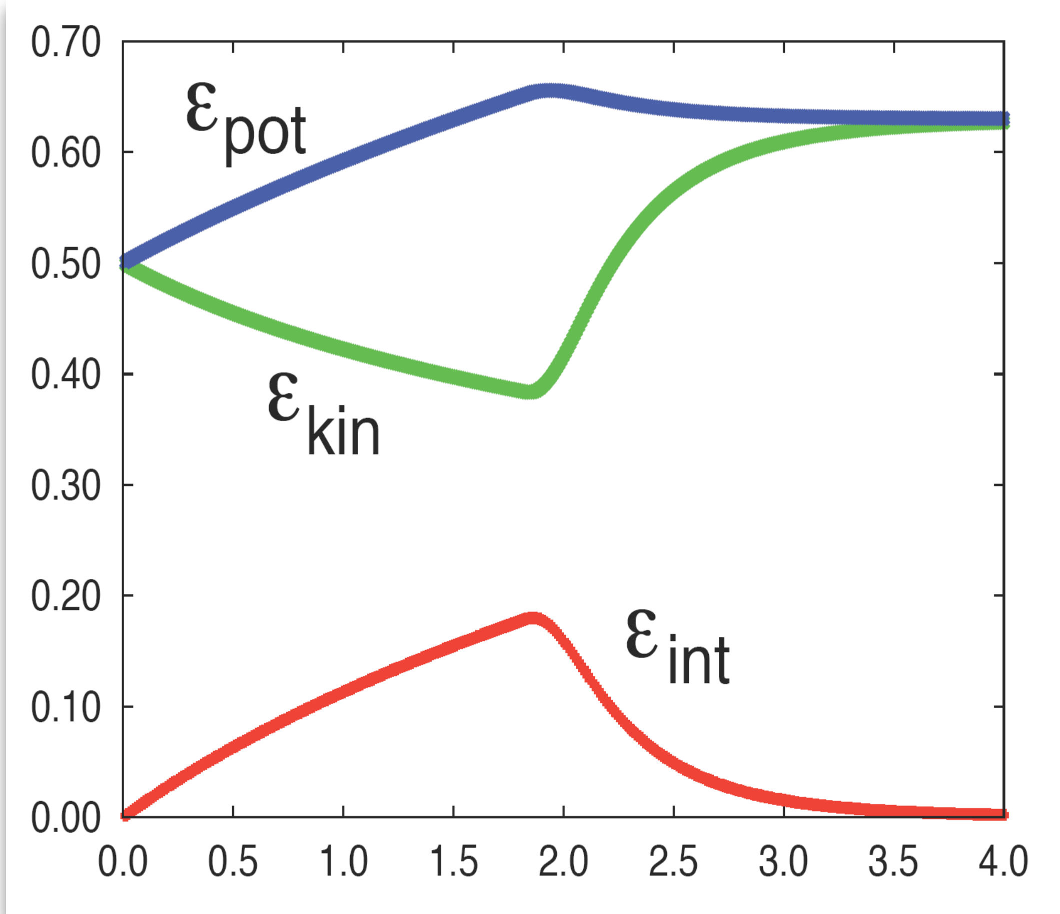
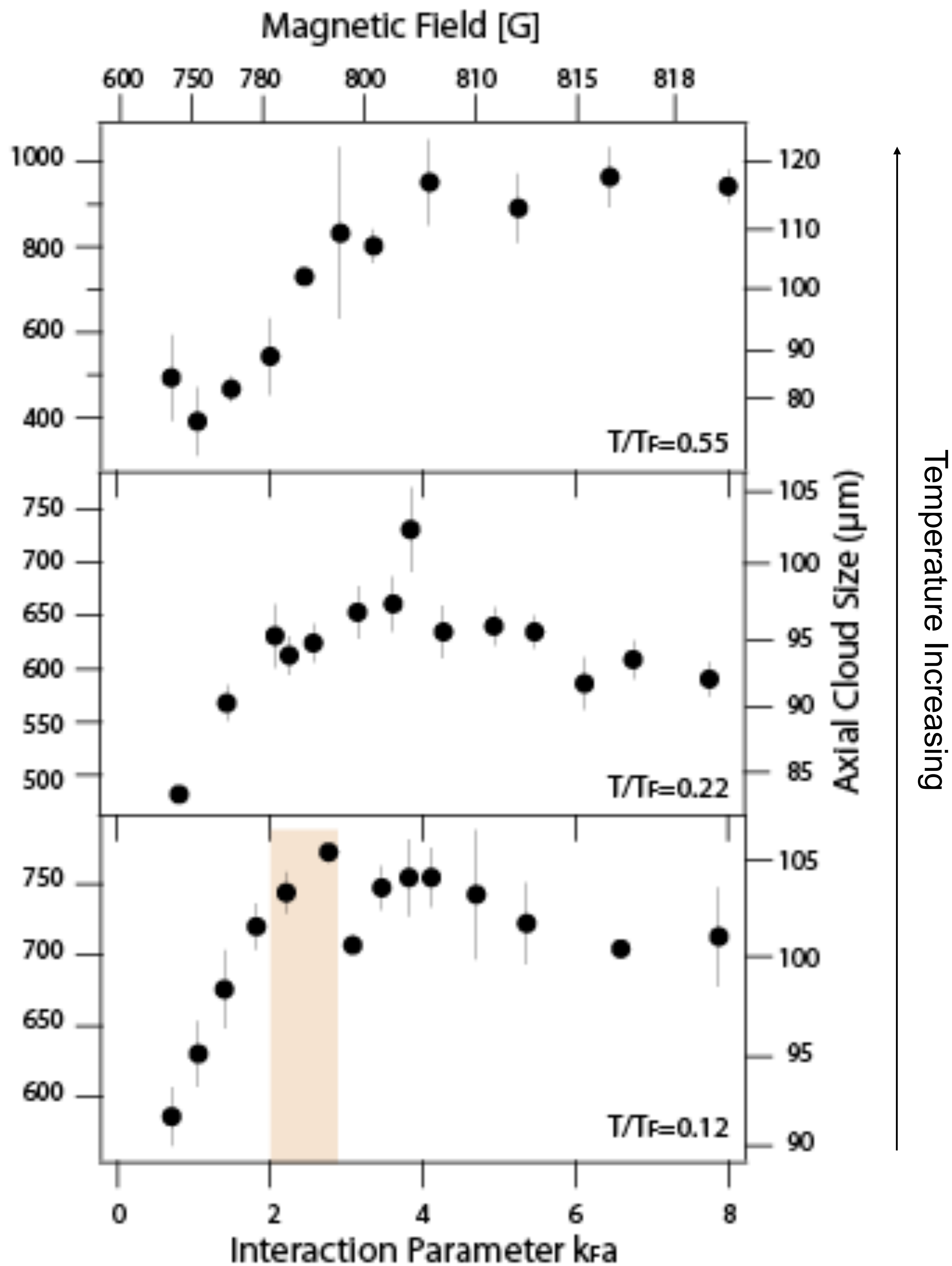


Chemical Potential



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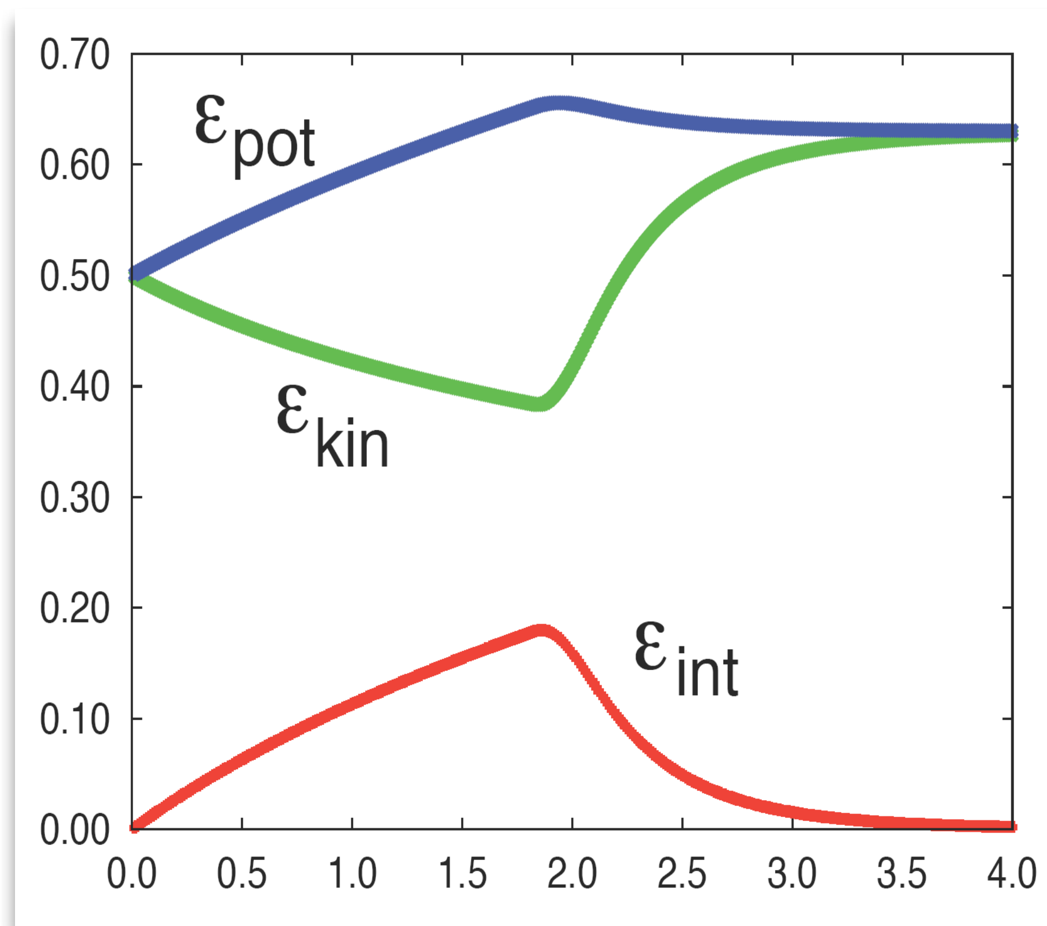


Further Discussion

non-observation of domains

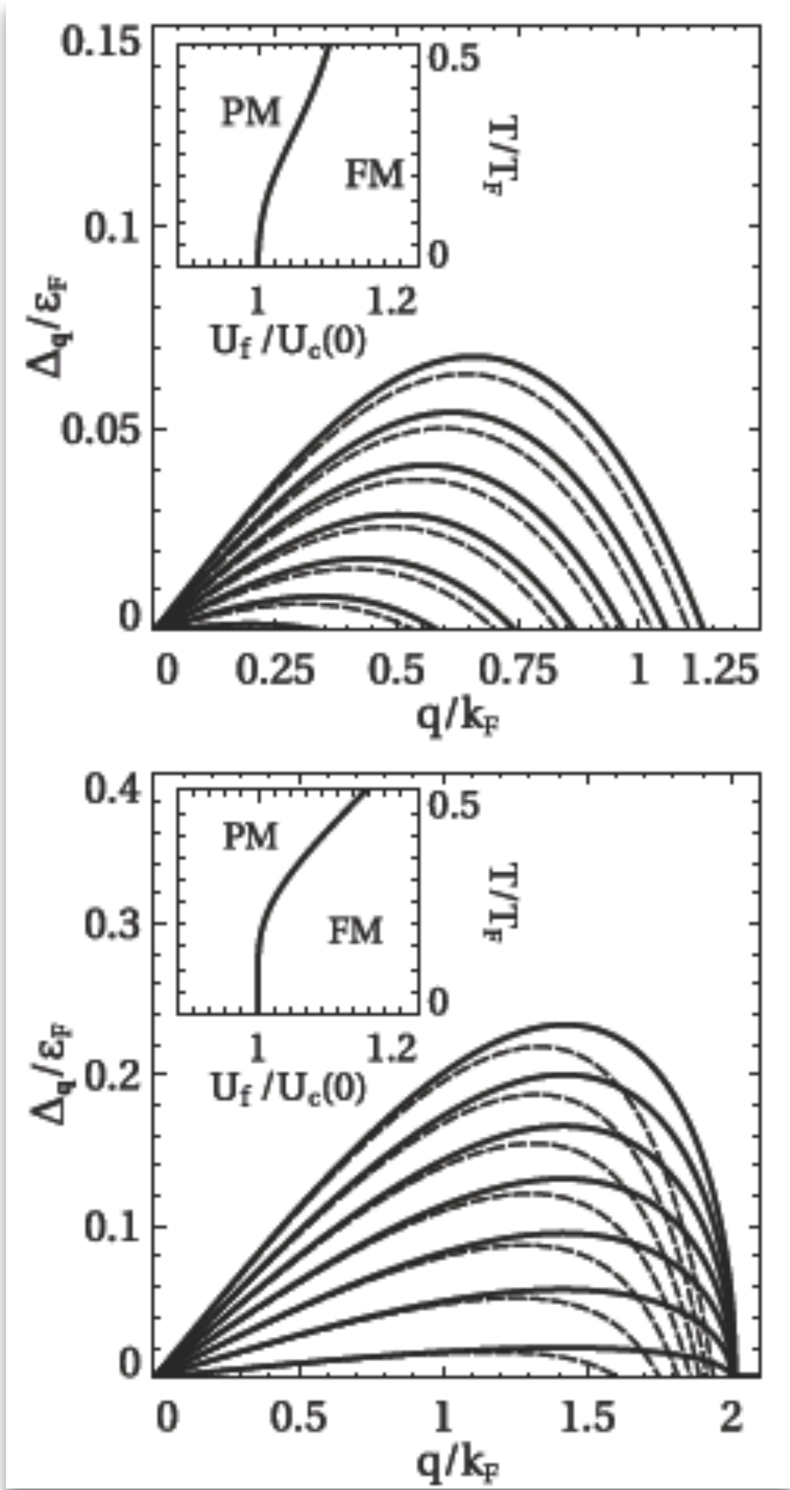
- imaging S/N ~ 10
- Domains hidden by shot noise if >100 in one pixel
- Given resolution of 3μm x 3μm x radius, implies domain size of <50 atoms.
- Domains also hidden if along |x> or |y>, which would increase possible hidden domain size (although a pi/2 pulse was tried)

Inconclusive.

Non-equilibrium growth of small domains

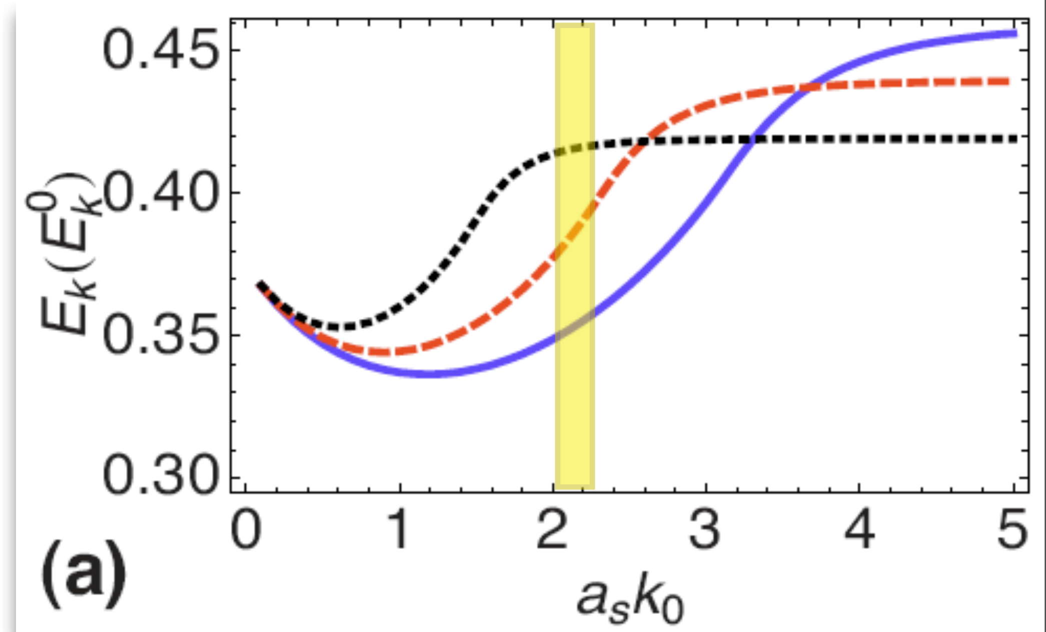
arXiv:0908.3483: Non-equilibrium dynamics of interacting Fermi systems in quench experiments M. Babadi, D. Pekker, R. Sensarma, A. Georges, E. Demler

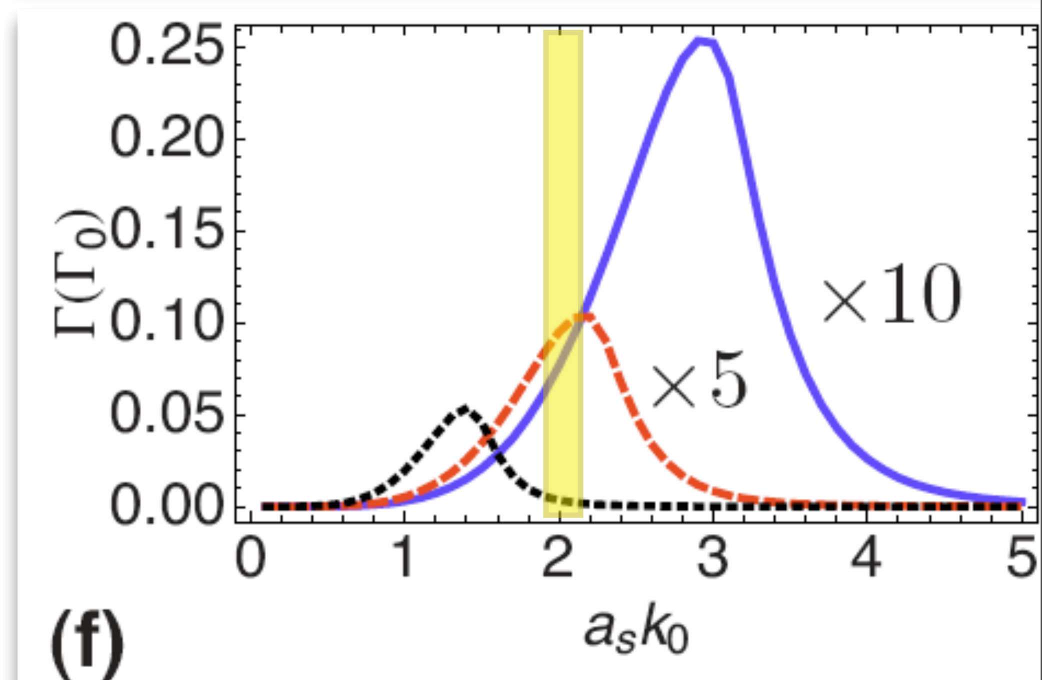
- Model experiment as quench
- Look at growth of modes
- Find ~ 2 k_F^{-1} … indeed smaller than resolution

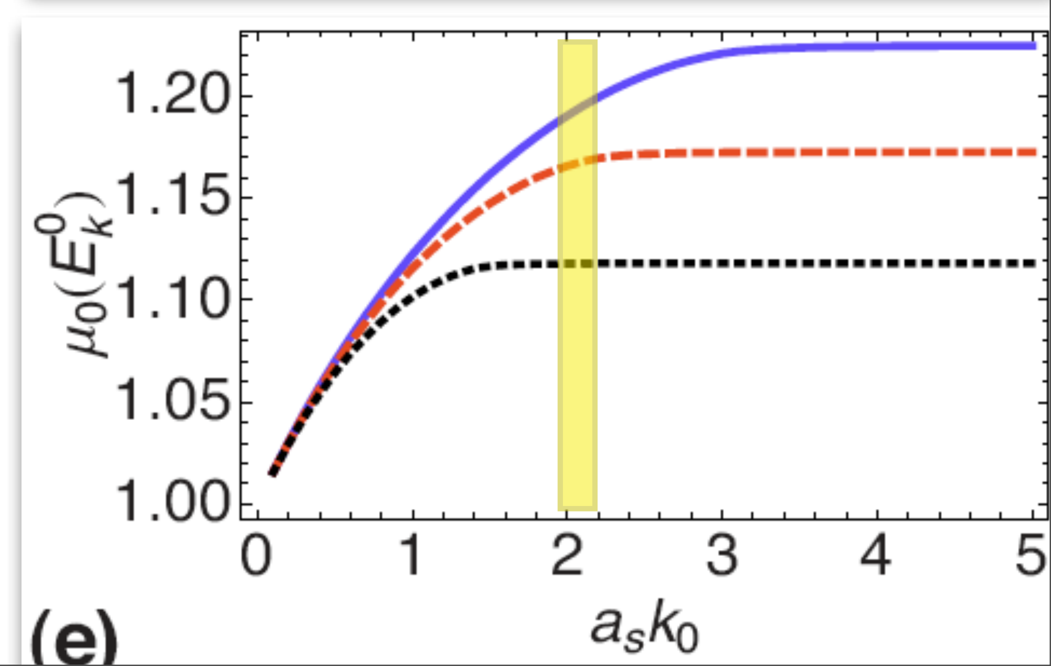


Correlated state

PRA **80**, 051605(R) (2009): Correlated vs Ferromagnetic State in Repulsively Interacting Two-Component Fermi Gases, Hui Zhai

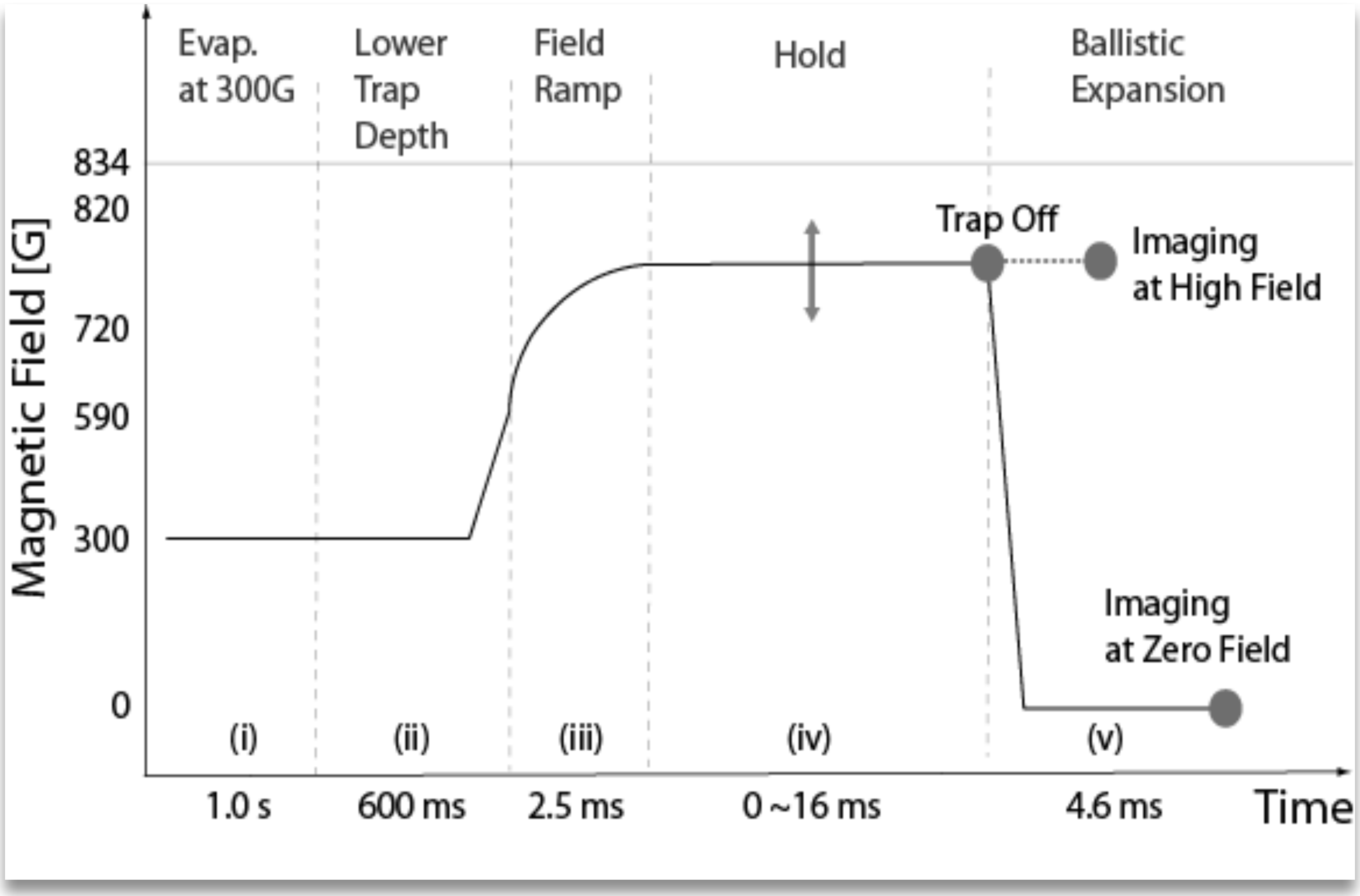




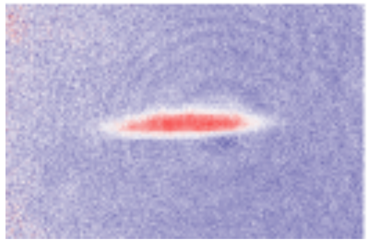


- Use a phenomenological equation of state with one parameter (alpha= 0.5,0.75,1 in figs)
- Observe many similar signatures:
  -minimum in KE
  -maximum in loss
- However
  -do not occur at same $k_F a$

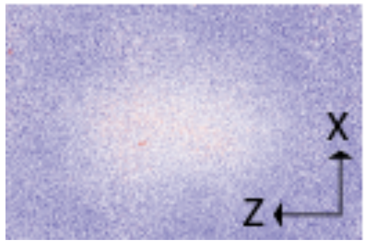
role of molecules



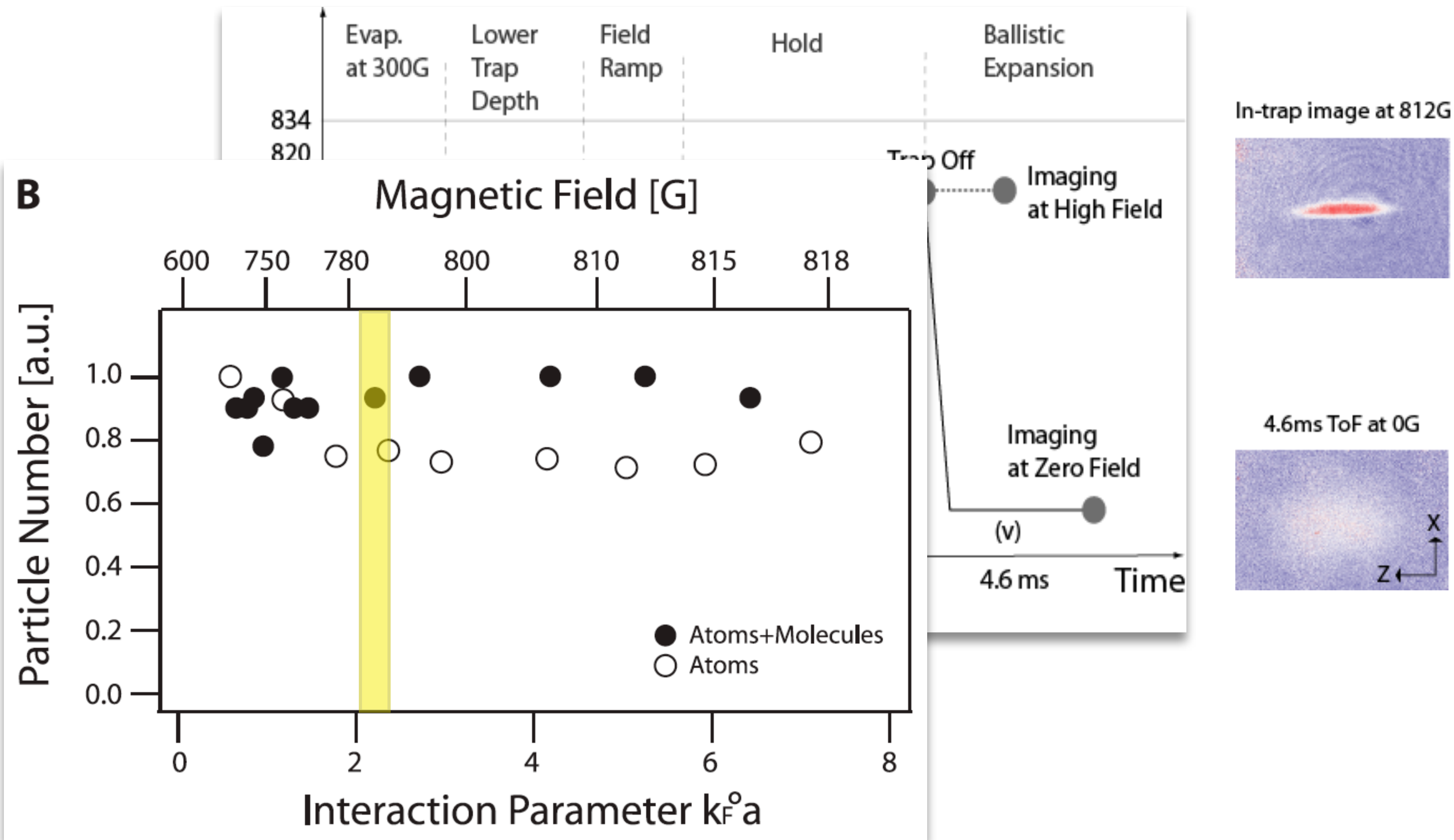
In-trap image at 812G



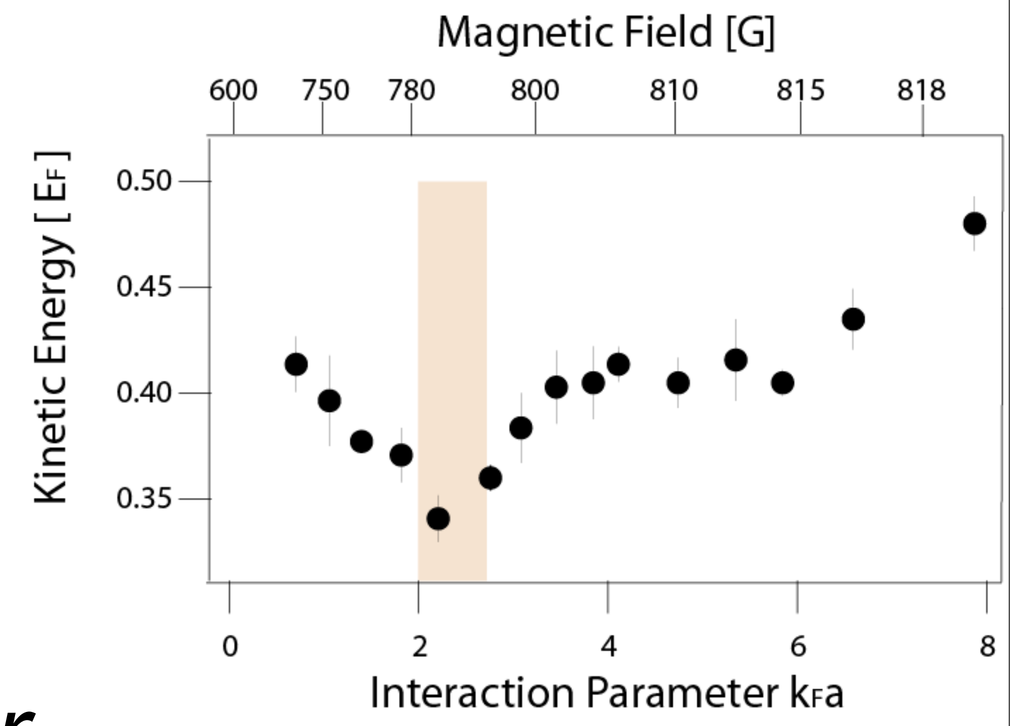
4.6ms ToF at 0G



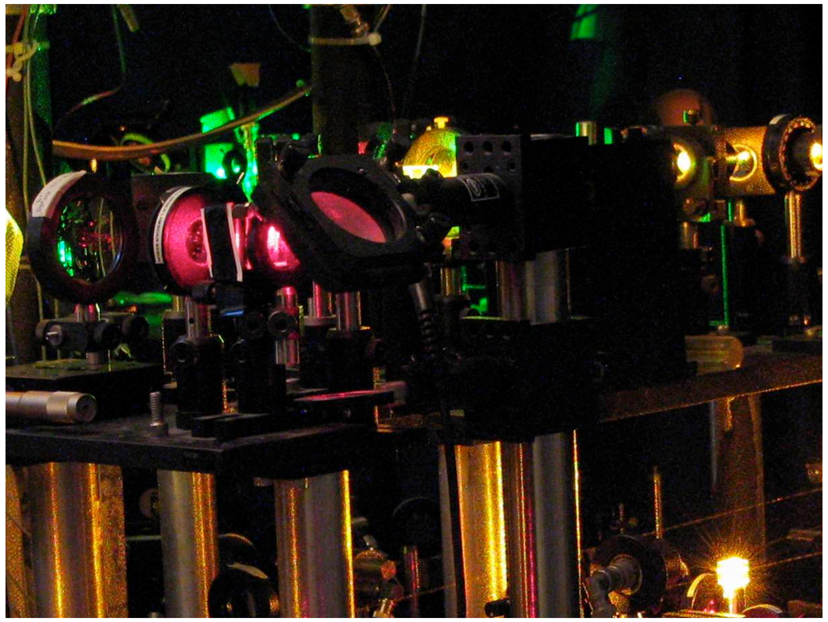
role of molecules

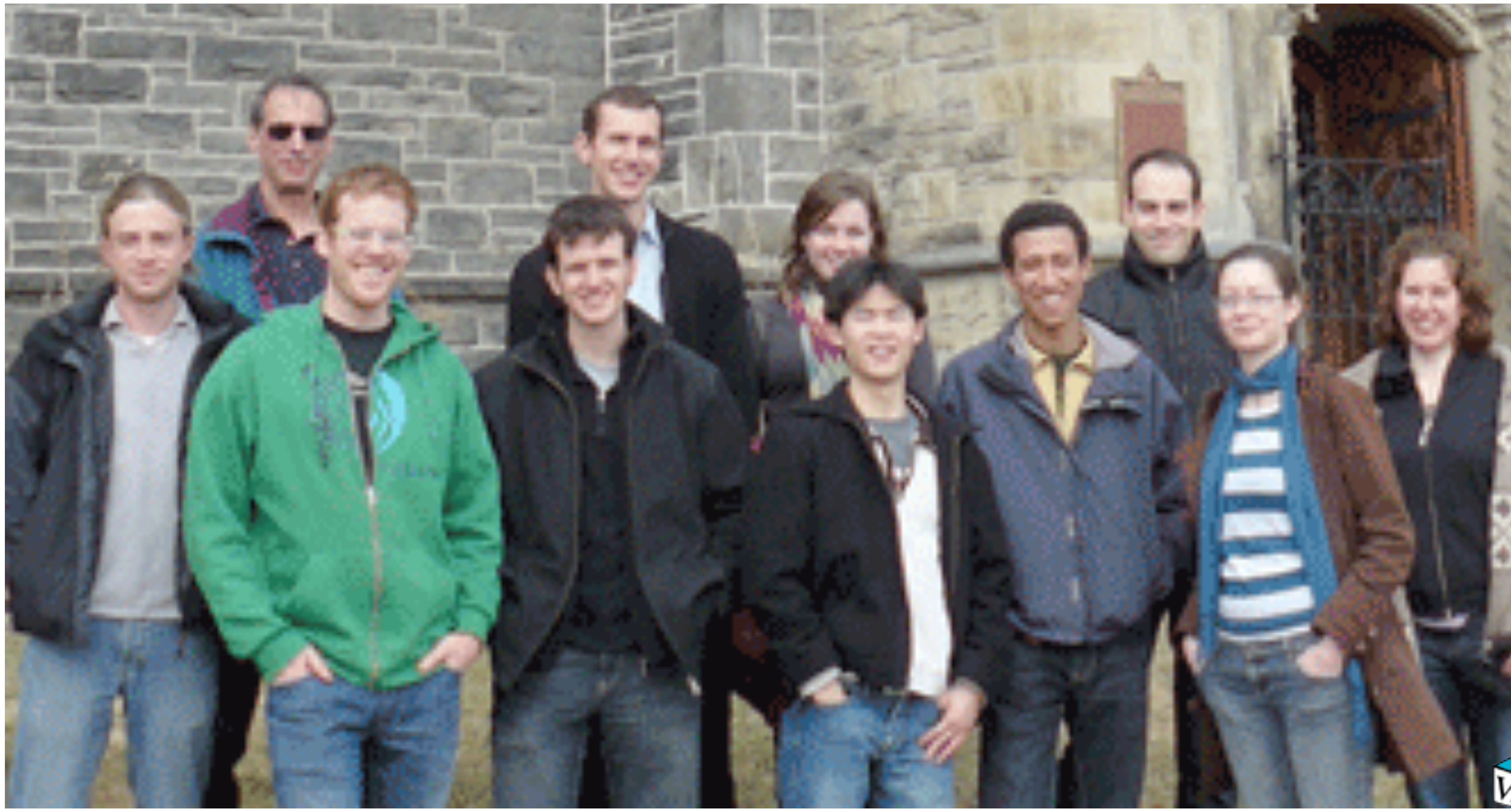


Conclusions



- **Three observations of non-monotonic behaviour** when approaching the Fesbach resonance.
  - –Implies that itinerant FM *can occur* for a free gas with short-range interactions
  - –First study of quantum magnetism in cold fermionic gases
- But:
  - –Lifetime only 10ms
  - –Magnetic domains not resolved
  - –Molecular fraction 25%

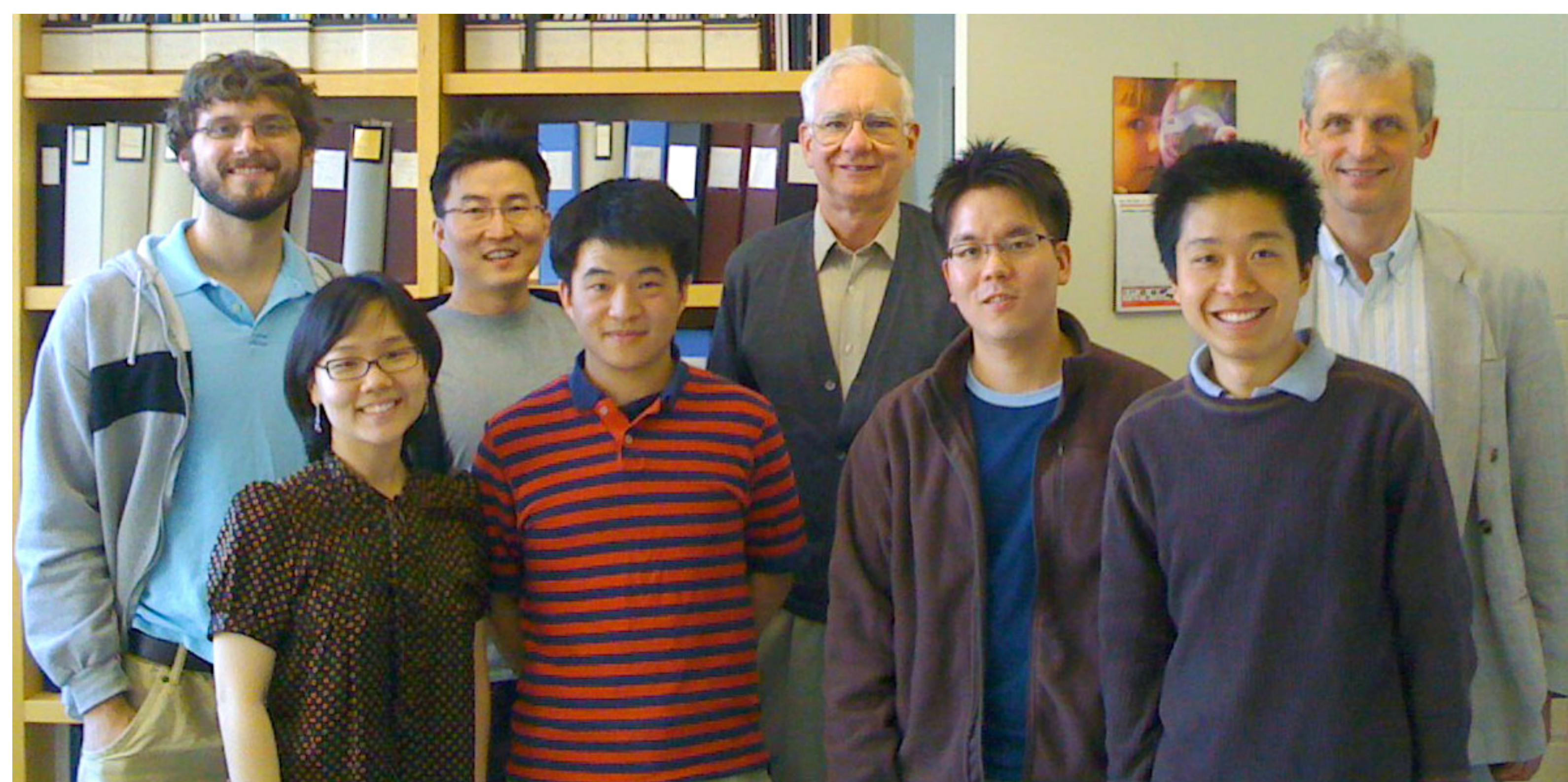




VELUT ARBOR ÆVO

CUA

Thank you!



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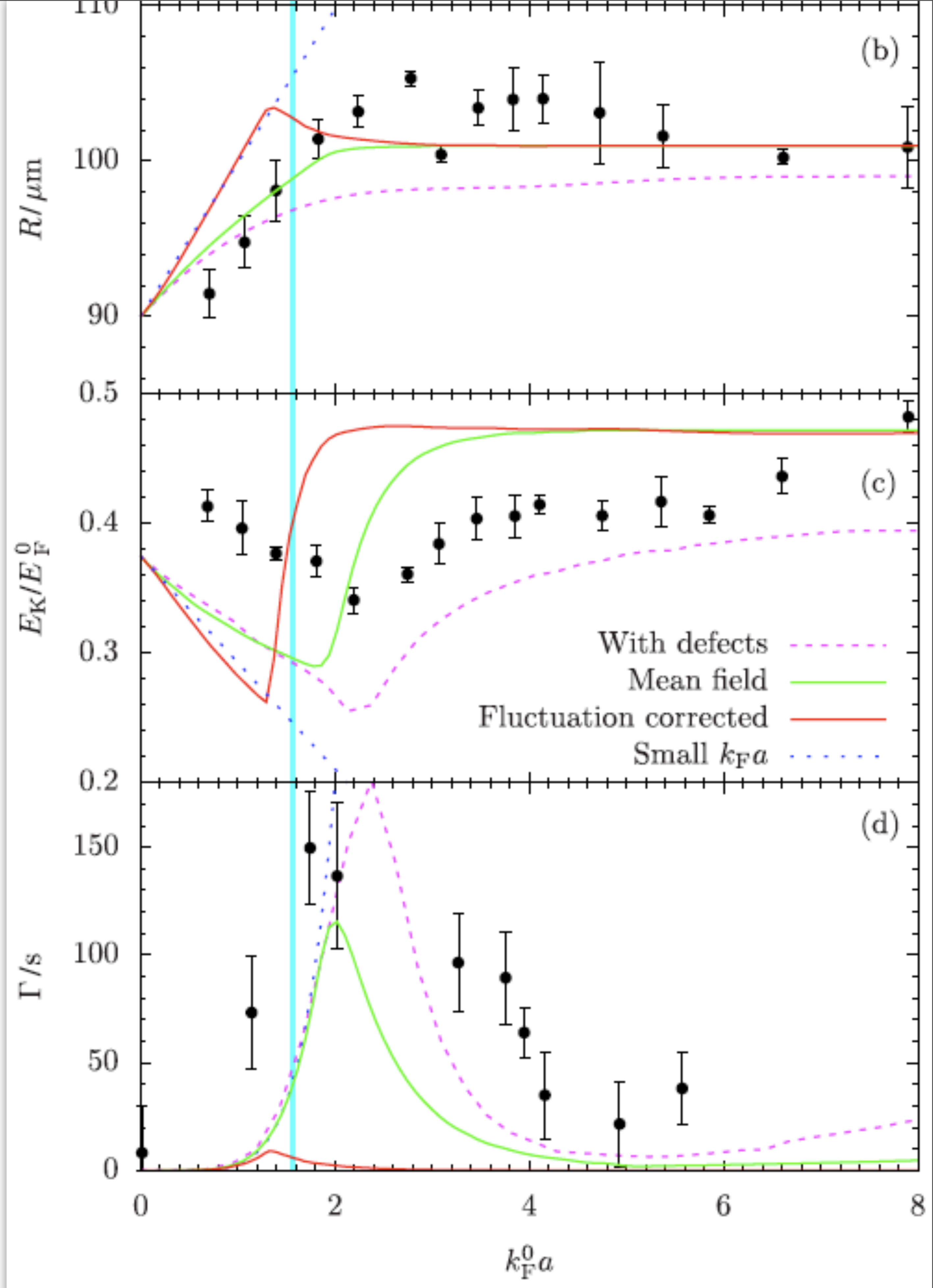
S. Zhang, H. Hung, C. Wu, arXiv:0805.3031v4 (2008)
M. Babadi, D. Pekker, R. Sensarma, A. Georges, E. Demler, arXiv:0908.3483 (2009)

PRL **103**, 200403 (2009): A repulsive atomic gas in a harmonic trap on the border of itinerant ferromagnetism, G.J. Conduit, B.D. Simons

Fluctuation corrections

PRL **103**, 200403 (2009): A repulsive atomic gas in a harmonic trap on the border of itinerant ferromagnetism, G.J. Conduit, B.D. Simons

- add 2nd-order term to include fluctuations.
- Consequences:
  -transition at lower $k_F a$
  -pk in chem potential



Magnetism without spin-orbit?

(important difference between UCA & CM)

- Moment along quantization axis (z) is difference between N_1 and N_2, where |1> is "spin up" and |2> is "spin down",
- Ferromagnetism is thus the observation of spontaneous (local) polarization "up" or "down"
- However *total* spin is conserved along z -- no spin-orbit interaction to equilibrate with some external field!
- If N_1=N_2, this cold atom experiment corresponds to a material with zero ambient field.