

# From molecules to behavior: *E. coli*'s memory, computation and chemotaxis

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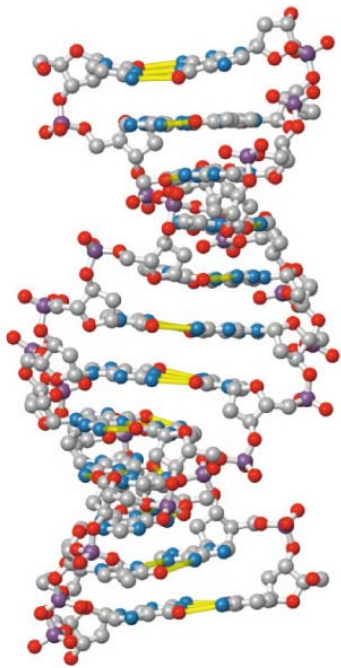
Victor Sourjik (now in Heidelberg)

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Howard Berg

**Physics**  **Biology**

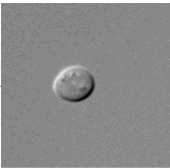
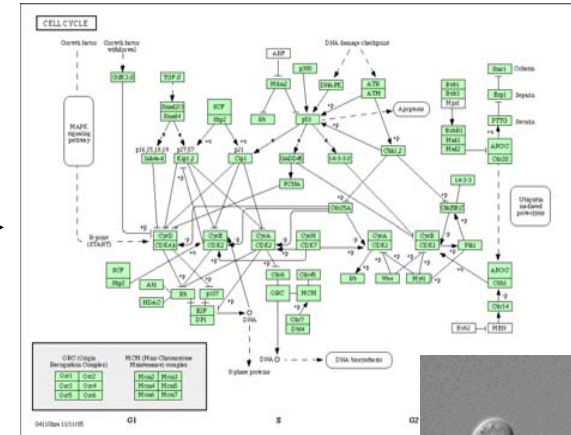
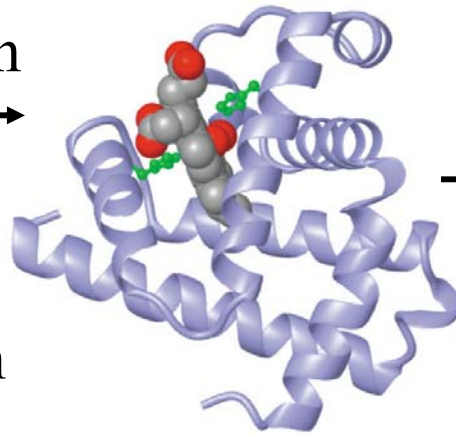
# From building blocks of biological systems to understanding of biological behaviors



transcription  
translation



regulation



**DNA**

**Proteins**

**Biological Network**

Genomic sequences

**Bioinformatics**

3-D structures

**Protein Folding**

Biological behaviors

**Systems Biology**

**How do bio-molecules interact to give rise to biological behaviors?**

# Biological systems are complex

- Many different types of molecules involved.
- Heterogeneous Interactions (temporally/spatially).
- Many missing elements (nodes)/links in the interaction network.
- Many kinetic constants are unknown.

**We need “Hydrogen atom” in systems biology!**

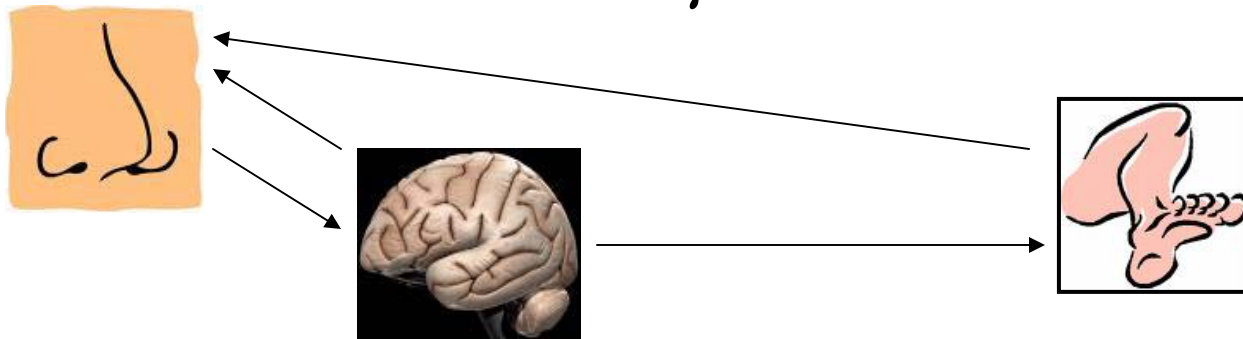
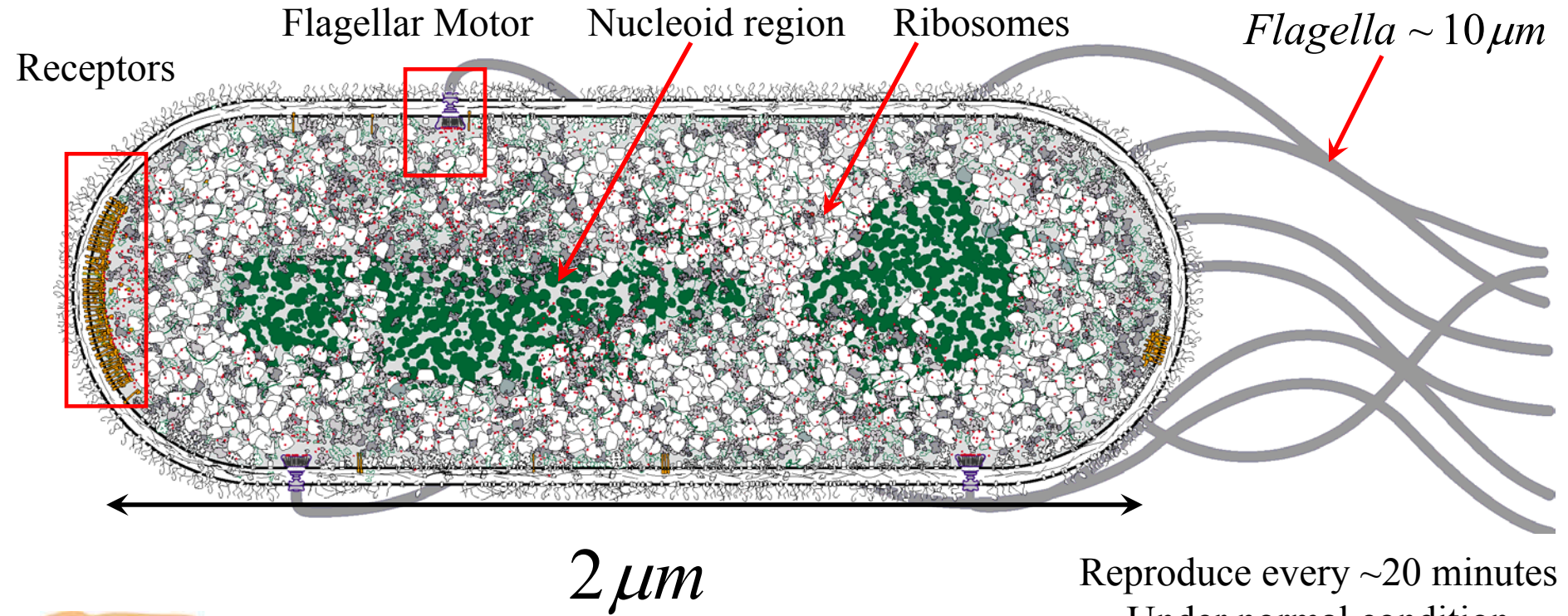
## **Chemotaxis in bacteria (E. coli)**

General behavior in simple model systems

- Important example of signal transduction and sensory system in biology
- Best chance in quantitatively understanding a complex biological system
- General principles in understanding complex biological systems

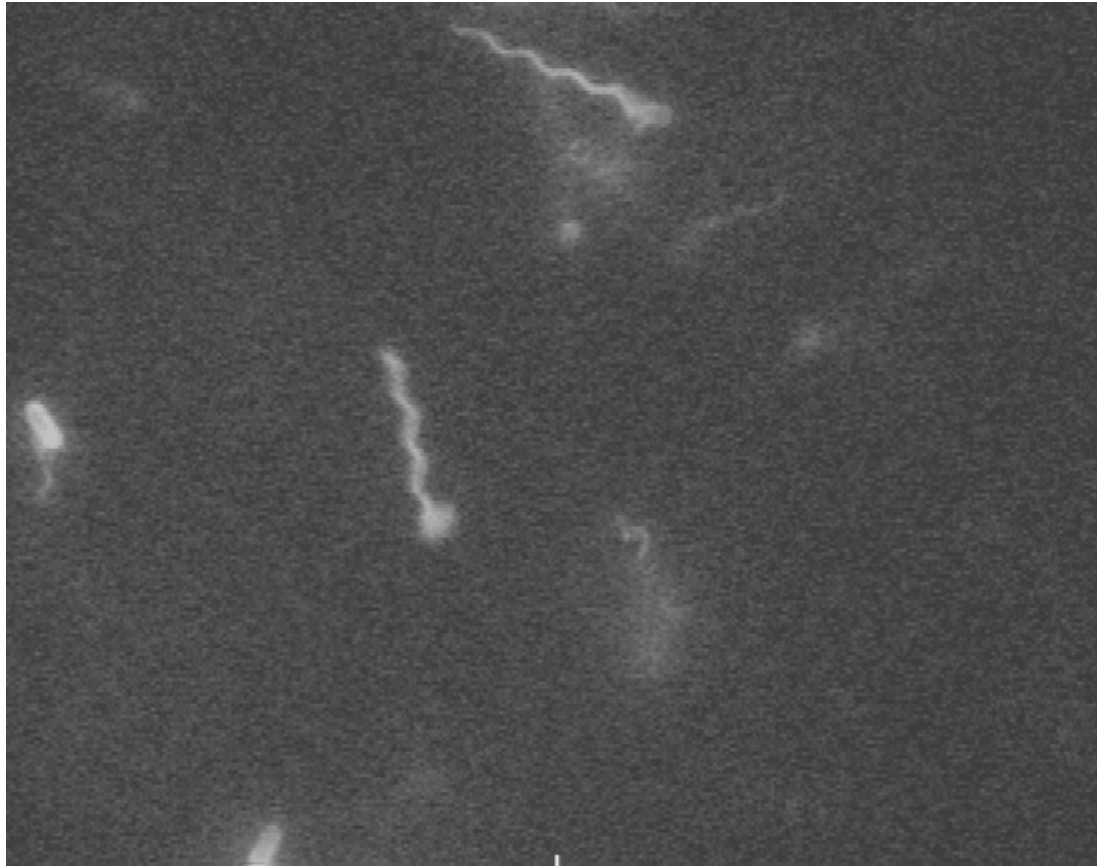
**Adaptation; Signal Processing; Robustness; Effect of Noise .....**

# *E. coli* anatomy and chemotaxis



How cells 1) receive signal; 2) process signal and 3) react to signal

# The biased random motion of *E. coli*: run & tumble

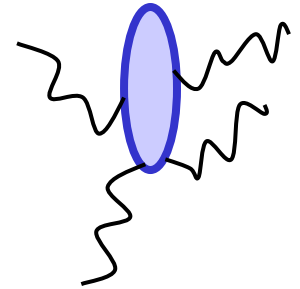


(From the Berg lab)

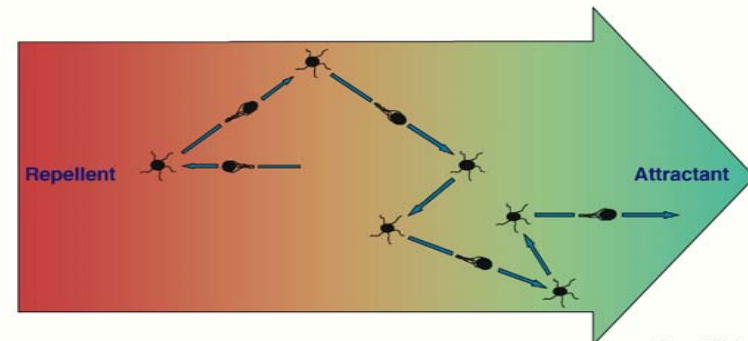
run



tumble



(Berg & Brown, 1972)



Biased random-walk

Current Biology

**Switch between tumble and run by comparing current environment with some memory encoded internally**

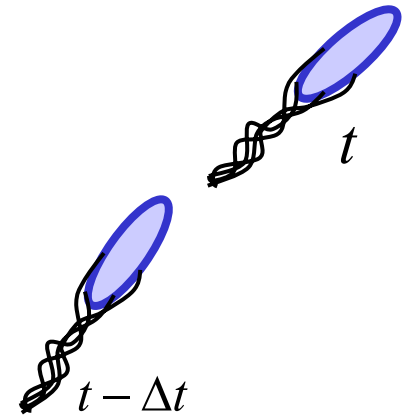
# The cell as a molecular information processor

Temporal comparison

$$\begin{array}{cc} \text{Current} & \text{Past} \\ \downarrow & \downarrow \\ \sim C(t) - C(t - \Delta t) \end{array}$$

Because the cell moves  $\Delta x = v\Delta t$

The cell effectively calculates the spatial gradient of C

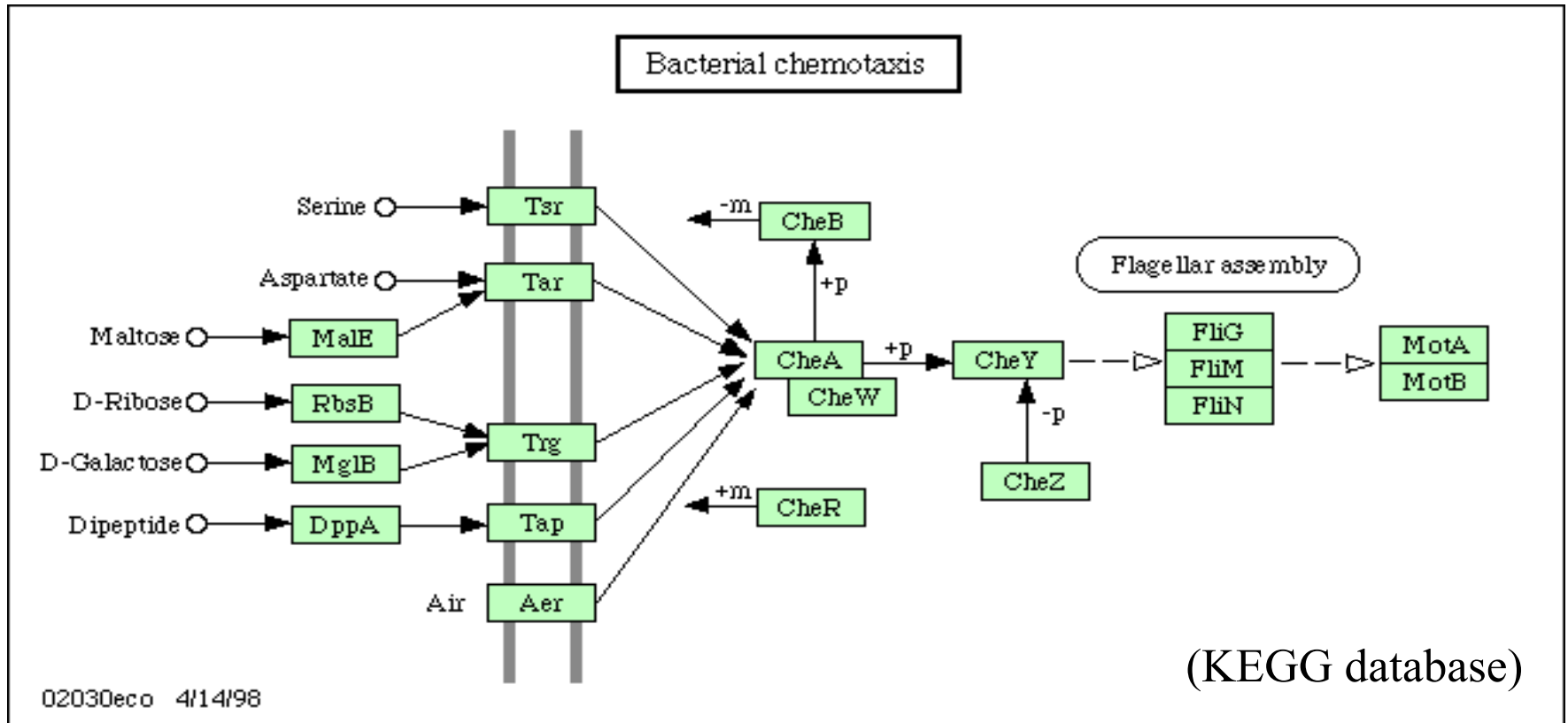


- How does cell keep a memory of its past history?
- How does it carry out the calculation of gradient?

How is the memory/computation performed by the molecules?



# The *E. Coli* chemotaxis signaling pathway (A molecular signal processing machine)



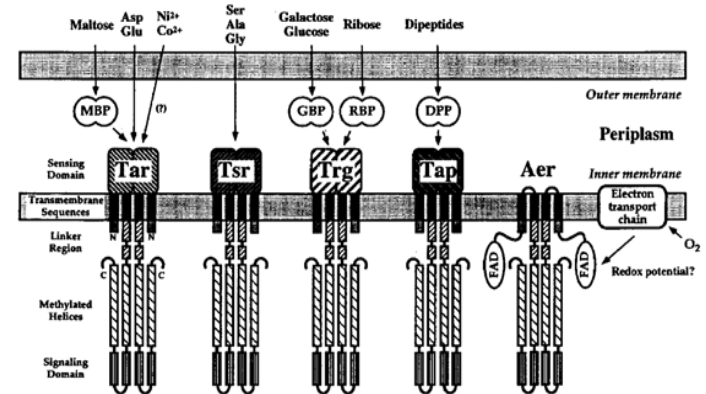
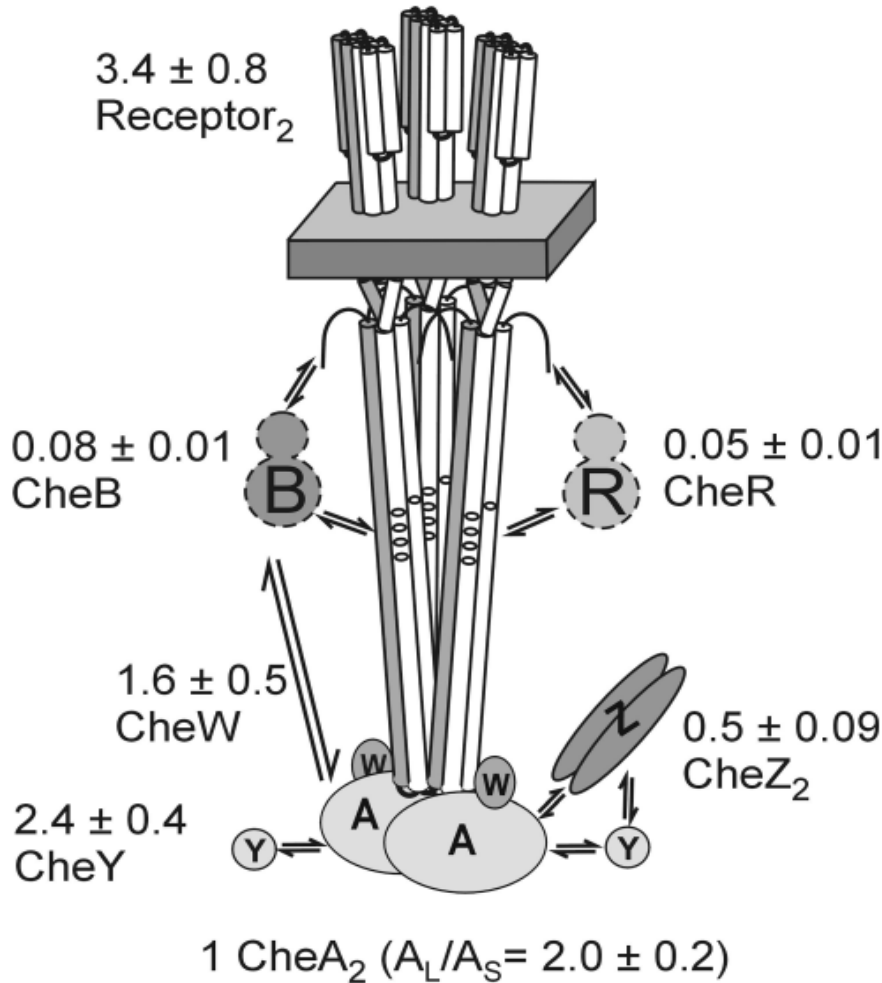
Signal transduction  
(sensing, amplification, adaptation.....)

Switch

Motor

# The key molecules for *E. Coli* chemotaxis signaling

## The chemo-receptor (sensor)



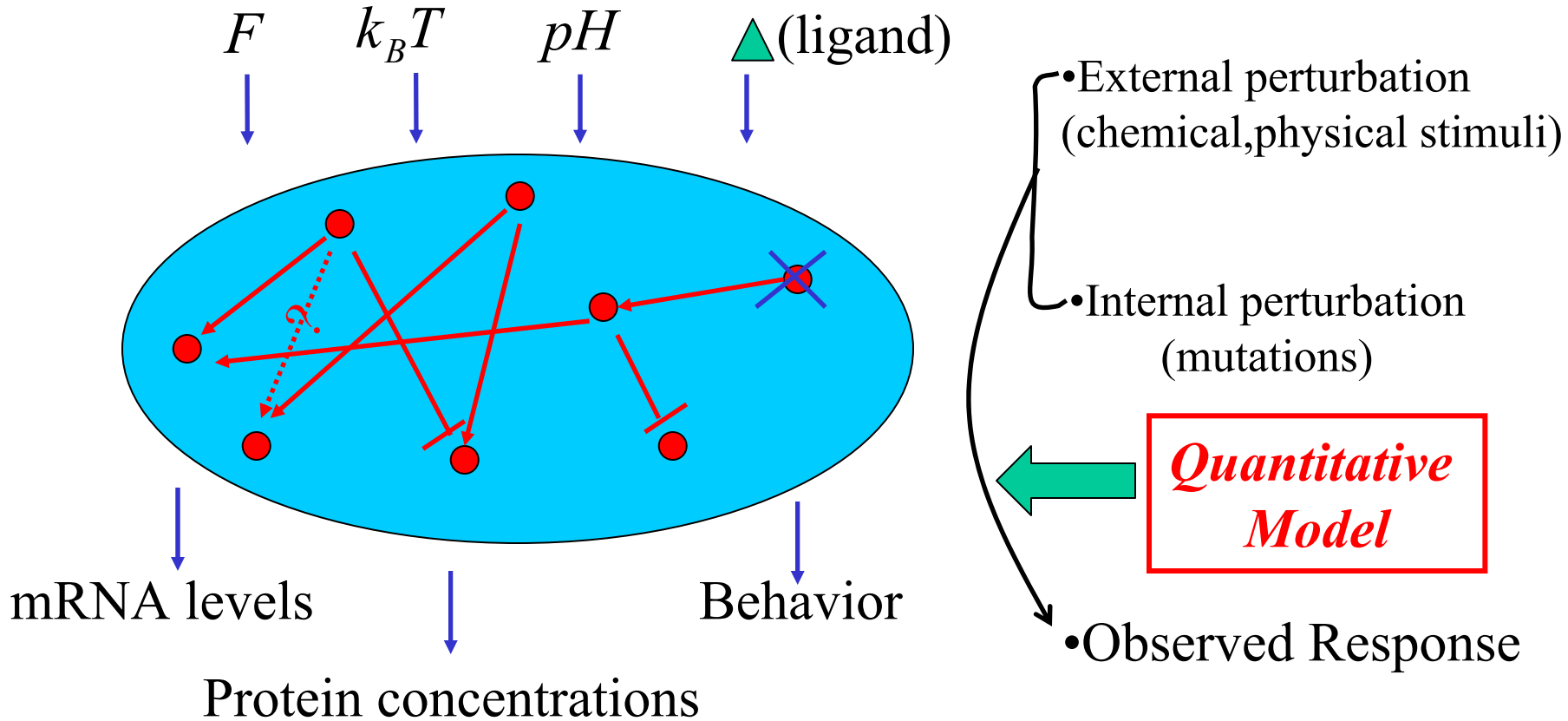
(5 types of chemoreceptor)

Total number of Receptors:  
15,000-26,000

**Tsr:Tar:Trg(Tap,Aer)~2:1:0.1**



# How can physics (modeling) help biology?



For complex systems,  
**quantitative modeling**  
is necessary to identify:



**Network properties**

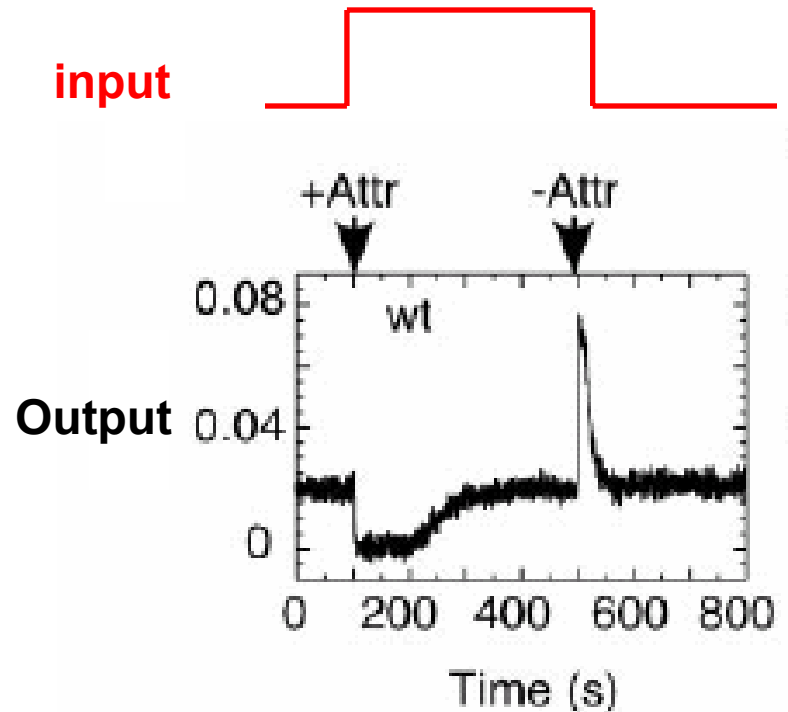
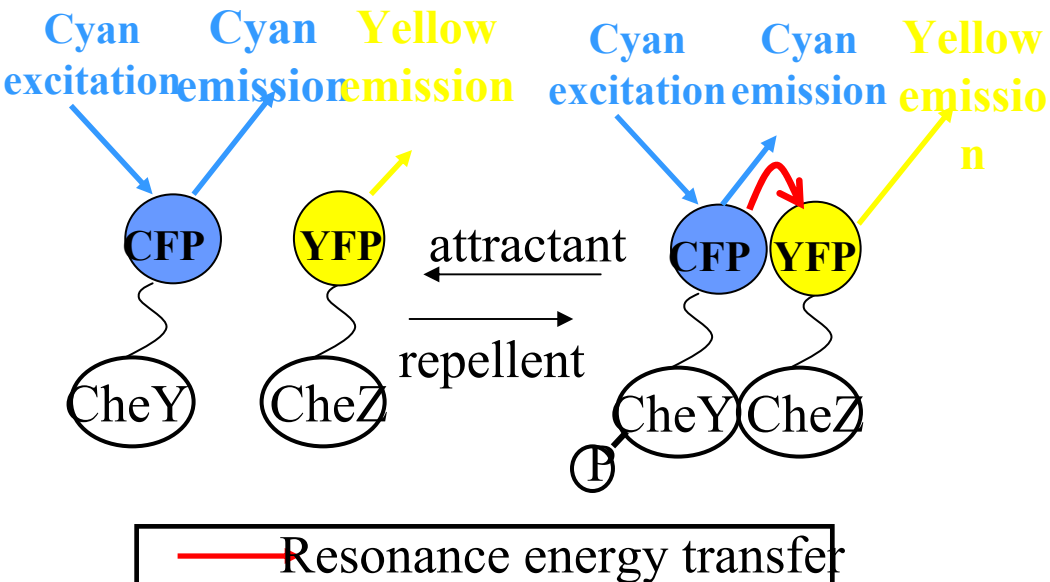
- Missing links & nodes
- Relation between links
- The numbers on the links



Understand and  
predict complex  
**Biological behavior**

# Probing the cell *in vivo* by perturbations

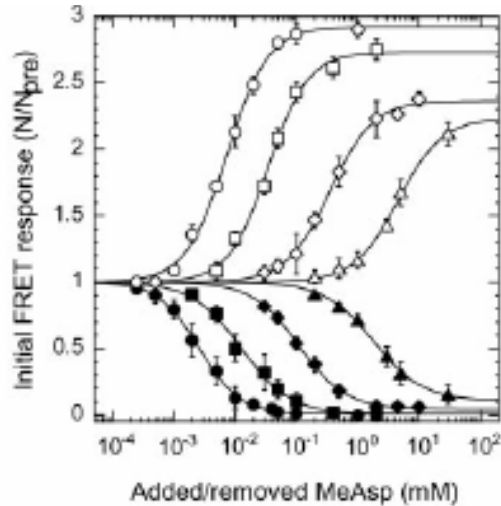
Direct *in vivo* measurement of CheY<sup>P</sup> level by FRET  
(Fluorescence Resonance Energy Transfer)  
(Sourjik&Berg, PNAS 99 123-127 (2002))



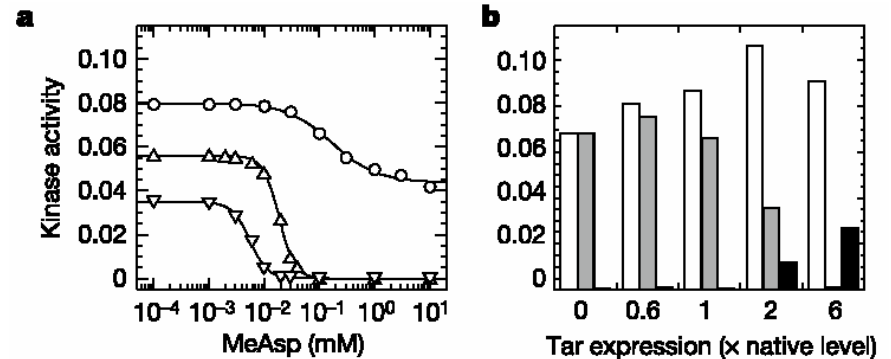
**Molecular level measurement while the cell is alive and behaving**

# The response data for wt and different mutants

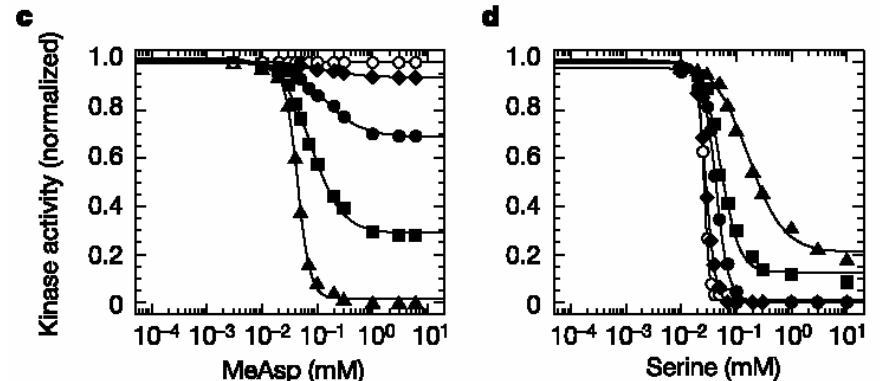
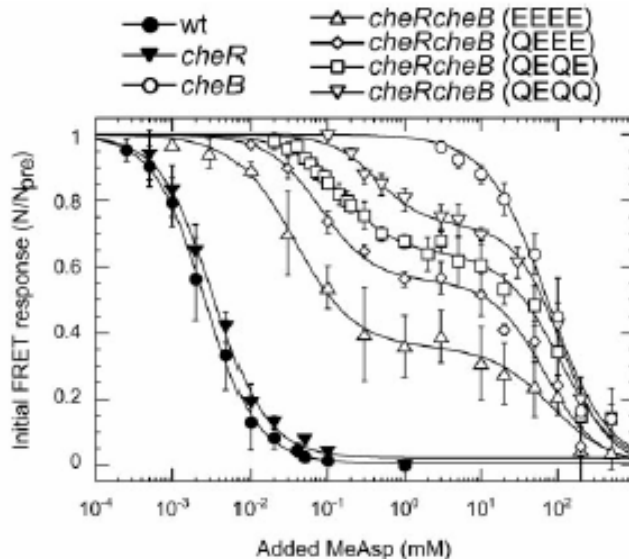
## WT in different background



## Different Tar/Tsr expression levels



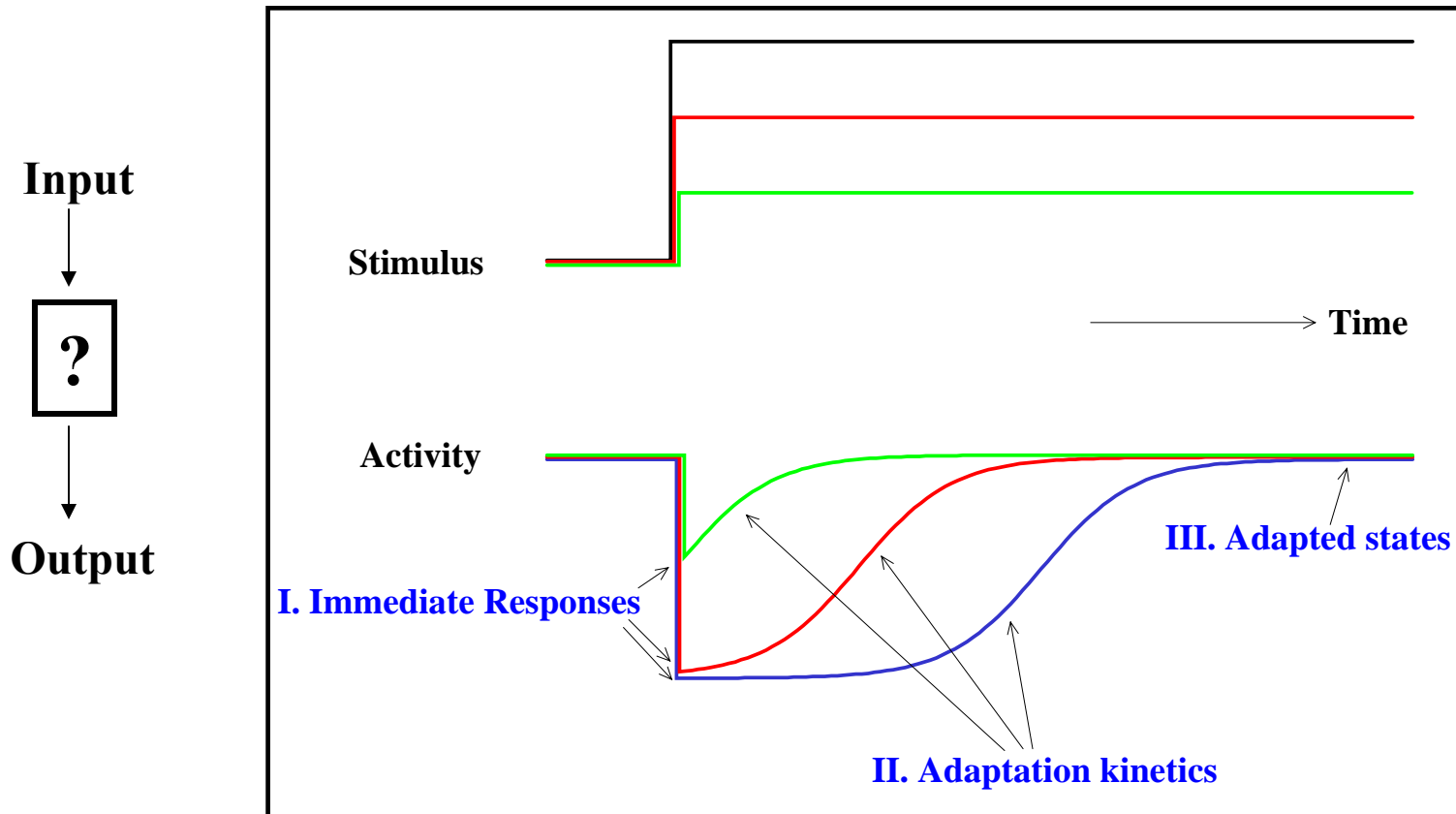
## Different methylation levels



..... (from the Berg Lab)

# High gain in a wide dynamic range for E. coli chemotaxis

- High sensitivity ( $\sim 10$ 's nM, a few ligand molecules)
  - Signal amplification ( $\sim 40X$ )
- High sensitivity exists in a wide range of backgrounds
  - Wide dynamic range (100nM  $\rightarrow$  1mM)
  - Near perfect adaptation



# **Energetics and the steady state behavior of the system**

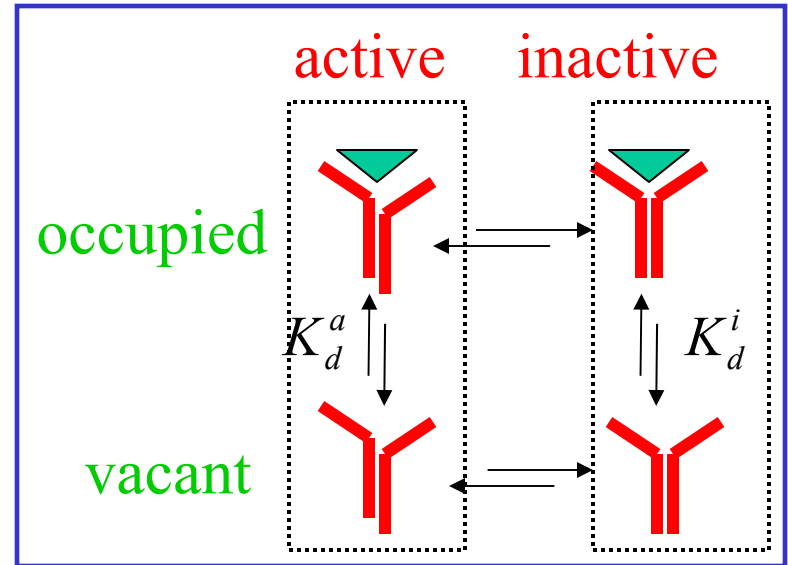
# The energetics of a receptor dimer

• 4 states for each individual receptor  $i$

• Kinase activity  $a_i = 0,1$

• Ligand binding  $l_i = 0,1$

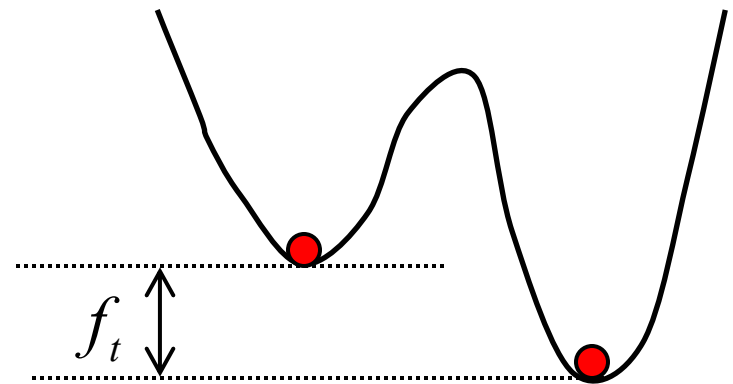
$$\langle a \rangle = (1 + \exp(-f_t))^{-1}$$



$$f_t = f_m(m) - \ln \frac{1 + [L]/K_d^i}{1 + [L]/K_d^a}$$

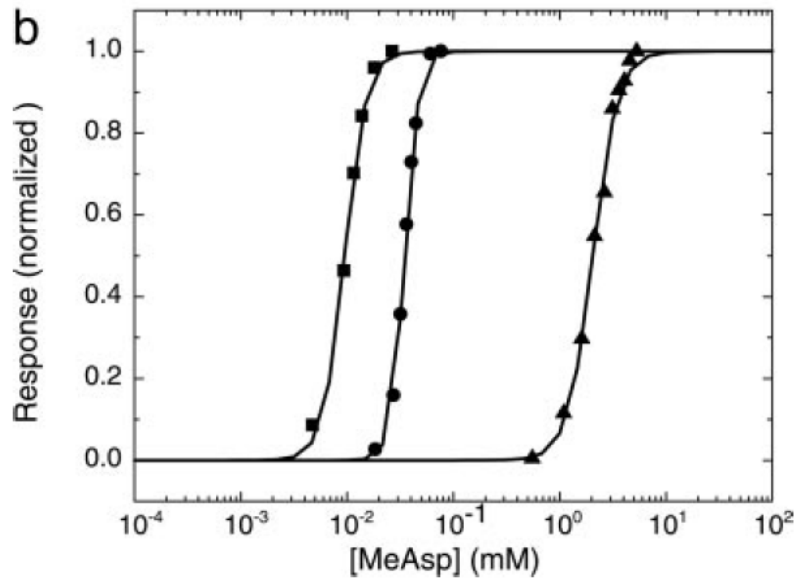
Methylation energy

Ligand binding energy

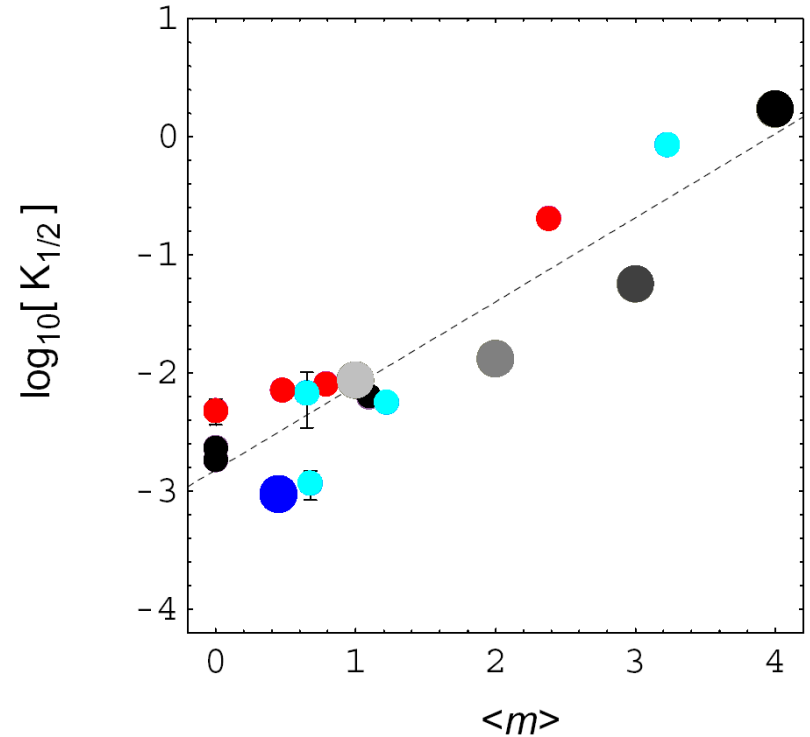




# The methylation energy function



(Shimizu et al. PNAS 2006)

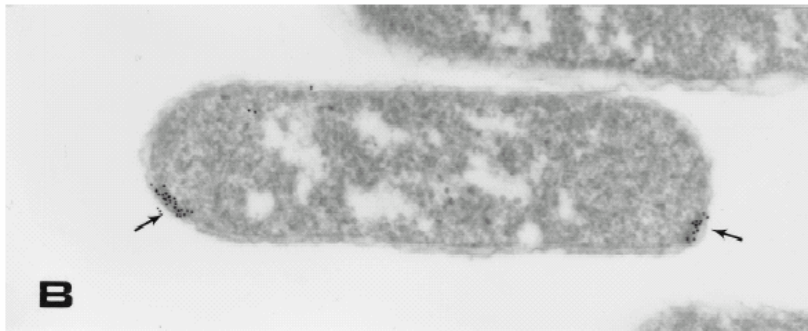


(Shimizu et al. unpublished)

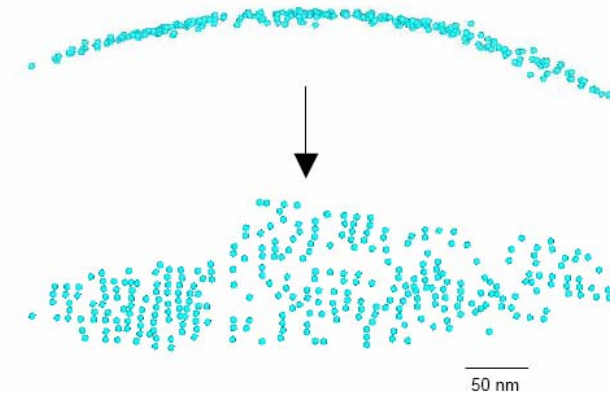
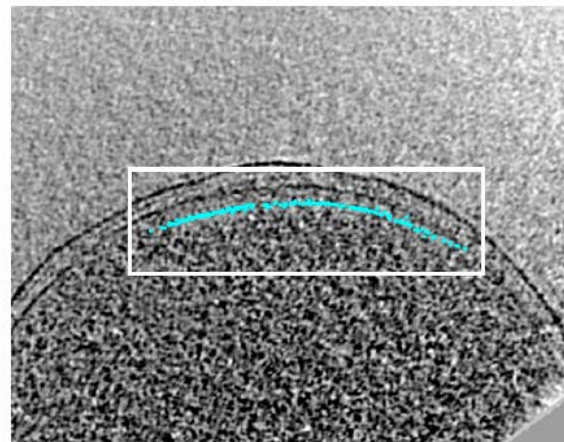
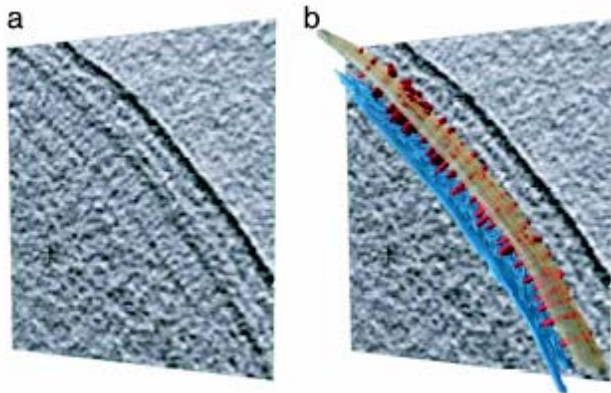
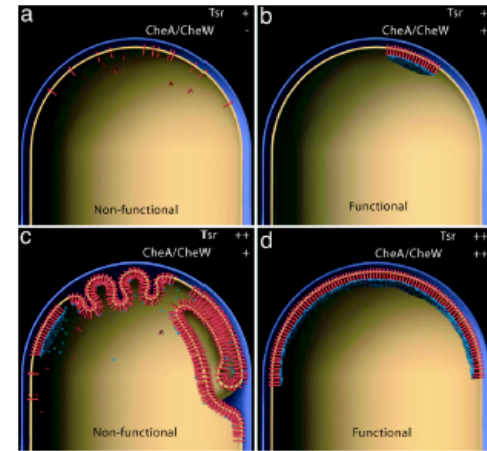
$$f_m(m) \approx \alpha(m - m_0)$$
$$\alpha \approx 1.5 - 2.0 ; m_0 \approx 1 - 2$$

# Chemotaxis receptors form clusters at the cell poles

Chemoreceptors cluster in bacteria  
(~20,000 chemo-receptors in a *E. Coli* cell)



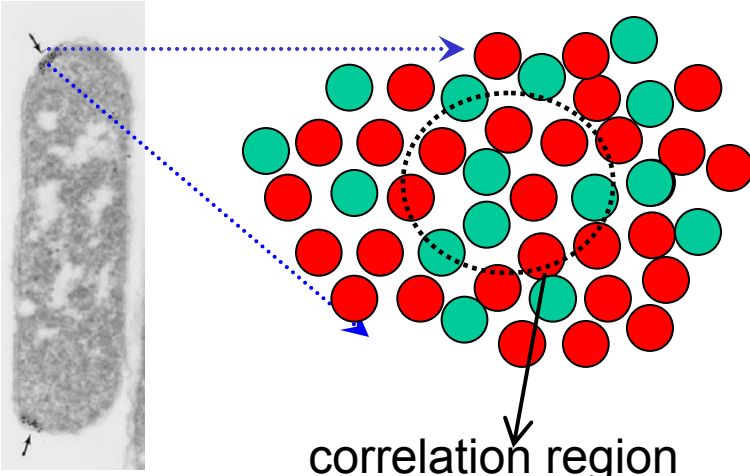
(Maddock & Shapiro, 1993)  
(Lybarger & Maddock)



(Subramanian Lab, NCI)

# The receptor-receptor interaction within the polar cluster

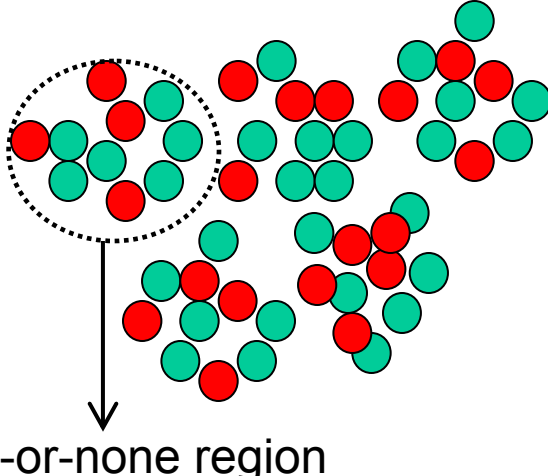
Ising type model (nearest neighbor)



$$+ a_i \sum_j C_{q_i q_j}^{(nn)} (a_j - \frac{1}{2})$$

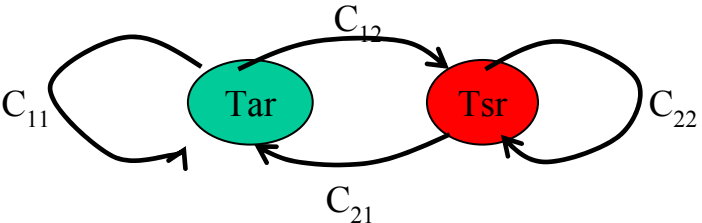
More realistic

MWC type model (all-or-none)



$C \rightarrow \infty$  within cluster size  $N$

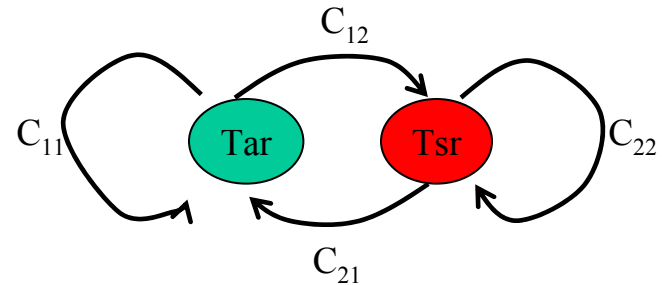
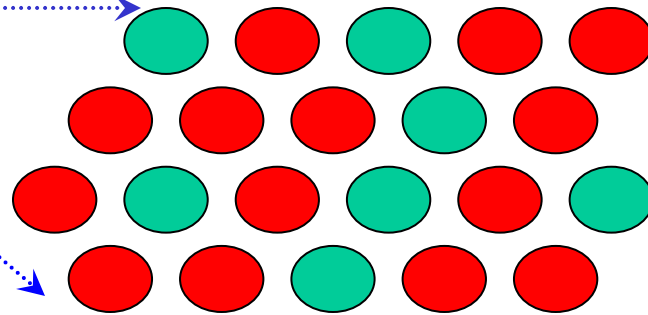
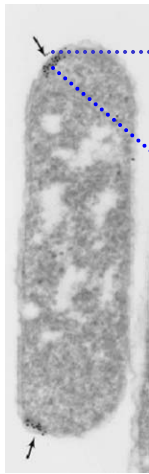
Simplicity (analytical solution)



**Non-discriminative Interaction between heterogeneous receptors**  
They act together

# The Ising-like model for receptor Interaction

- Activity of a receptor affected by the activities of its neighbor in the receptor cluster. Cooperativity in a continuum lattice.



$$\text{Interaction energy} = a_i \sum_j C_{q_i q_j} \left( a_j - \frac{1}{2} \right)$$

- $j$  labels all the “neighboring” receptors of  $i$ 'th receptor

$$H = \sum_i a_i \left[ \underbrace{E_m(m_i) + E_L(m_i)l_i}_{\text{“local magnetic field”}} + \sum_{j(i)} C_{q_i q_j} \left( a_j - \frac{1}{2} \right) \right] + \mu_l(m_i)l_i$$

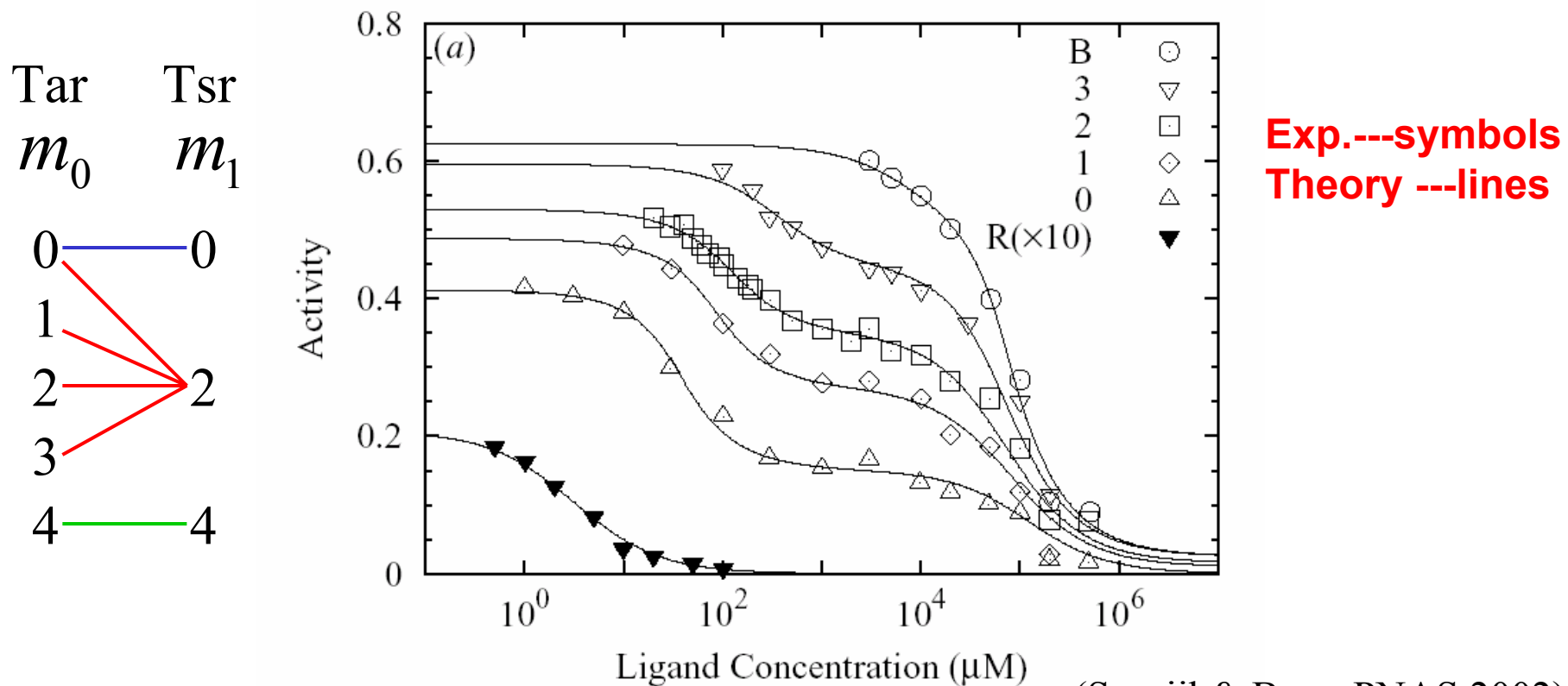
“Spin”
“coupling to neighbors”

**Analogous to the Ising model for magnetism in physics**

# The model results for the cheRB- mutant strains

Adaptation disabled: Receptor methylation level fixed

Data can only be explained with interaction between Tar and Tsr receptors  
**Different types of chemo-receptors act together**



# of parameters in the model:  $3 \times 8 + 4 = 28$

# of **independent** data points:  $\sim 6 \times 7 = 42$

(Sourjik & Berg, PNAS 2002)

(Mello & Tu, PNAS, 2003)

# Responses in wild-type (wt) *E. coli* cells that can adapt accurately

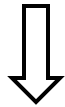
Monod-Wyman-Changeaux (MWC) model for N highly correlated receptors:

$$a_{wt}([L],[L]_0) = \frac{L(1+[L]/K_a)^N}{(1+[L]/K_i)^N + L(1+[L]/K_a)^N}$$

$[L]_0$  -- background ligand signal concentration;  $K_{i,a}$  -- dissociation constants

WT cell adapt perfectly:

$$a_{wt}([L]_0,[L]_0) = a_0$$

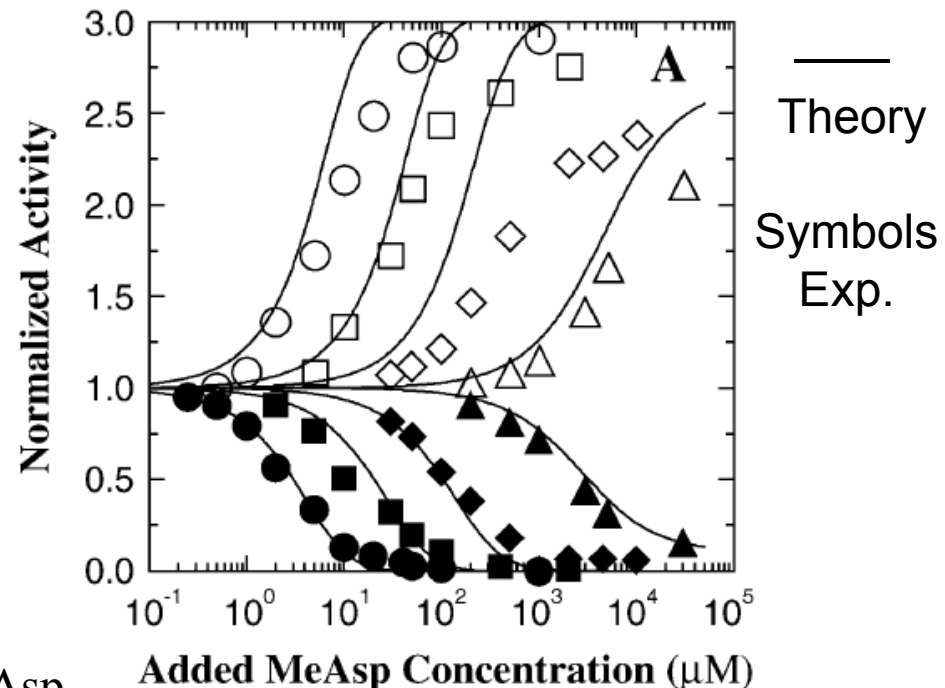


$$L = \frac{a_0}{1-a_0} \left( \frac{1+[L]_0/K_i}{1+[L]_0/K_a} \right)^N$$

Microscopic parameters determined

$N_i = 3N \approx 20$ ;  $K_i \approx 18\mu M$ ;  $K_a \approx 3mM$  for MeAsp

(Mello&Tu, BioPhys. J. 2007)

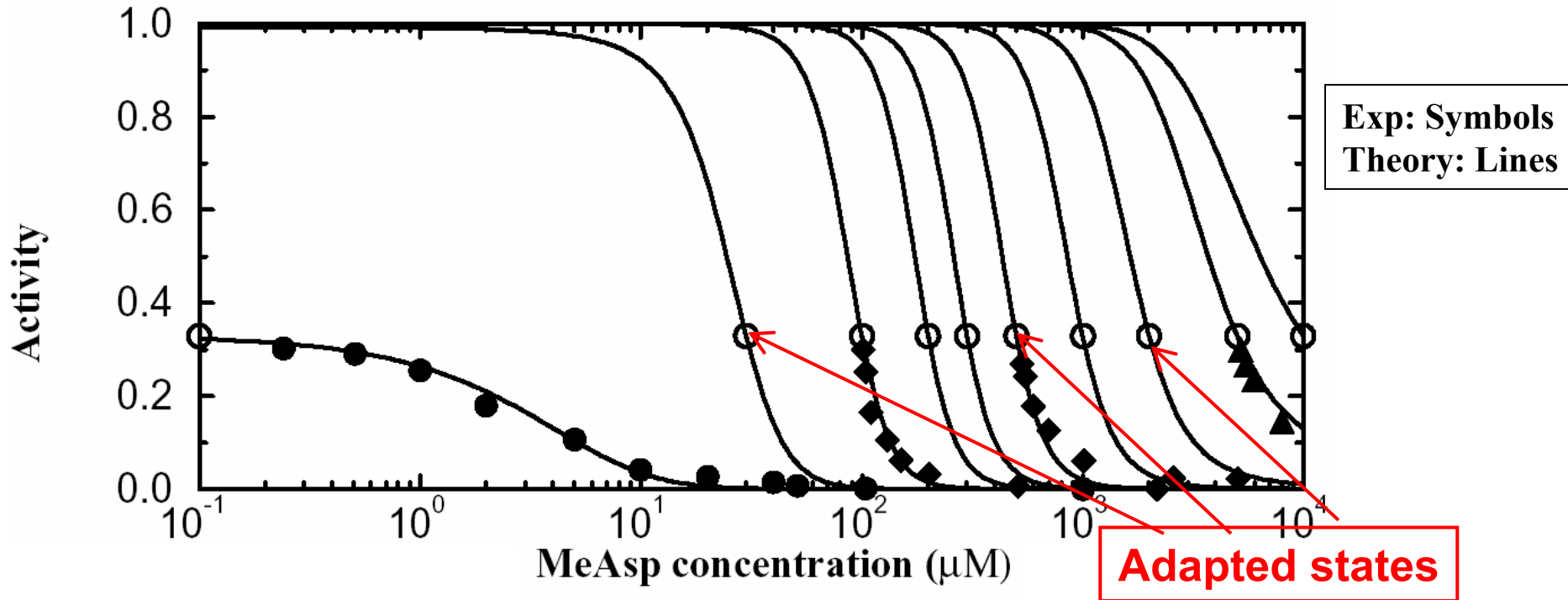


( $[L]_0 = 0, 0.1, 0.5, 5mM$ )



# Adaptation enables high sensitivity over a wide range of backgrounds

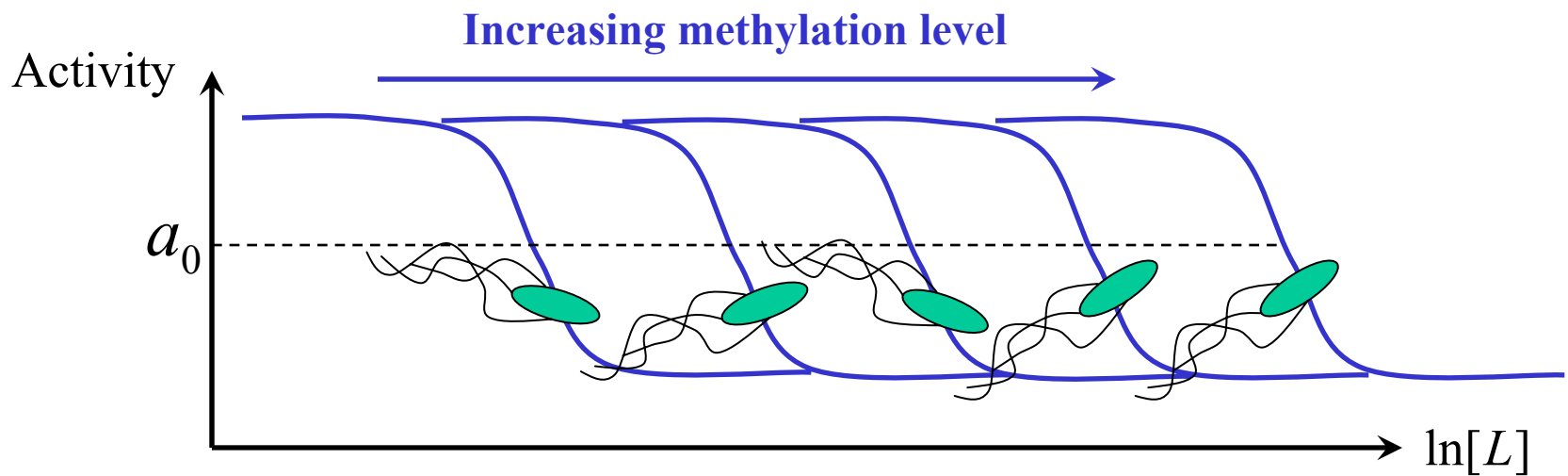
Increasing receptor methylation level  $\longrightarrow$   
(increasing background concentrations  $[L]_0 = 0, 0.1, 0.5, 5\text{mM}$ )



Mechanism for sustained high gain:

Self-tuned near-critical behavior: the “smart” Ising model

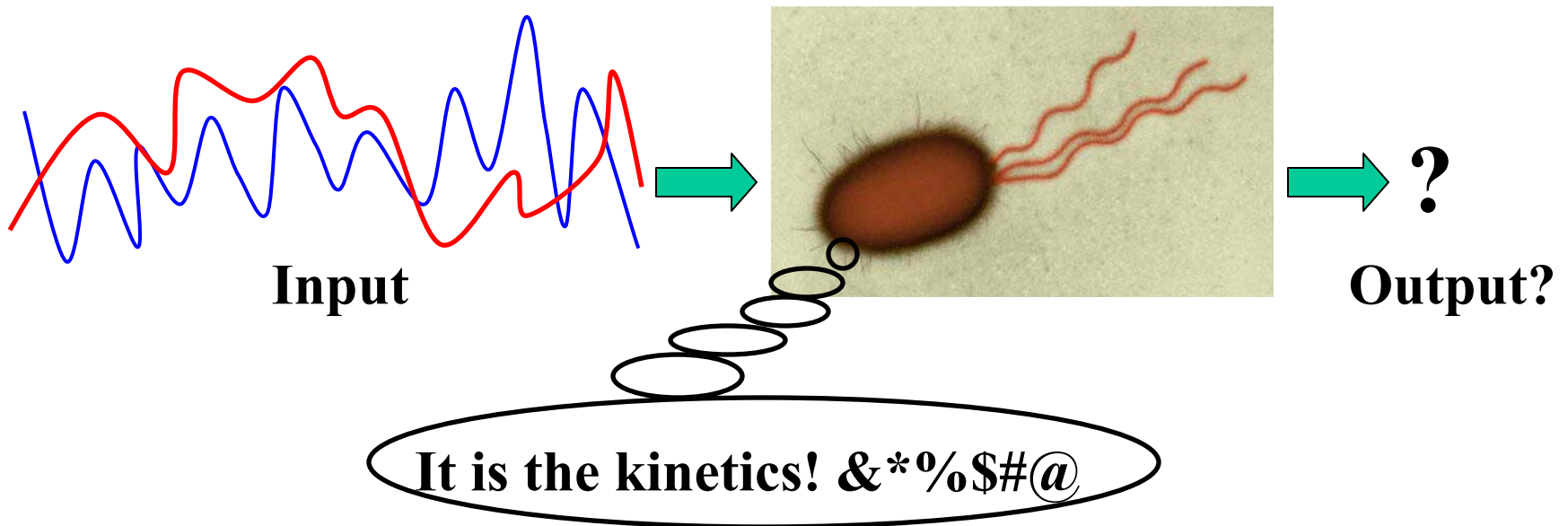
# E. Coli surfing the adaptation wave



# **Kinetics and responses to time varying signals**

# Responses to time varying signals

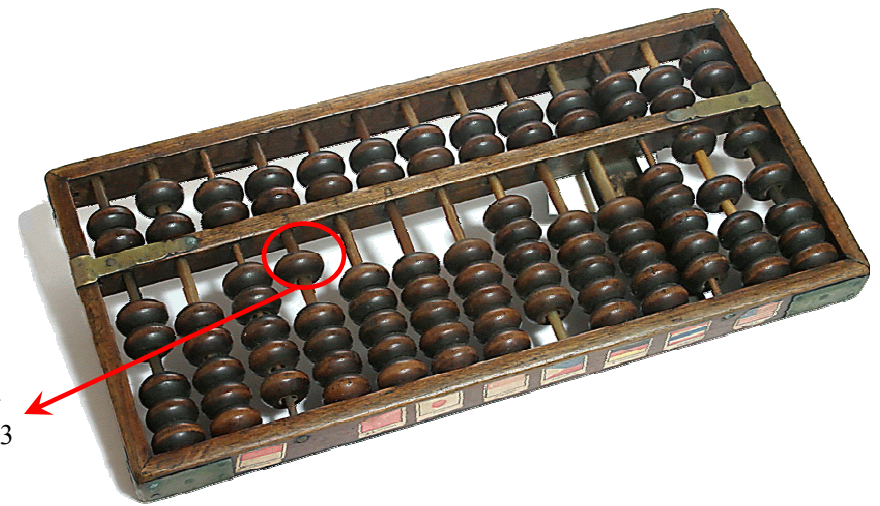
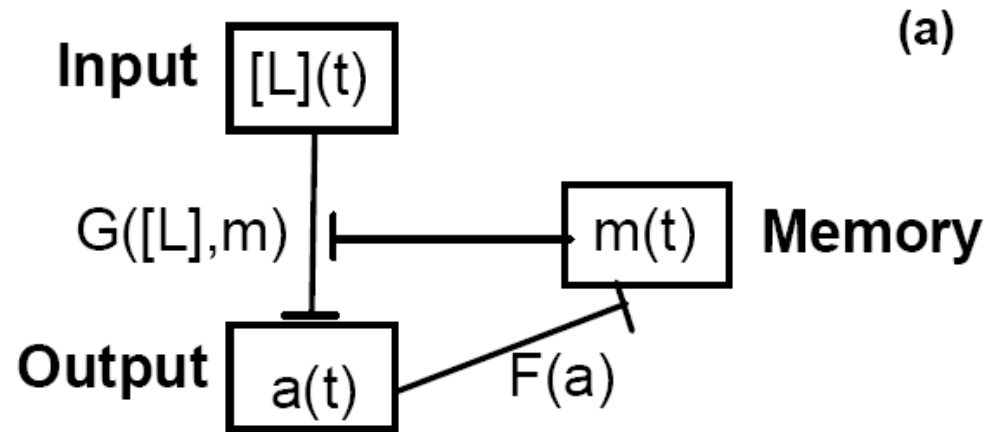
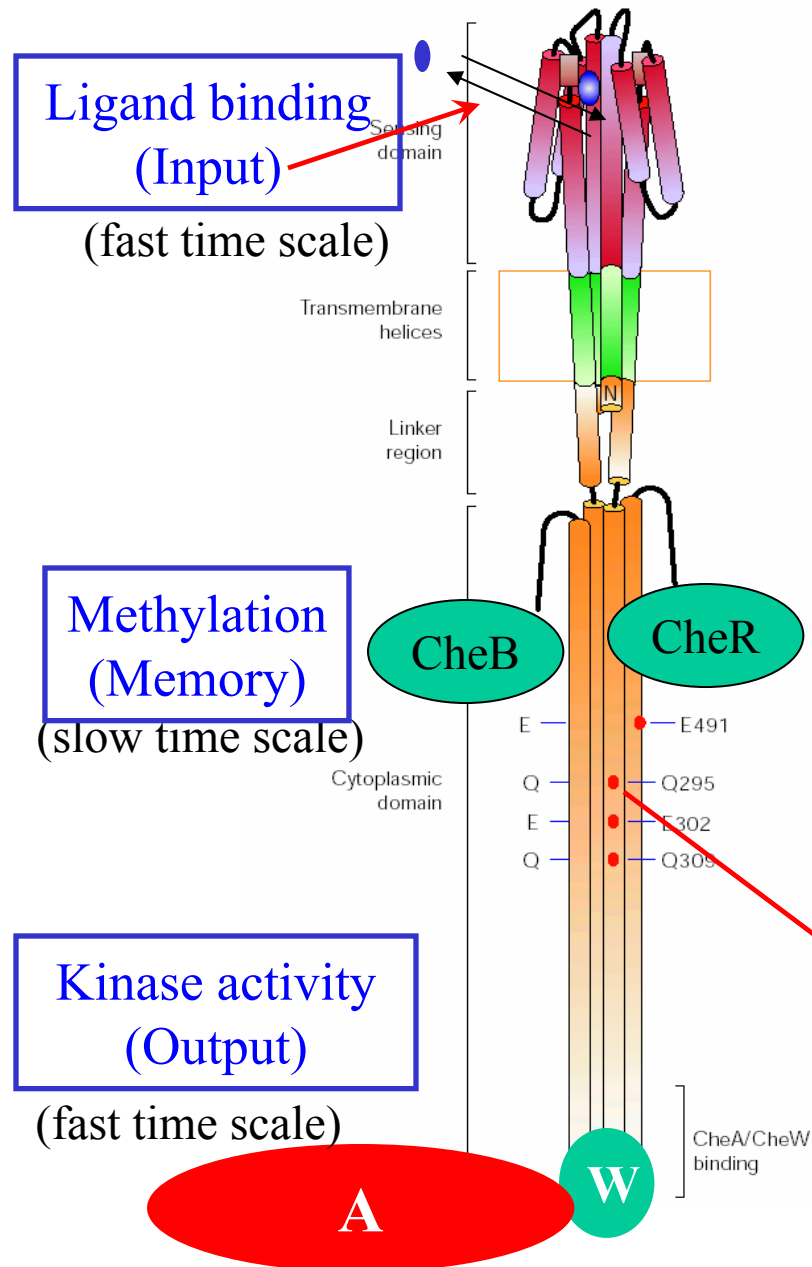
Simple step function stimulus is useful to understand the pathway.  
But, such simple stimuli is un-physiological.



**What type of signal processor is bacterial chemotaxis pathway?**

Amplifier; filter; nonlinear effects; signal integration/differentiation

# The dynamics of the receptor complex



# A coarse-grained dynamical model for chemotaxis

Quasi-equilibrium  $a = G(m, [L]) = [1 + \exp(-\Delta E(m, [L]))]^{-1}$

Slow methylation  $\frac{dm}{dt} = F(a, \cancel{m}, \cancel{[L]})$  **Perfect adaptation**

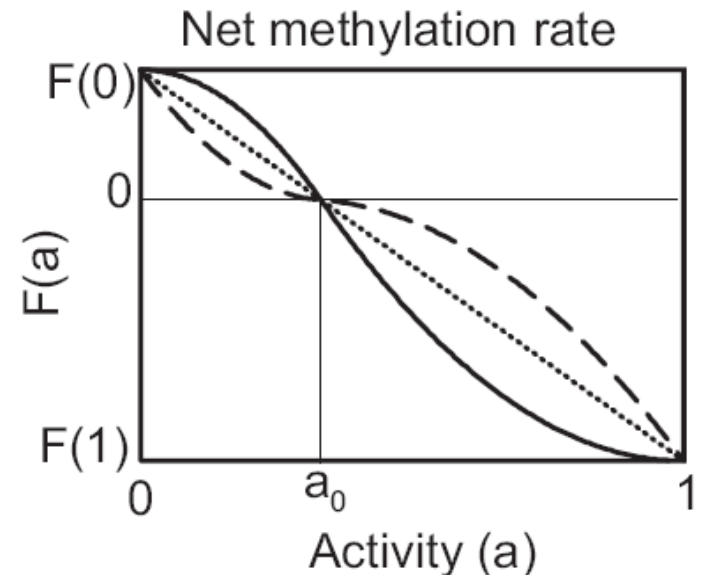
$\frac{dm}{dt} = 0 \Rightarrow F(a) = 0 \Rightarrow a = a_0$   
Independent of  $[L]$ : perfect adaptation

$$\Delta E = N \left[ \alpha(m - m_0) - \ln \frac{1 + [L]/K_d}{1 + C[L]/K_d} \right]$$

Number of receptor in the all-or-none cluster

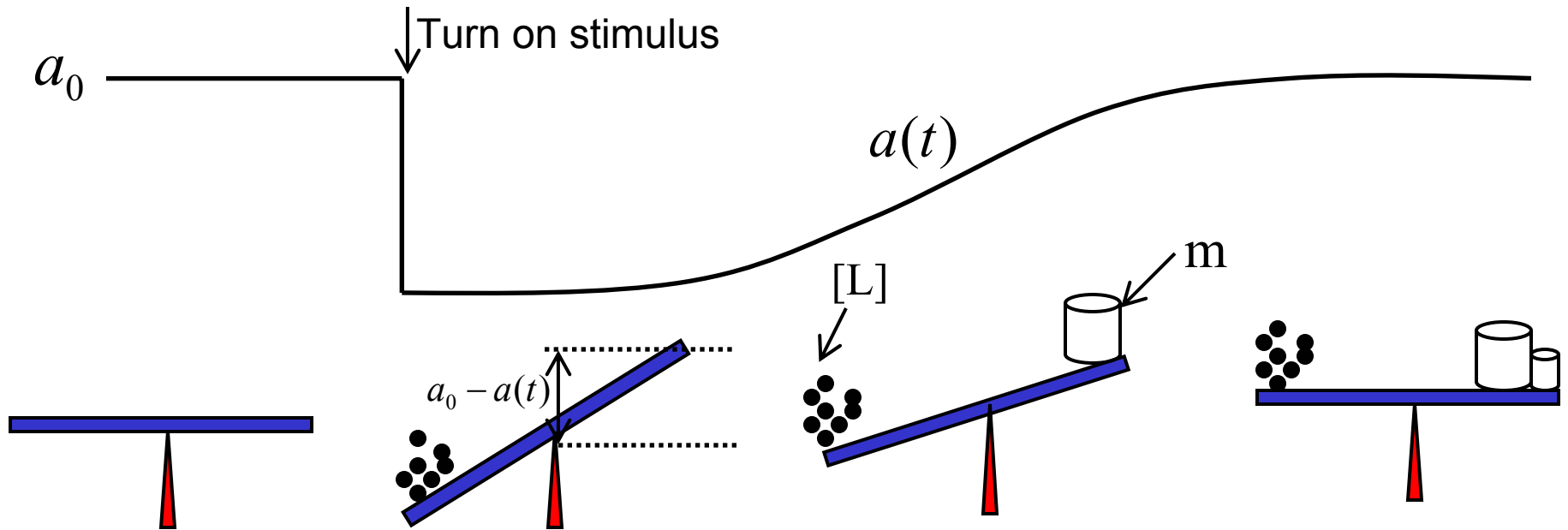
$$F(a_0) = 0, F(0) > 0, F(1) < 0$$

$$-F(1) > F(0)$$





# The operation of a measurement device



- Measure external object by balancing it with internal weights.
- Operate by feedback: add or subtract weights based on the imbalance.

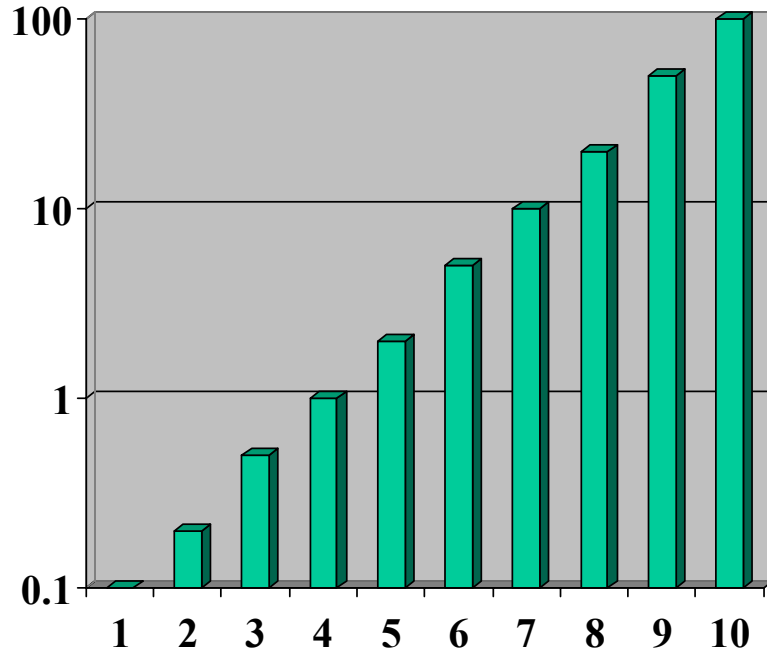
**Methylation level measure external ligand concentration in log-scale**

$$a = a_0 \Rightarrow m \approx \alpha^{-1} \ln[L] + \text{const.}$$

(“Logarithmic-sensing in E. coli chemotaxis”, Kalinin et al, Bio. J. 2009)

# The measuring the outside world in log-scale

Log-scale weight is an efficient way of representing a wide range of values by a small number of units: e.g., the Chinese currency units



Chinese  
Currency



# Some “forgotten” experiments and its recent incarnation: response to time varying signals

Experiments done in the 80's by Howard Berg's group

JOURNAL OF BACTERIOLOGY, Apr. 1983, p. 312-323  
0021-9193/83/040312-12\$02.00/0  
Copyright © 1983, American Society for Microbiology

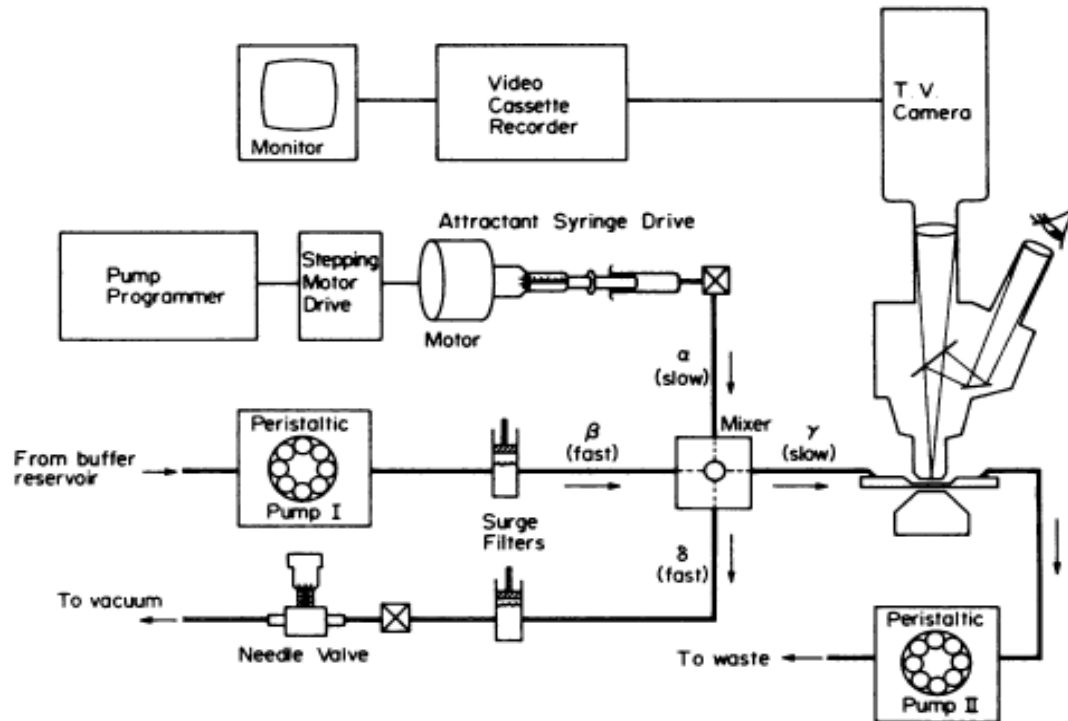
Vol. 154, No. 1

## Adaptation Kinetics in Bacterial Chemotaxis

STEVEN M. BLOCK, JEFFREY E. SEGALL, AND HOWARD C. BERG\*

*Division of Biology, California Institute of Technology, Pasadena, California 91125*

Received 18 October 1982/Accepted 21 January 1983



- Exponential ramp
- Exponentiated sine wave
- Steps and impulses

The response are now measured  
by using FRET now.  
(by T. Shimizu, H. Berg)

# Theory prediction: constant activity shift in response to exponential ramp

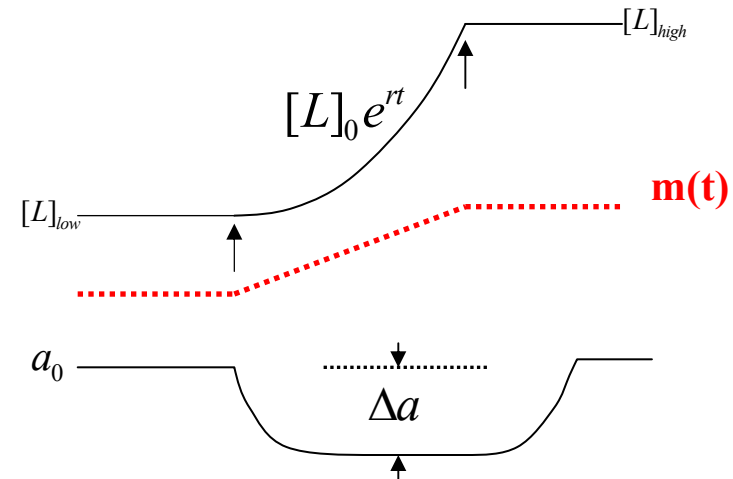
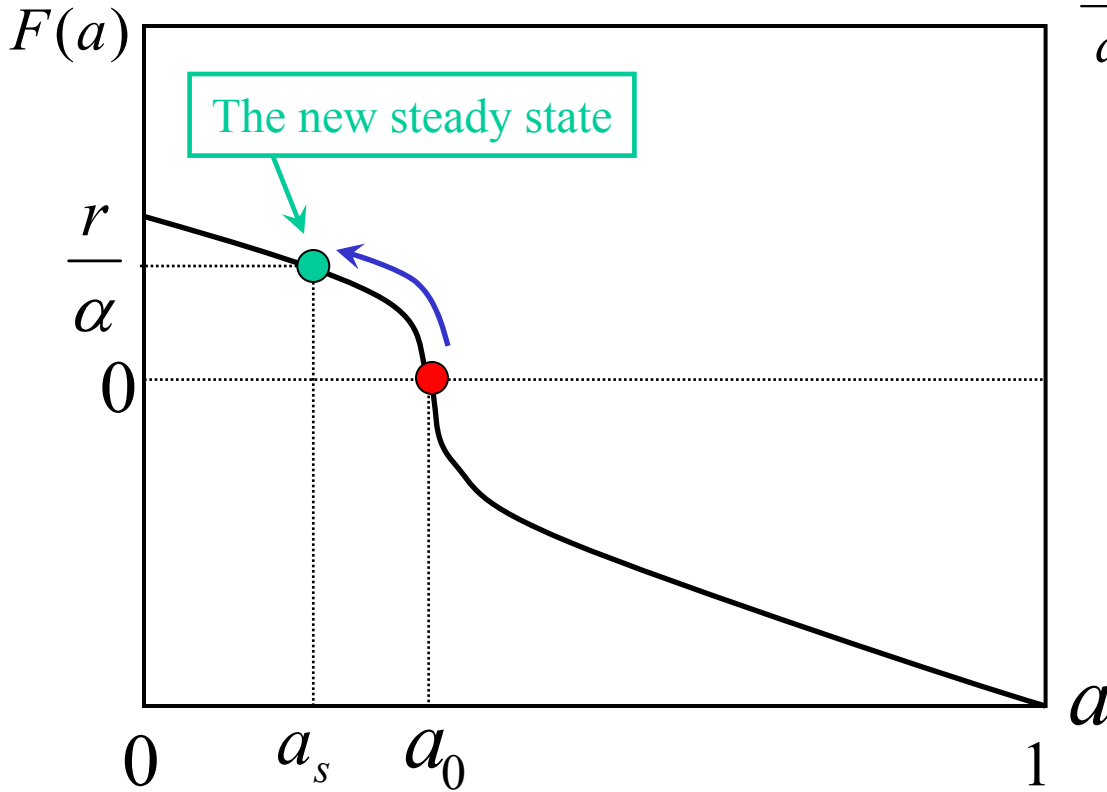
$$\Delta E \sim N[\alpha m - \ln([L]/K_d)] + const.$$

$$\ln[L] = rt$$

$$\frac{dm}{dt} = r / \alpha$$

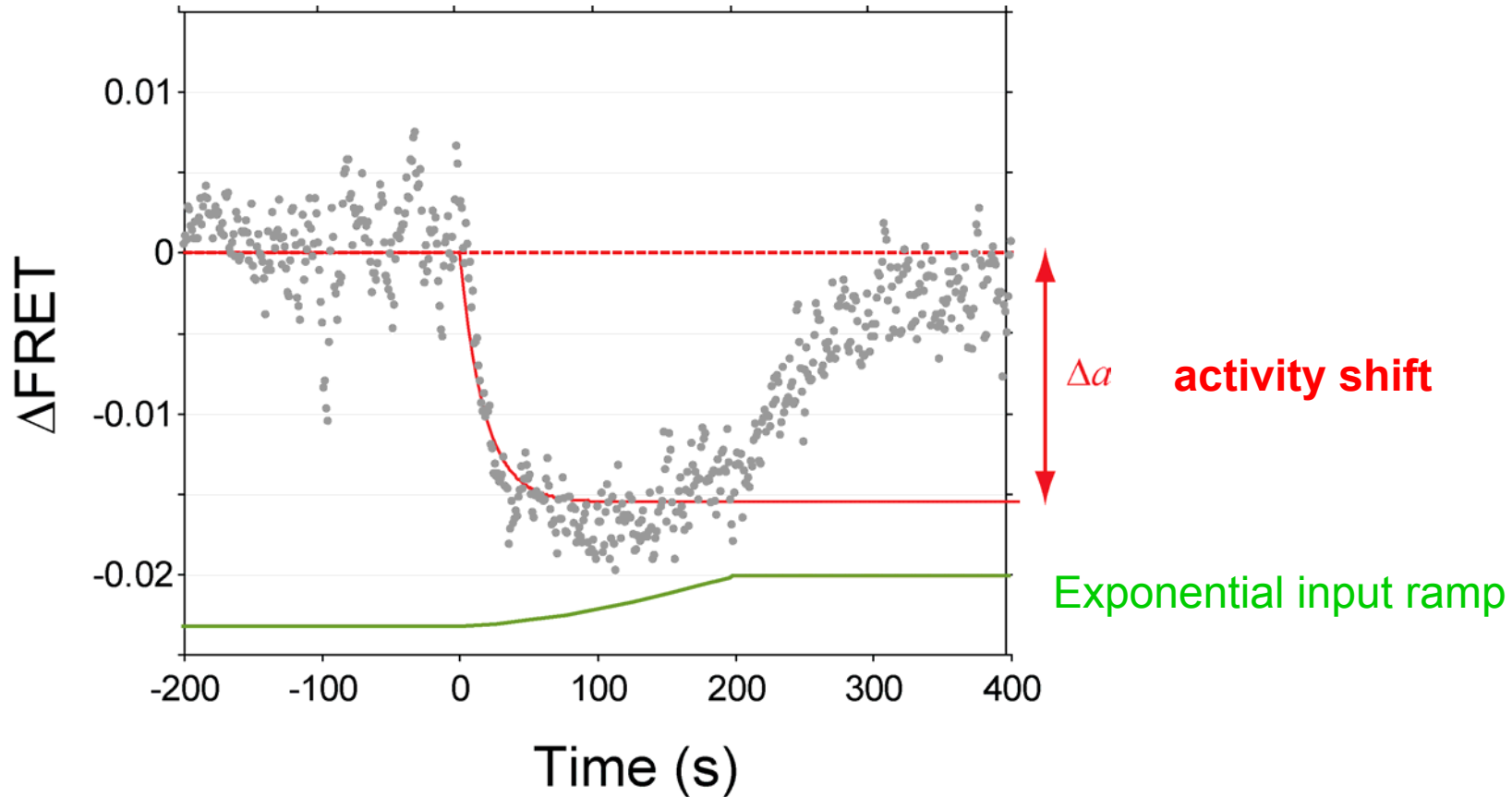
$$\frac{dm}{dt} = F(a)$$

$$F(a_s) = \frac{r}{\alpha}$$



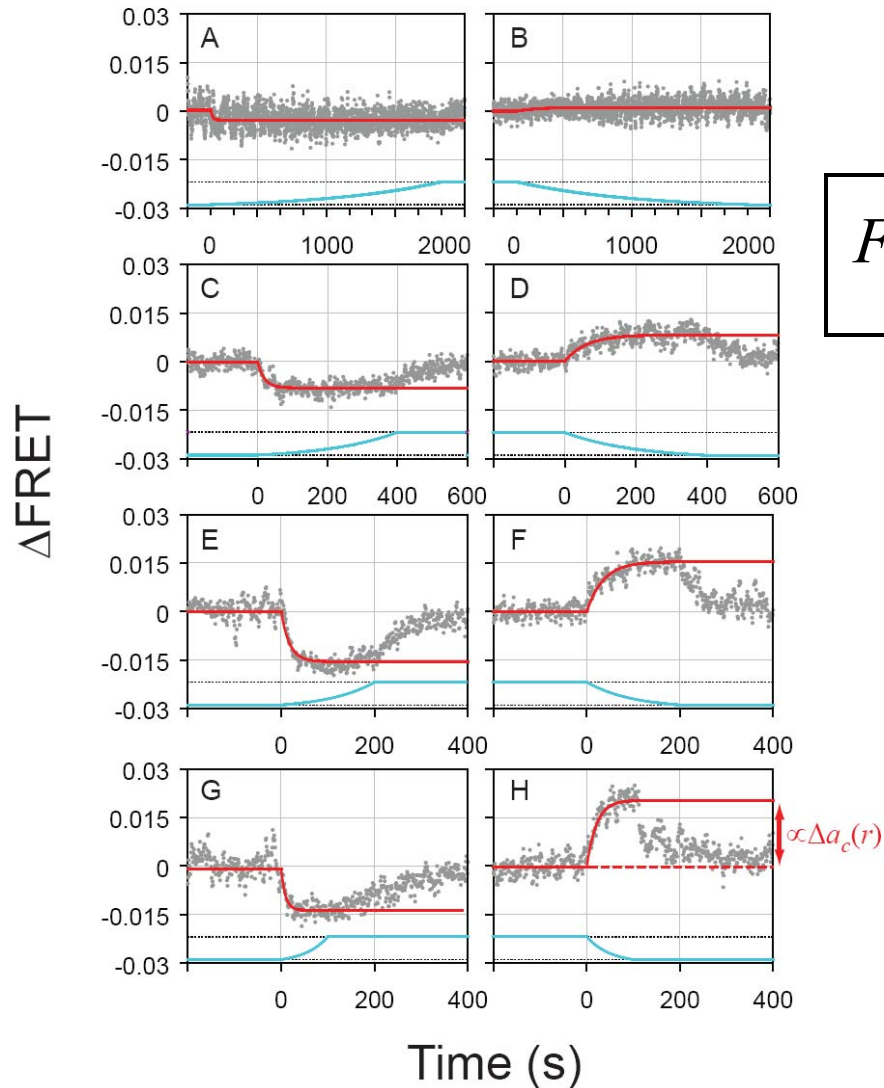
**Methylation tries to catch up with the exponentially changing external stimulus  
But it lag behind it, which leads to the activity shift**

# Response to exponential ramps: FRET experiments

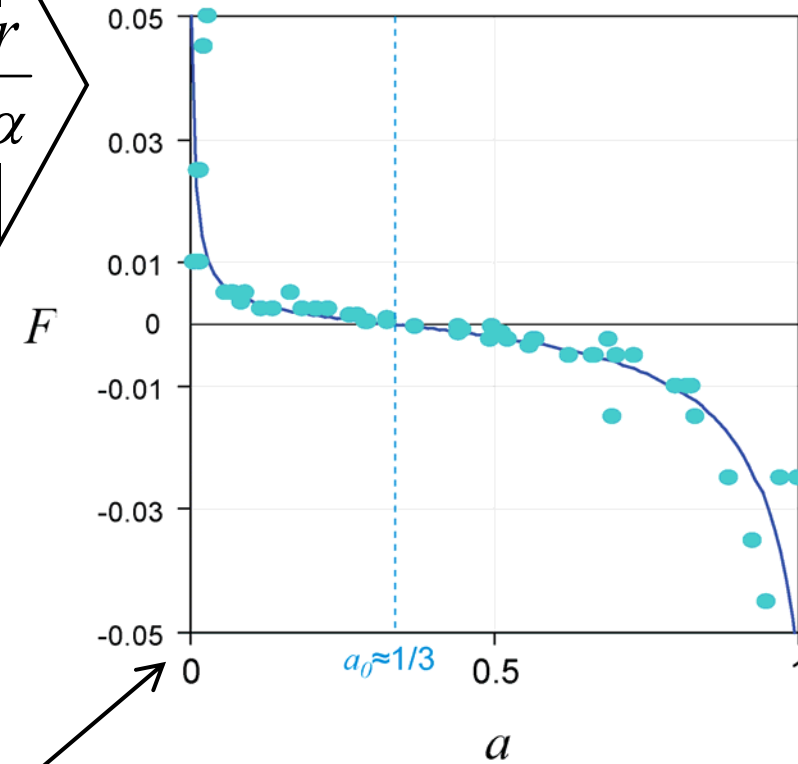


Constant activity shift in response to exponentially increasing signal

# The dependence of the activity shift on ramp rate



$$F(a_s) = \frac{r}{\alpha}$$



The methylation rate function  $F(a)$  revealed: soft control at  $a_0$



# Response to sine waves: the spectral analysis

— **Input:**  $[L](t) = [L]_0 e^{\beta \sin^2(\pi ft)}$

— **Memory (Control):**  
methylation  $m(t)$

— **Output:** kinase activity  $a(t)$

$$\Delta a = G'(f_t^0) \alpha \left[ \Delta a - \frac{A_L}{\alpha} \cos(2\pi ft) \right]$$

$$\frac{d\Delta m}{dt} = F'(a_0) \Delta a$$

$$A_a = \frac{ifc_a}{if + f_m} A_L, \quad A_m = \frac{f_m \alpha^{-1}}{if + f_m} A_L$$

**The chemotaxis  
transfer functions**

$$c_a = Na_0(a_0 - 1); \quad f_m = -(\alpha F'(a_0) Na_0(1 - a_0)) / 2\pi$$

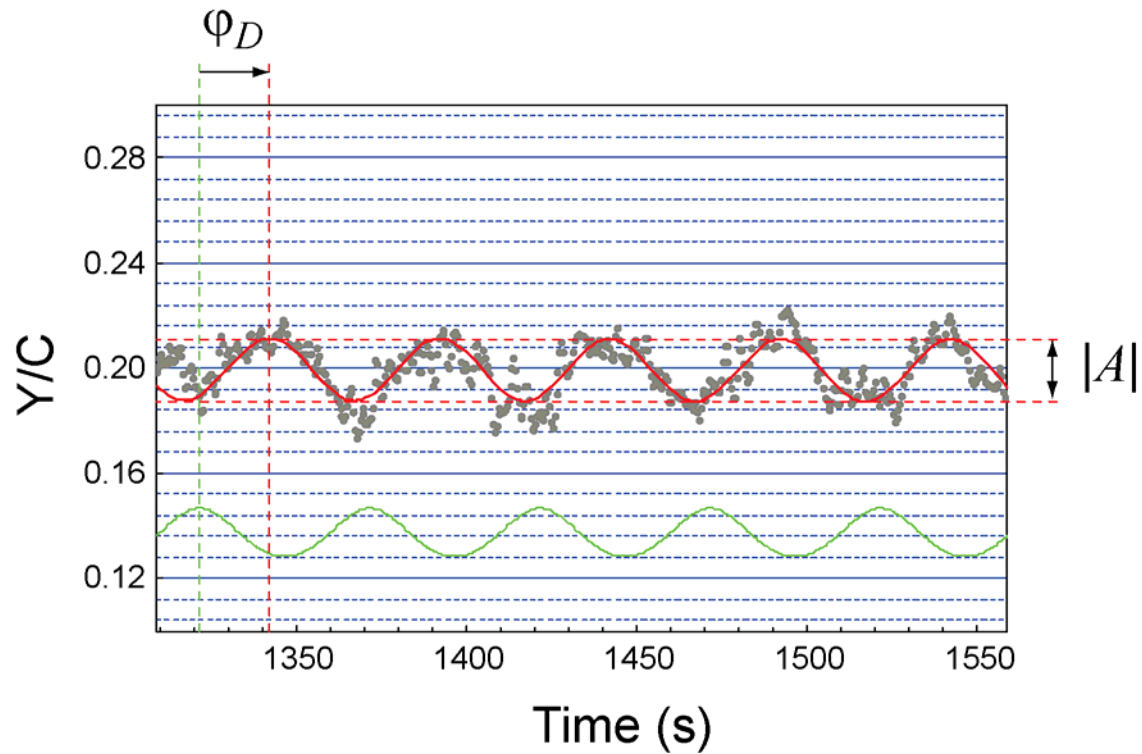
Low frequency:  $f \ll f_m$

$$A_a \sim ifcA_L, \quad A_m \sim A_L$$

High frequency:  $f \gg f_m$

$$A_a \sim A_L, \quad A_m \sim A_L / if$$

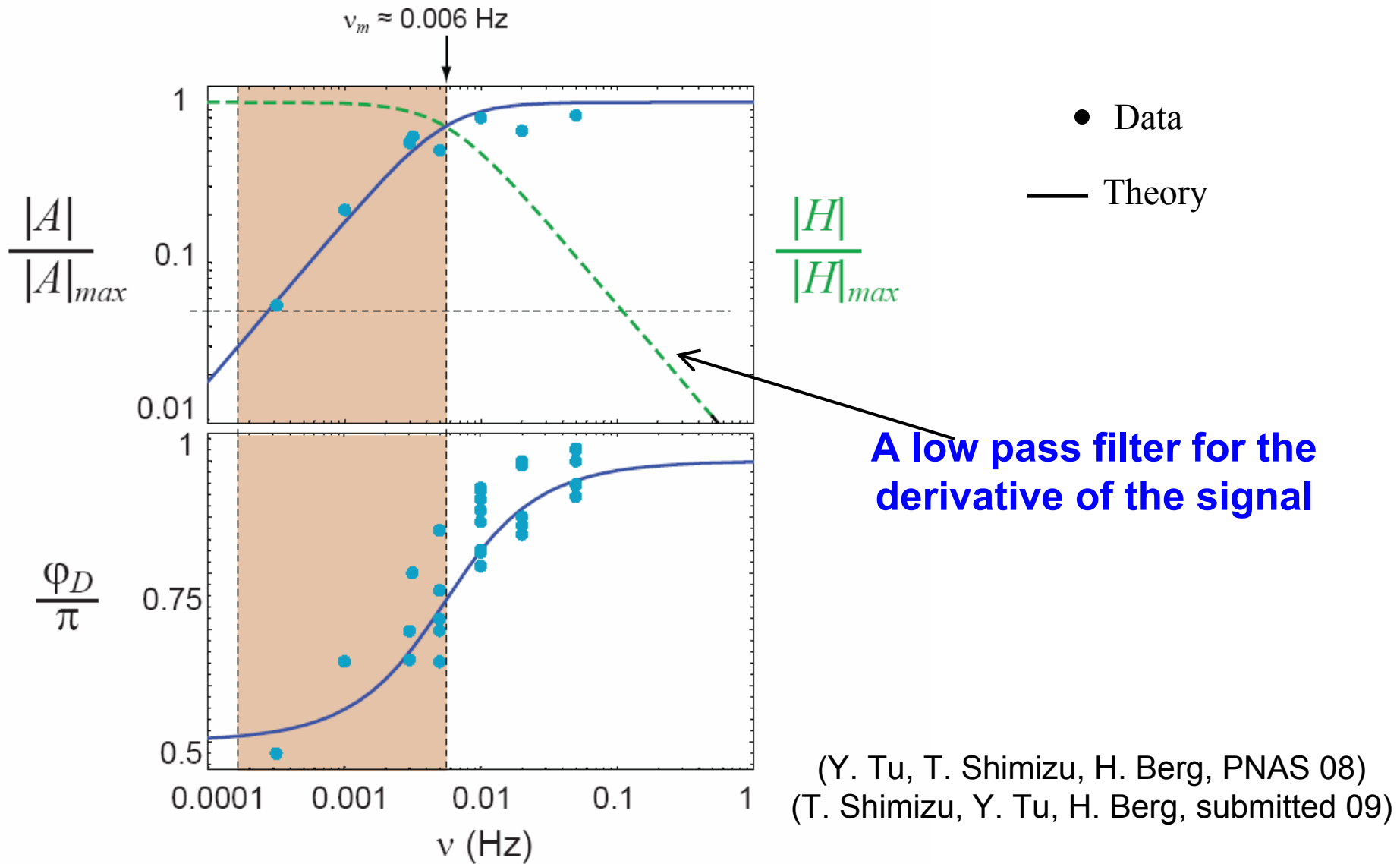
# The phases and amplitudes of the responses and their dependences on frequency



$$a(t) = |A| \cos(2\pi\nu t - \varphi_D)$$

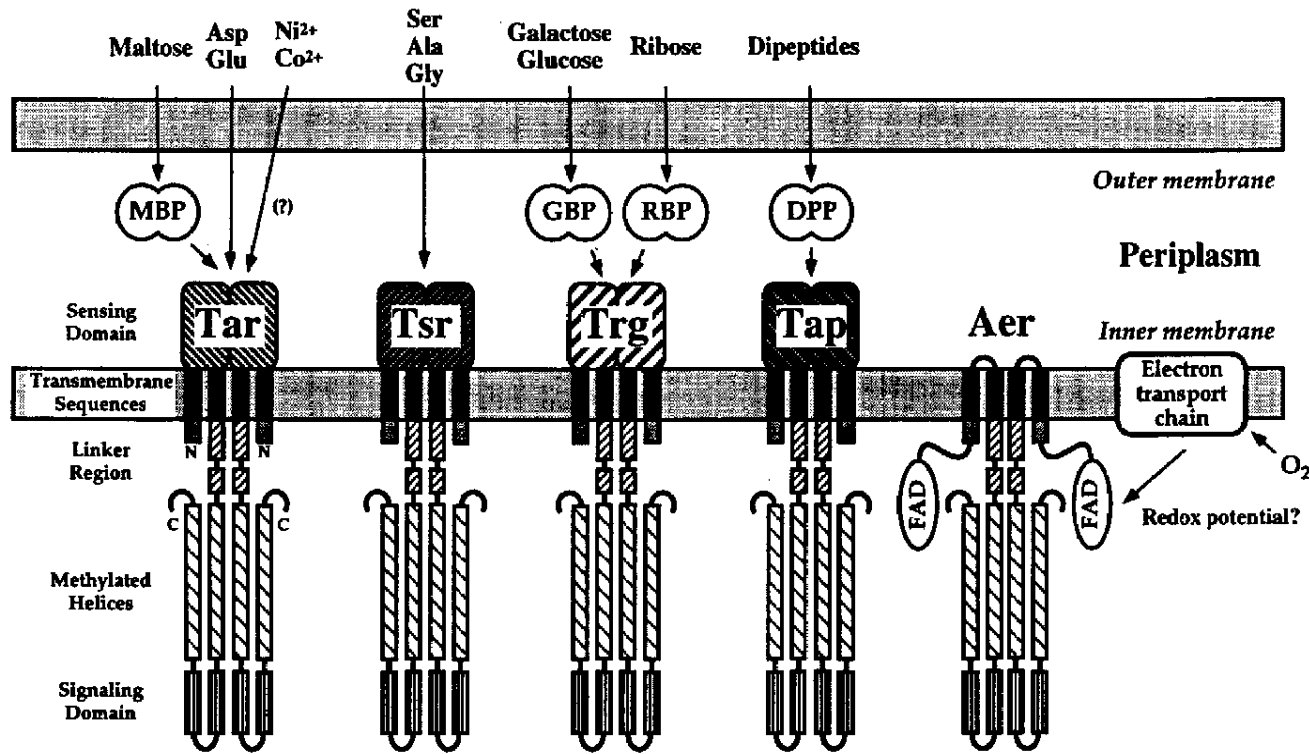
$$[L](t) = [L]_0 \exp[A_L \sin 2\pi\nu t]$$

# Theoretical predictions and experiments



# **Signal differentiation: Adaptation and response to mixed signals**

# There five different types of chemo-receptors forming mixed receptor clusters



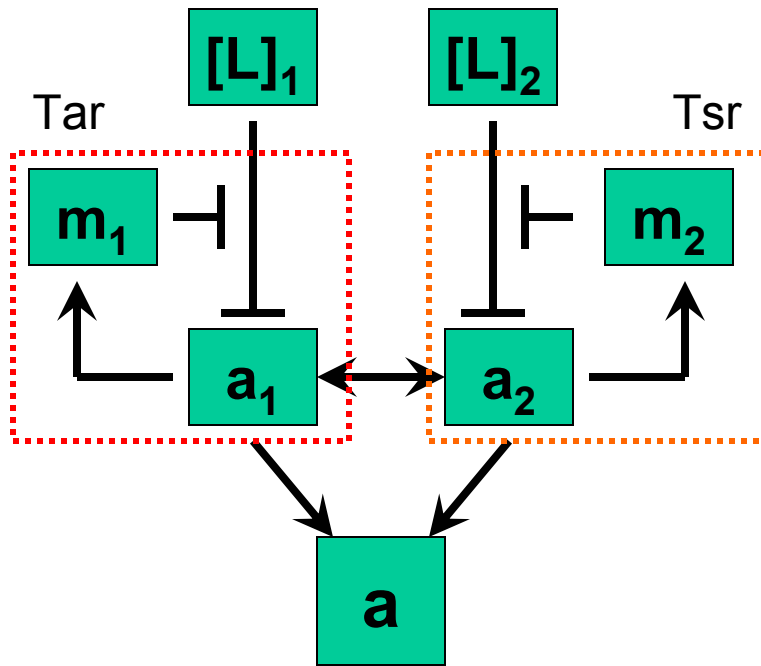
(5 types of chemoreceptor, each sensing different signals)

Total number of Receptors: 15,000-26,000. **Tsr:Tar:Trg(Tap,Aer)~2:1:0.1**

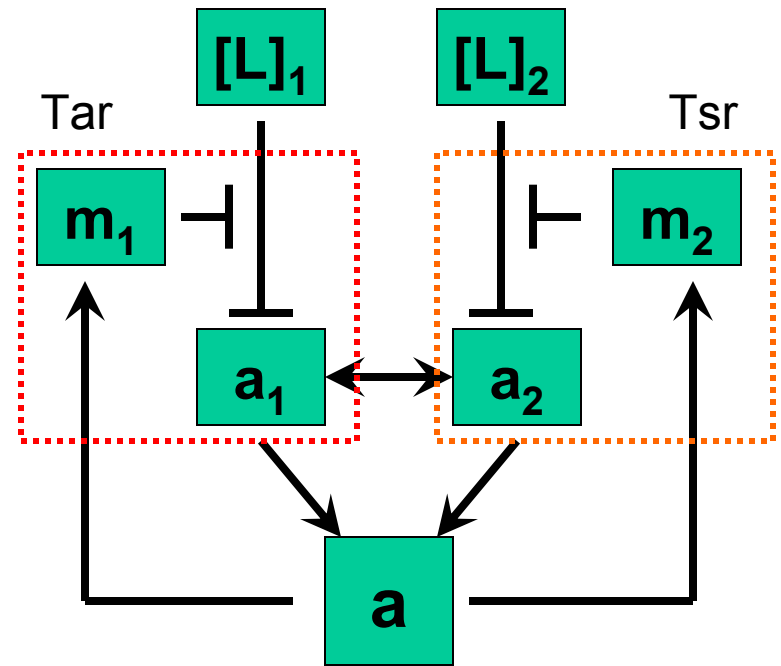
**Can the cell tell different signals apart? How? and Why?**

# The local versus global methylation dynamics

The local adaptation model



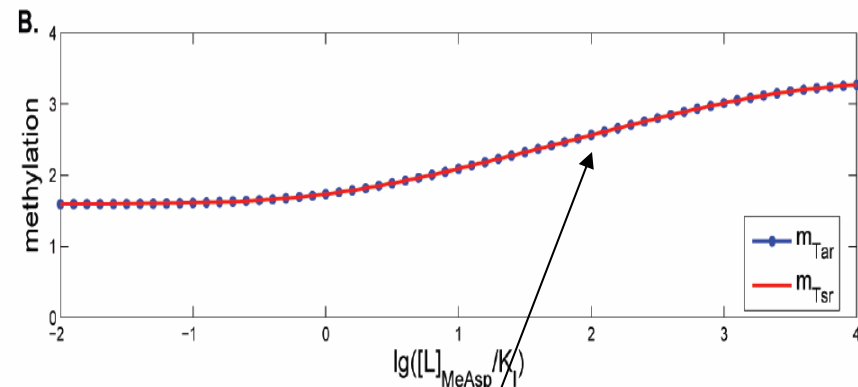
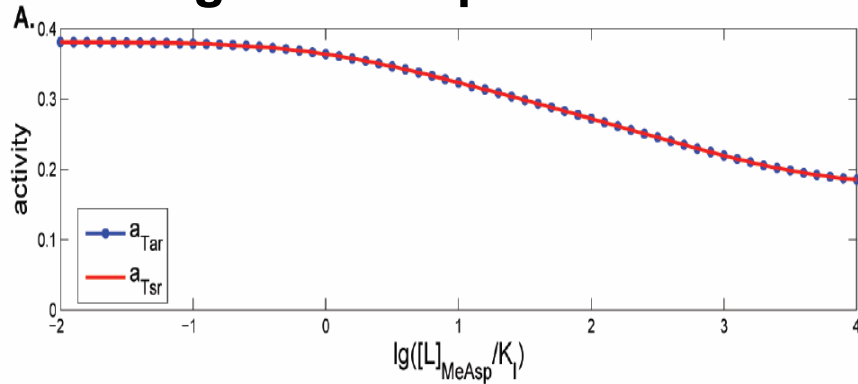
The global adaptation model



1—Tar/Asp; 2—Tsr/Serine

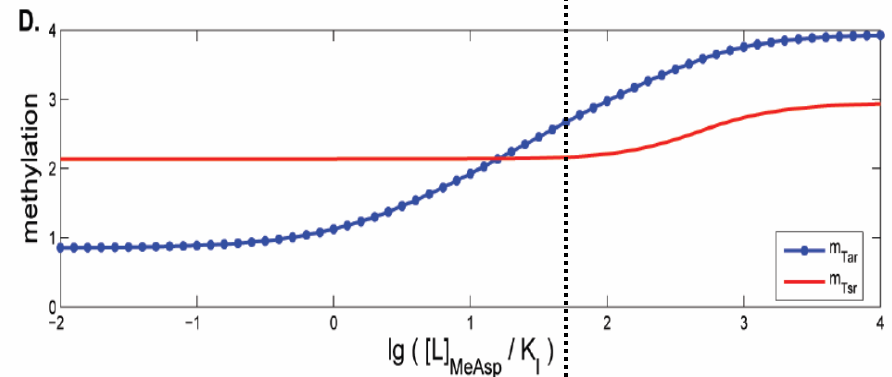
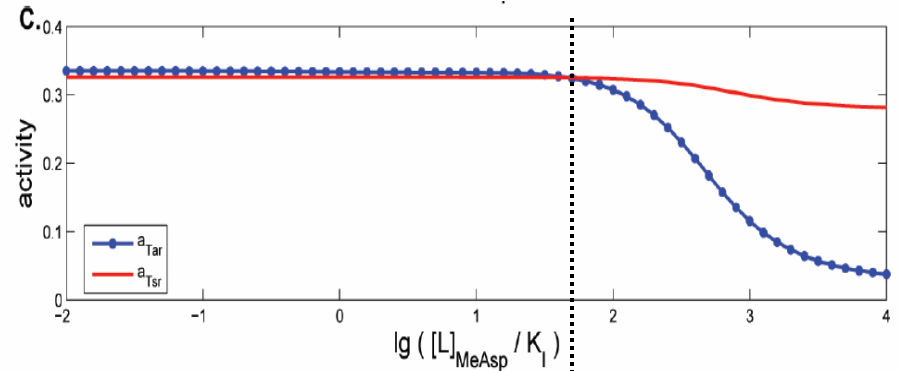
# The properties of the adapted (steady) state

## The global adaptation model



Methylation level of both Tar and Tsr respond (equally) to either MeAsp or Serine—always crosstalk.

## The local adaptation model



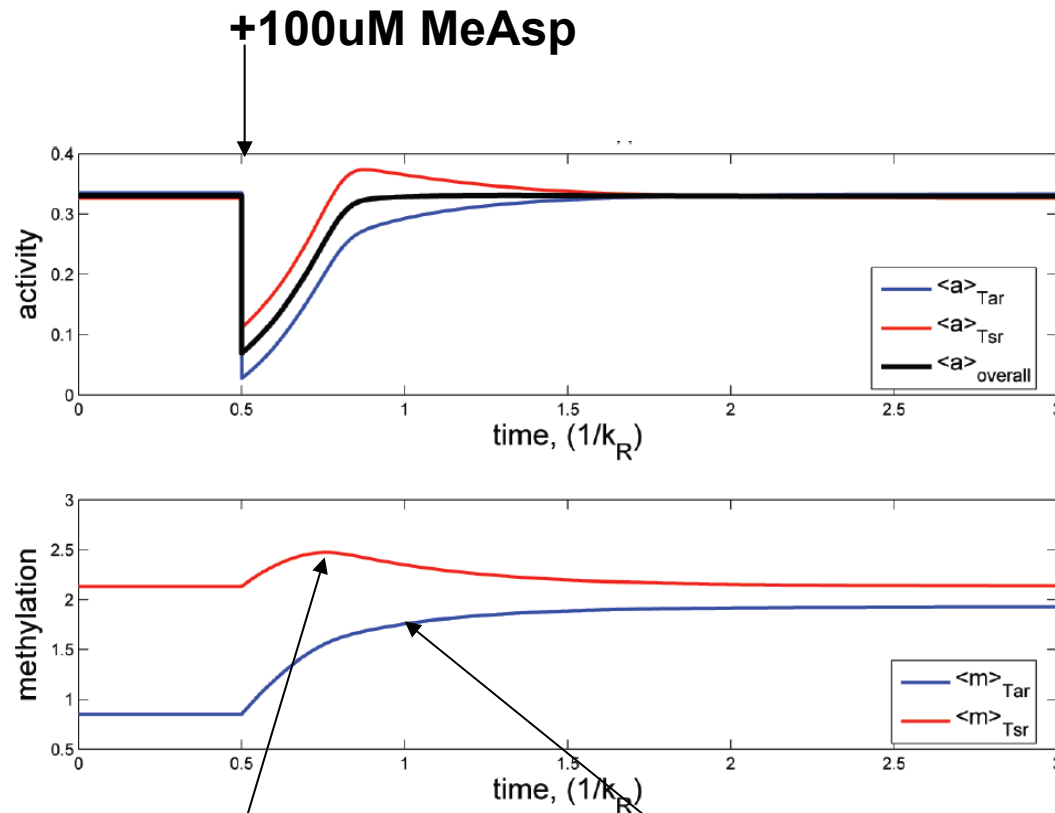
Perfect adaptation  
No methyl-crosstalk

Inaccurate adap.  
Methyl-crosstalk



- Only methylation level of Tar changes in response to MeAsp when adaptation is accurate, no crosstalk.
- Methylation crosstalk occurs only when accurate adaptation fails.

# The adaptation kinetics in the local model



Methylation of the un-hit receptor Tsr increases initially before going back to its original value.

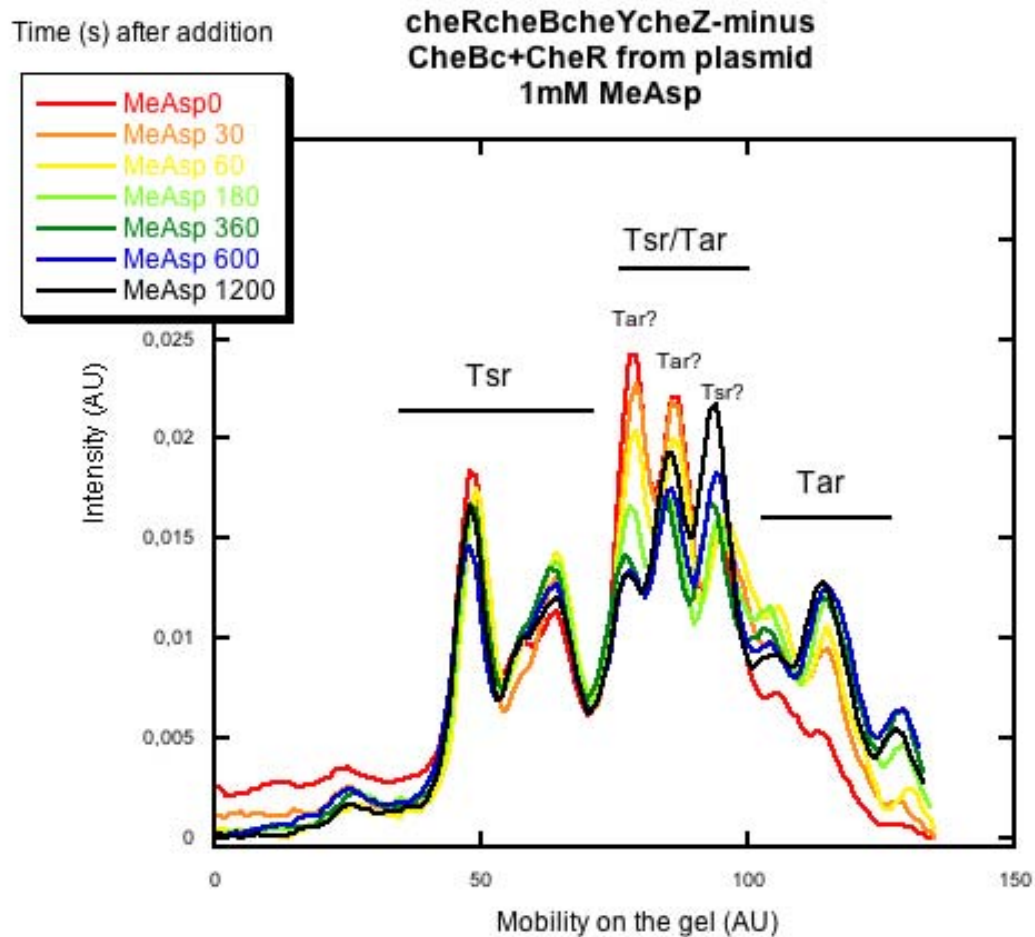
Methylation of the hit receptor Tar increases monotonically to its final value.



# Some experimental evidence

Time series of receptor methylation states after addition of stimuli  
(From Sourjik lab)

+1mM MeAsp

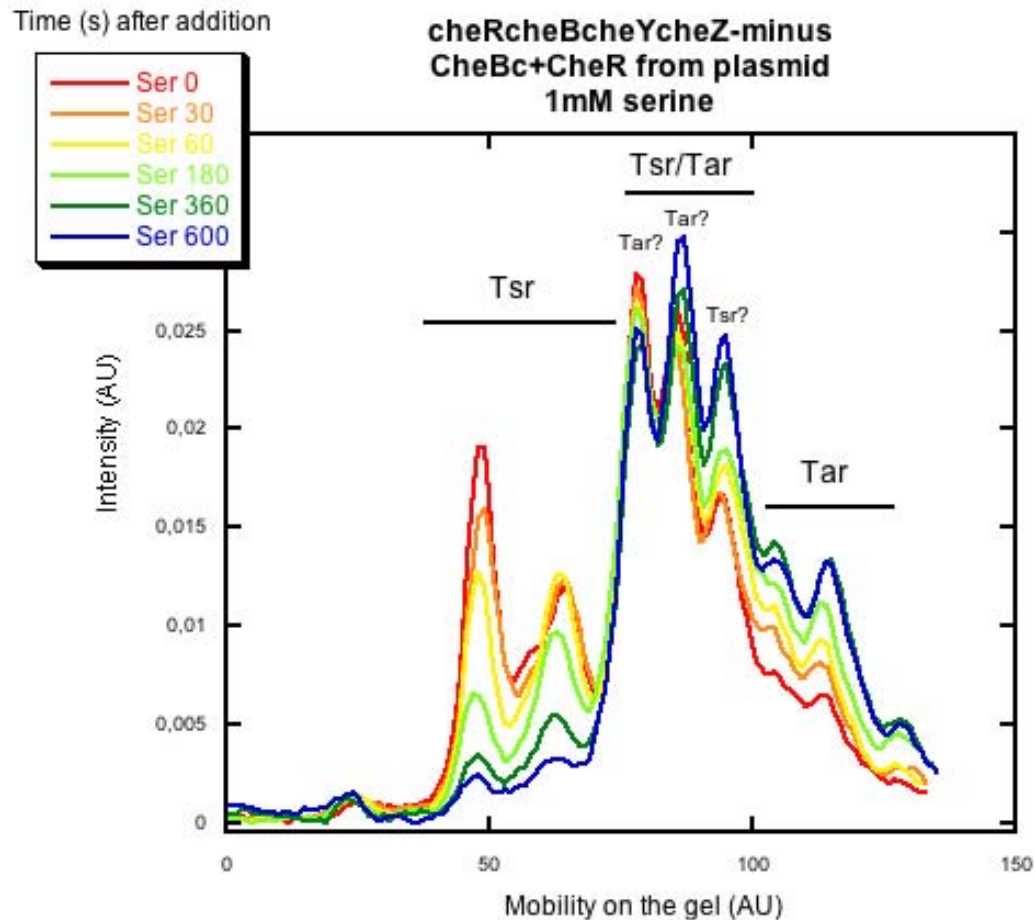


No methylation cross talk by adding MeAsp

# More experimental evidence

Time series of receptor methylation states after addition of stimuli

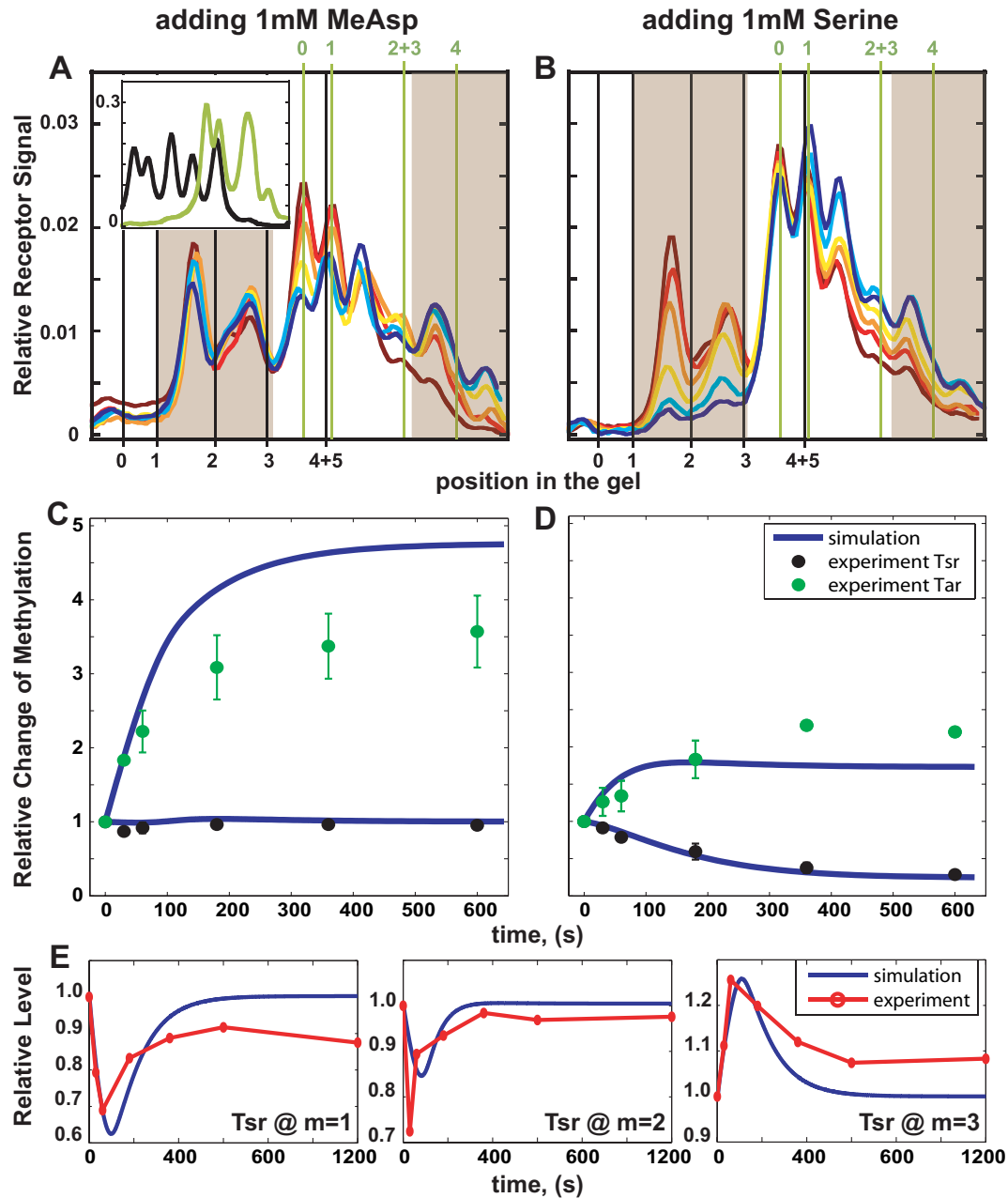
**+1mM serine**



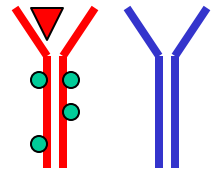
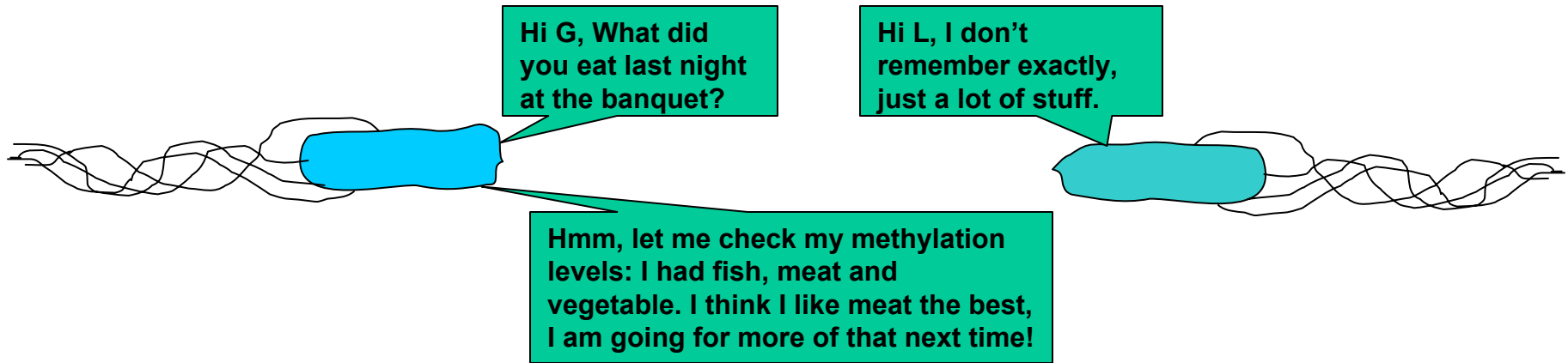
**Tsr responds primarily (strongly).**

**Tar also responds, because the cell does not adapt perfectly to Serine.**

# Comparison with the local adaptation theory (model)



# The advantage of having a clear memory



0 ● 0

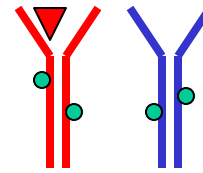
Local field in Ising model

Average Activity  $\langle a \rangle$

$$\frac{1}{2} (\because h = 0)$$

Sensitivity (susceptibility)  $d\langle a \rangle / dh$

$$\frac{1}{4}$$



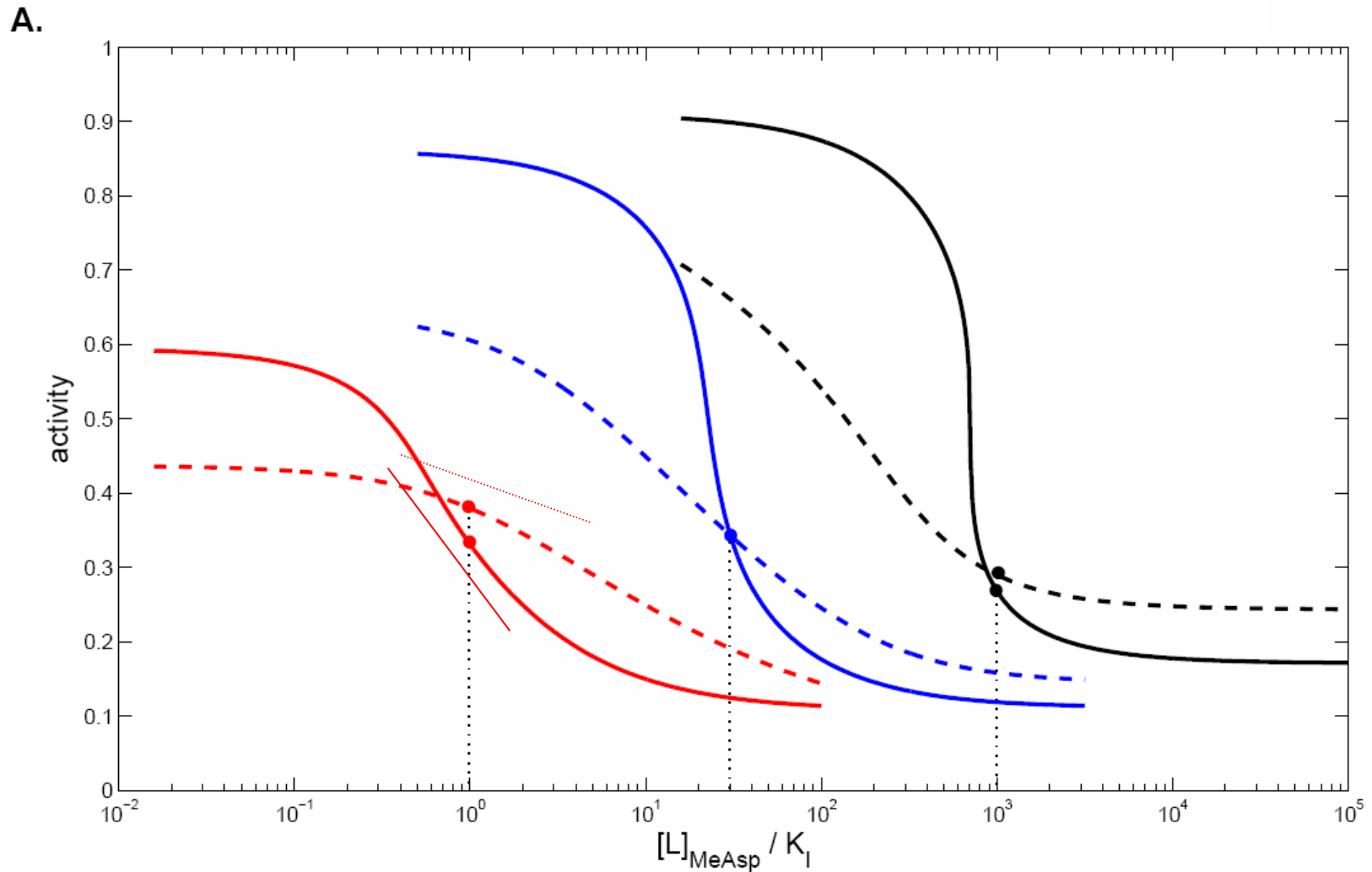
$h \uparrow \downarrow -h$

$$\frac{1}{2} \left[ \frac{e^{-h}}{e^h + e^{-h}} + \frac{e^h}{e^h + e^{-h}} \right] = \frac{1}{2}$$

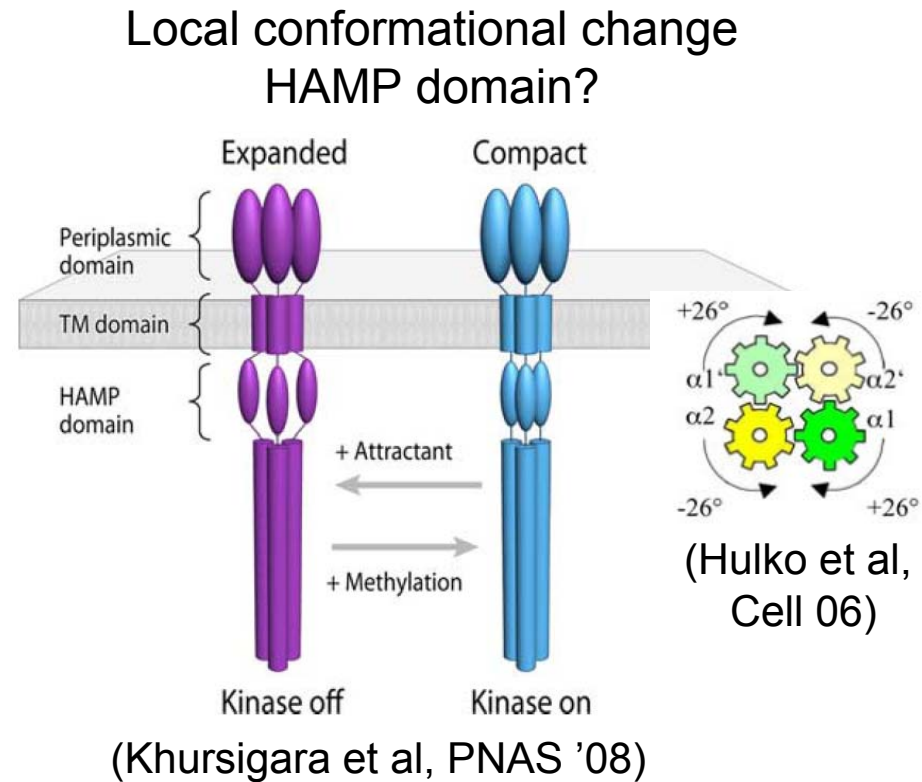
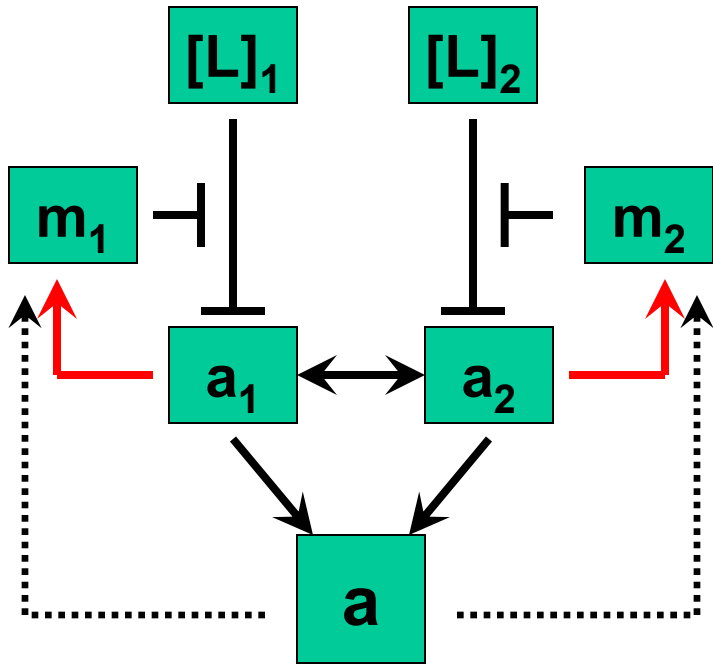
$$(e^h + e^{-h})^{-2} \leq \frac{1}{4}$$

# Heightened sensitivity with local memory

Solid lines --- Local model; Dashed lines – Global model  
(3 different background MeAsp concentrations)



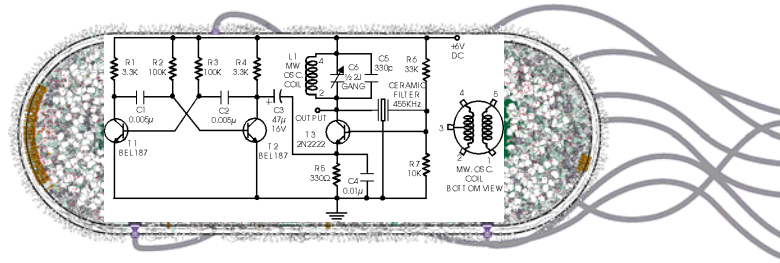
# Local conformational change of individual receptor controls its methylation dynamics



**Ising model**, where each receptor is assigned a local order parameter, is better suited to describe the methylation dynamics for mixed signals, than the all-or-none MWC model.

# Summary

# Chemotaxis pathway as a information processor



1) It amplifies the signal in a wide range of background receptor-receptor interaction in receptor cluster near perfect adaptation

2) It senses the concentration in log-scale

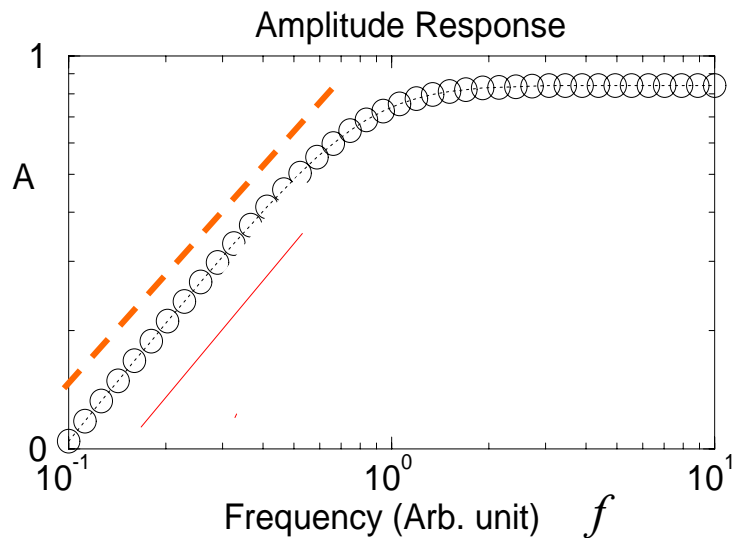
➤ Responses depend on  $\Delta[L]/[L] = \Delta(\ln[L])$   
The Weber-Fechner Law in sensory system

➤ Information compression: wide range of concentration, limited scale of methylation levels.



### 3) It is a low pass filter for the derivative of the signal

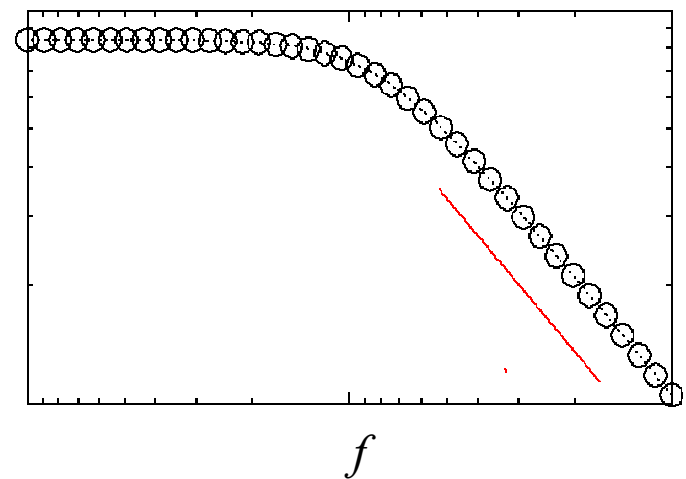
Compute **derivative of the input** in low frequency regime



$A(f) / f$

Divided by  
 $f$

Detailed description: A large white arrow with a black outline points from the left plot to the right plot, indicating the operation of dividing the amplitude response by frequency.



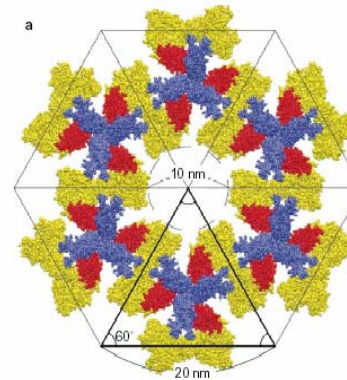
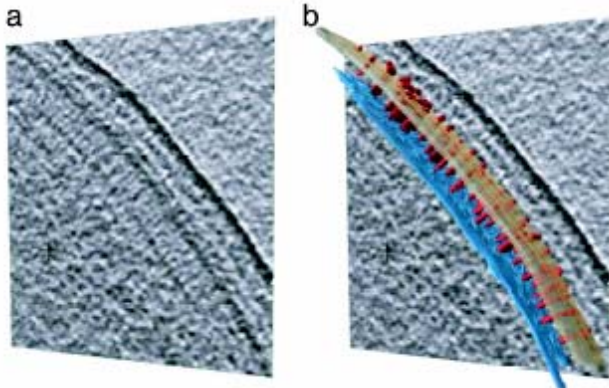
4) It records information on different ligand by the methylation levels of the corresponding receptors

**Local memory; Global action**

**Some remaining challenges**

# At the molecular level

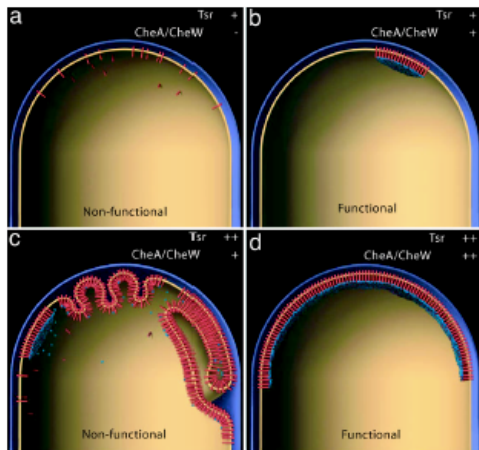
1) What's the structure of the cluster? How do they form?  
What affects the formation of the functional complex? What's the role of Cell membrane? Role of CheW and CheA?



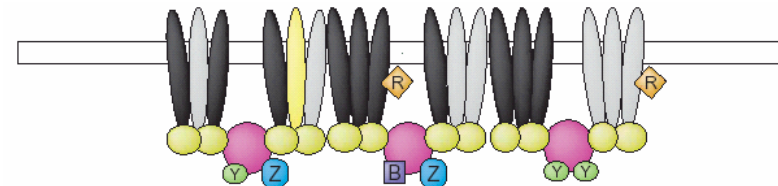
Ordered?

(shimitzu et al, 2000)

Disordered?



(S. Subramaniam Lab)



2) What is the molecular basis for the detailed methylation/demethylation dynamics? How is perfect adaptation achieved?

# At the systems level

- **How does the system differentiate different signal?**

**How the cell distinguish between different signal?**

**How smart is the bacteria?**

**Can the system be “rewired” (changing “coupling”) due to learning (exposure to some stimulus)**

- **Can the same pathway be used to perform other task?**

**e.g. Thermotaxis---going to a particular temperature**

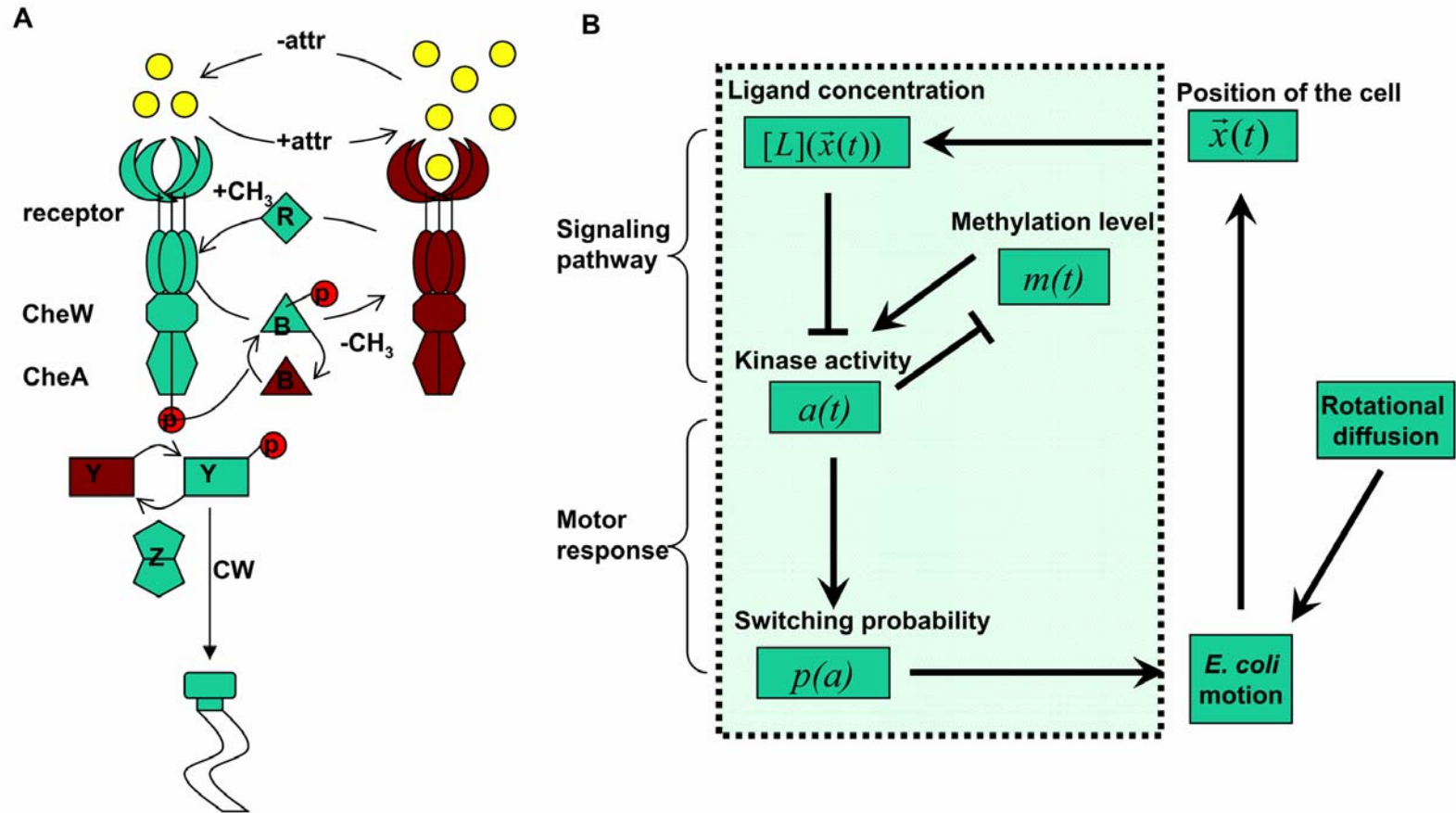
- **Why such a large gain? What about noise?**

**What about signal gain in response to real stimulus encountered in the wild , e.g., as the bacterium (biased) random walking towards a nutrient source.**

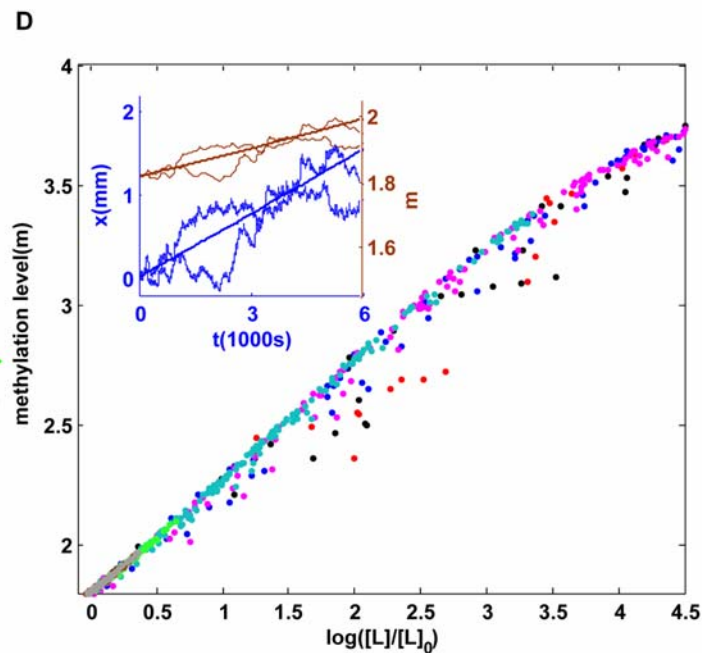
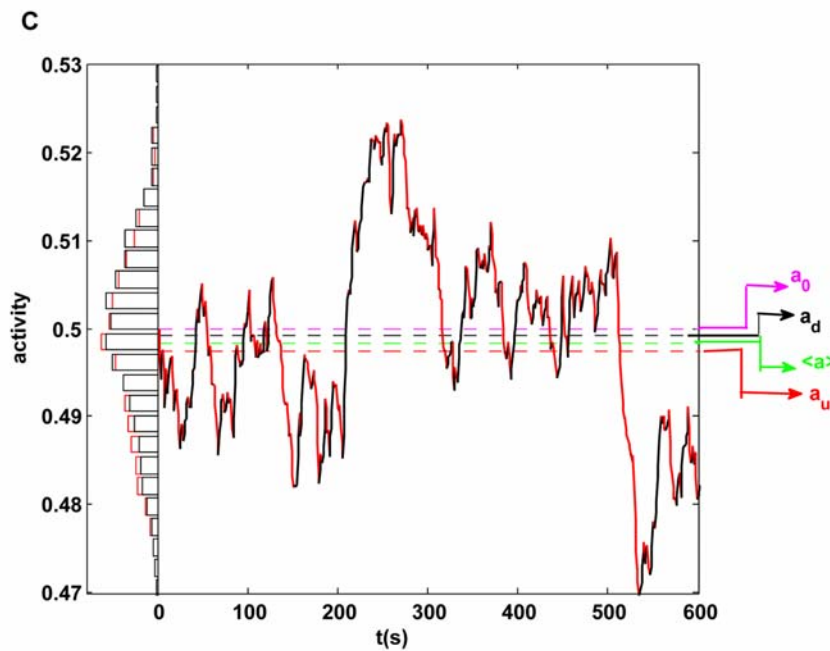
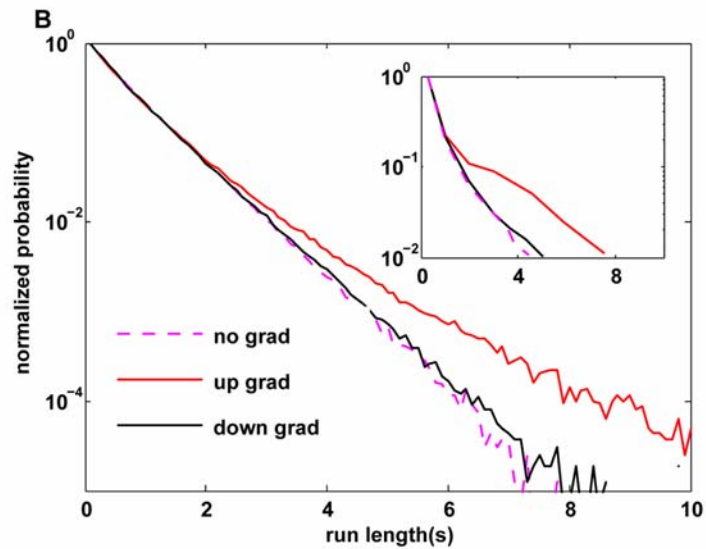
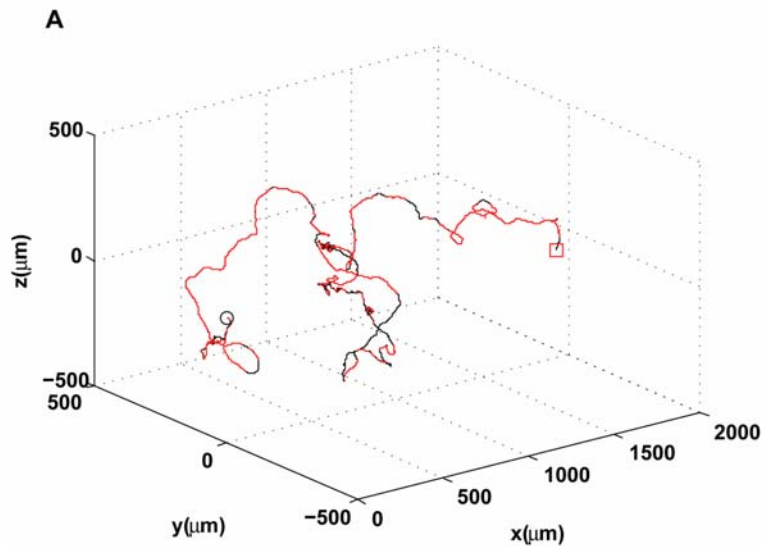
**Thank You**

**From molecular pathway to  
understand behavior (cell motion)**

# From molecules to behaviors: A *E. coli* chemotaxis model based on intracellular signaling pathway dynamics



# The single cell behavior



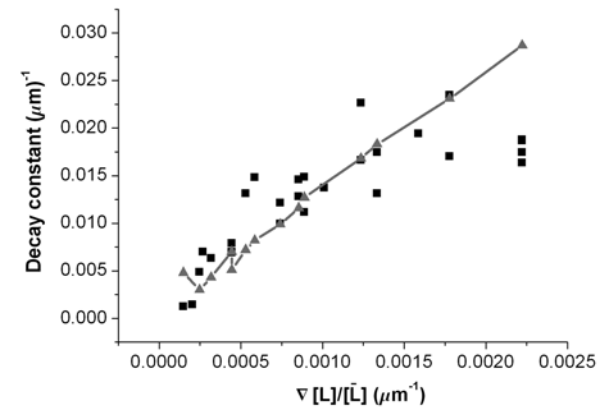
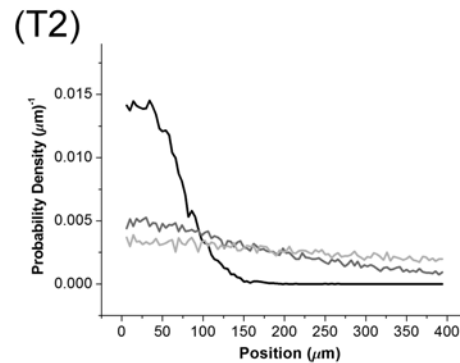
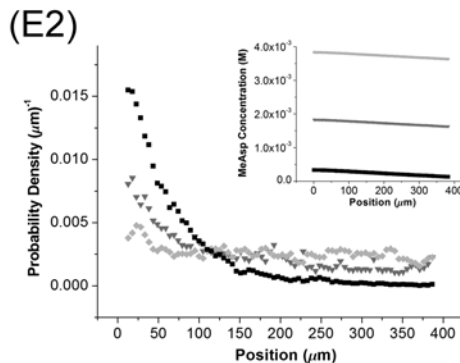
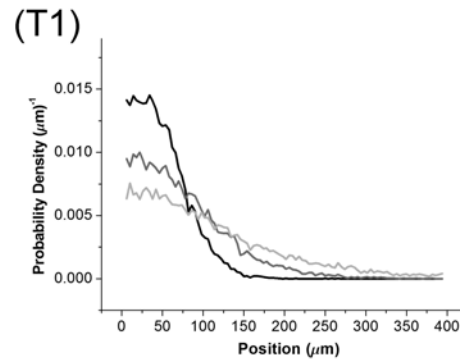
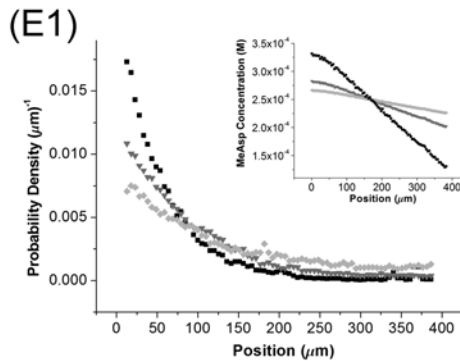
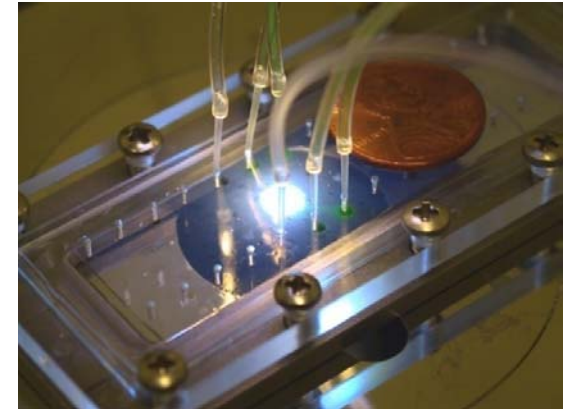
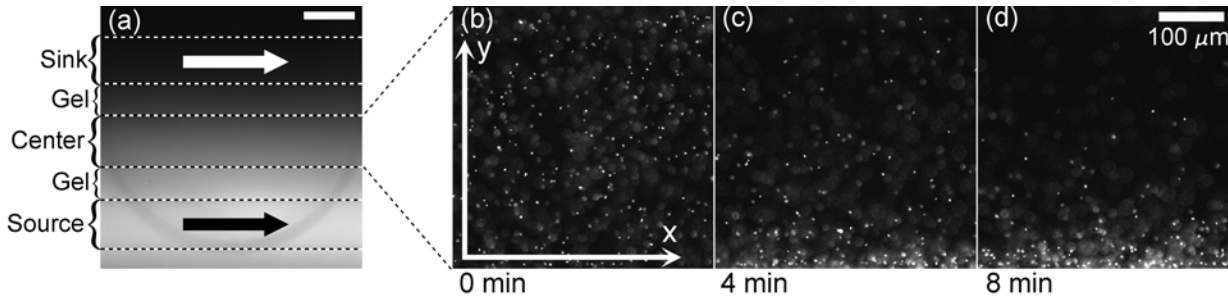


# Comparison with microfluidics experiments

What happens to a cell when it is moving in a spatial profile (gradient, traps, etc.)

Direct comparison with quantitative microfluidics experiments

(Done with M. Wu lab in Cornell)



# Comparison with the classical capillary experiments

