Effective Field Theory and Ultracold Atoms

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Effective Field Theory and Ultracold Atoms

- What is Effective Field Theory?
- Effective Field Theory for QED
- Ultracold Atoms
- Effective Field Theory for Ultracold Atoms

Effective Field Theory

in High Energy Physics,
EFT is the universal framework for

- low-energy approximations
  to the Standard Model of particle physics
  (see Mike Luke)

- model-independent analyses
  of new (higher-energy) physics
  beyond the Standard Model
  (see Bob Holdom)

EFT is also proving to be a powerful method
in Cold Atom Physics

Effective Field Theory

general setup

- low-energy degrees of freedom
- higher-energy degrees of freedom

basic principles

- can describe low-energy behavior
  with arbitrarily high accuracy
  using only low-energy d.o.f.

- effects of higher-energy d.o.f.
  on low-energy d.o.f.
  is smooth (i.e. analytic)

Michelson-Morely experiment: 1887

implies that light travels at the same speed c
independent of velocity of emitter
or observer

suggests that Newton's laws
should be modified to take into account
this new constant of nature c

expect dramatic effects at high velocity ~ c
but small effects even at low velocity

Kinetic energy

Newton: $E = \frac{1}{2}\, m\, v^2$

modified: $E(v) = ?$

EFT approach

- rotational symmetry
  E is function of $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ only

- effects of high energy on low energy are smooth
  E should be analytic function of v_x, v_y, v_z
  must have expansion in powers of v^2

- dimensional analysis
  higher powers of v^2 must be compensated by $1/c^2$

modified kinetic energy:

$$E = \tfrac{1}{2}\, m\, v^2 + c_1\, m\, v^4/c^2 + c_2\, m\, v^6/c^4 + \ldots$$

numerical coefficients: $c_1, c_2, \ldots$

Kinetic energy at not-so-low velocity

modified kinetic energy:

$$E = \tfrac{1}{2}\, m\, v^2 + c_1\, m\, v^4/c^2 + c_2\, m\, v^6/c^4 + \ldots$$

numerical coefficients: $c_1, c_2, \ldots$

- systematically improvable approximation at low energy
- effects of high energies reduced to constants with hierarchy of importance
- model-independent framework for analyzing corrections

As we know, there are known knowns.
There are things we know we know.
We also know there are known unknowns.
That is to say,
we know there are some things we do not know.
But there are also unknown unknowns,
the ones we don't know we don't know.

Donald Rumsfeld
US Secretary of Defense
February 2002

Albert Einstein 1905

special theory of relativity

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

$$E = m\,c^2 + \tfrac{1}{2}\,m\,v^2 + 3/8\;m\,v^4/c^2 + 5/16\;m\,v^6/c^4 + \ldots$$

predictions

- numerical coefficients: $c_1 = 3/8$, $c_2 = 5/16$, ...
- rest mass! $m\,c^2$

Classical Field Theory

classical field $\varphi(x,t)$:
function of space that evolves with time

<u>classic example</u>
electric and magnetic fields: $E(x,y,z,t)$, $B(x,y,z,t)$
time evolution: Maxwell equations

<u>other examples</u>
density, magnetization, ...
mean field of Bose-Einstein condensate, ...

What is Quantum Field Theory?

field theory with fields that do not commute

commutation relations

$$\varphi(x,t)\,\varphi(x',t)^{\dagger} - \varphi(x',t)^{\dagger}\,\varphi(x,t) = i\,\hbar\,\delta(x-x')$$

$\varphi(x,t)^{\dagger}$ creates particle at point x
$\varphi(x,t)$ annihilates particle at point x
and the particles are identical bosons

anti-commutation relations

$$\varphi(x,t)\,\varphi(x',t)^{\dagger} + \varphi(x',t)^{\dagger}\,\varphi(x,t) = i\,\hbar\,\delta(x-x')$$

$\varphi(x,t)^{\dagger}$ creates particle at point x
$\varphi(x,t)$ annihilates particle at point x
and the particles are identical fermions

What is Quantum Field Theory?

Quantum Field Theory
describes quantum mechanics of point particles
automatically taking into account
behavior of identical bosons
and identical fermions

Local Quantum Field Theory
describes point particles with point interactions

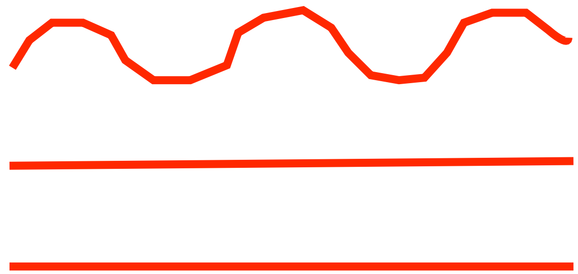
Quantum ElectroDynamics

degrees of freedom

photons

electrons

positrons

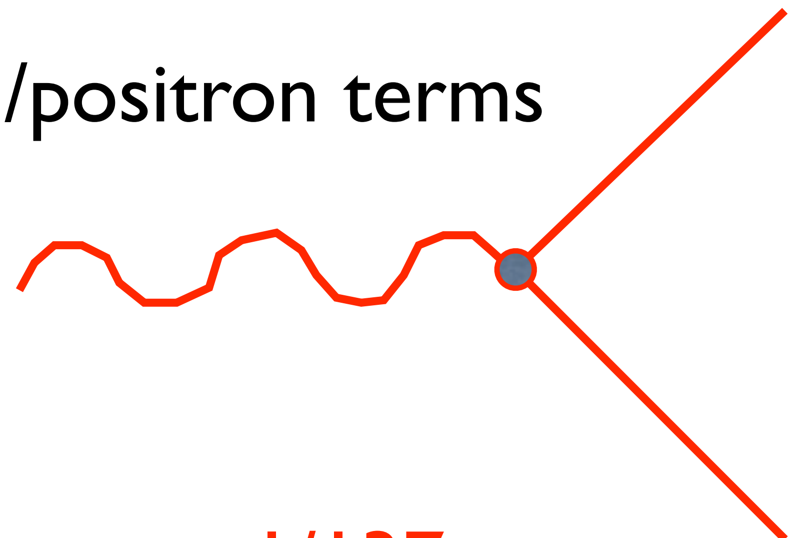


(each with 2 spin states)

<u>Lagrangian</u>

$L = \frac{1}{2}(E^2 - B^2)$ + electron/positron terms

point interaction:



one interaction parameter: $\alpha \cong 1/137$

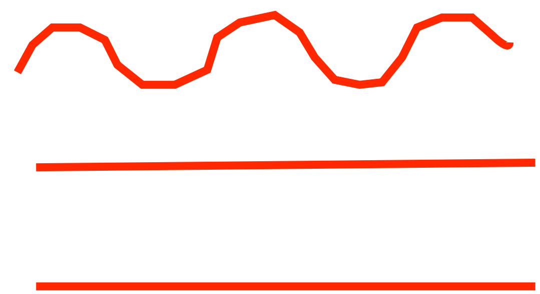
Quantum ChromoDynamics

<u>degrees of freedom</u>

quarks (2 spins, 3 colors, 6 flavors)

antiquarks (2 spins, 3 colors, 6 flavors)

gluons (2 spins, 8 colors)



point interactions

one interaction parameter: α_s

"runs" with momentum scale: $\alpha_s \cong 1/8$ at 100 GeV

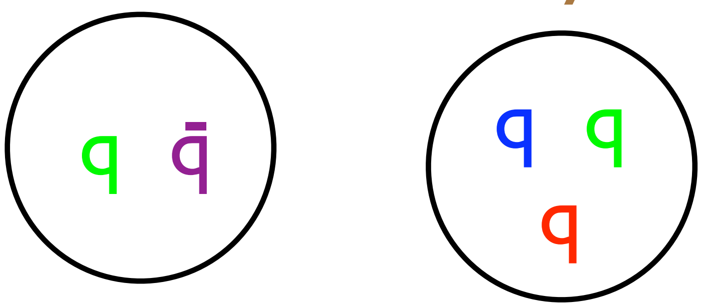
Quantum ChromoDynamics

<u>fundamental degrees of freedom</u>

quarks, antiquarks, gluons

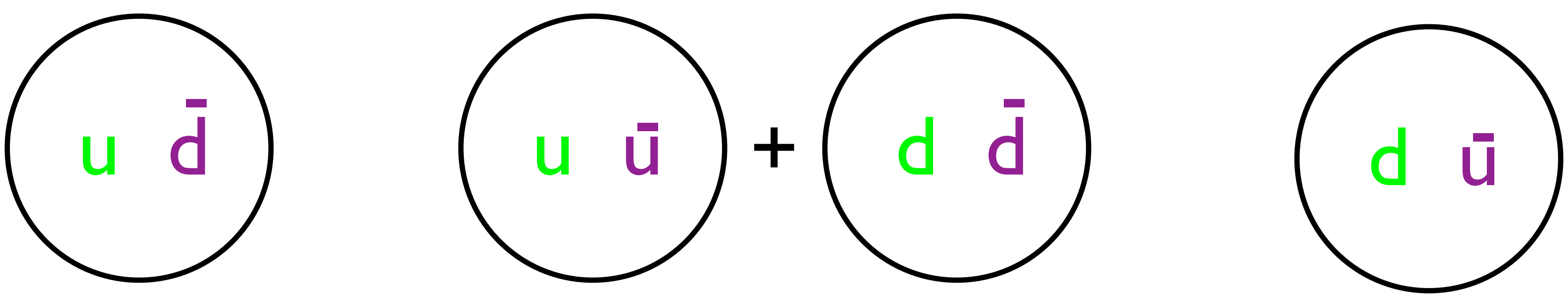
physical particles are bound states:

mesons baryons glueballs? hybrids? ...?



lightest particles are pions: mass $\cong$ 140 MeV

π^+ π^0 π^-



Effective Field Theory

- low-energy degrees of freedom
  momentum scale p

- higher-energy degrees of freedom
  momentum scales $\geq \Lambda$

describe low-energy d.o.f. using local QFT

- infinitely many parameters: $C_1, C_2, C_3, \ldots$
- choose parameters so they scale
  as definite powers of Λ: $C_n \sim 1/\Lambda^{d_n}$
- effects of C_n are suppressed by $(p/\Lambda)^{d_n}$

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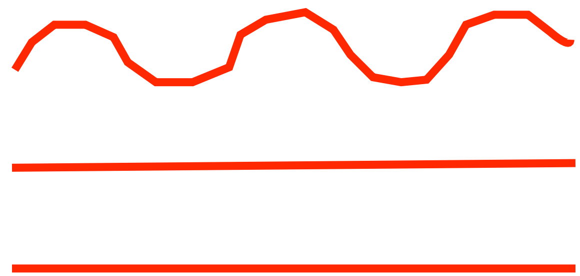
Quantum ElectroDynamics

degrees of freedom

photons
electrons
positrons

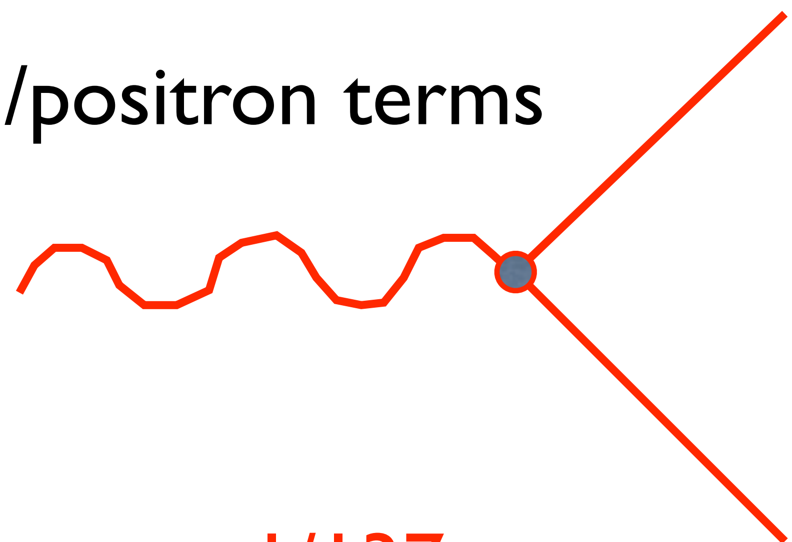


(each with 2 spin states)

Lagrangian for QED

$L = \frac{1}{2}(E^2 - B^2)$ + electron/positron terms

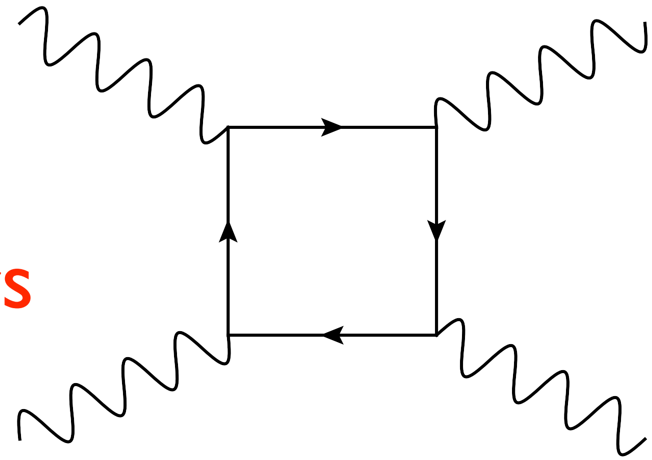
point interaction:



one interaction parameter: $\alpha \cong 1/137$

Quantum ElectroDynamics

at energies below $2m_e c^2 = 10^6$ eV
electrons and positrons cannot be created



but low-energy photons can scatter
through virtual electron-positron pairs

low-energy photons can be described by QED
but they can described more simply
(and to arbitrarily high accuracy)
by an EFT with photons only

Quantum Photon Dynamics

describe low-energy photons
by EFT with photons only

fields: $\vec{E}, \ \vec{B}$

Lagrangian for EFT: L_{eff}

must respect symmetries of QED

gauge invariance: L_{eff} is function of $\vec{E}, \vec{B}$ only

Lorentz invariance: L_{eff} is function of E^2-B^2 and $\vec{E} \cdot \vec{B}$

parity: L_{eff} is even function of $\vec{B}$

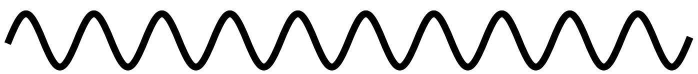
$$L_{eff} = \tfrac{1}{2}(E^2-B^2) + c_1(E^2-B^2)^2 + c_2\,(E \cdot B)^2 + \ldots$$

Quantum Photon Dynamics

effective Lagrangian: $L_{\text{eff}} = L_0 + L_1 + L_2 + \ldots$

<u>0th approximation</u> Maxwell (1861)

$L_0 = \frac{1}{2}(E^2 - B^2)$

free photons! 

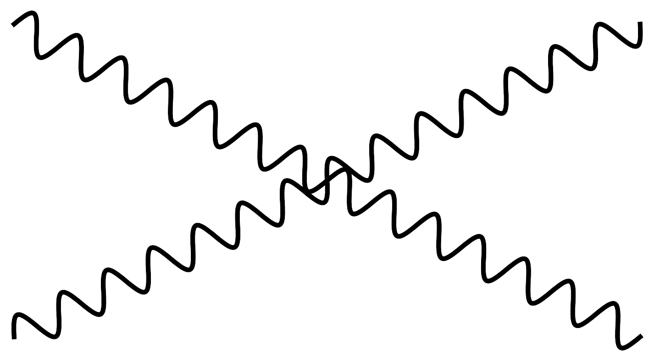
Quantum Photon Dynamics

effective Lagrangian: $L_{eff} = L_0 + L_1 + L_2 + \ldots$

$$L_0 = \tfrac{1}{2}(E^2 - B^2)$$

1st approximation   Euler and Heisenberg (1936)

add $L_1 = c_1(E^2\text{-}B^2)^2 + c_2\,(E\cdot B)^2$



photon-photon scattering!

determine coefficients by matching to QED

$$c_1 = (2/45)\alpha^2/m_e^2 \qquad c_2 = (14/45)\alpha^2/m_e^2$$

amplitude ~ p^2/m_e^2

Quantum Photon Dynamics

effective Lagrangian: $L_{\mathrm{eff}} = L_0 + L_1 + L_2 + \ldots$

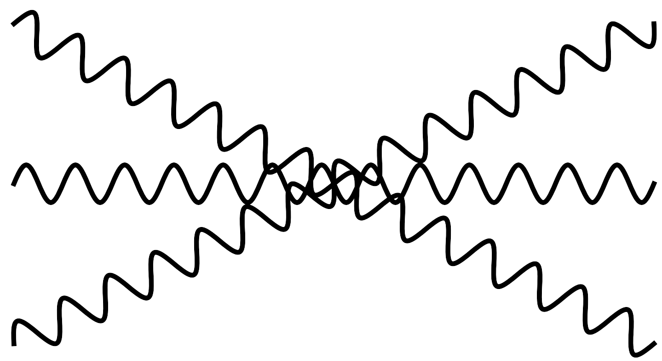
$L_0 = \frac{1}{2}(E^2 - B^2)$

$L_1 = c_1(E^2 - B^2)^2 + c_2\,(E \cdot B)^2$

2nd approximation

add $L_2 = c_3(E^2-B^2)^3 + c_4\,(E^2-B^2)(E \cdot B)^2$

6-photon amplitudes



coefficients: $c_n \sim 1/m_e^4$

amplitude $\sim p^4/m_e^4$

Quantum Photon Dynamics

effective Lagrangian: $L_{eff} = L_0 + L_1 + L_2 + \ldots$

$$L_0 = \tfrac{1}{2}(E^2 - B^2)$$
$$L_1 = c_1(E^2 - B^2)^2 + c_2\,(E \cdot B)^2$$
$$L_2 = c_3(E^2 - B^2)^3 + c_4\,(E^2 - B^2)(E \cdot B)^2$$

- infinitely many interaction parameters
- scaling of parameters: $c_n \sim 1/m_e^{d_n}$
- suppression factor for c_n: $(p/m_e)^{d_n}$
- arbitrarily high accuracy

Cosmic Microwave Background

energy scale of photons: $T \approx 3^{\circ}K \approx 10^{-4}$ eV

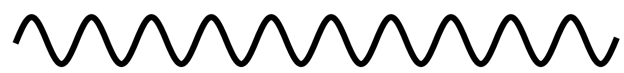
energy scale of virtual electron-positron pairs
$m_e \approx 10^{6}$ eV

large hierarchy: $T/m_e \approx 10^{-10}$

can be described to arbitrarily high accuracy
using Quantum Photon Dynamics

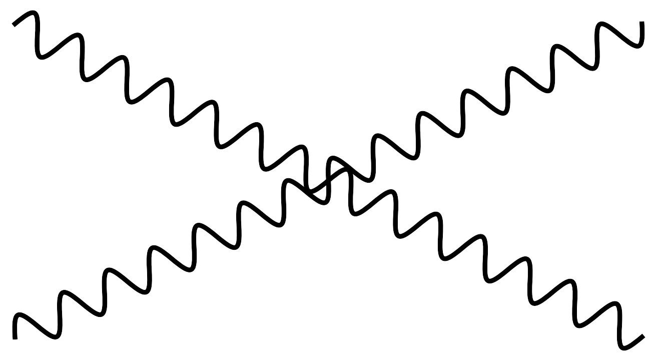
Cosmic Microwave Background

effective Lagrangian: $L_{\text{eff}} = L_0 + L_1 + L_2 + \ldots$



0th approximation: L_0

ideal gas of photons



1st approximation: add L_1

photon-photon scattering!

corrections ~ $T^2/m_e^2 \approx 10^{-20}$

2nd approximation: add L_2

corrections ~ $T^4/m_e^4 \approx 10^{-40}$

Beyond the Standard Model

<u>high energy frontier</u>
create new heavy particles at the LHC (Atlas, CMS)

<u>precision frontier</u>
observe effects of virtual heavy particles
through precision measurements at low energy

EFT: effects of virtual heavy particles
can be described to <u>arbitrary accuracy</u>
in terms of Standard Model particles only

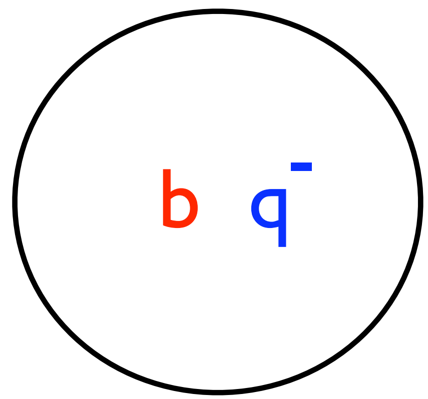
Standard Model

EFT: systematically improvable
low-energy approximations

<u>Heavy Quark Effective Theory</u> (see Mike Luke)

EFT for sector of Quantum Chromodynamics
with one heavy quark (bottom or charm)

energy scale of light quark: Λ_{QCD}
heavy quark mass: M_Q
systematic expansion in Λ_{QCD}/M_Q

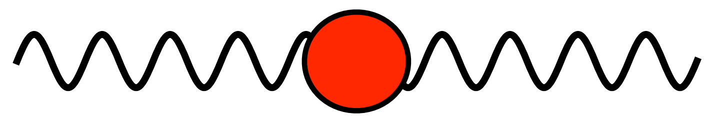


basis for analysis of experiments at B factories
that established CKM mechanism for CP violation

Beyond the Standard Model

EFT for effects of virtual heavy particles
on Standard Model particles

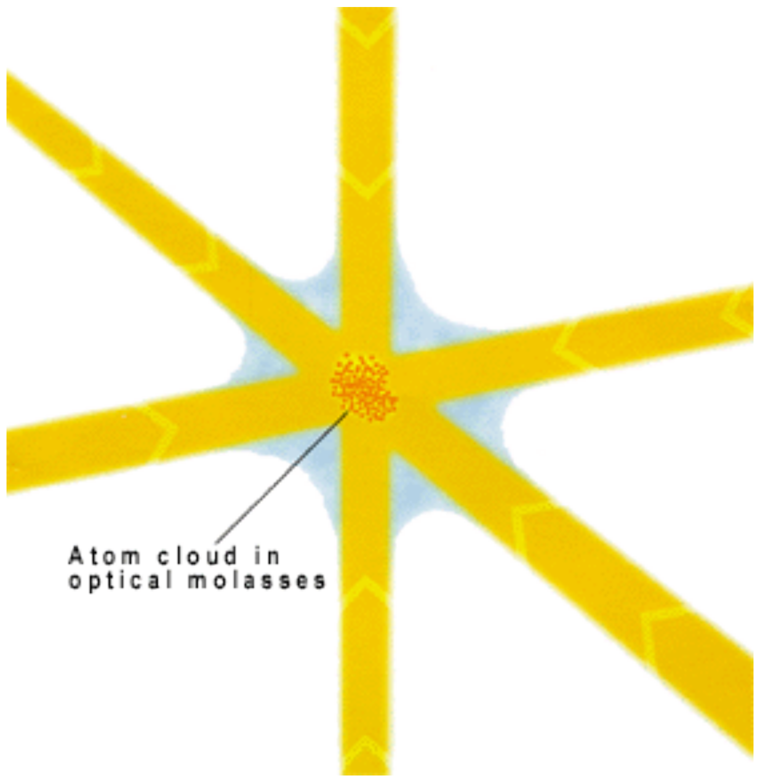
leading propagator corrections for
Z^0 boson
W boson
γ-Z^0 mixing



determined by 3 constants: *S, T, U*
strongly constrained
by precision electroweak measurements
e.g. $S = -0.04 \pm 0.09$
must be respected by any model
for new heavy particles at the LHC
(see Bob Holdom)

Cold Atom Physics

Atoms trapped and cooled using lasers

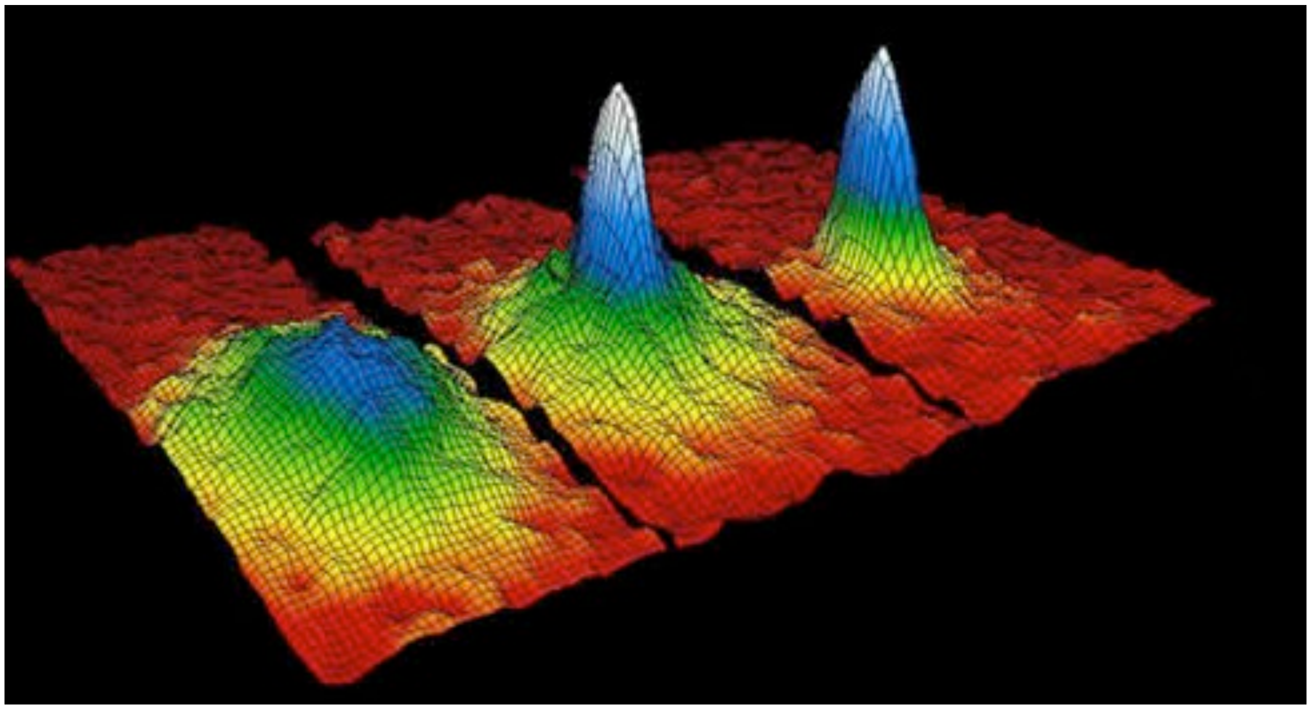


Nobel Prize 1997: Chu, Cohen-Tannoudji, Phillips

Cold Atom Physics

Bose-Einstein condensation of atoms!

^{87}Rb atoms	JILA (Cornell, Wieman)	1995
^{7}Li atoms	Rice (Hulet)	1995
^{23}Na atoms	MIT (Ketterle)	1995

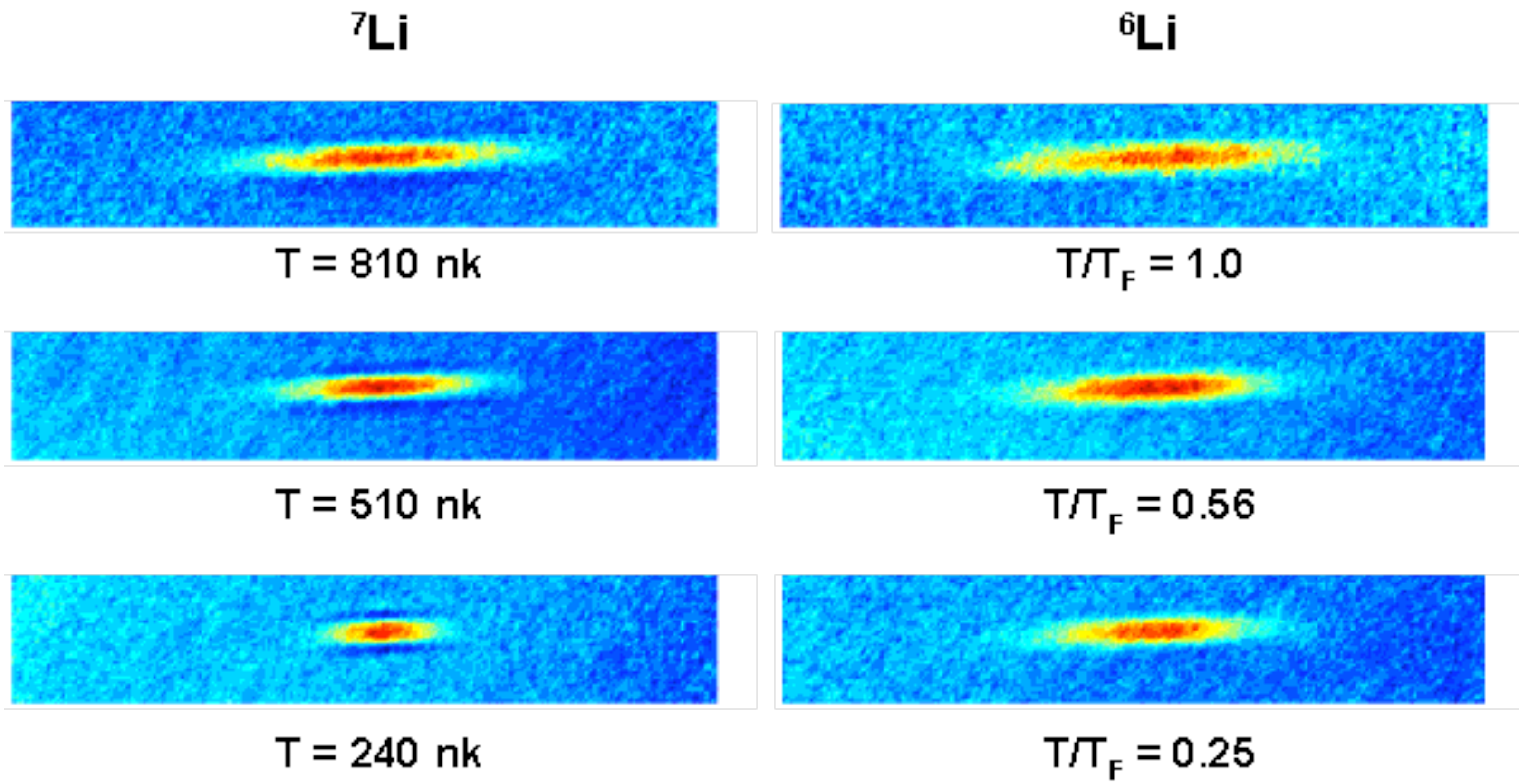


Nobel Prize 2001: Cornell, Wieman, Ketterle

Cold Atom Physics

Cooling of fermions to quantum degeneracy!

^{40}K atoms	JILA (Jin)	Jan 2001
^{6}Li atoms	Ecole Normale (Salomon)	July 2001
^{6}Li atoms	Rice (Hulet)	Aug 2001



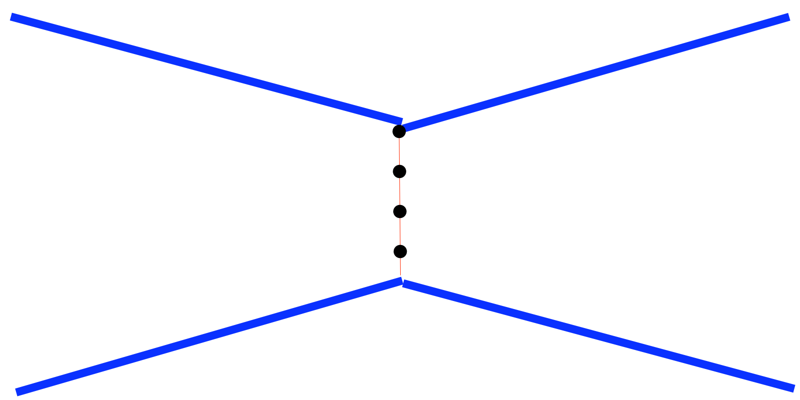
Ultracold Atoms

(sufficiently) Fundamental Theory

many-body Schroedinger equation

for atoms in a trapping potential V(r)

interacting through interatomic potential U(r-r')

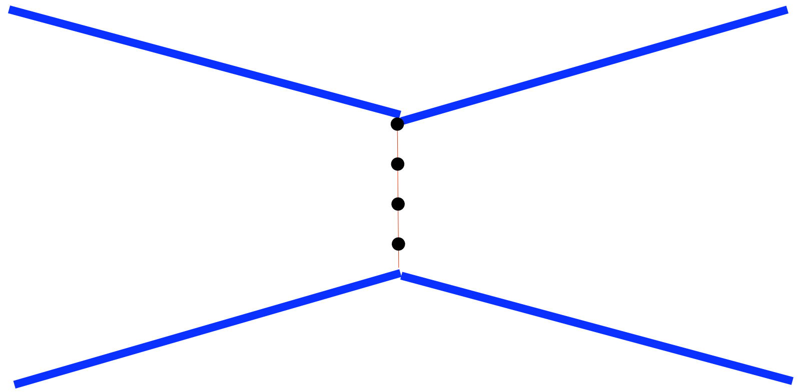


complications: hyperfine spin states

multiple scattering channels

Ultracold Atoms

atoms interacting through potential $U(r-r')$



solve Schroedinger equation

to obtain scattering phase shifts $\delta(k)$

low-energy expansion: $\delta(k) = -1/a + \frac{1}{2} r_e k^2 + \ldots$

a = scattering length

r_e = effective range

Effective Field Theory

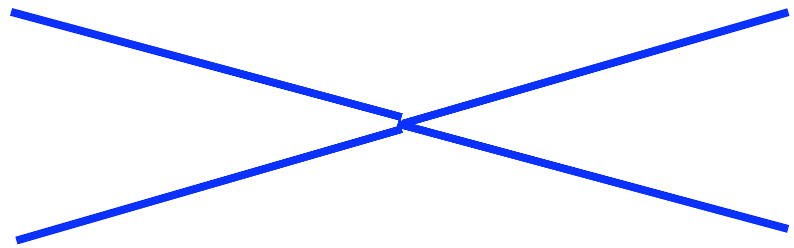
low-energy expansion: $\delta(k) = -1/a + \frac{1}{2} r_e k^2 + \ldots$

a = scattering length

r_e = effective range

construct EFT with point interactions

that reproduces low-energy expansion



most important parameter: a

next most important: r_e

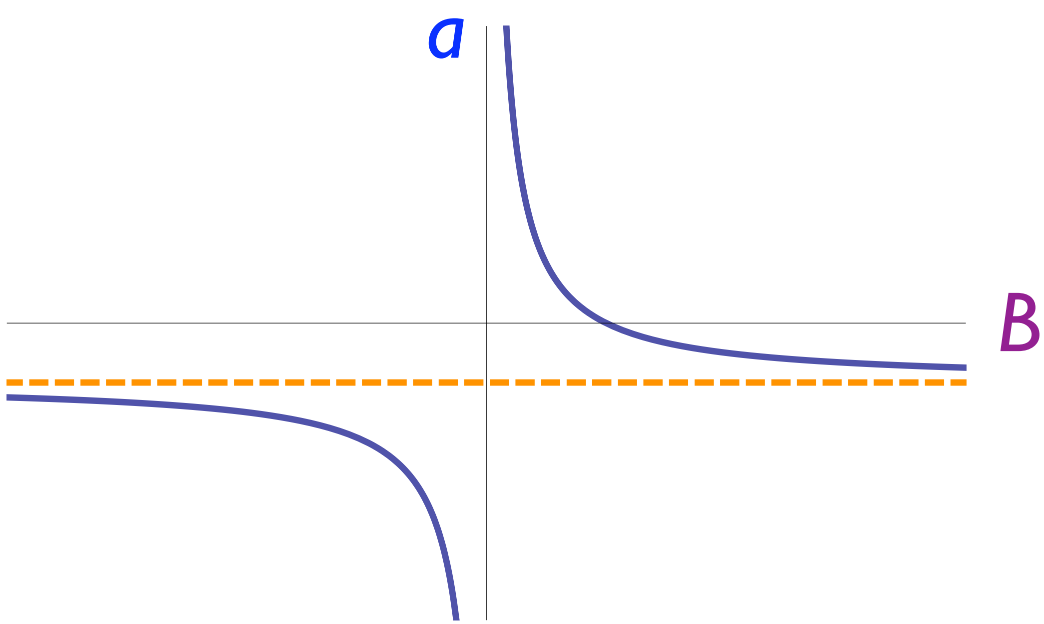
effects suppressed by $k^2 \times \text{range}^2$

Atoms with large scattering length

experimental fine-tuning using magnetic field *B*

alkali atom near Feshbach resonance

$$a(B) = a_{\mathrm{bg}} + \frac{\Delta}{B - B_{\mathrm{res}}}$$



- $|a|$ becomes arbitrarily large as $B \to B_{\mathrm{res}}$

Unitary limit $a = \pm\infty$

- scattering cross section saturates unitarity bound
  strongest interaction allowed
  by quantum mechanics!

- infinitely strong interactions provide no length scale!
  scale invariant interactions!

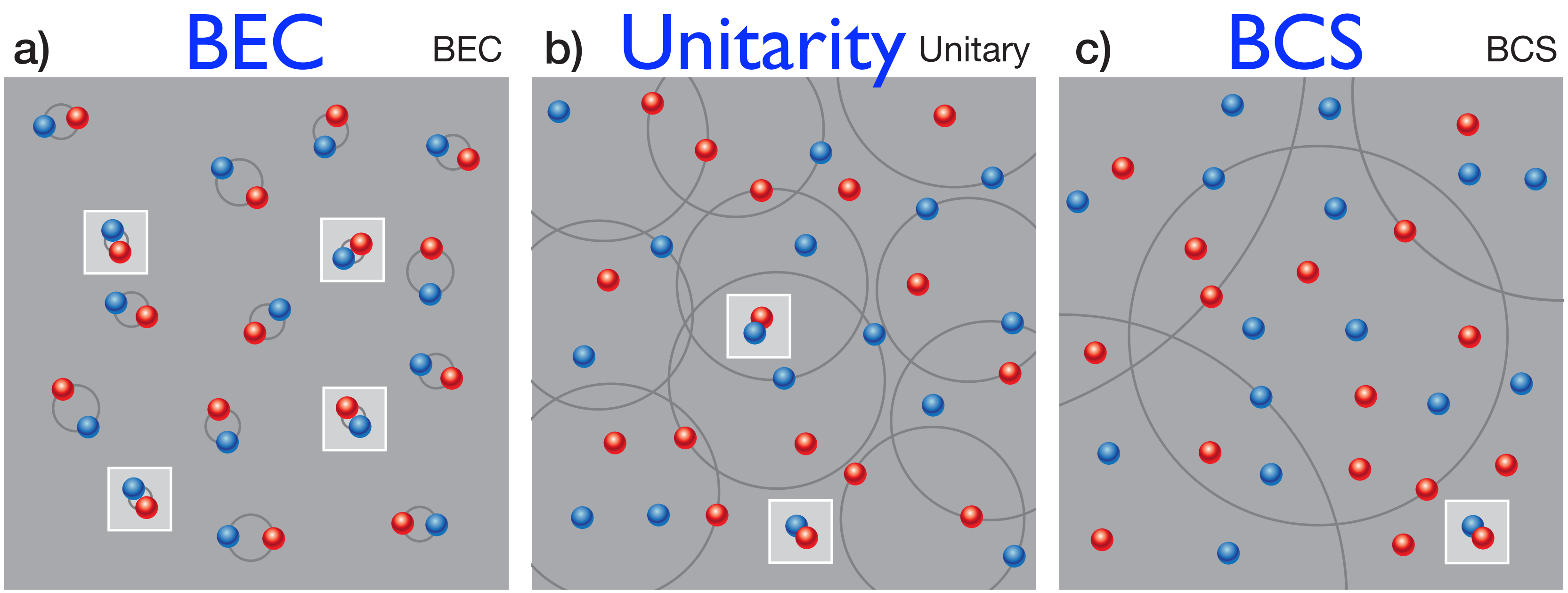
What is the bevavior of condensed matter
with infinitely-strong scale-invariant interactions?

What is mechanism for superfluidity
in the ground state?

mechanism for superfluidity?

BEC mechanism: Bose-Einstein condensation of diatomic molecules

BCS mechanism: formation of Cooper pairs near Fermi surface



experimental verification of smooth crossover from BEC to BCS

as a goes from positive to negative values through $\pm\infty$

2-Body Quantum Mechanics

Universal behavior at large scattering length a

<u>Cross section</u>

low energy: $8\pi\, a^2$

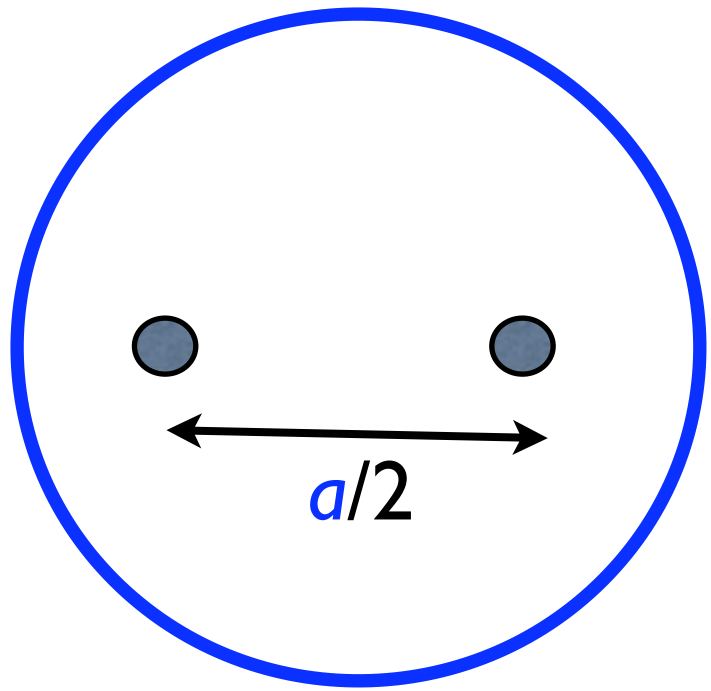
high energy: $8\pi\hbar^2/(m\, E)$

<u>Diatomic molecule</u> if $a > 0$

binding energy: $\hbar^2/(m\, a^2)$

mean radius: $a/2$

"halo dimer", "giant dimer"

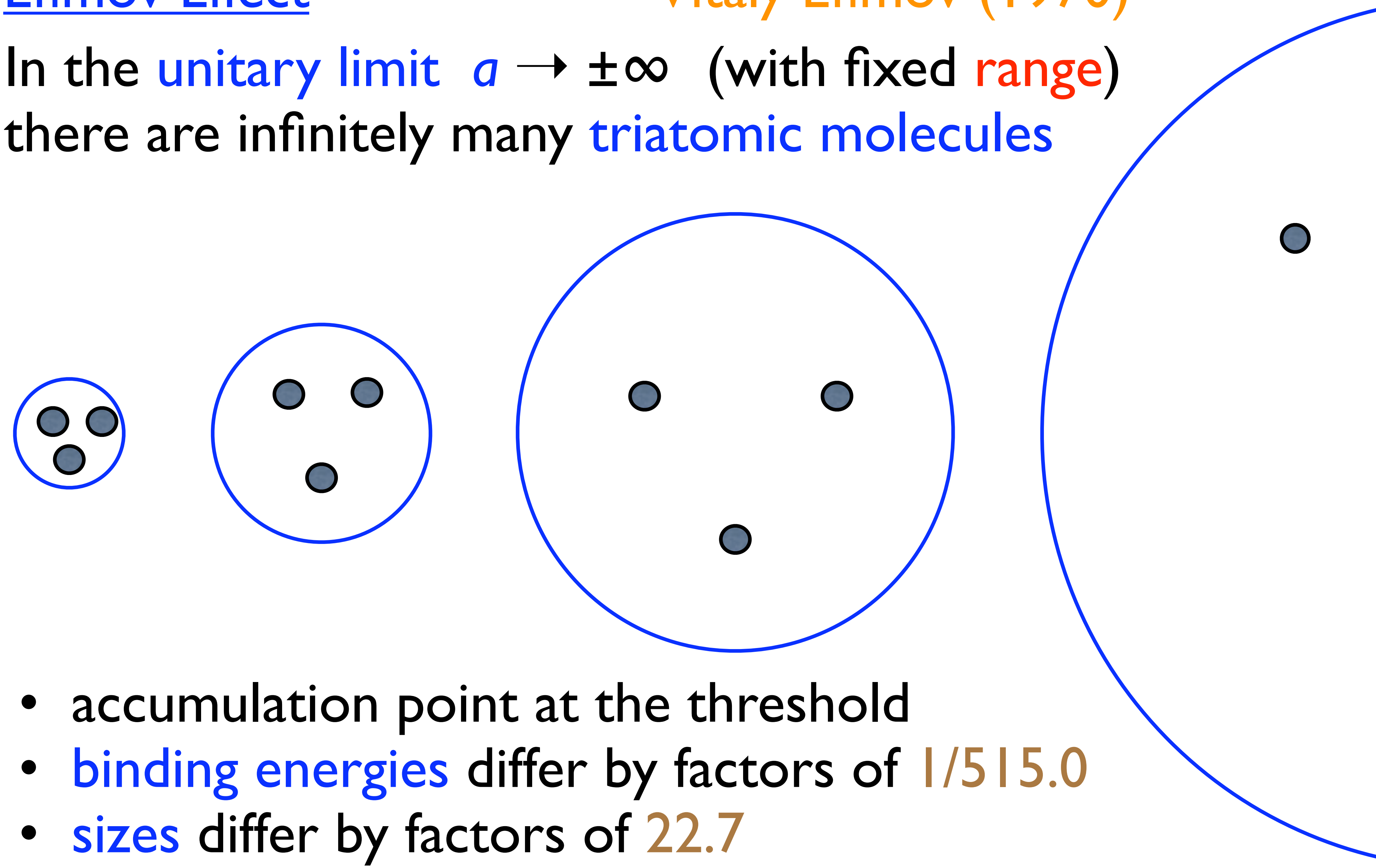


3-Body Quantum Mechanics

Efimov Effect

Vitaly Efimov (1970)

In the unitary limit $a \to \pm\infty$ (with fixed range)
there are infinitely many triatomic molecules

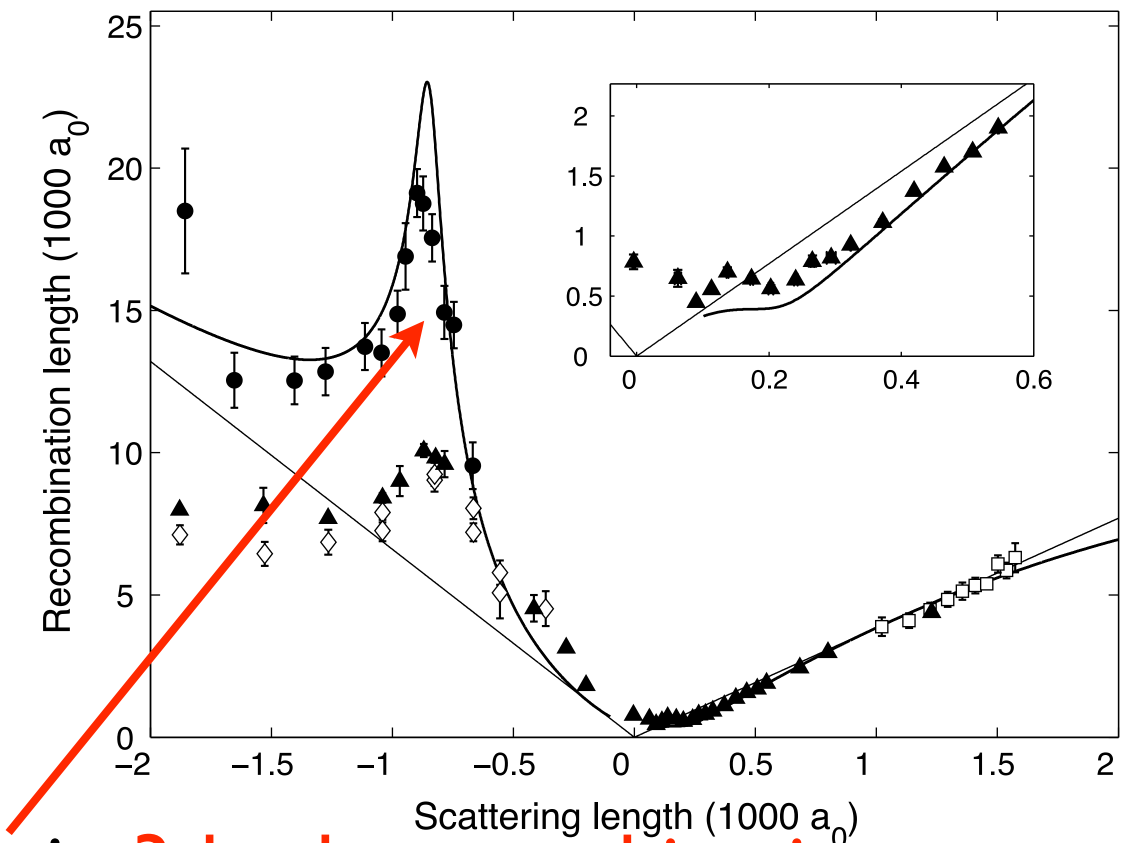


- accumulation point at the threshold
- binding energies differ by factors of 1/515.0
- sizes differ by factors of 22.7

Efimov Physics in Cold Atoms

discovery of Efimov trimer of ^{133}Cs atoms

Innsbruck group Nov 2005



resonance in 3-body recombination rate near -850 a_0

agrees with universal line shape from EFT

Braaten & Hammer 2003

Bose-Einstein Condensation

Critical temperature Einstein (1925)

homogeneous ideal gas with number density n

thermal deBroglie wavelength: $\lambda_{th} = (2\ \pi\ m\ kT/m)^{1/2}$

critical phase space density: $n\ \lambda_{th}^3 = 2.612$

critical temperature: $kT_c = 3.31\ h^2 n^{2/3}/m$

What is the shift in Tc from interactions?

Bose-Einstein Condensation

Critical temperature

in homogeneous gas with number density n

$kT_c = 3.31\ h^2 n^{2/3}/m$

What is the shift in T_c
from weak interactions with scattering length a?

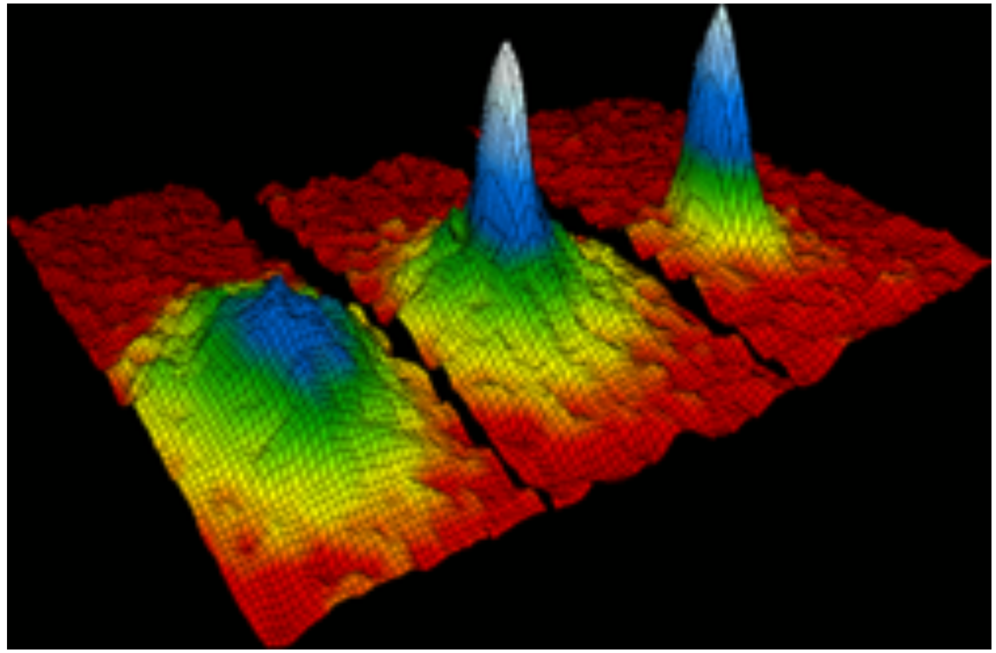
Solution

EFT for fluctuations near critical point:
statistical field theory of scalar field in 3D

$\Delta T_c / T_c = 1.8\ a/\lambda_{th}$

(coefficient from Monte Carlo calculations)

Bose-Einstein Condensation



Trapped gas

behavior near critical point

is different

but described by same EFT

shift in T_c is not sensitive to critical fluctuations

but condensate fraction is

precise measurements of condensate fraction

Cambridge group July 2011

good agreement with coefficient in EFT!