Quantum entanglement and the phases of matter

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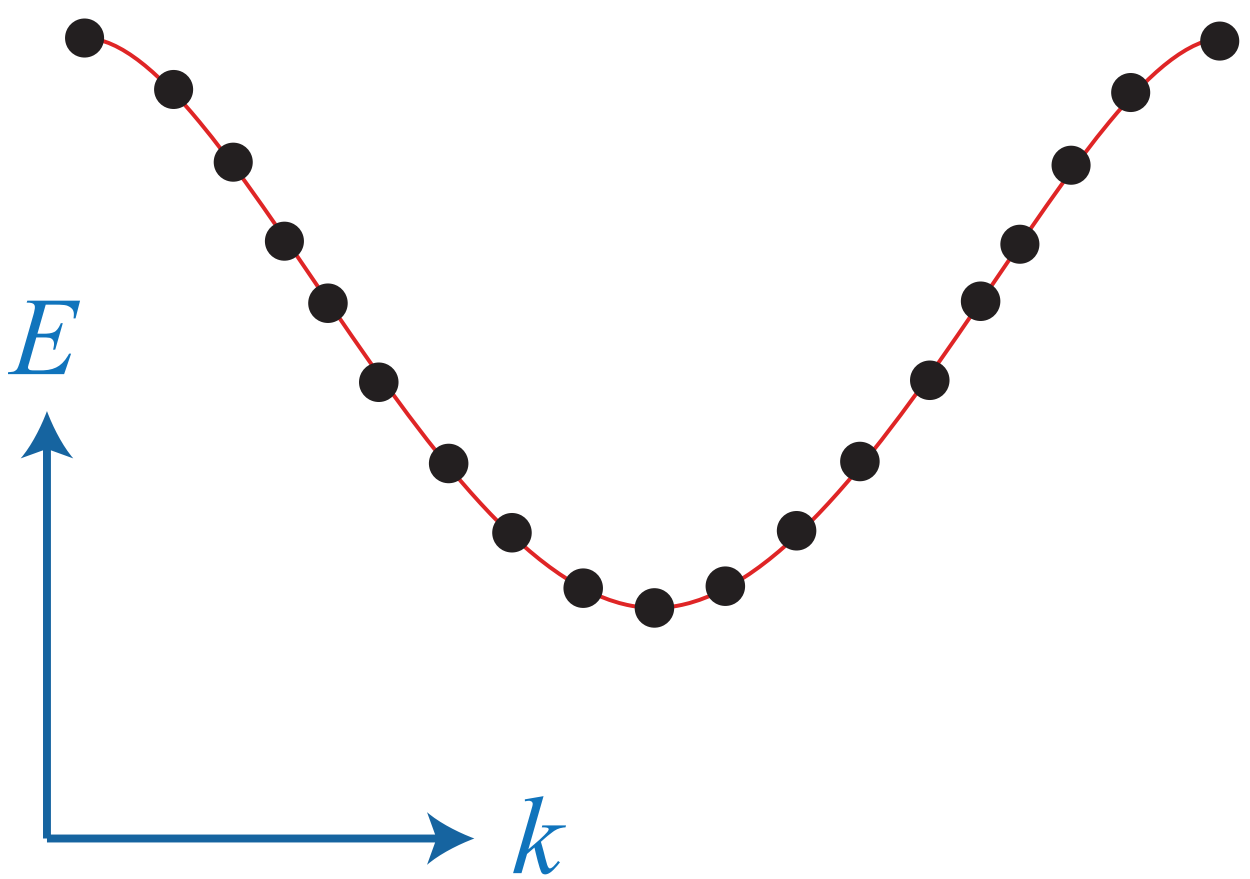
Sommerfeld-Bloch theory of
metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically connected to independent electron states

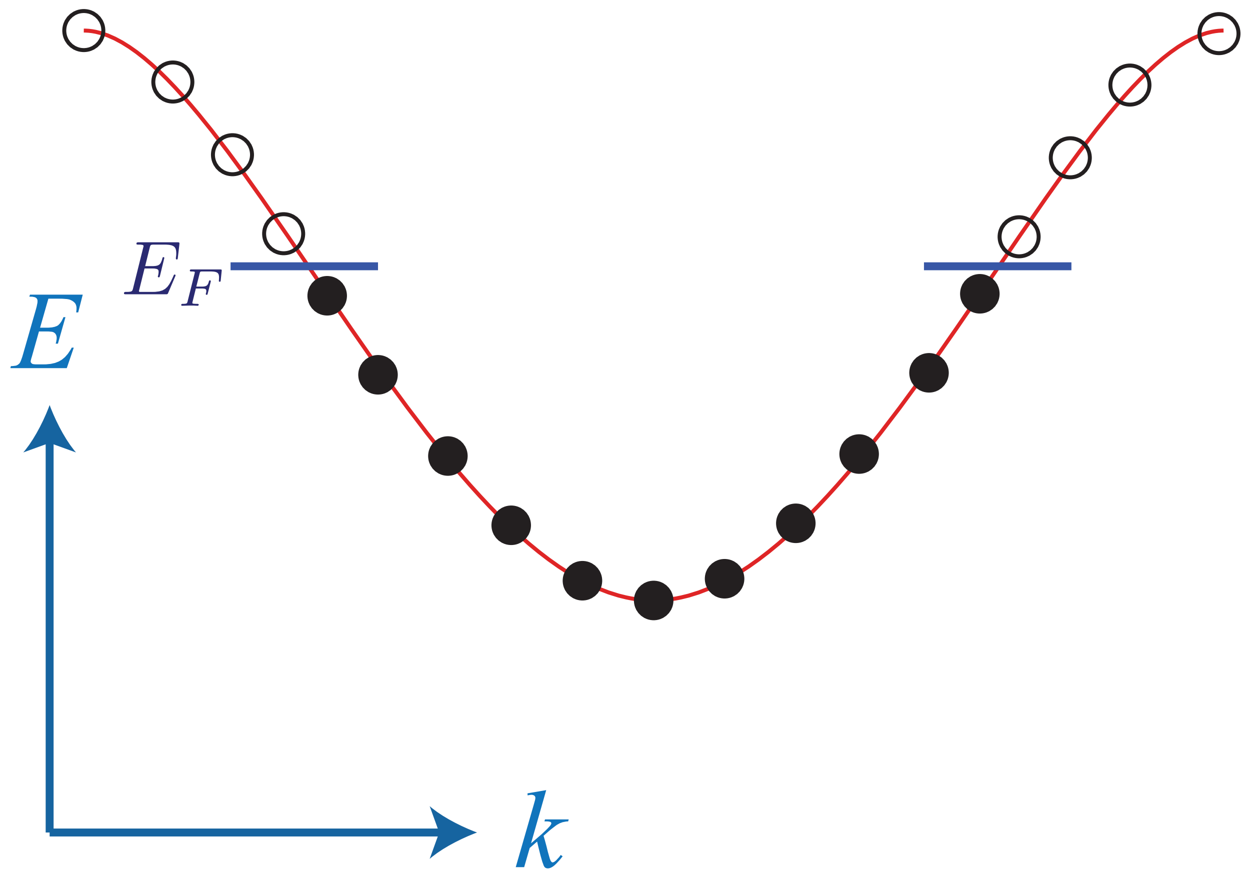
Band insulators



An even number of electrons per unit cell

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically connected to independent electron states

Metals



An odd number of electrons per unit cell

Sommerfeld-Bloch theory of
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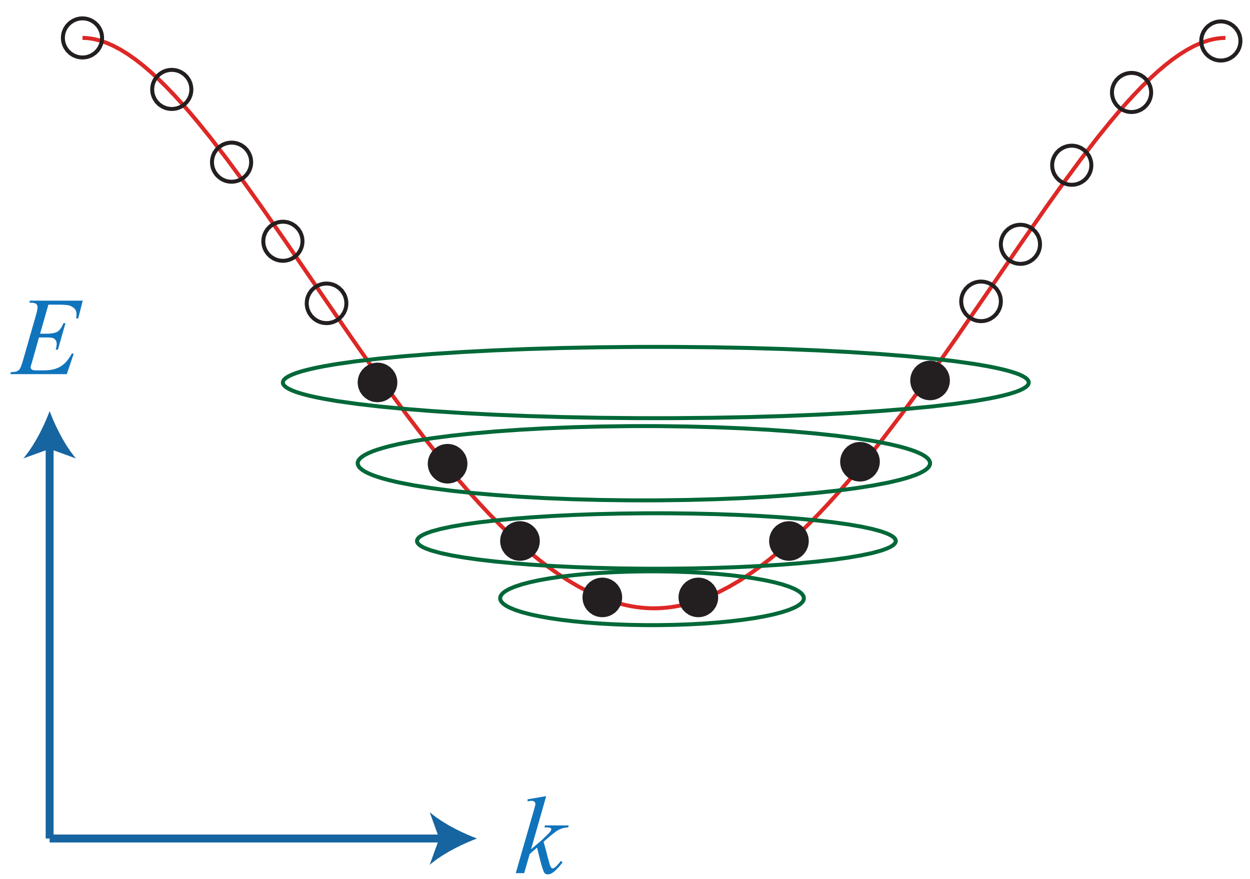
Superconductors



Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

Modern phases of quantum matter

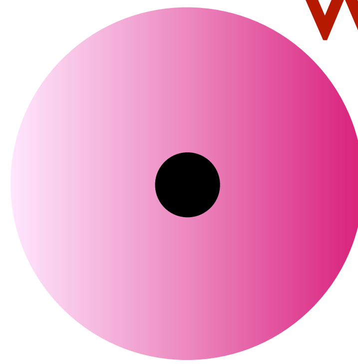
Not adiabatically connected to independent electron states:

many-particle quantum entanglement

Quantum superposition and entanglement

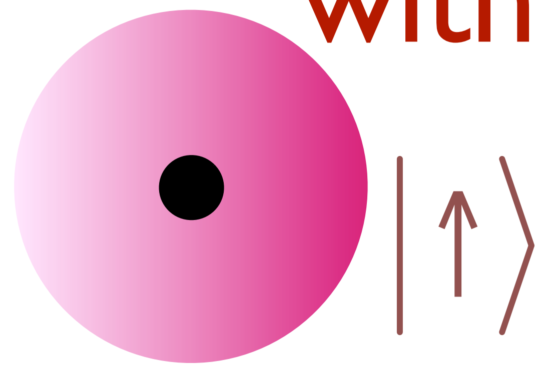
Quantum Entanglement: quantum superposition with more than one particle

Quantum Entanglement: quantum superposition with more than one particle

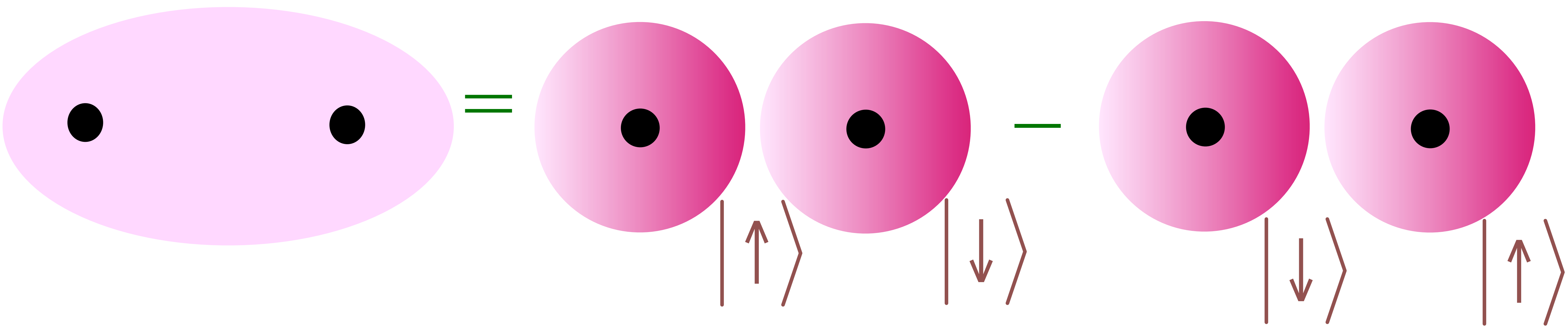
Hydrogen atom:  $|\uparrow\rangle$

<u>Quantum Entanglement</u>: quantum superposition with more than one particle

Hydrogen atom:



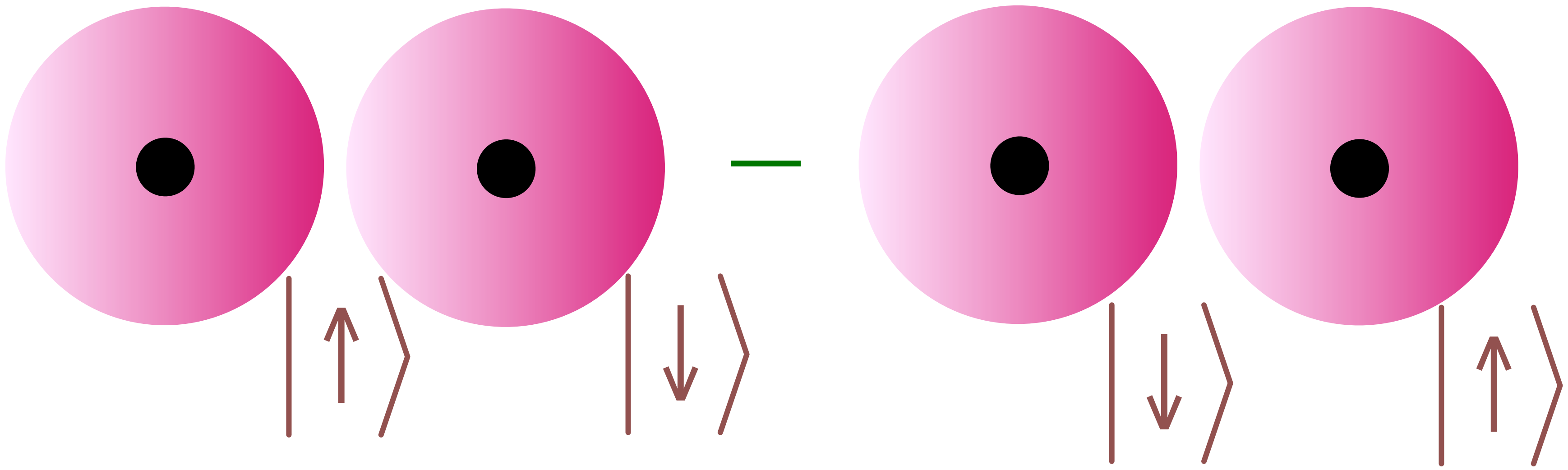
Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

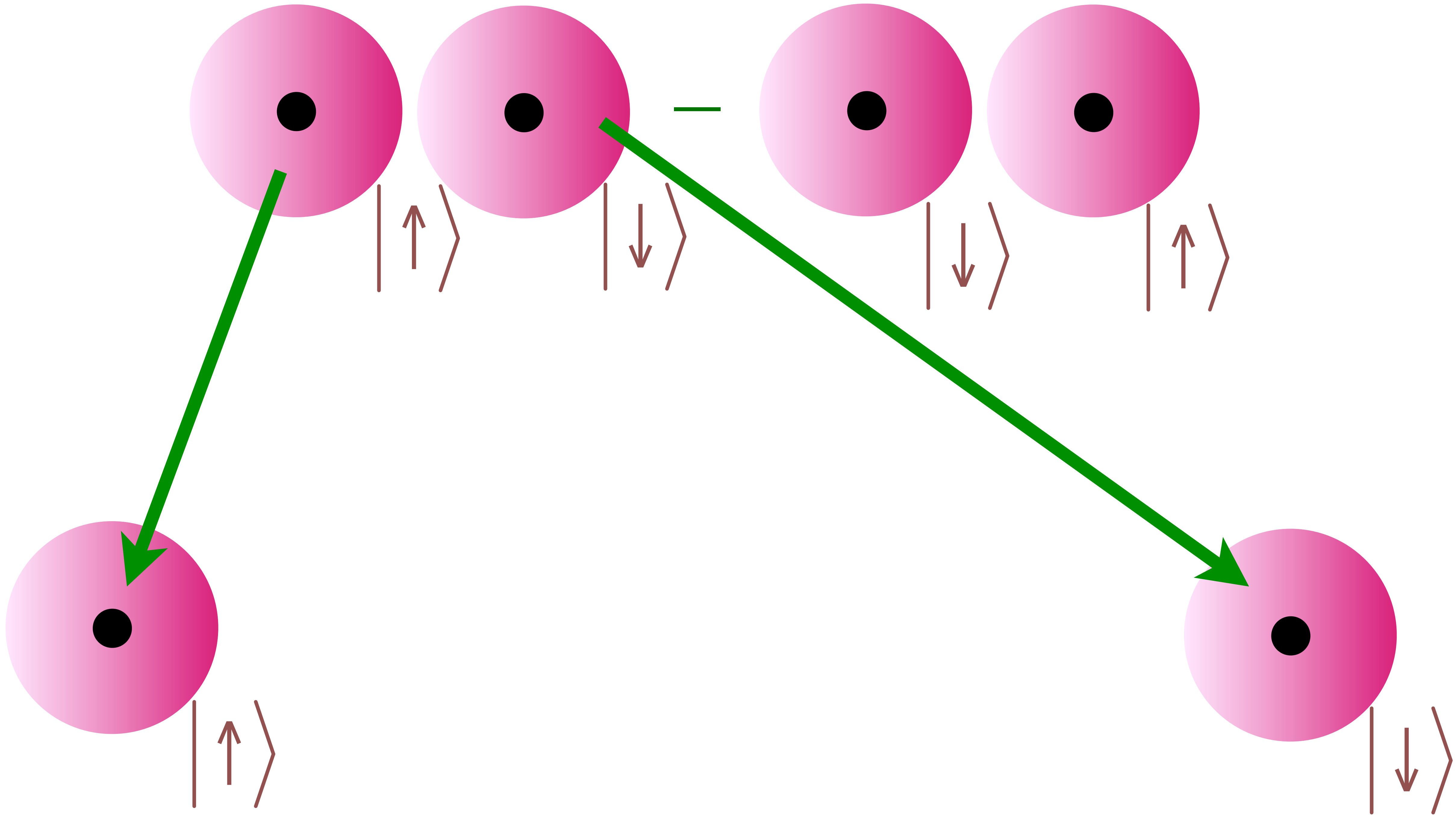
Superposition of two electron states leads to non-local correlations between spins

Quantum Entanglement: quantum superposition with more than one particle

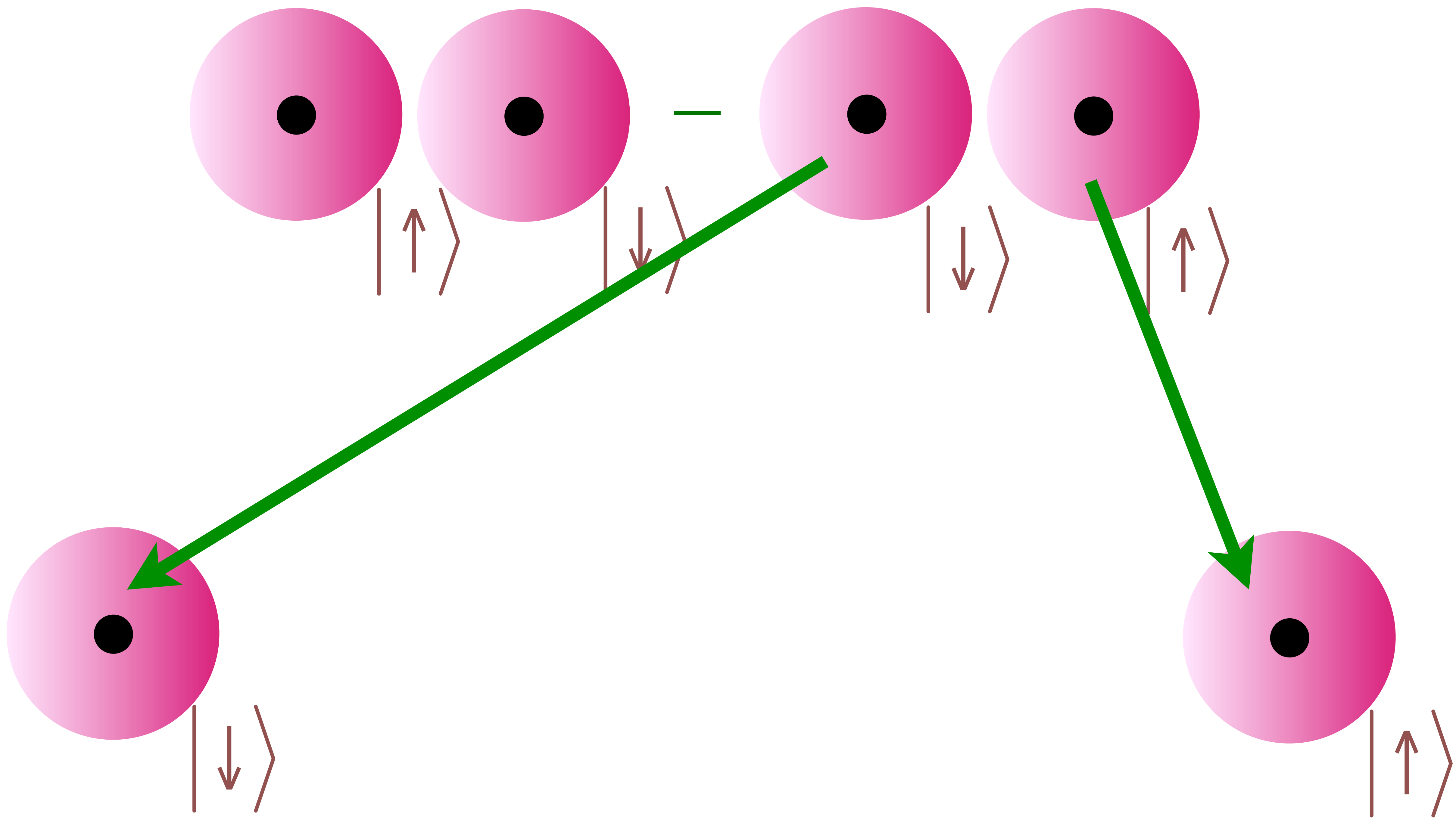


$$|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle$$

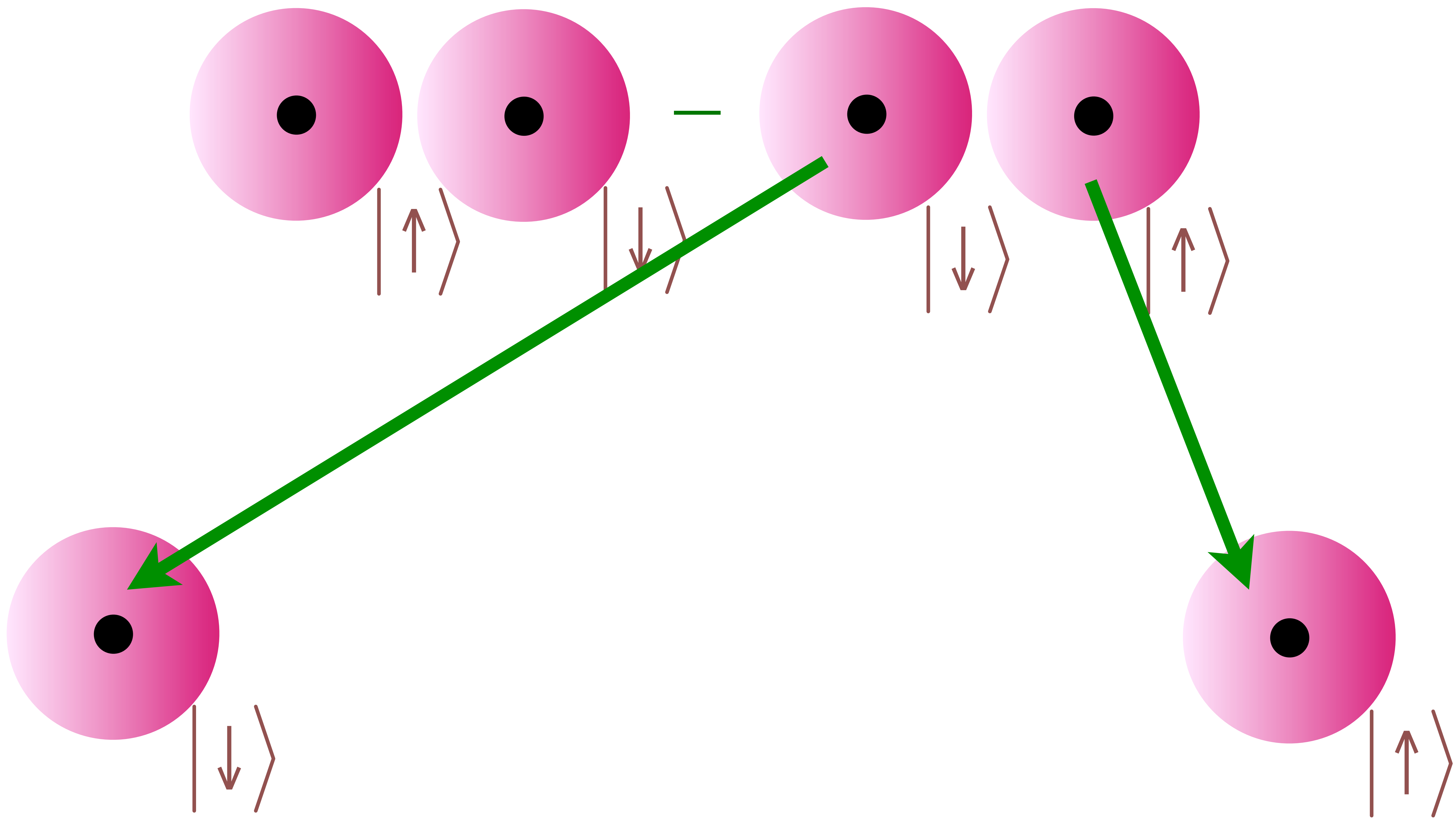
Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



<u>Quantum Entanglement</u>: quantum superposition with more than one particle



Einstein-Podolsky-Rosen "paradox": Non-local correlations between observations arbitrarily far apart

Quantum superposition and entanglement

Quantum superposition and entanglement

Quantum critical points of electrons in crystals

String theory and black holes

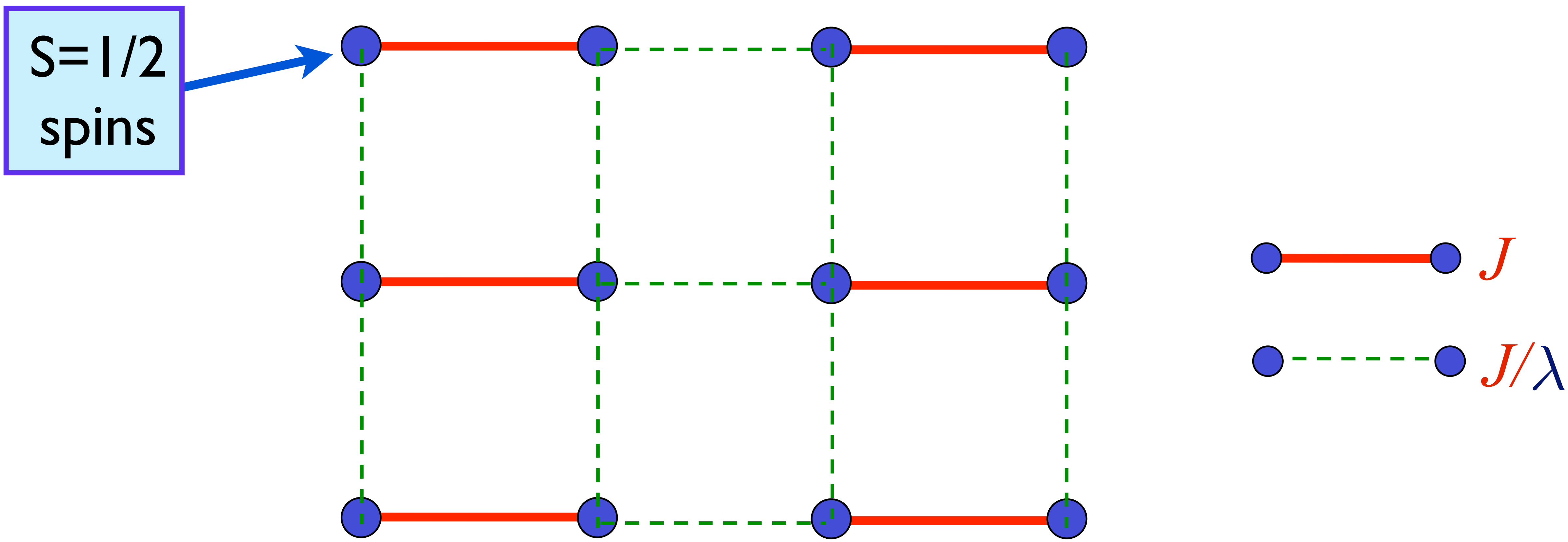
Quantum superposition and entanglement

Quantum critical points of electrons in crystals

String theory and black holes

Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

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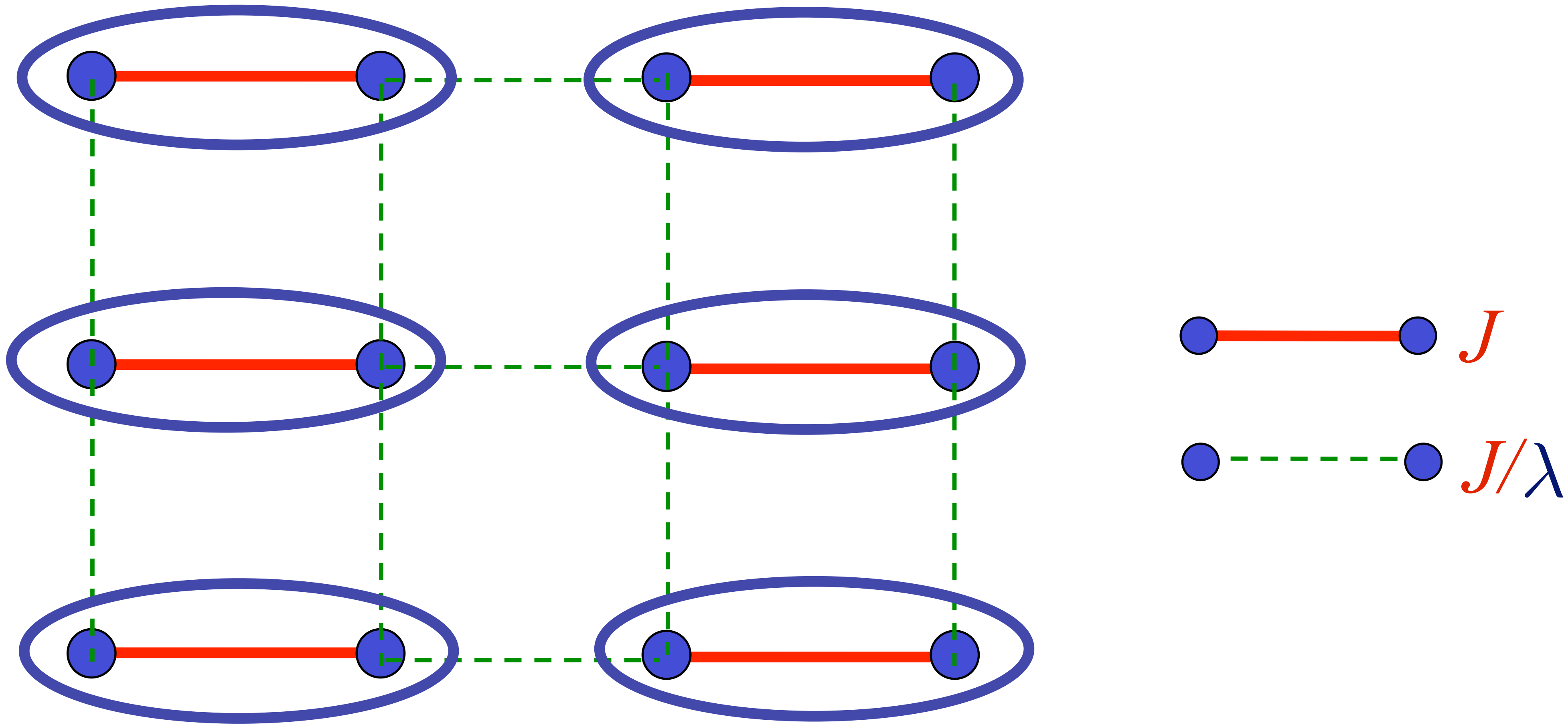

$$= \frac{1}{\sqrt{2}} \Big(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \Big)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

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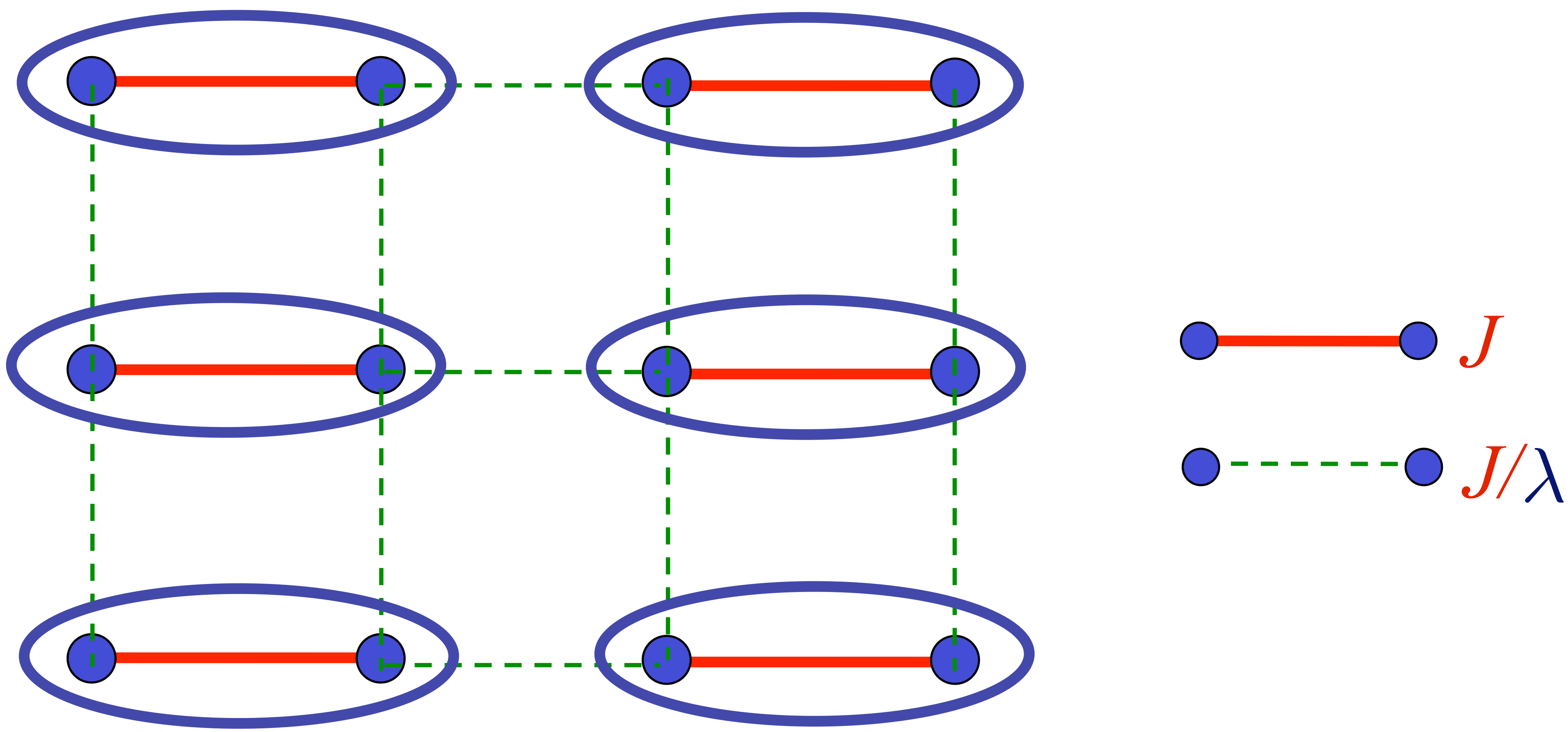
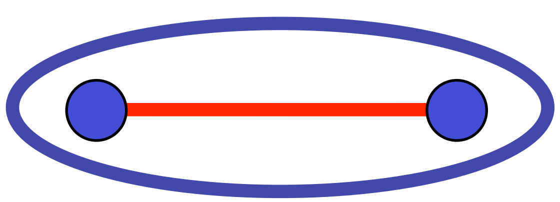


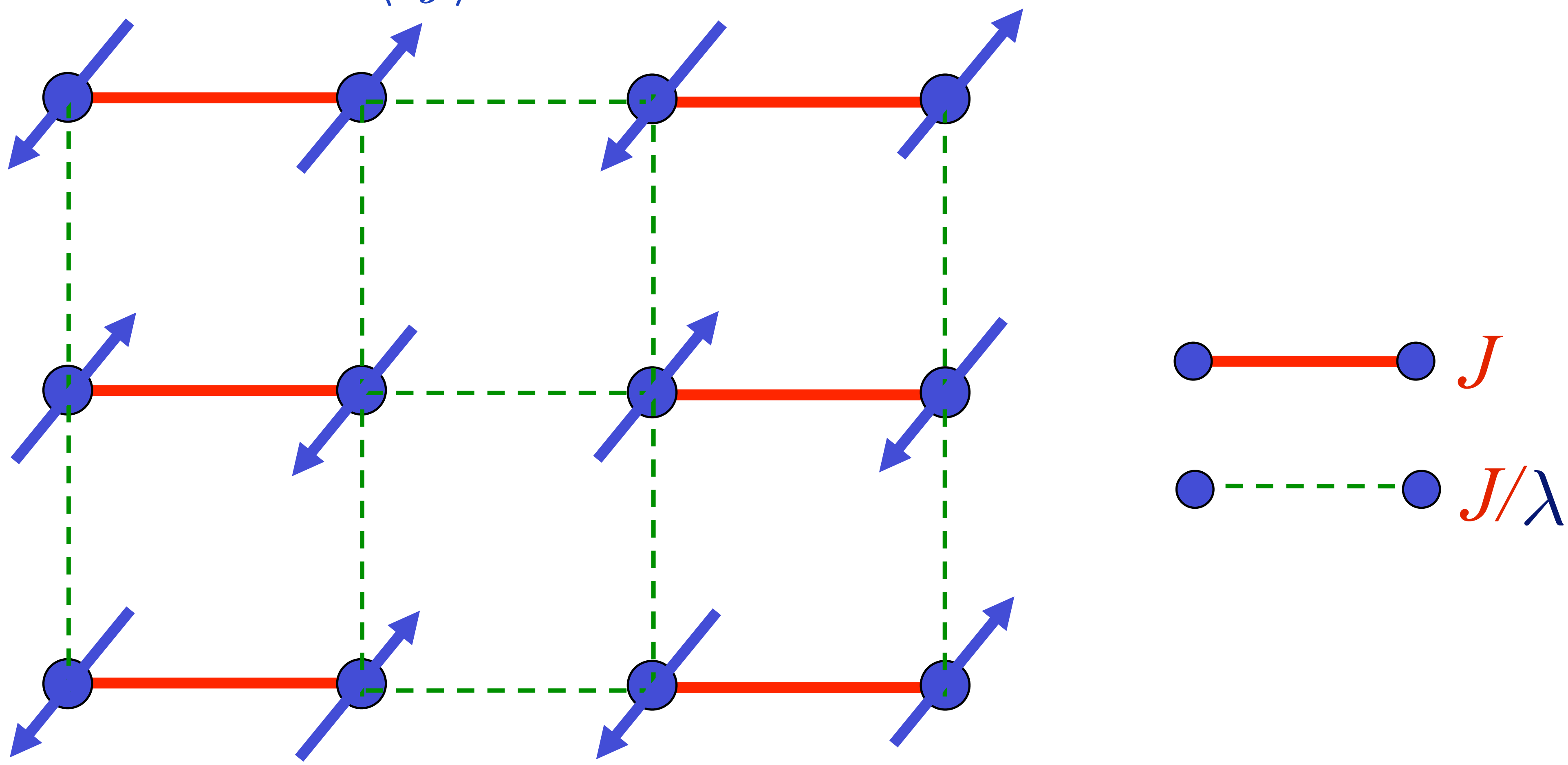


$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

Spinning electrons localized on a square lattice

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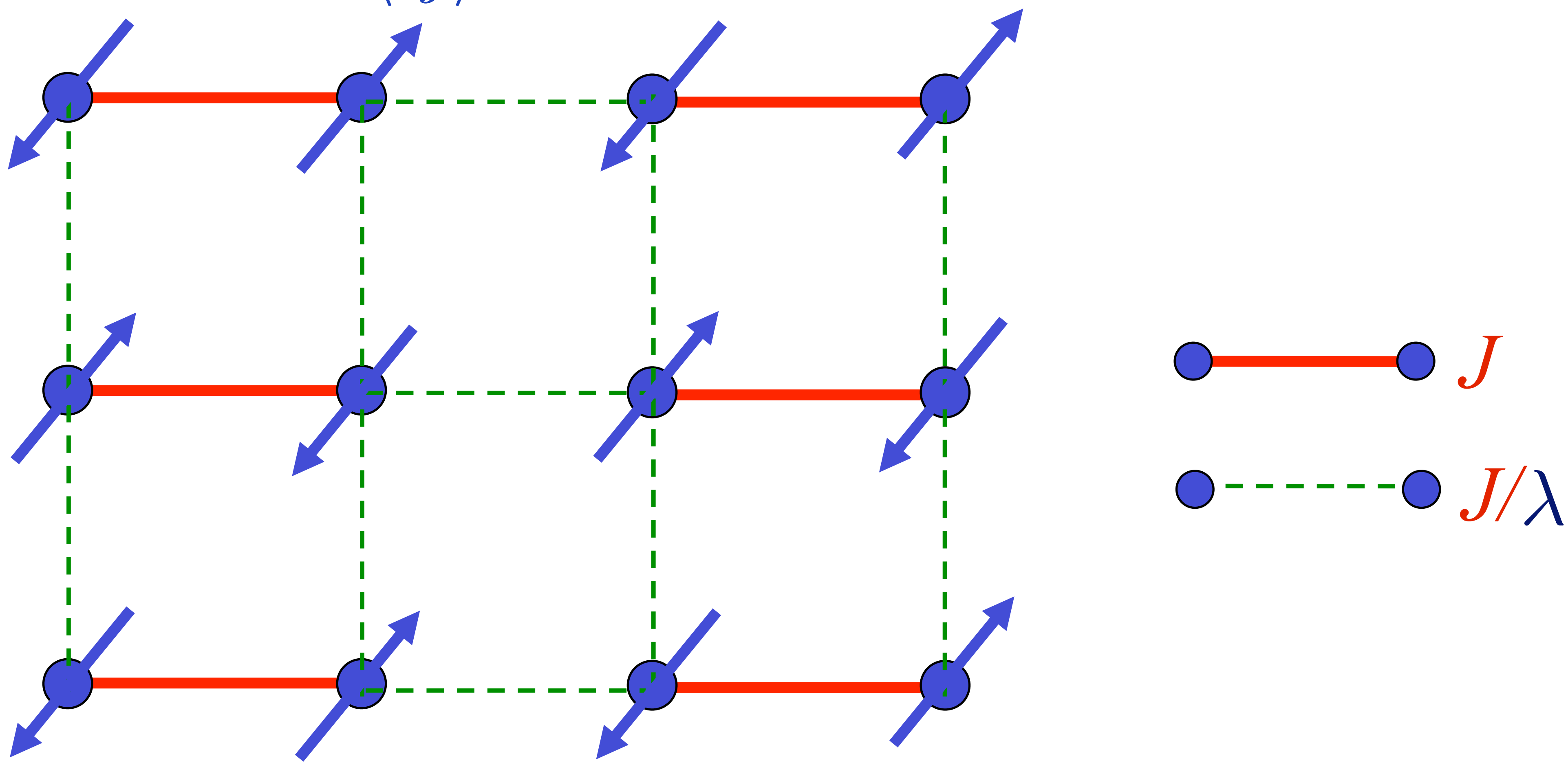

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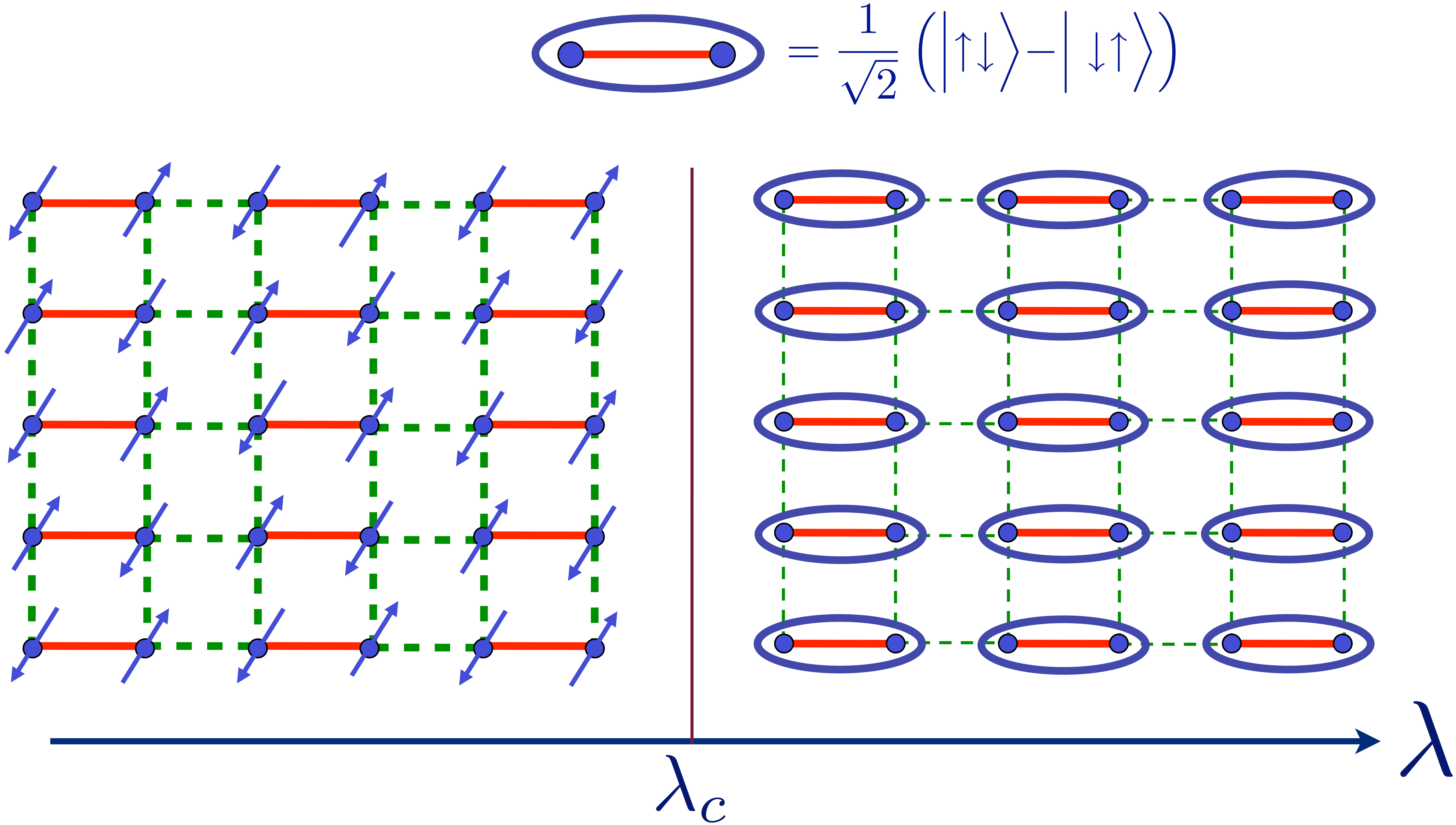
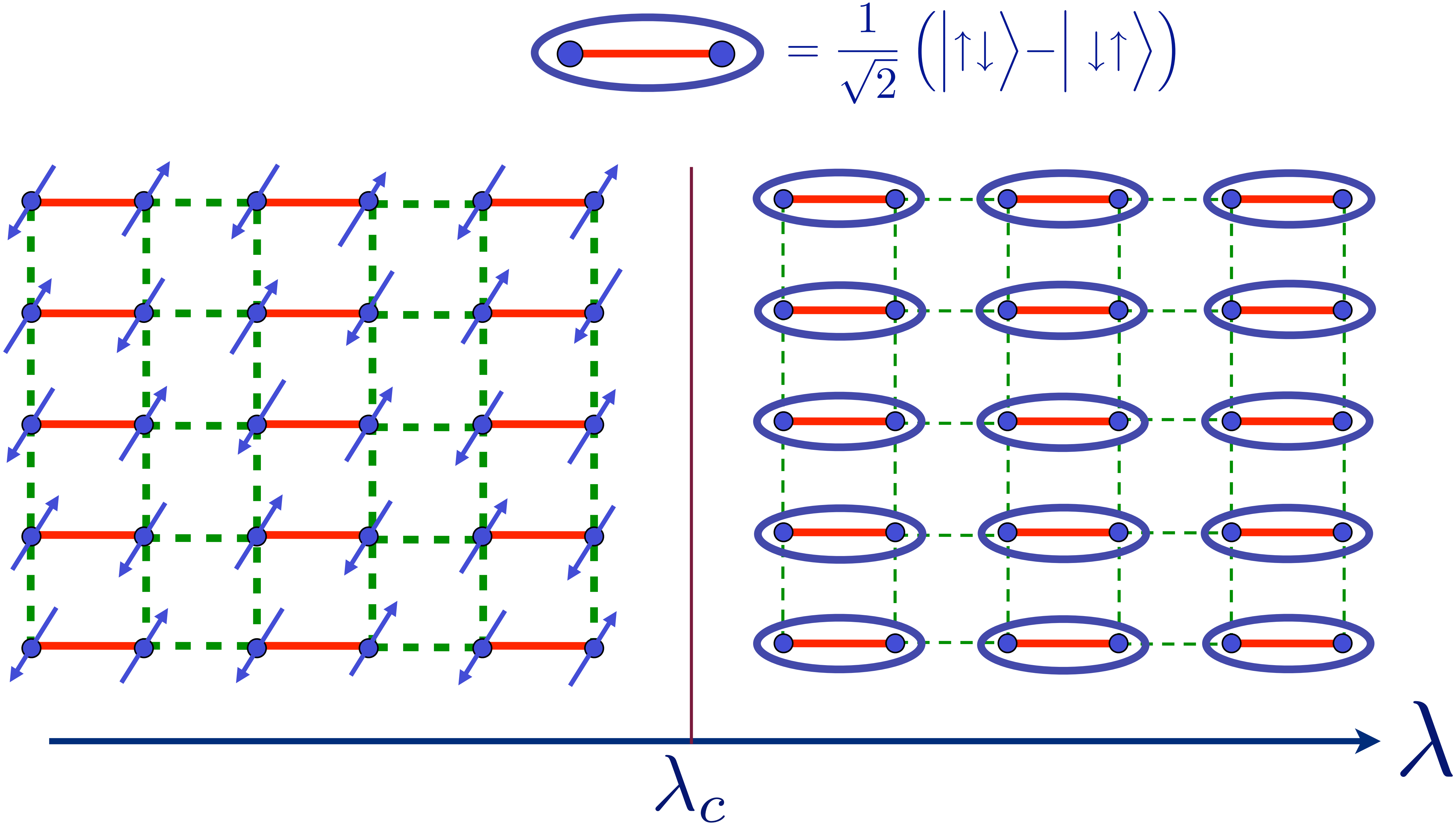
No EPR pairs

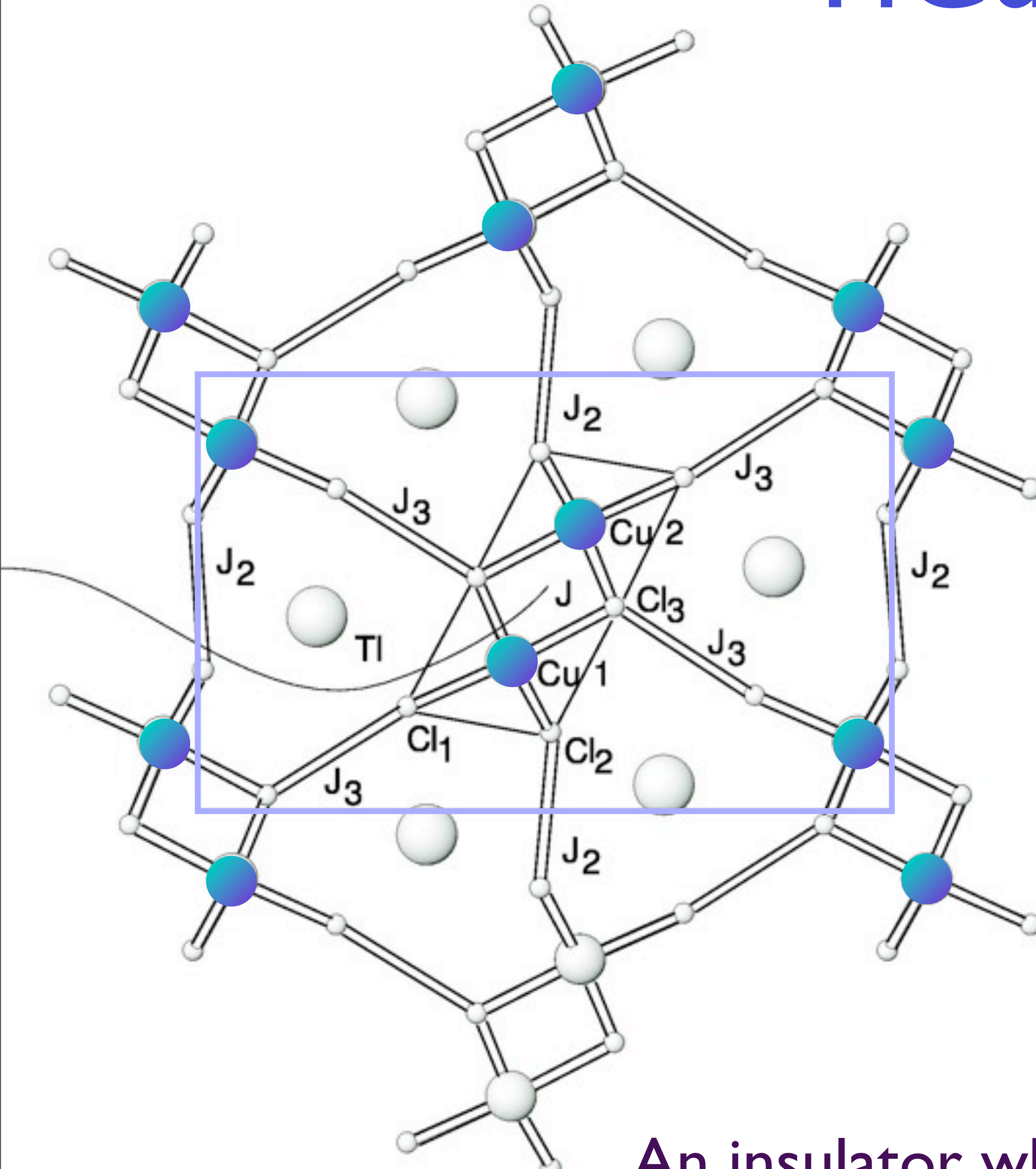

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ

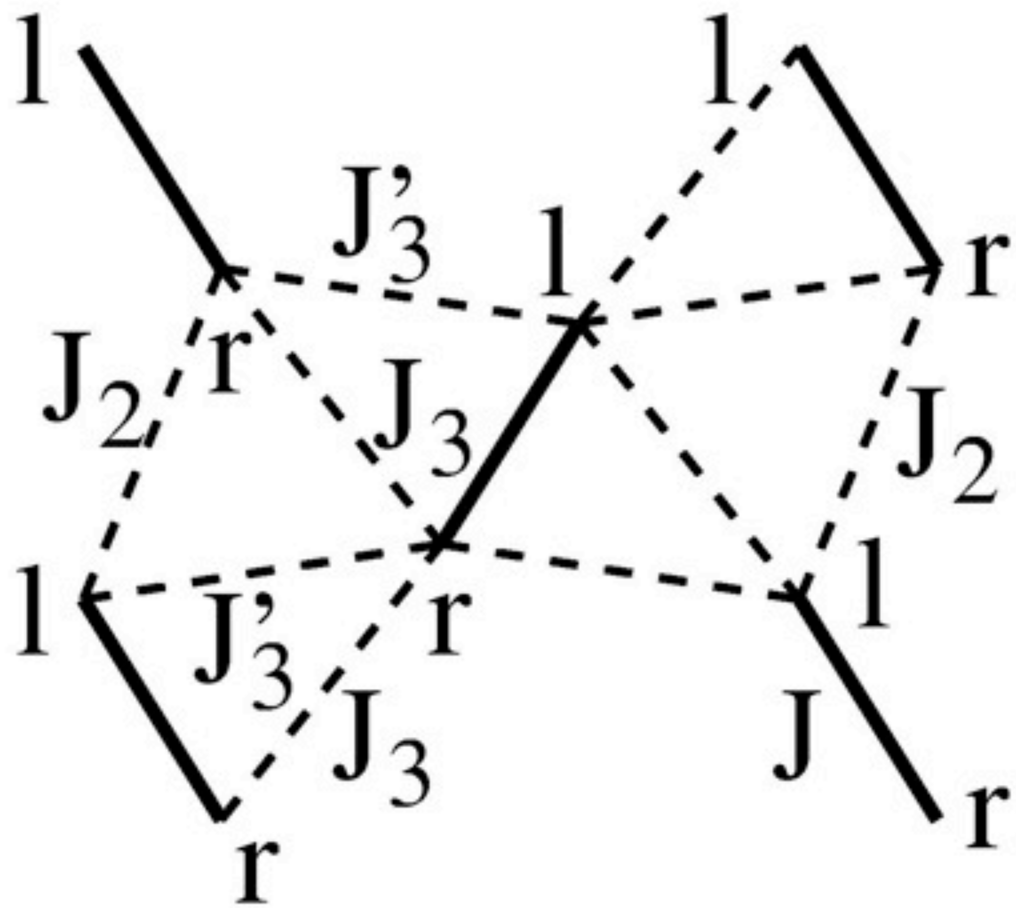


Pressure in $TlCuCl_3$

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

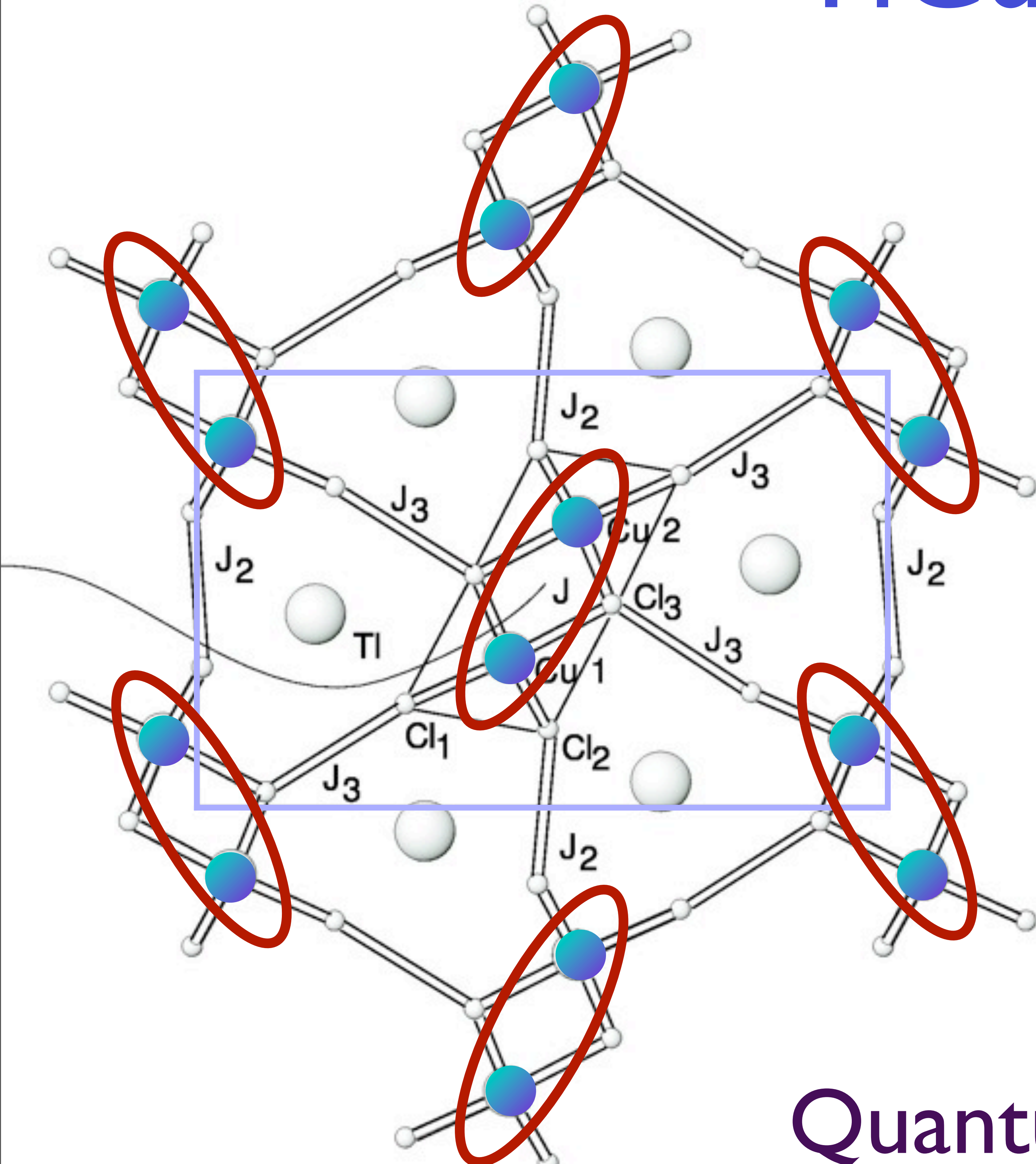
$TlCuCl_3$

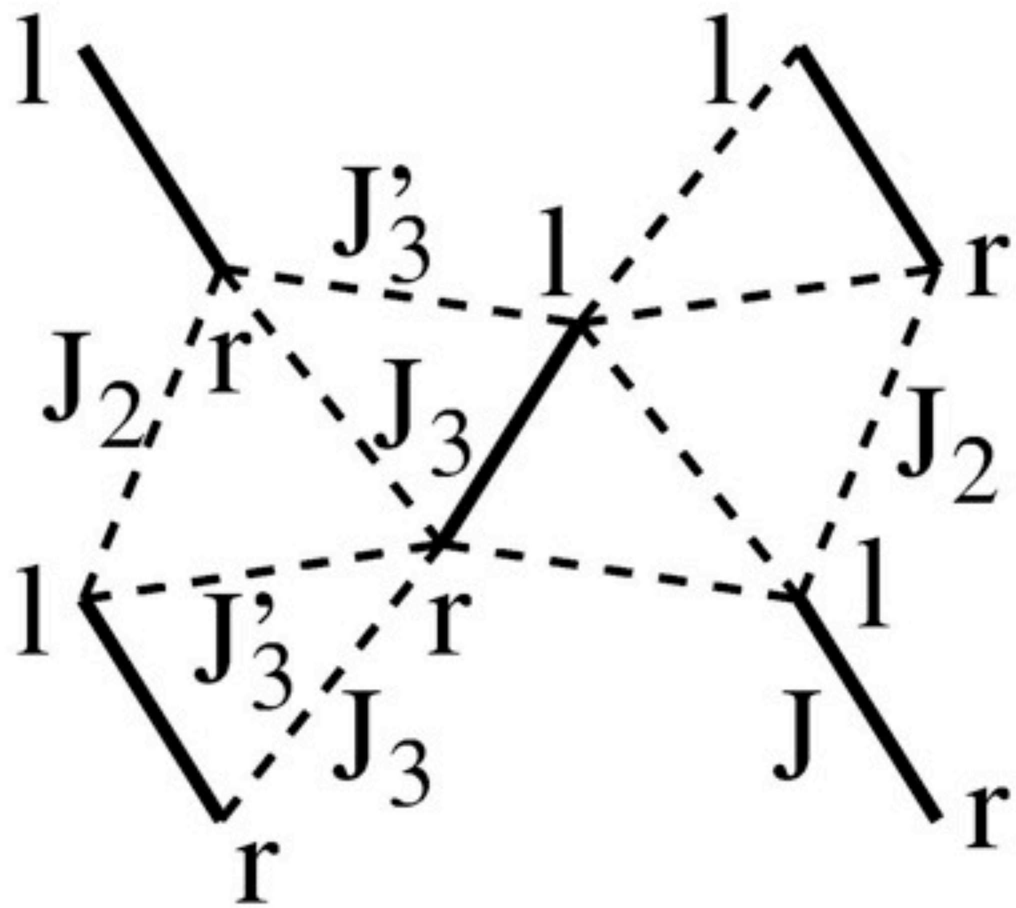




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

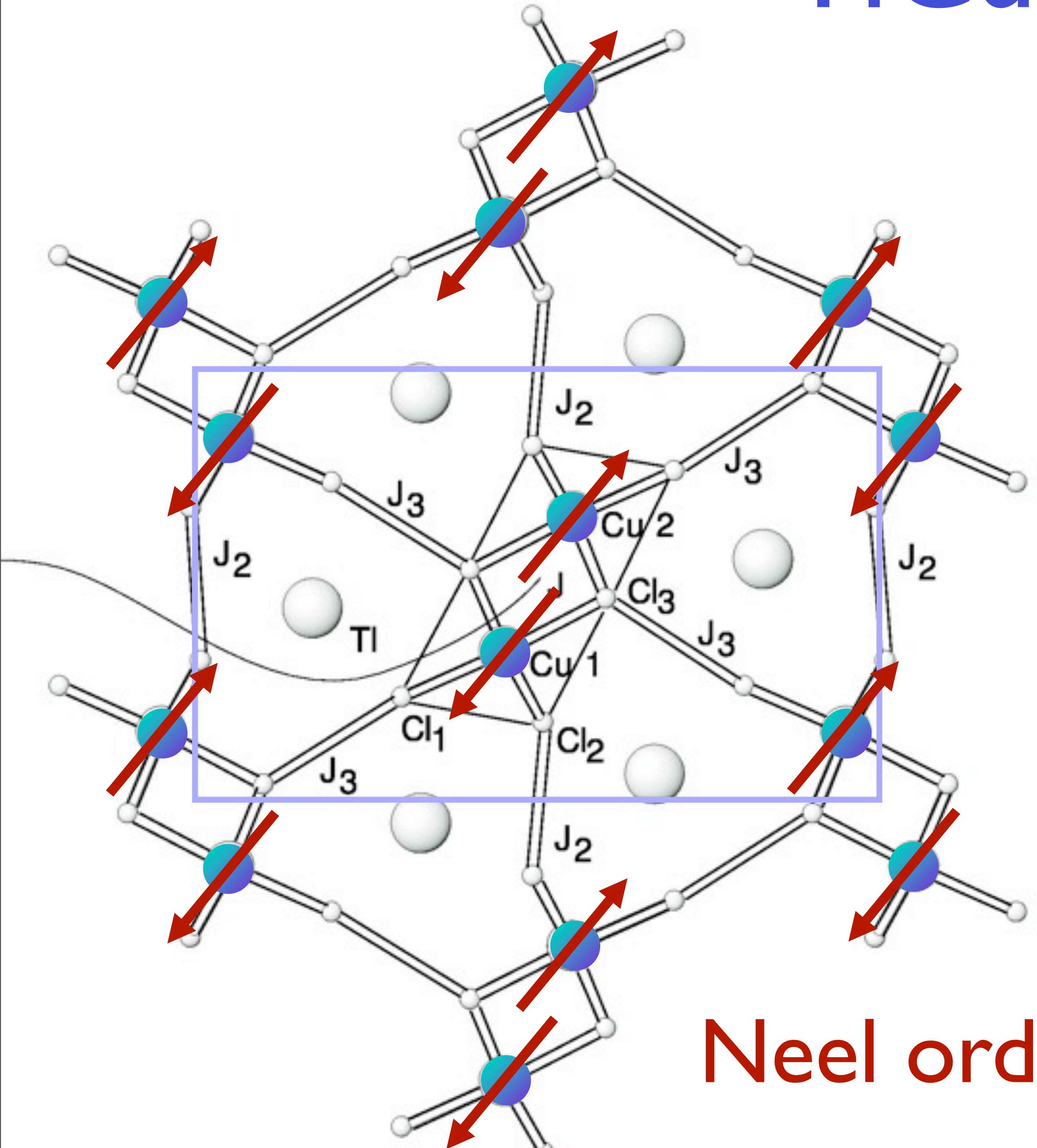
$TlCuCl_3$

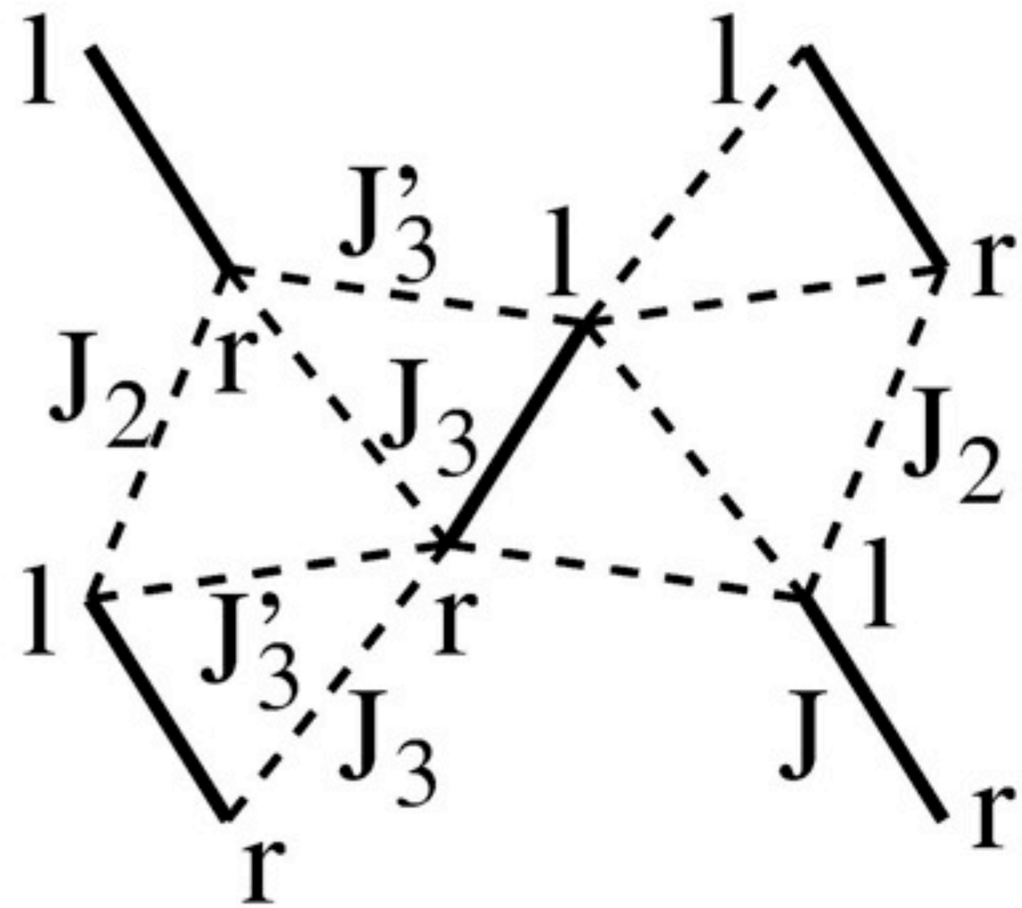




Quantum paramagnet at ambient pressure

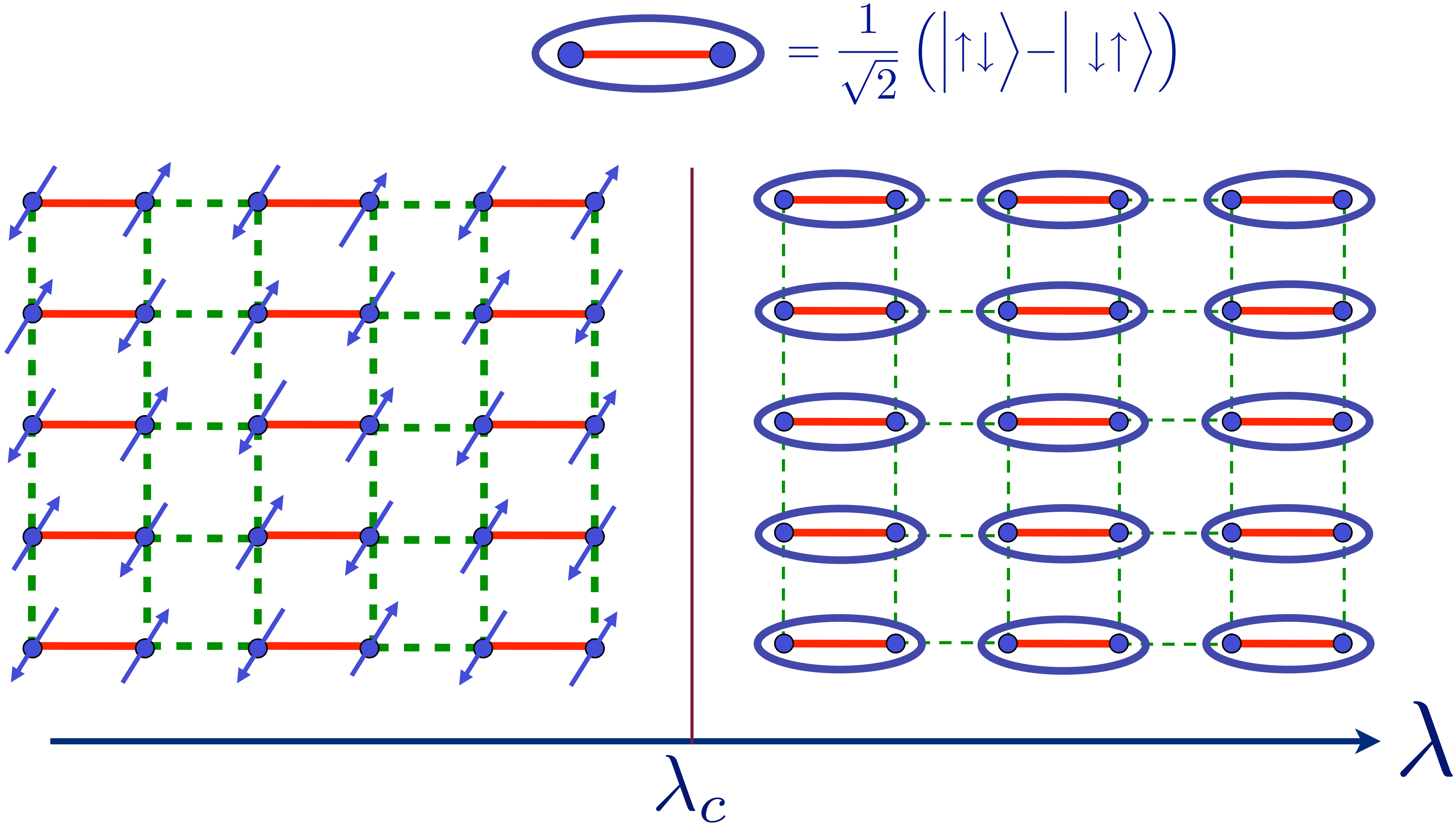
$TlCuCl_3$



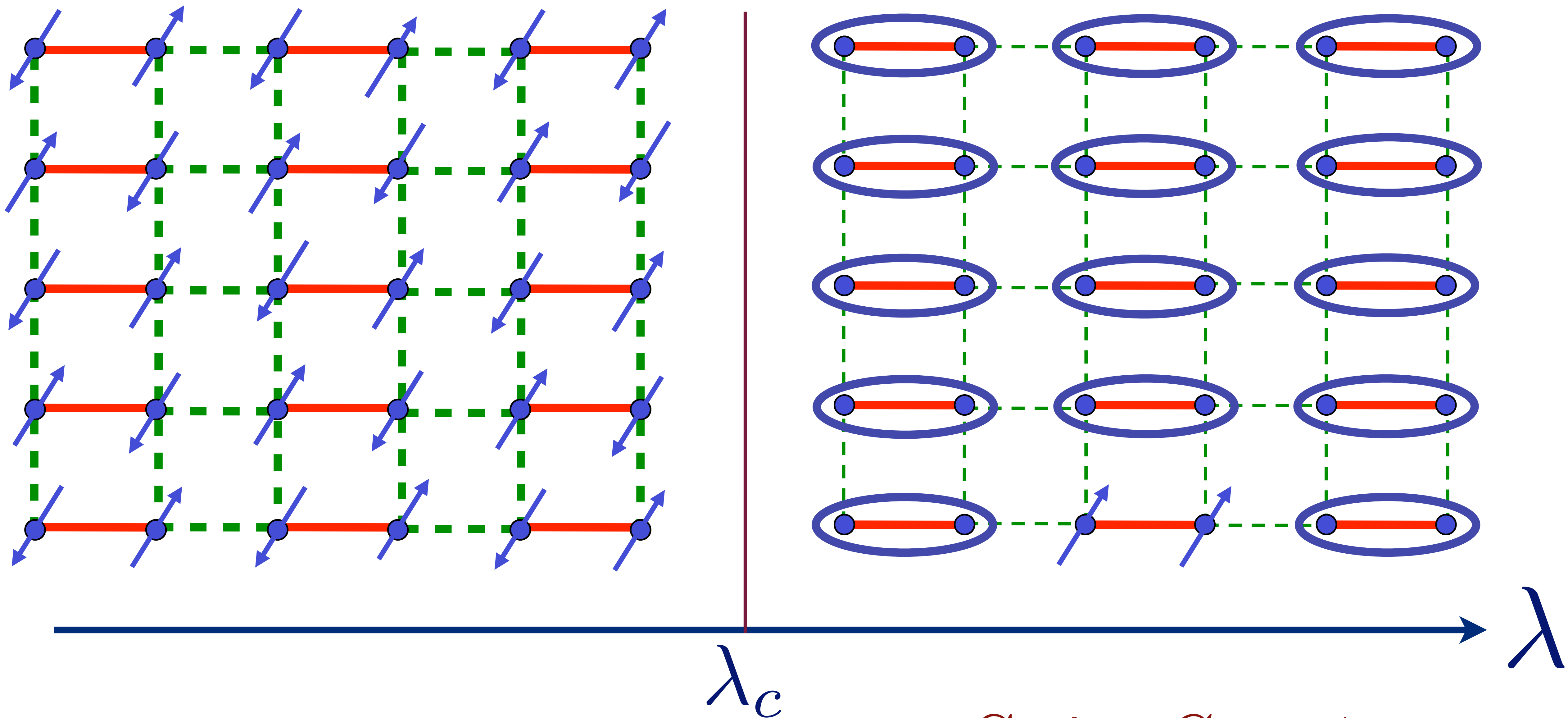


Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).


$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ
λ_c

Excitation spectrum in the paramagnetic phase



Spin $S = 1$
"triplon"

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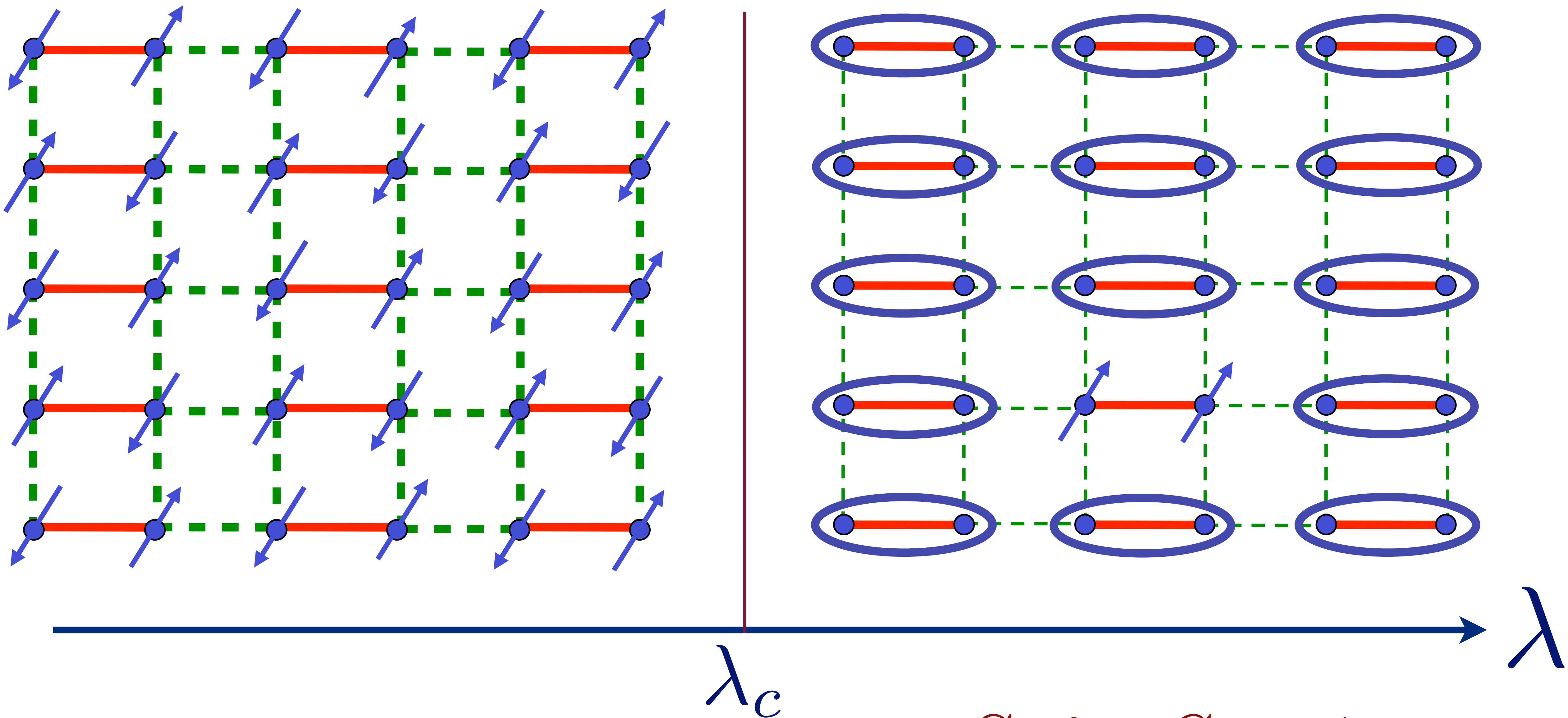

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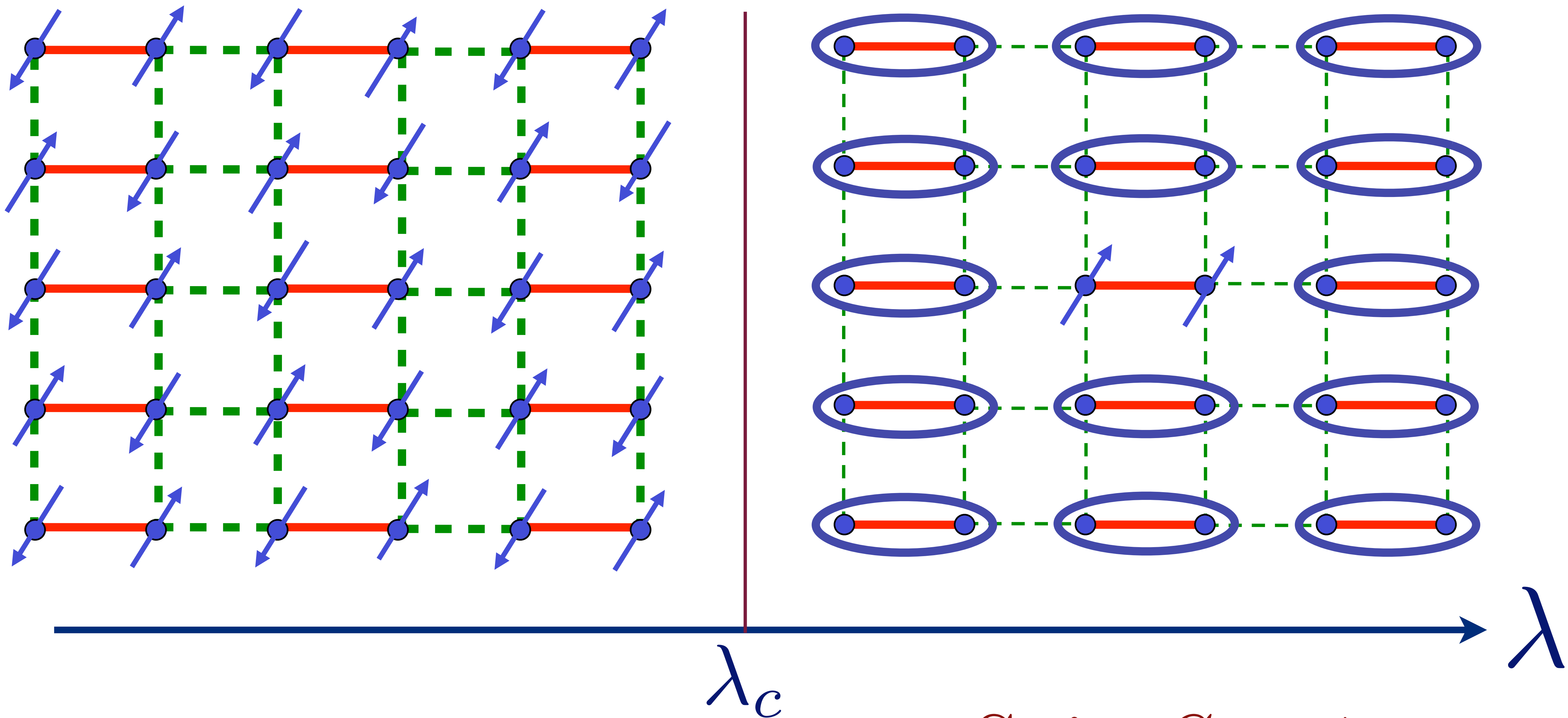

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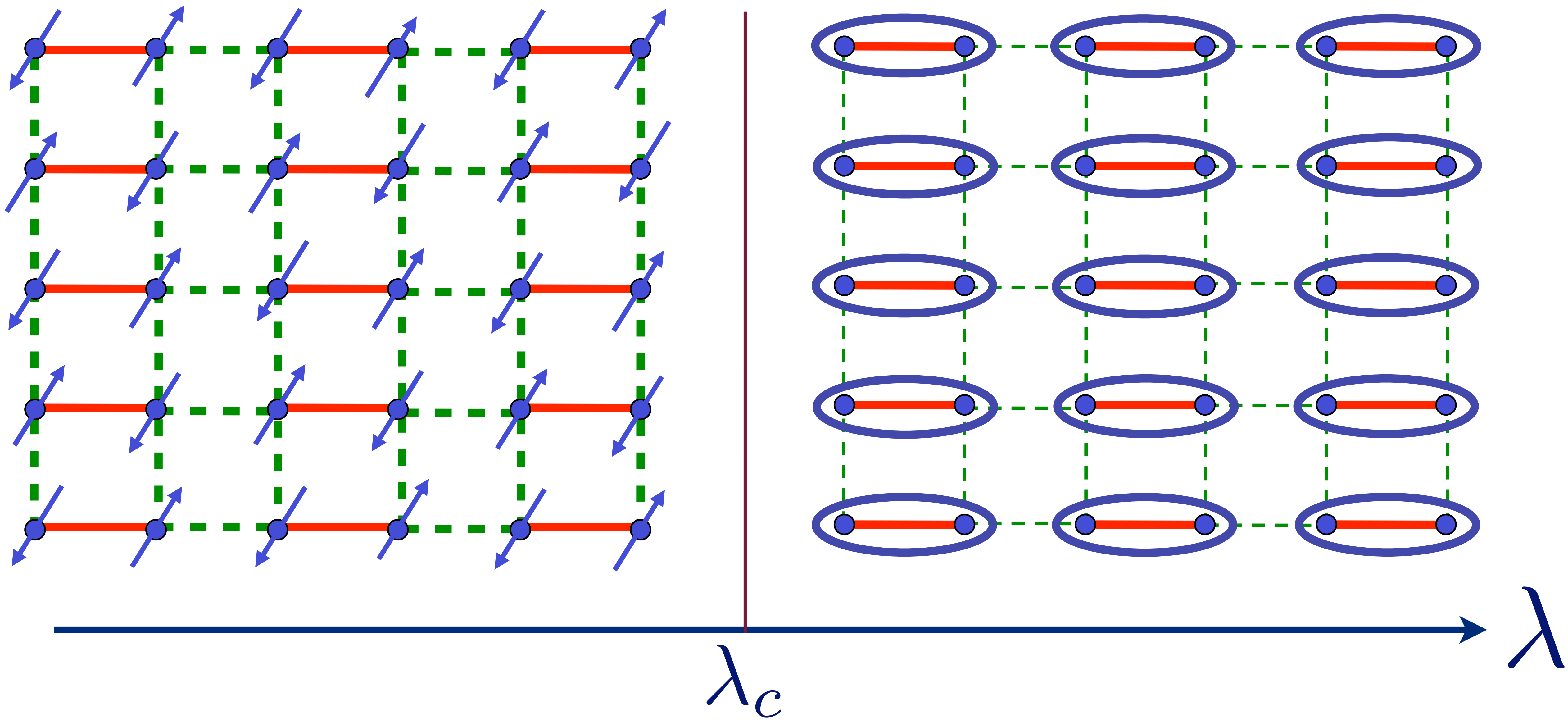
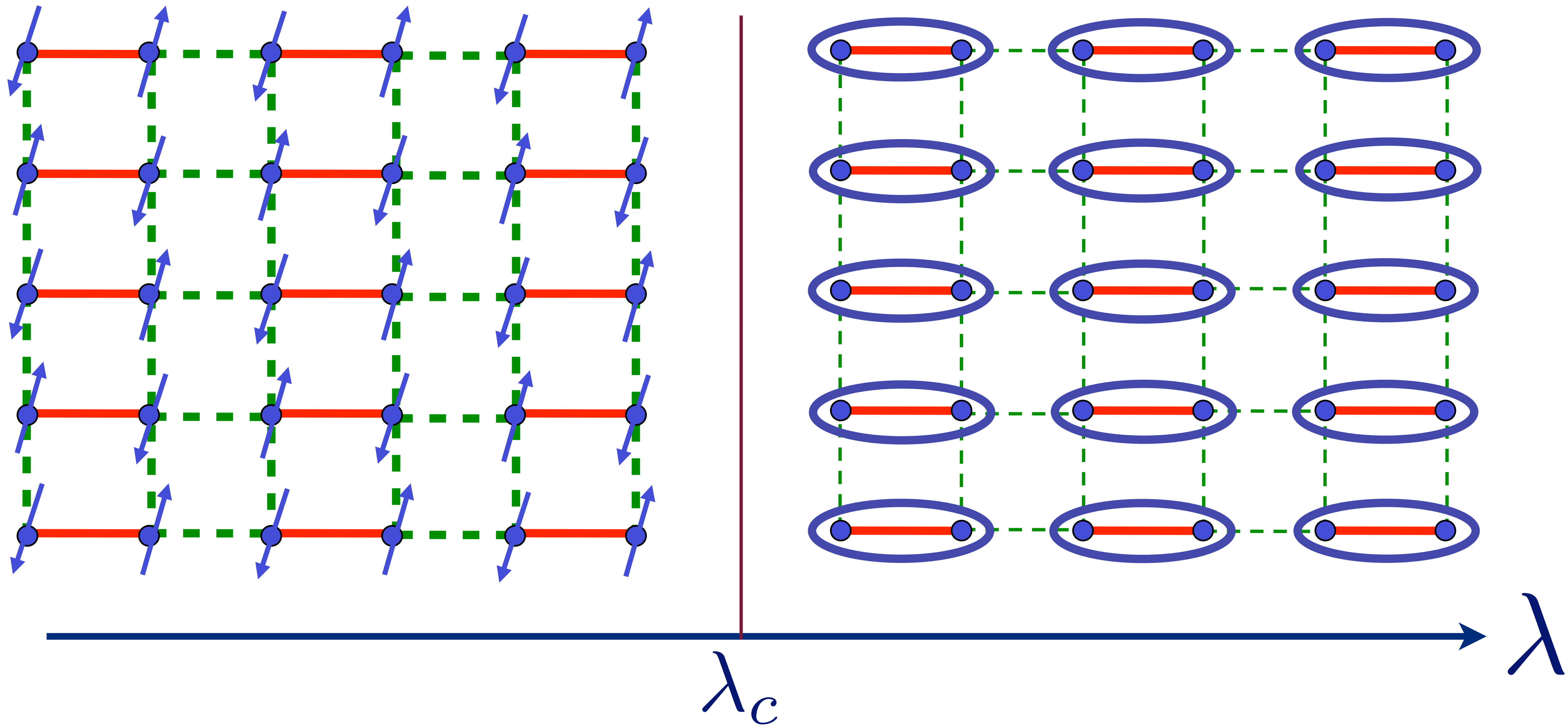
Excitation spectrum in the Néel phase



Spin waves

Excitation spectrum in the Néel phase



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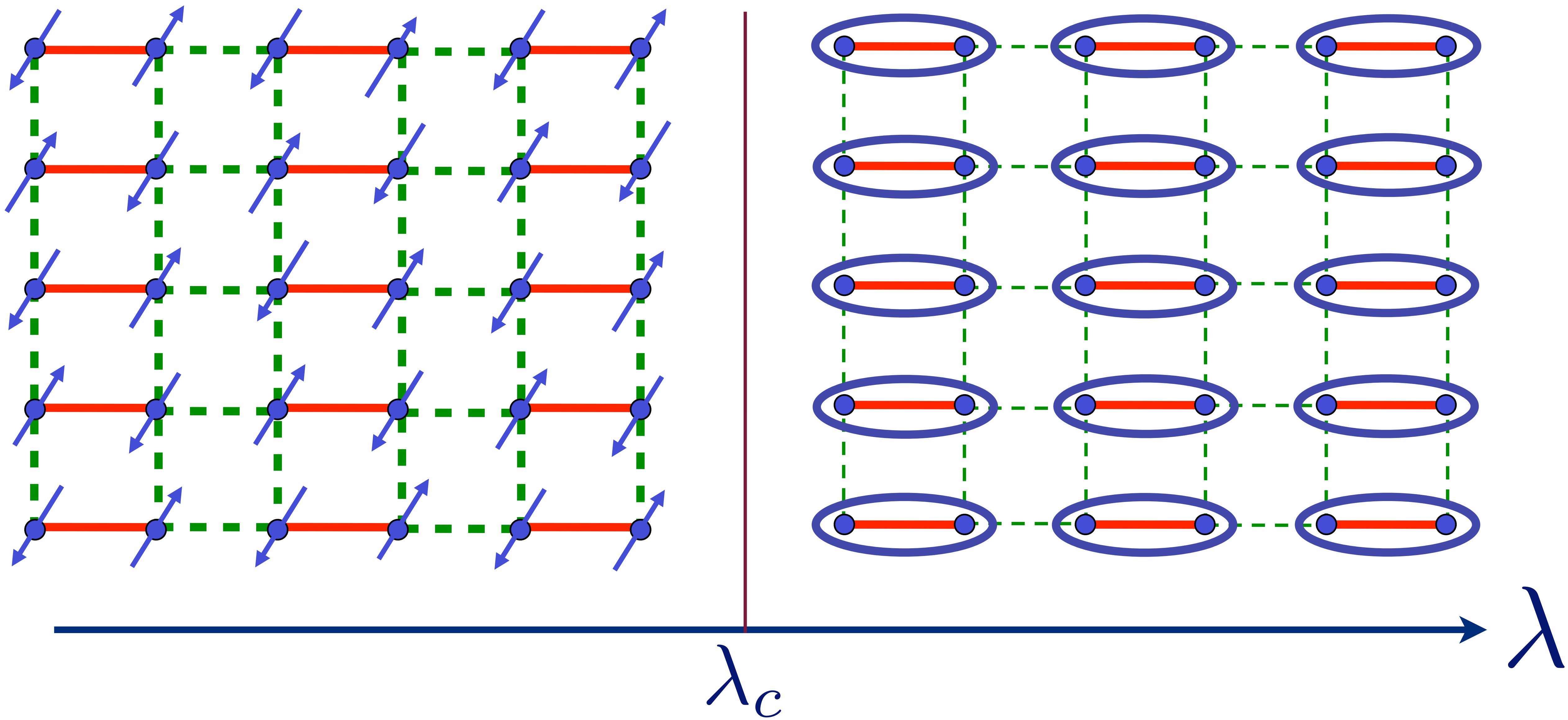
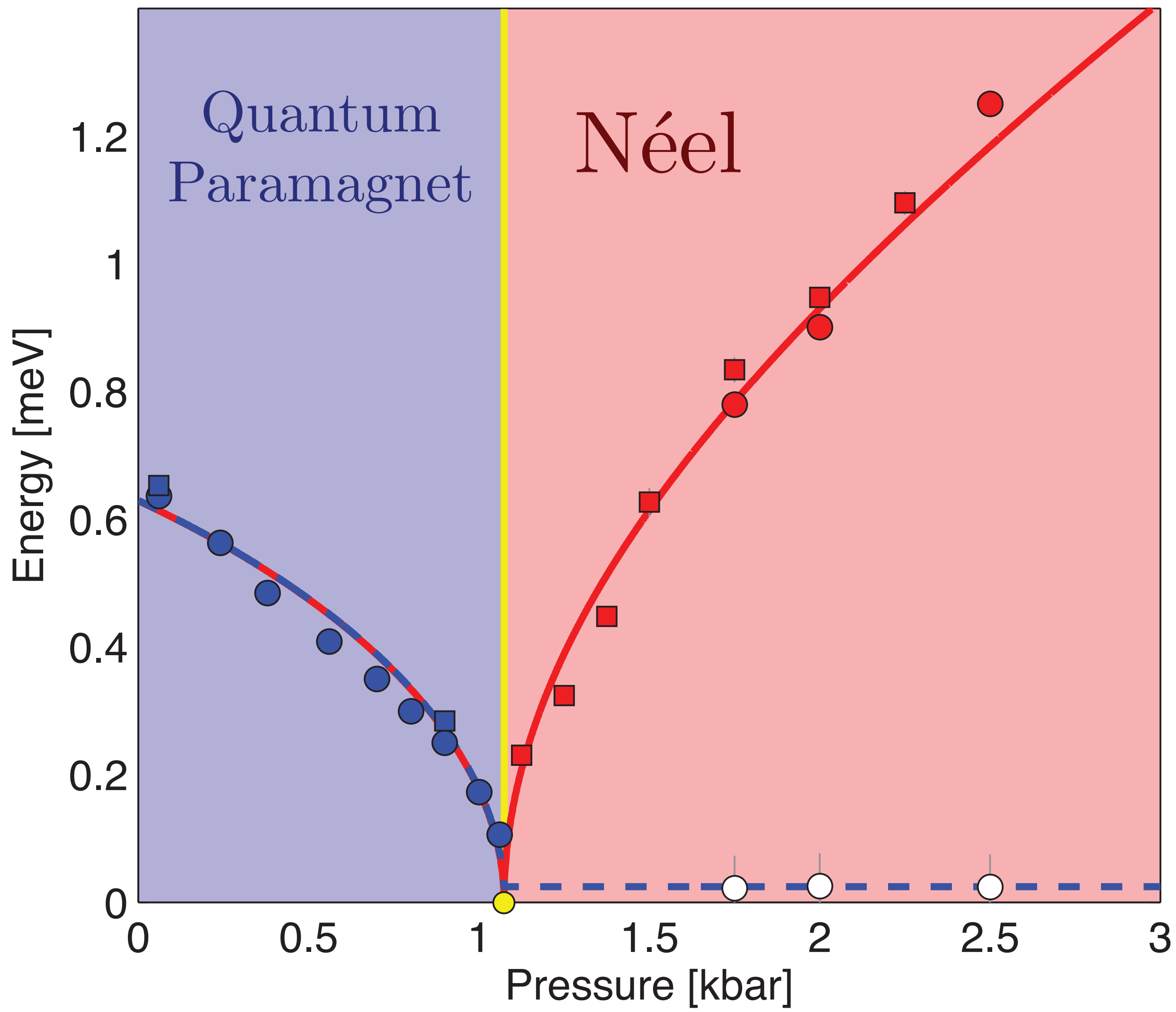


Spin waves

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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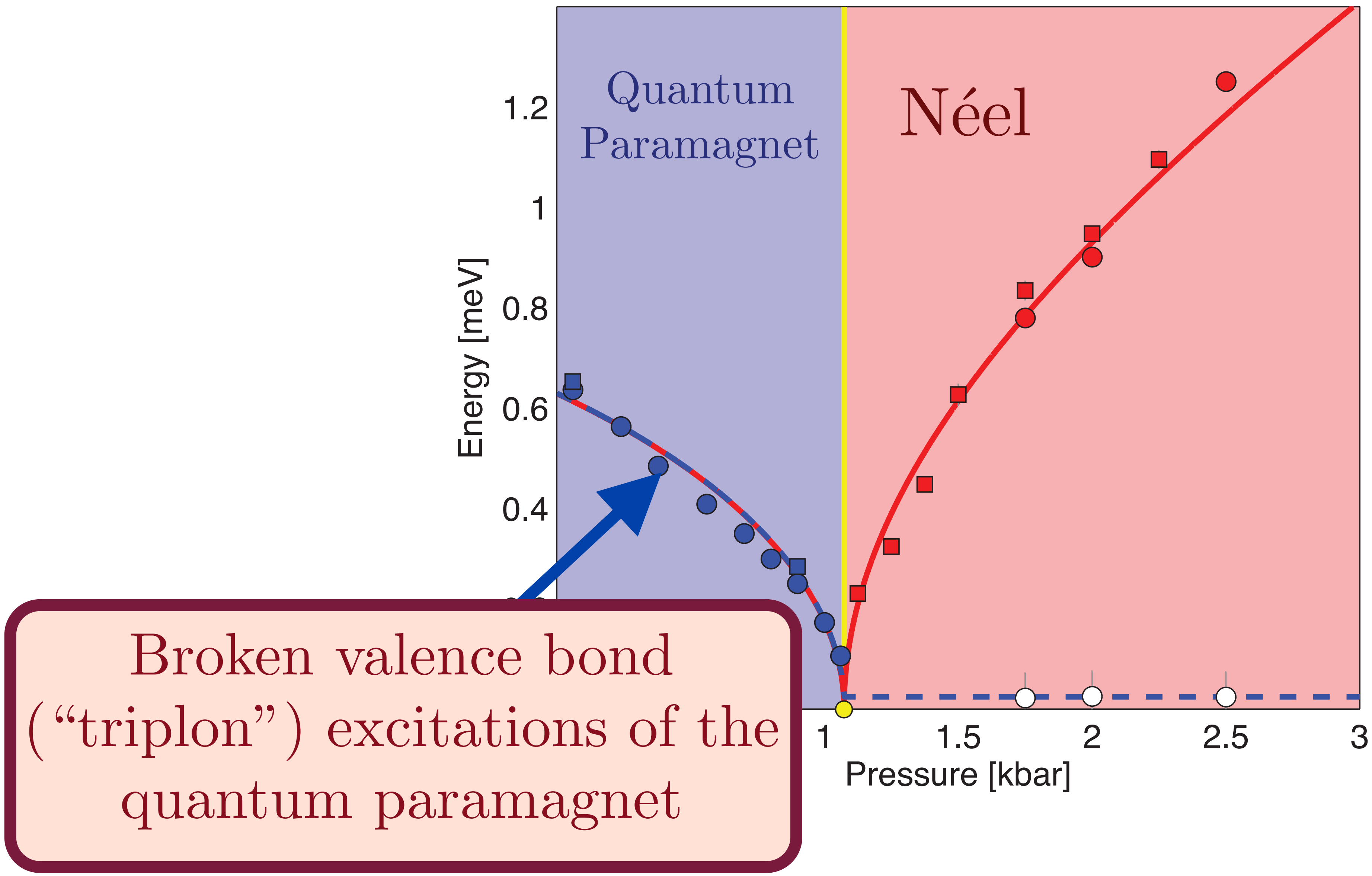

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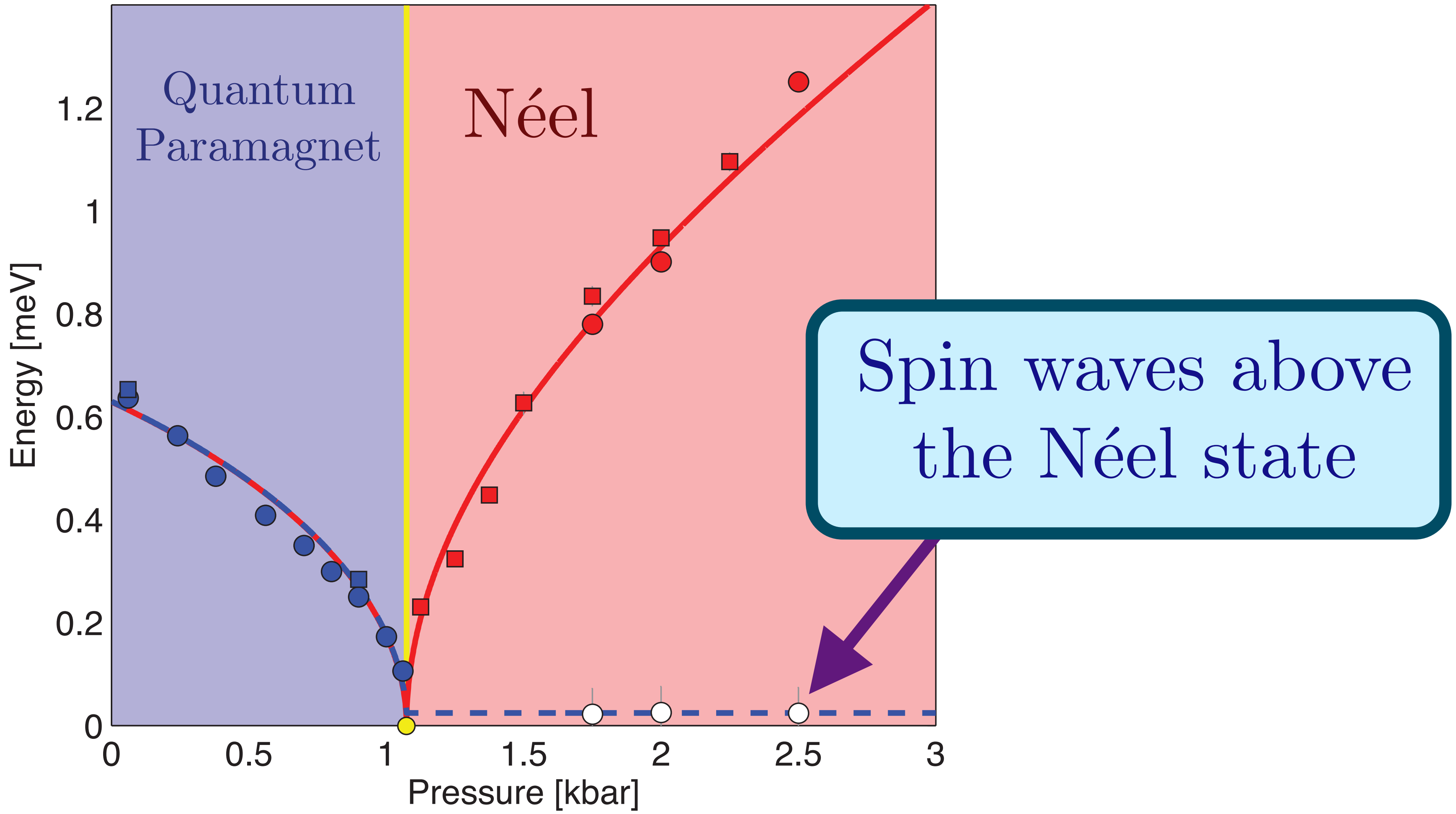

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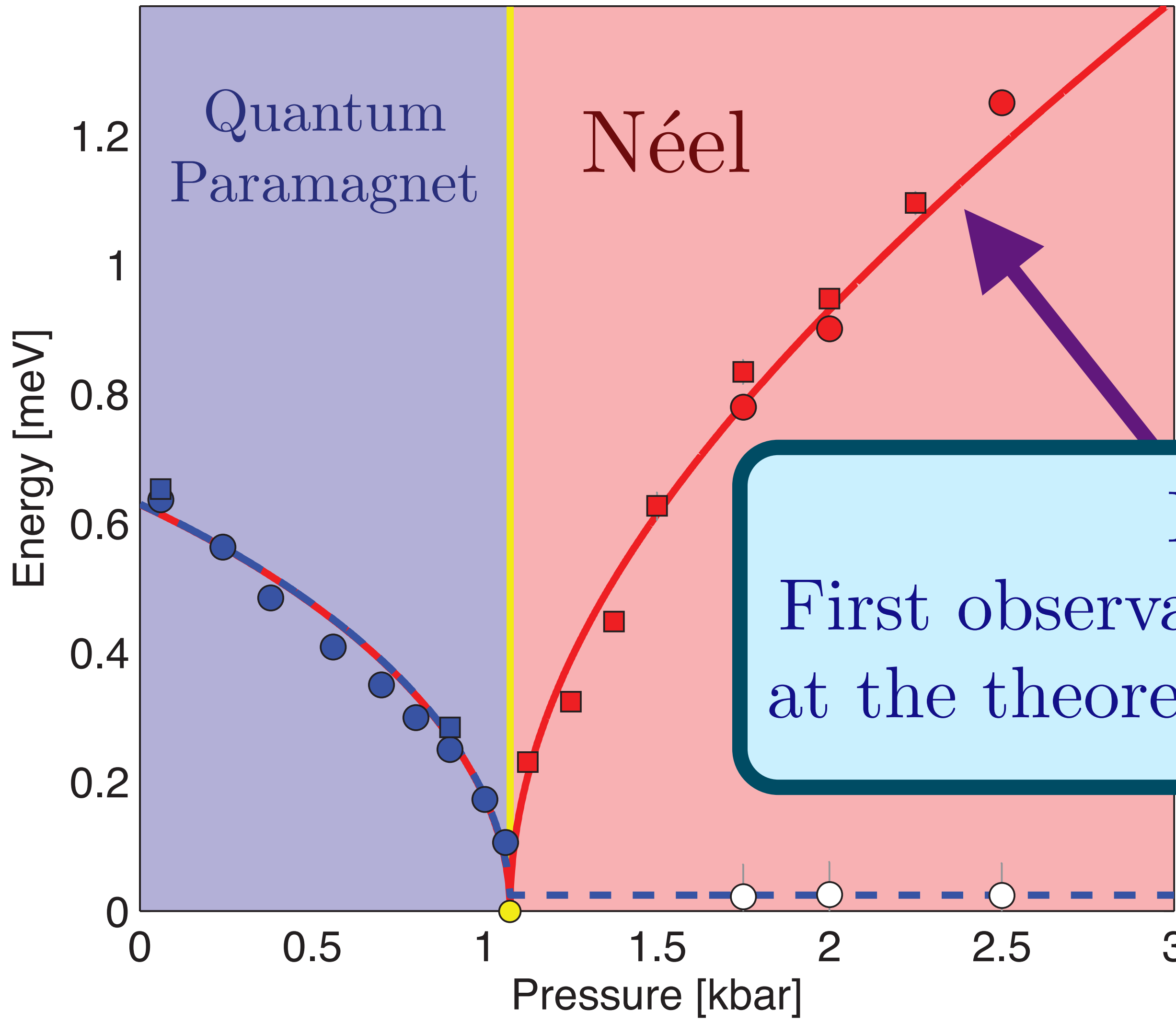
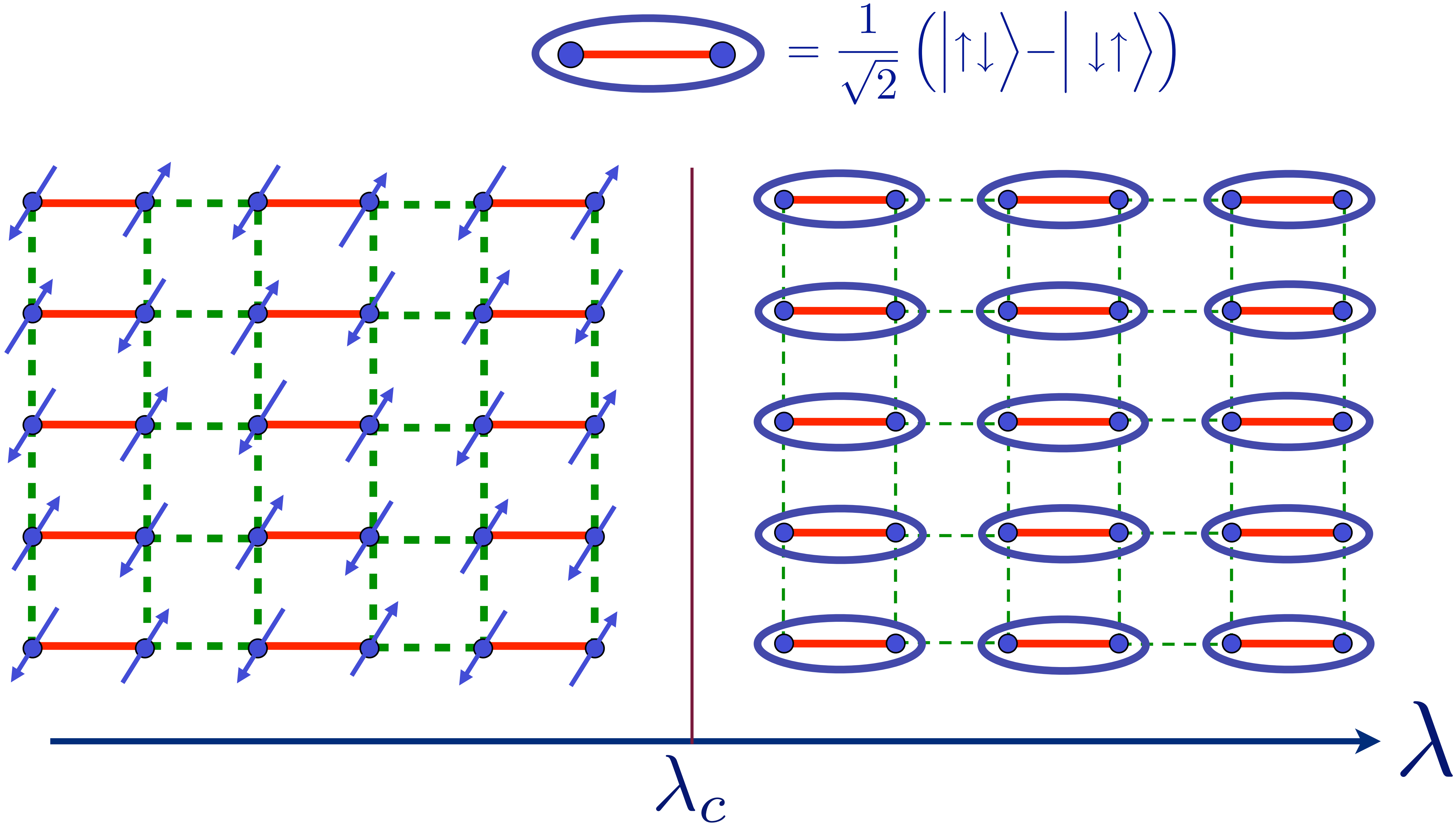


Higgs boson
First observation of the Higgs boson
at the theoretically predicted energy!

S. Sachdev,
arXiv:0901.4103

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)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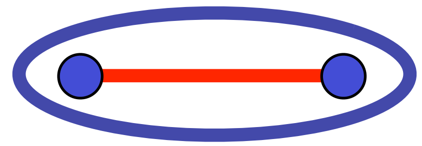

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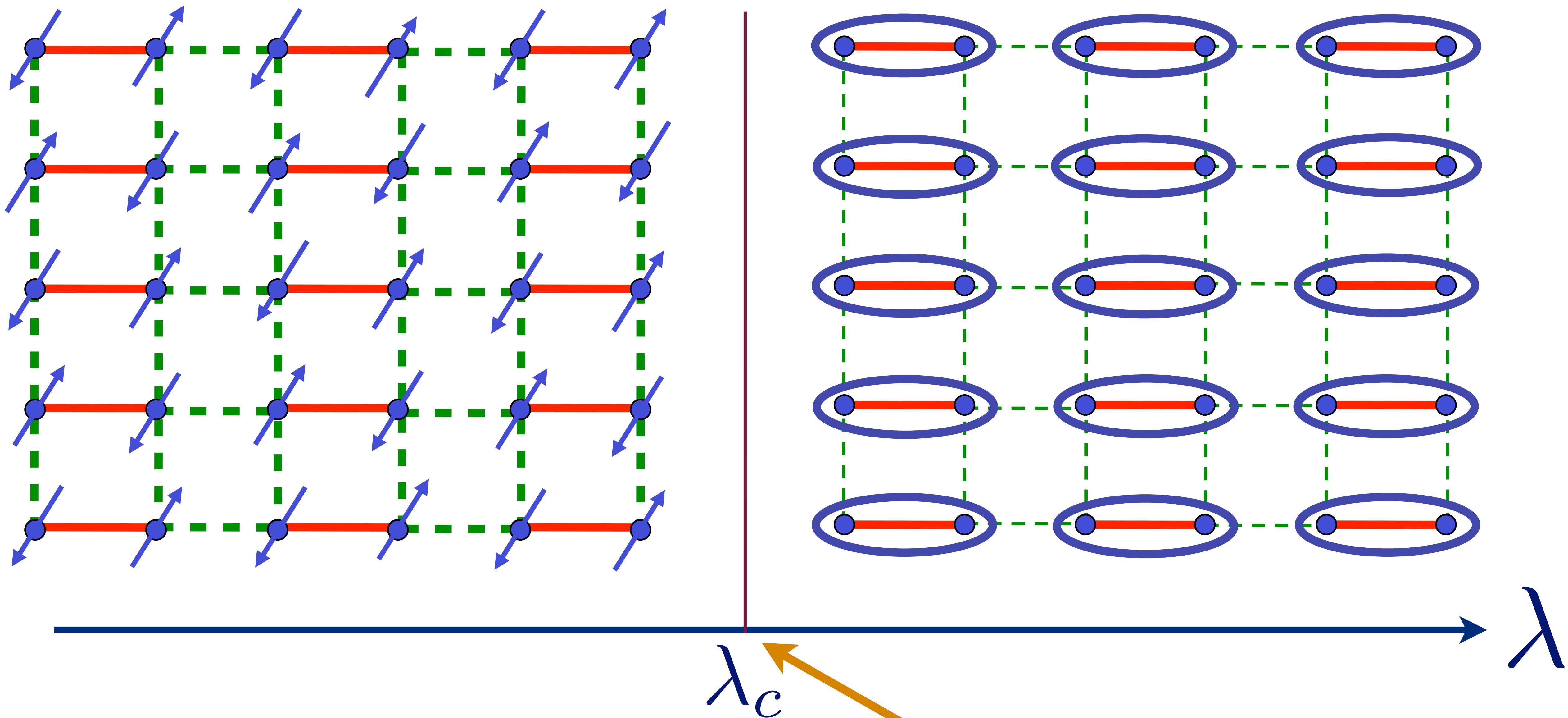
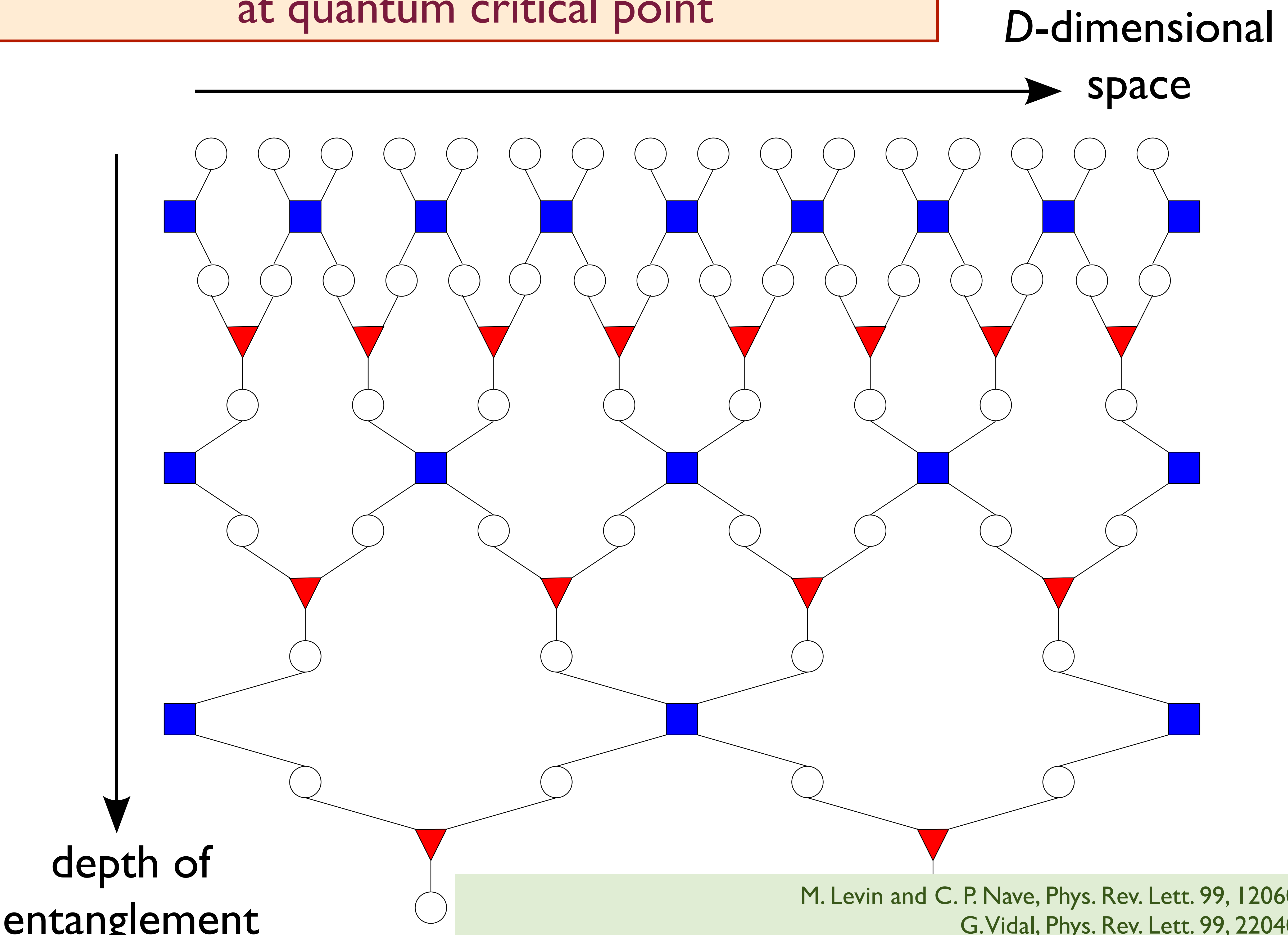


Quantum critical point with non-local entanglement in spin wavefunction

A. W. Sandvik and D. J. Scalapino, Phys. Rev. Lett. **72**, 2777 (1994).

Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

Characteristics of quantum critical point

- Long-range entanglement

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Quantum superposition and entanglement

Quantum critical points of electrons in crystals

String theory and black holes

Quantum superposition and entanglement

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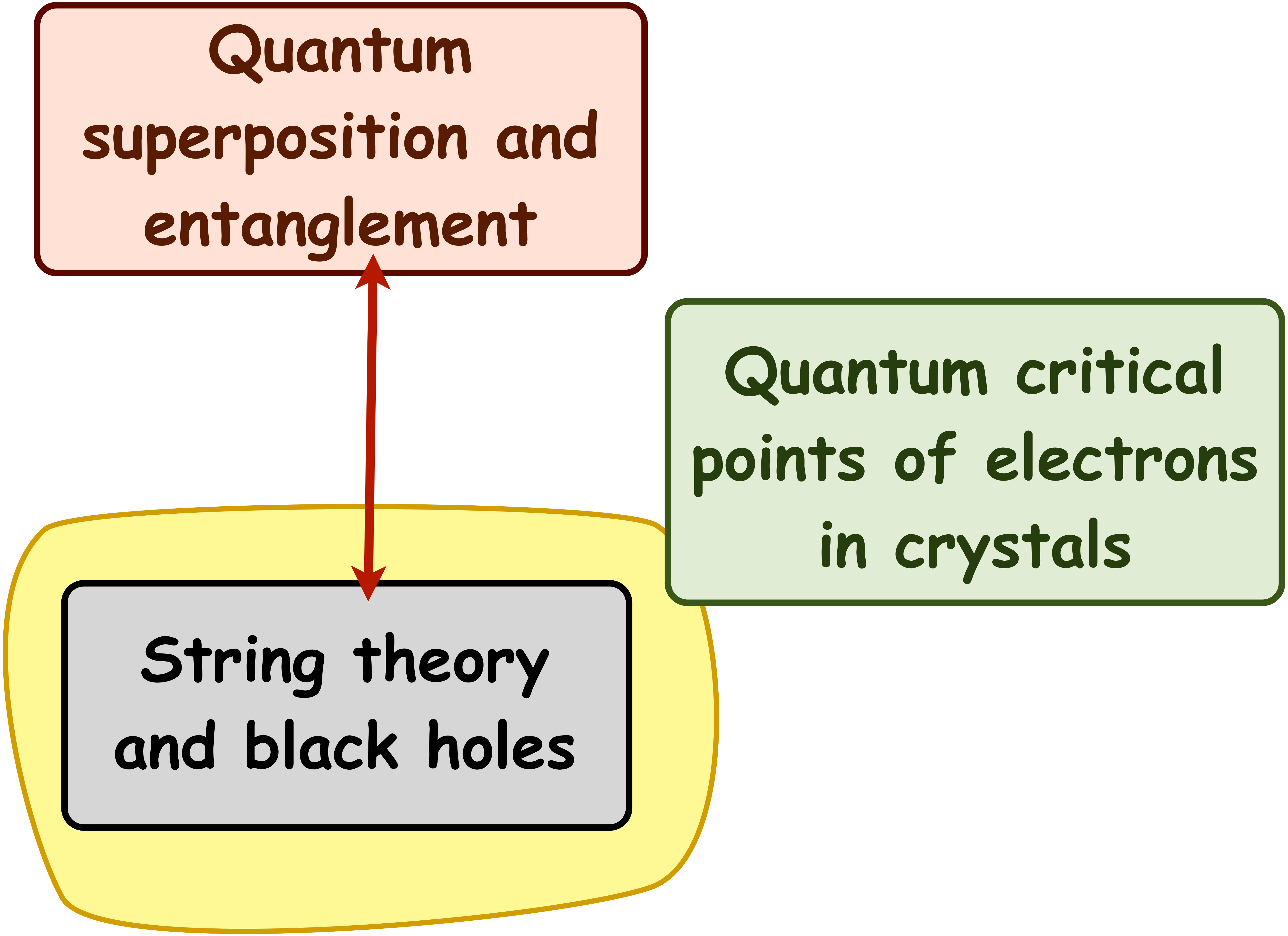
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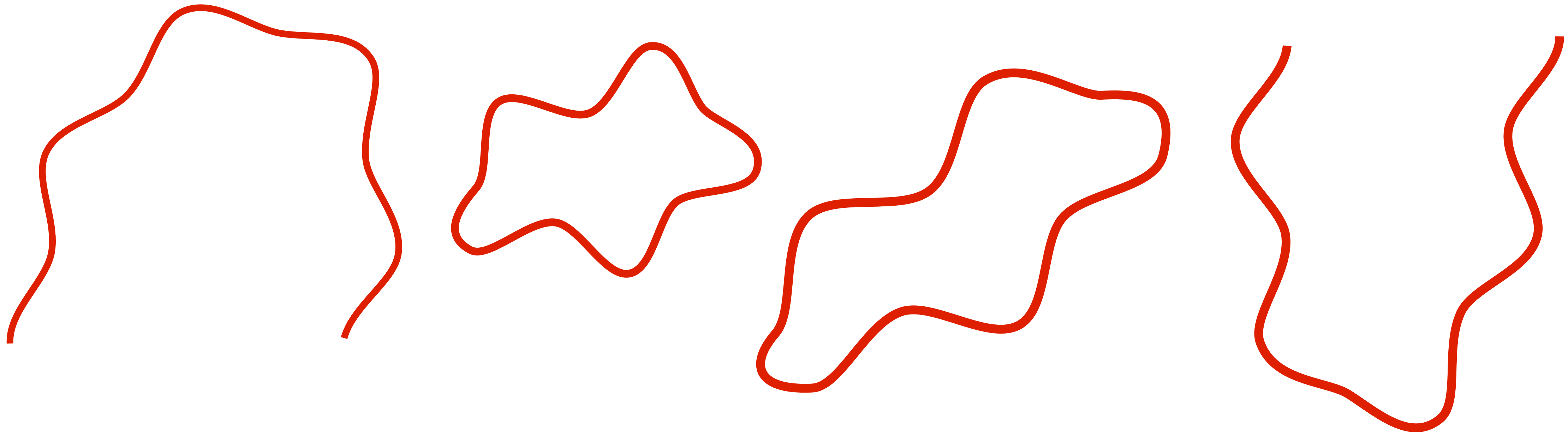
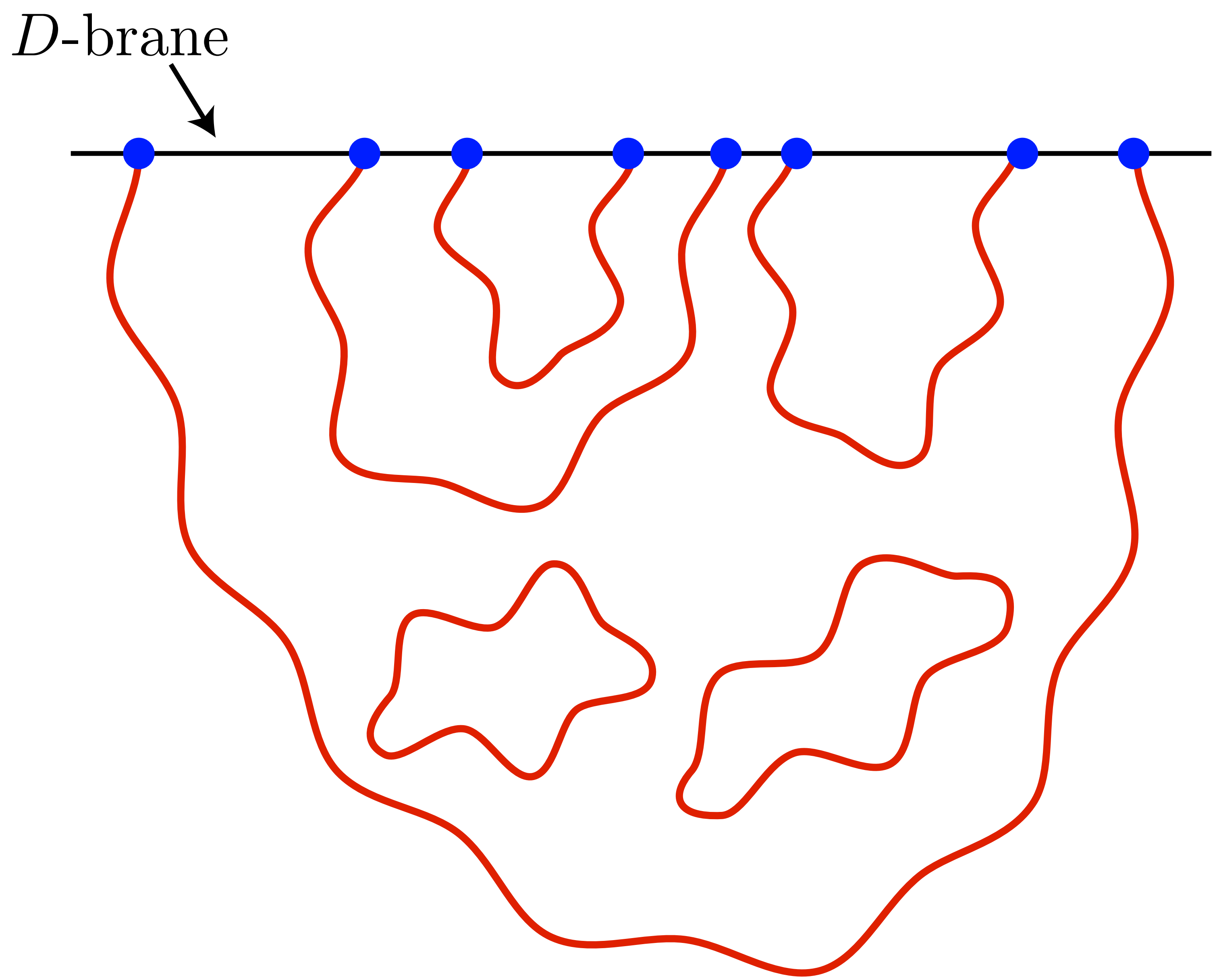
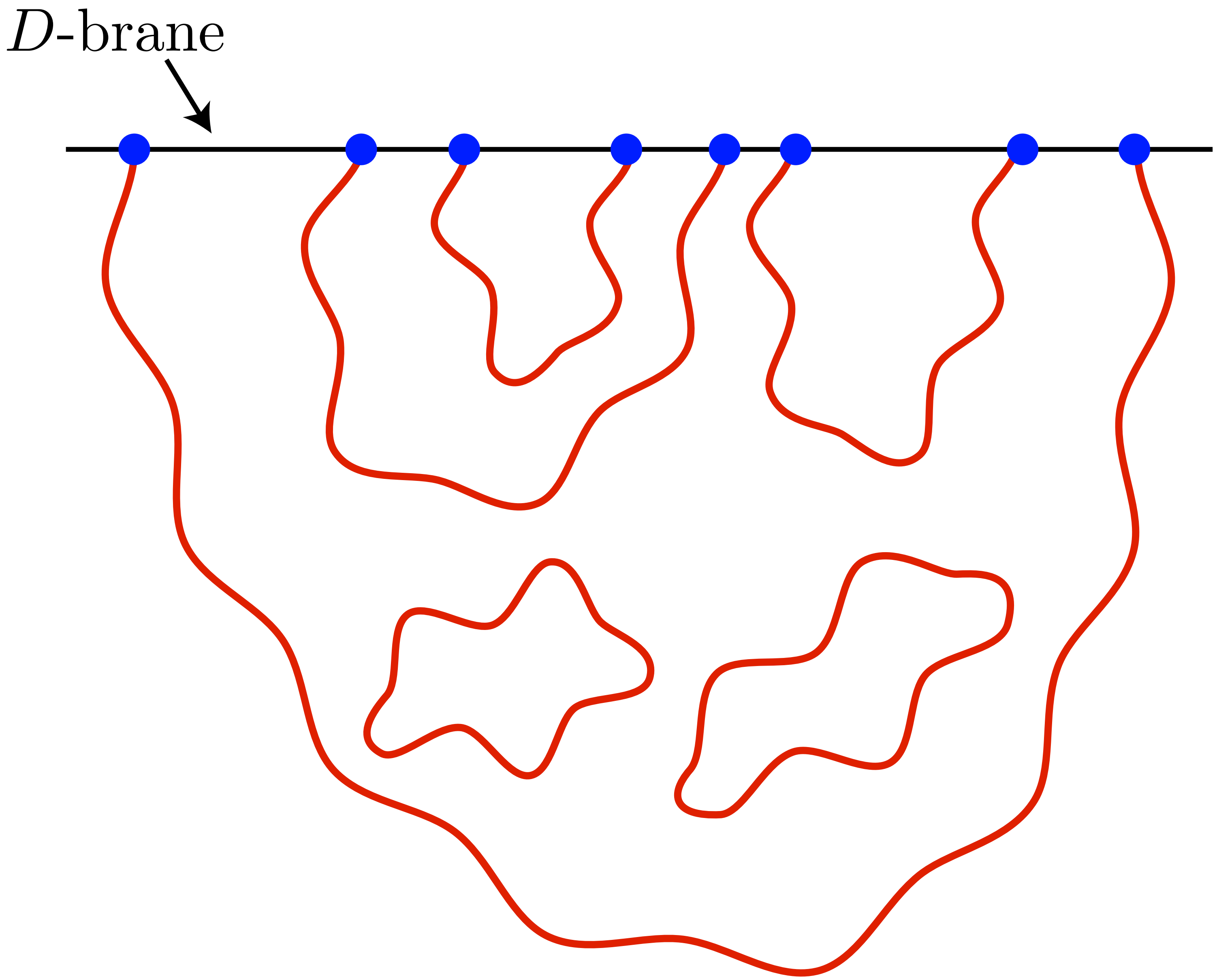
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



- A D-brane is a d-dimensional surface on which strings can end.



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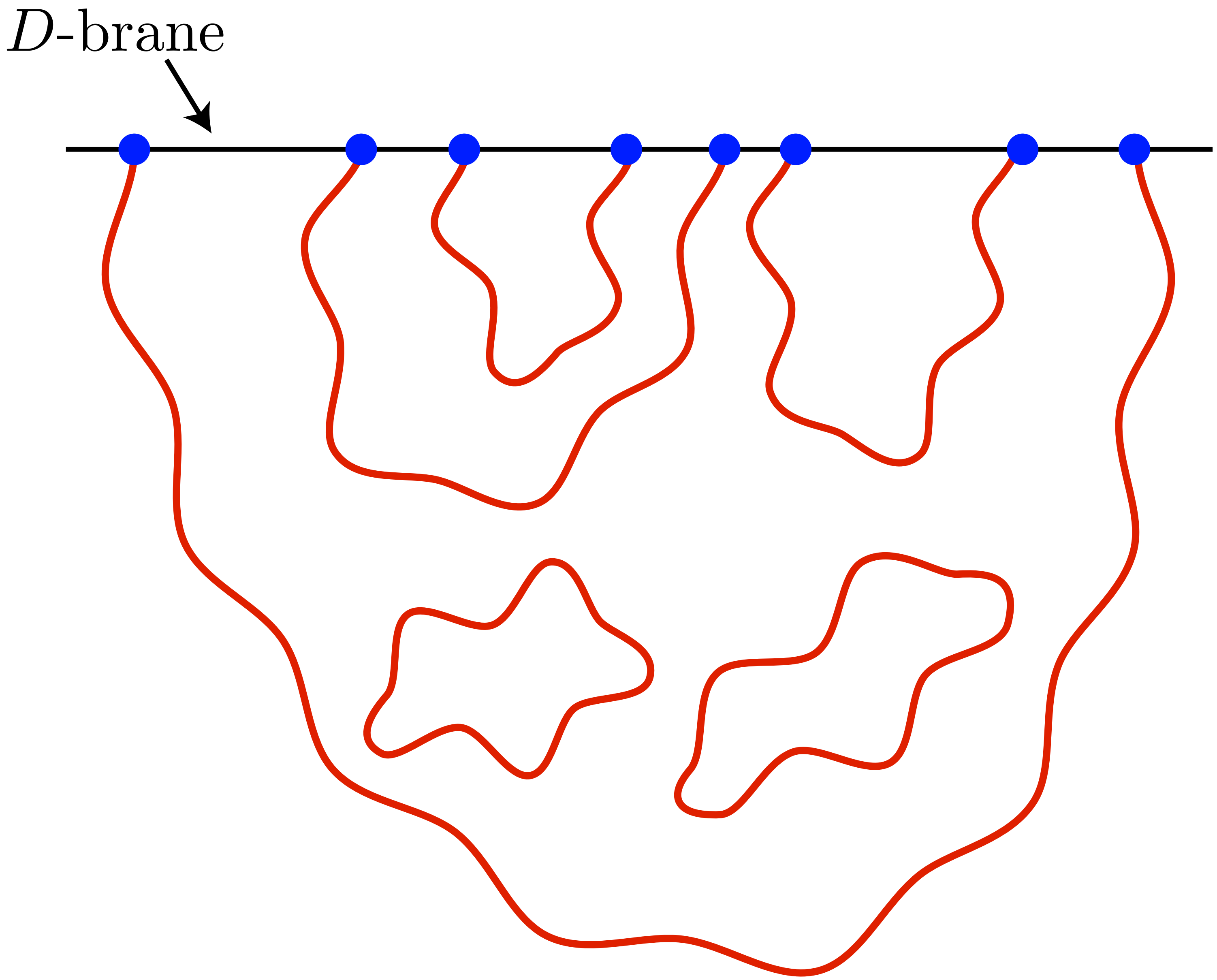
- The low-energy theory on a D-brane has no gravity, similar to theories of entangled electrons of interest to us.



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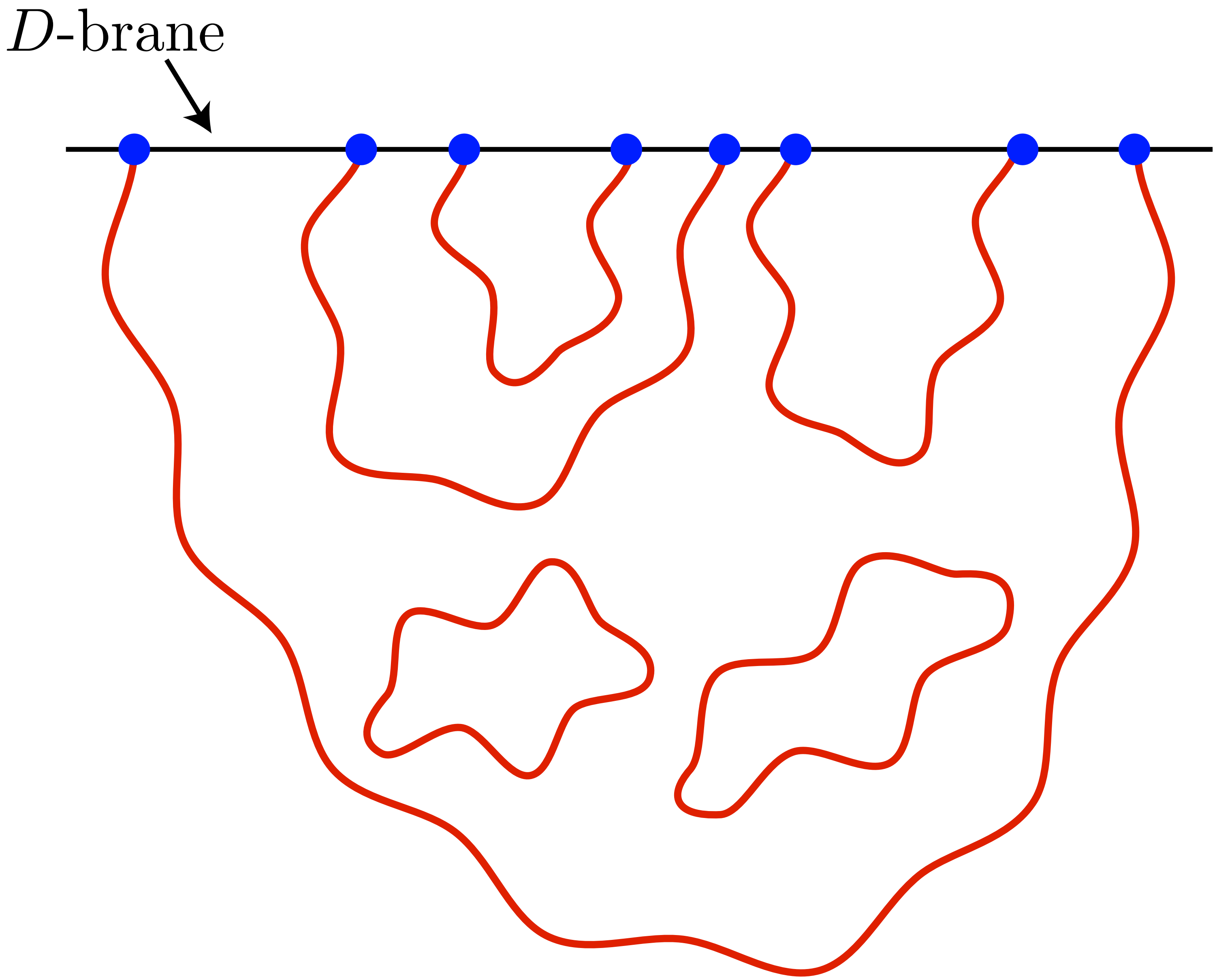

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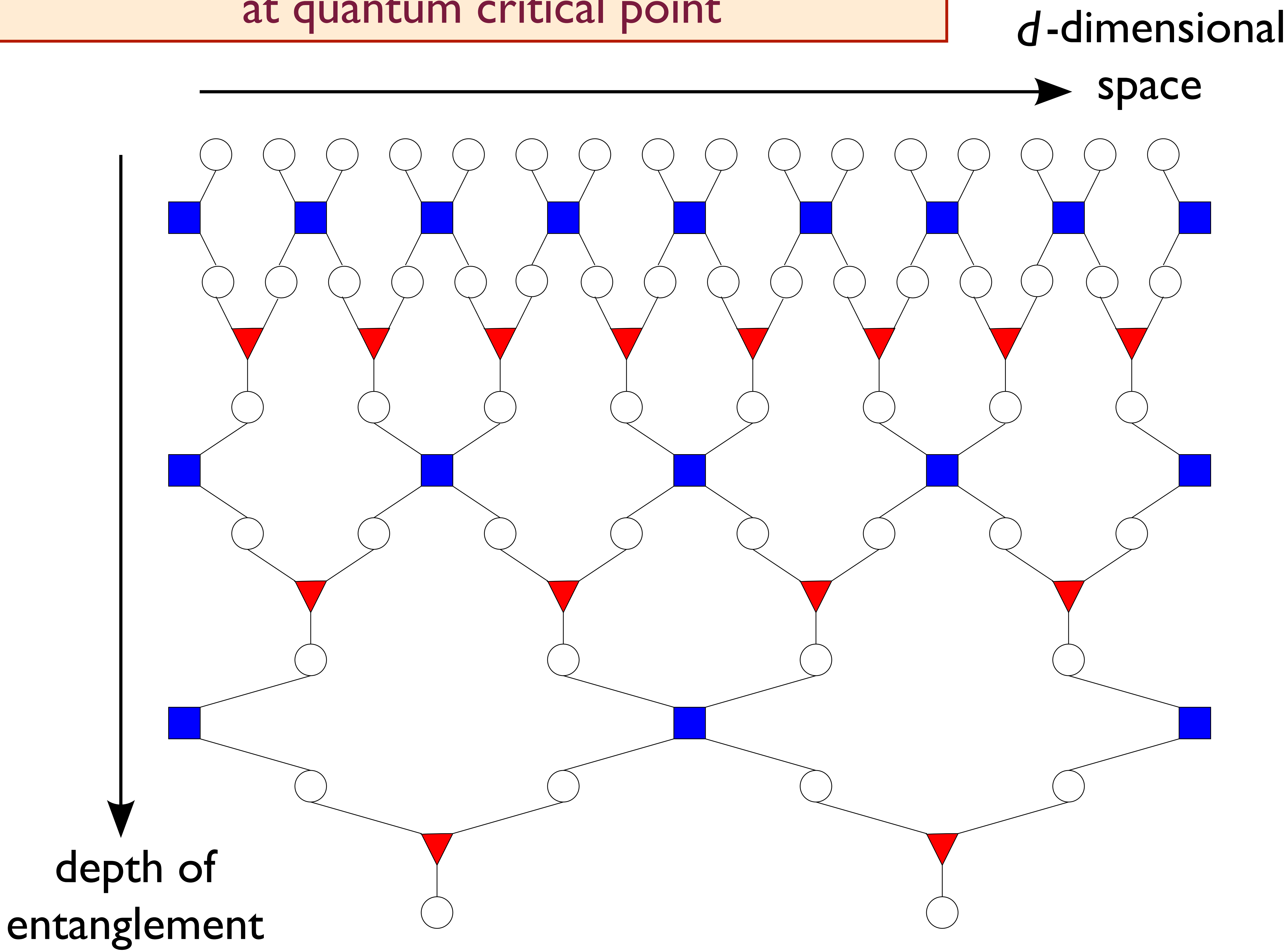
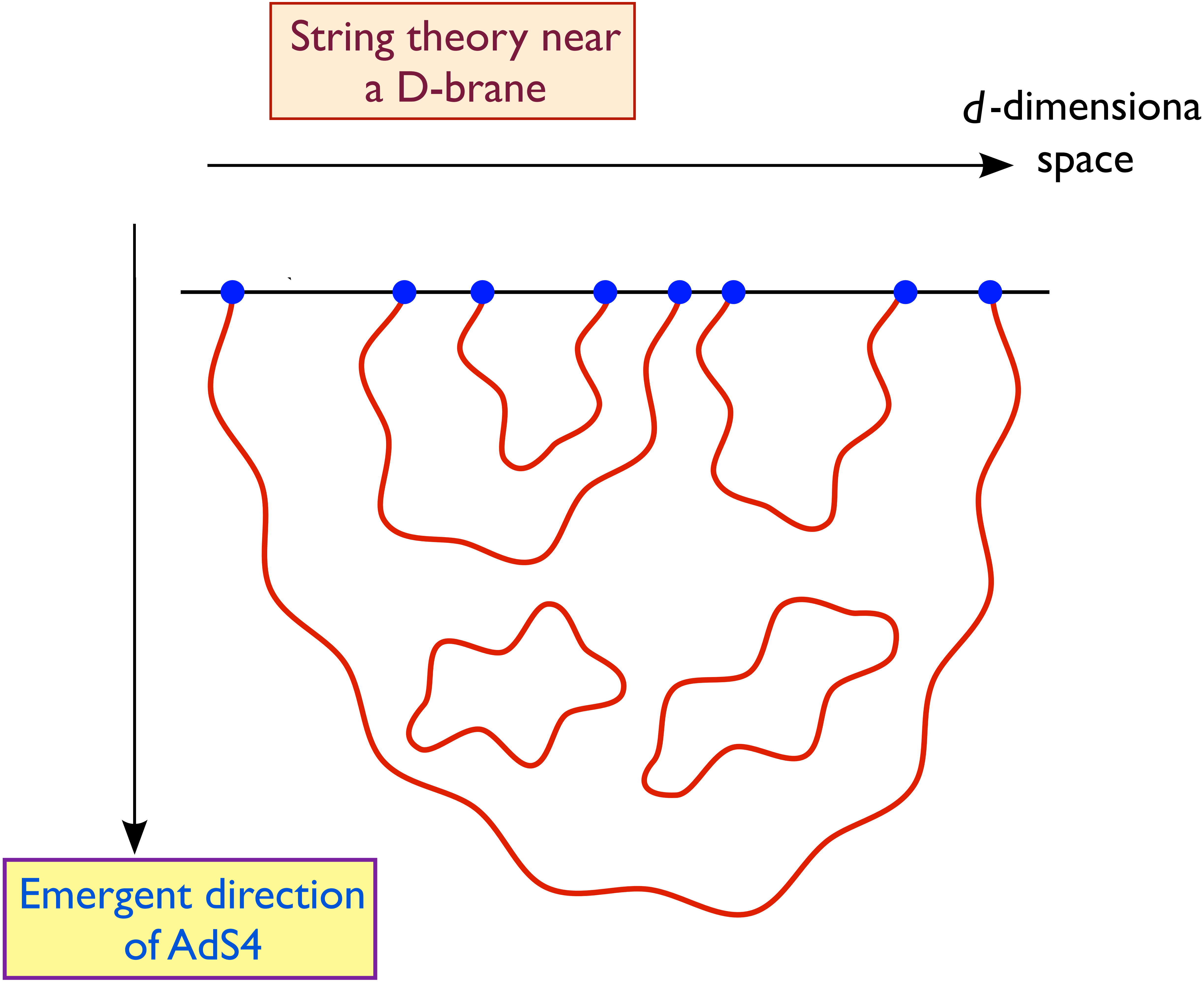
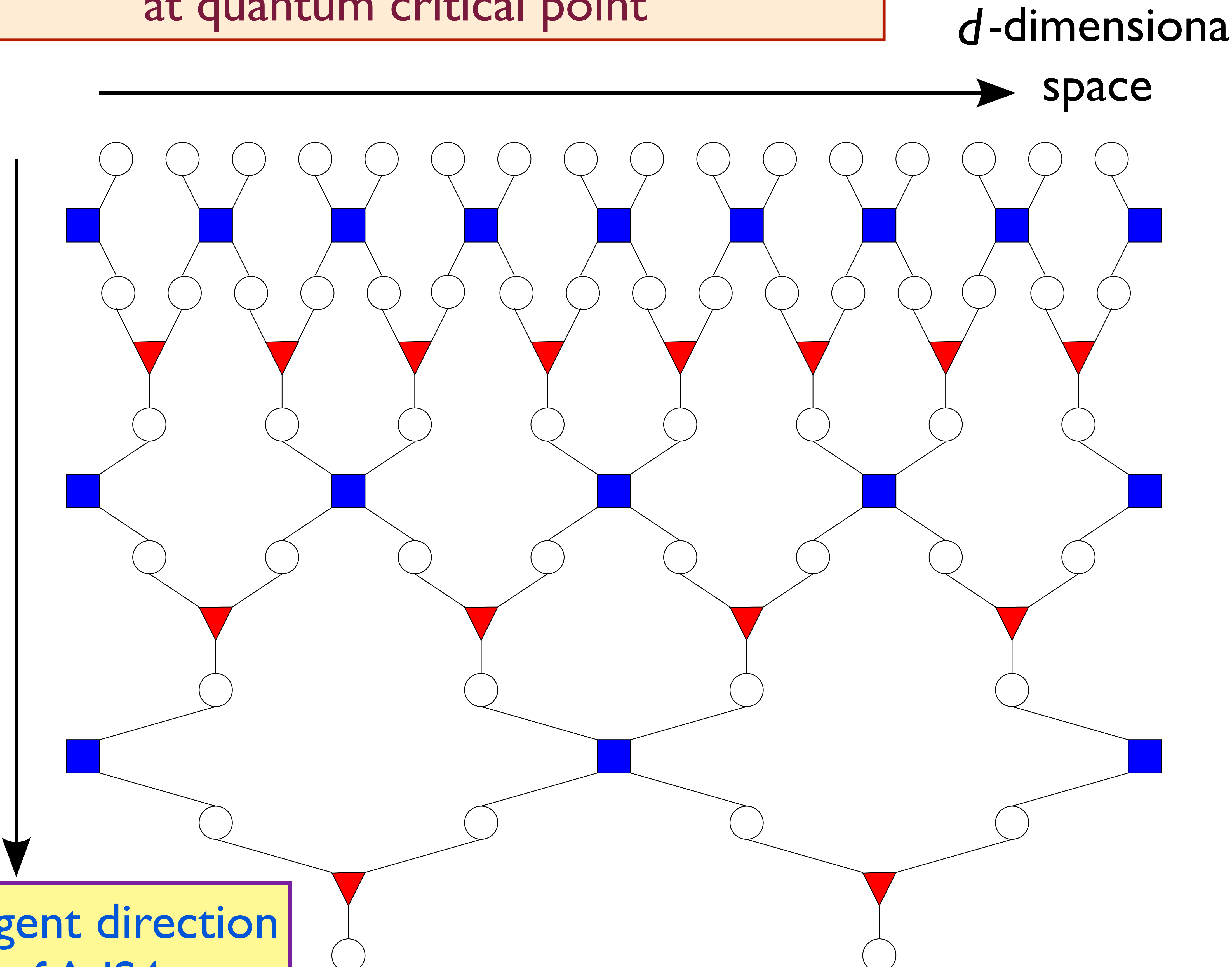
Tensor network representation of entanglement at quantum critical point



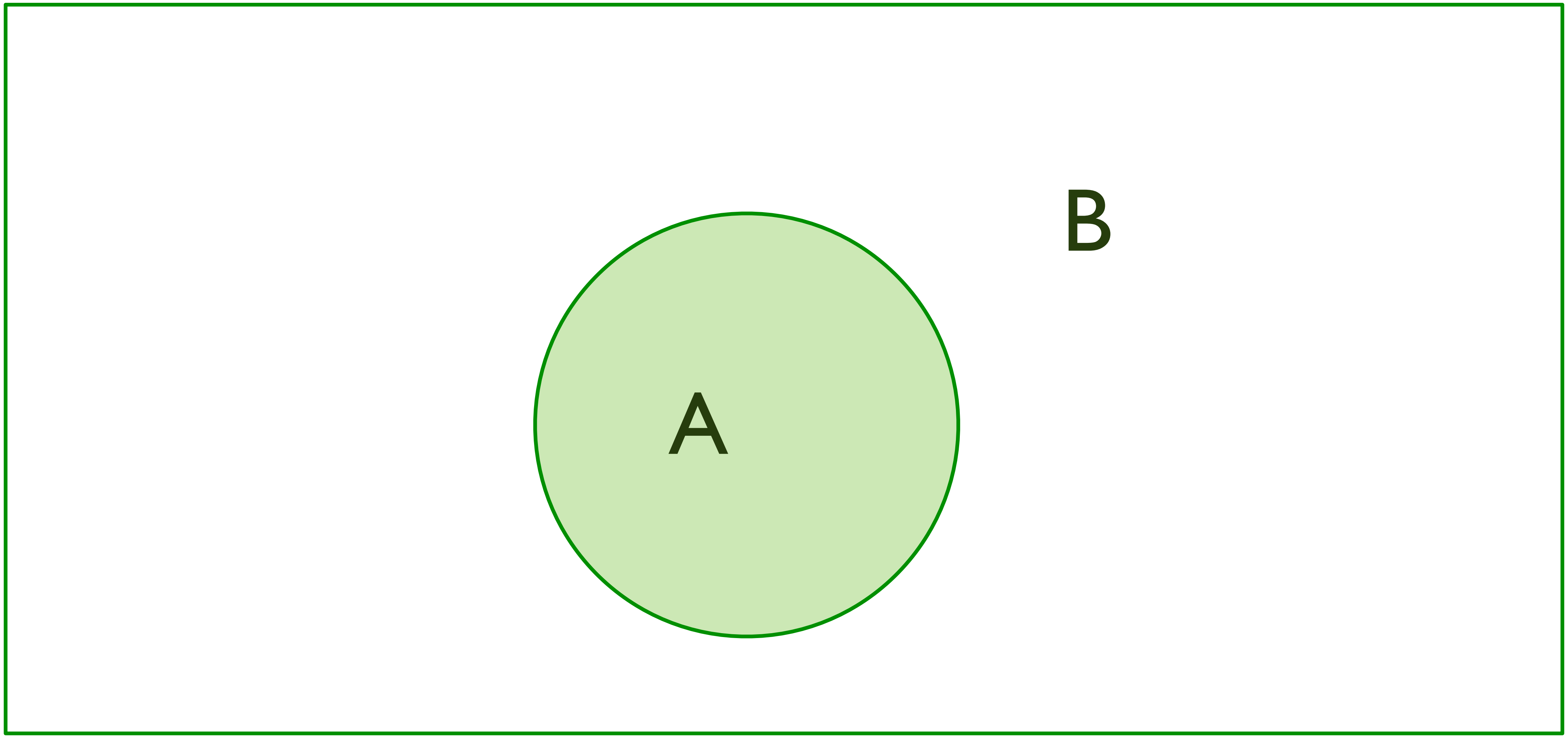

String theory near a D-brane
d-dimensional space
Emergent direction of AdS4

Tensor network representation of entanglement at quantum critical point



Brian Swingle, arXiv:0905.1317

Entanglement entropy

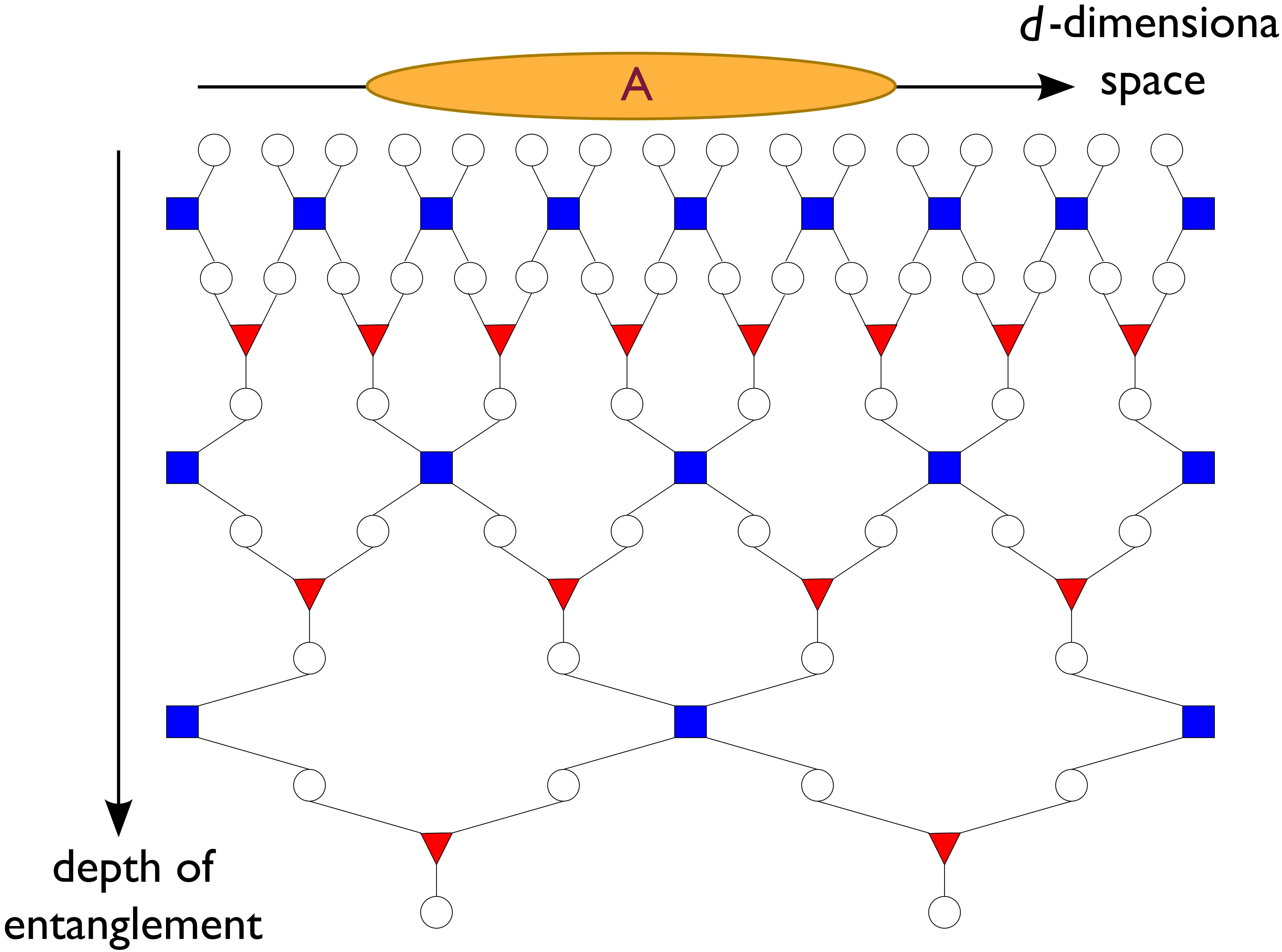


Measure strength of quantum entanglement of region A with region B.

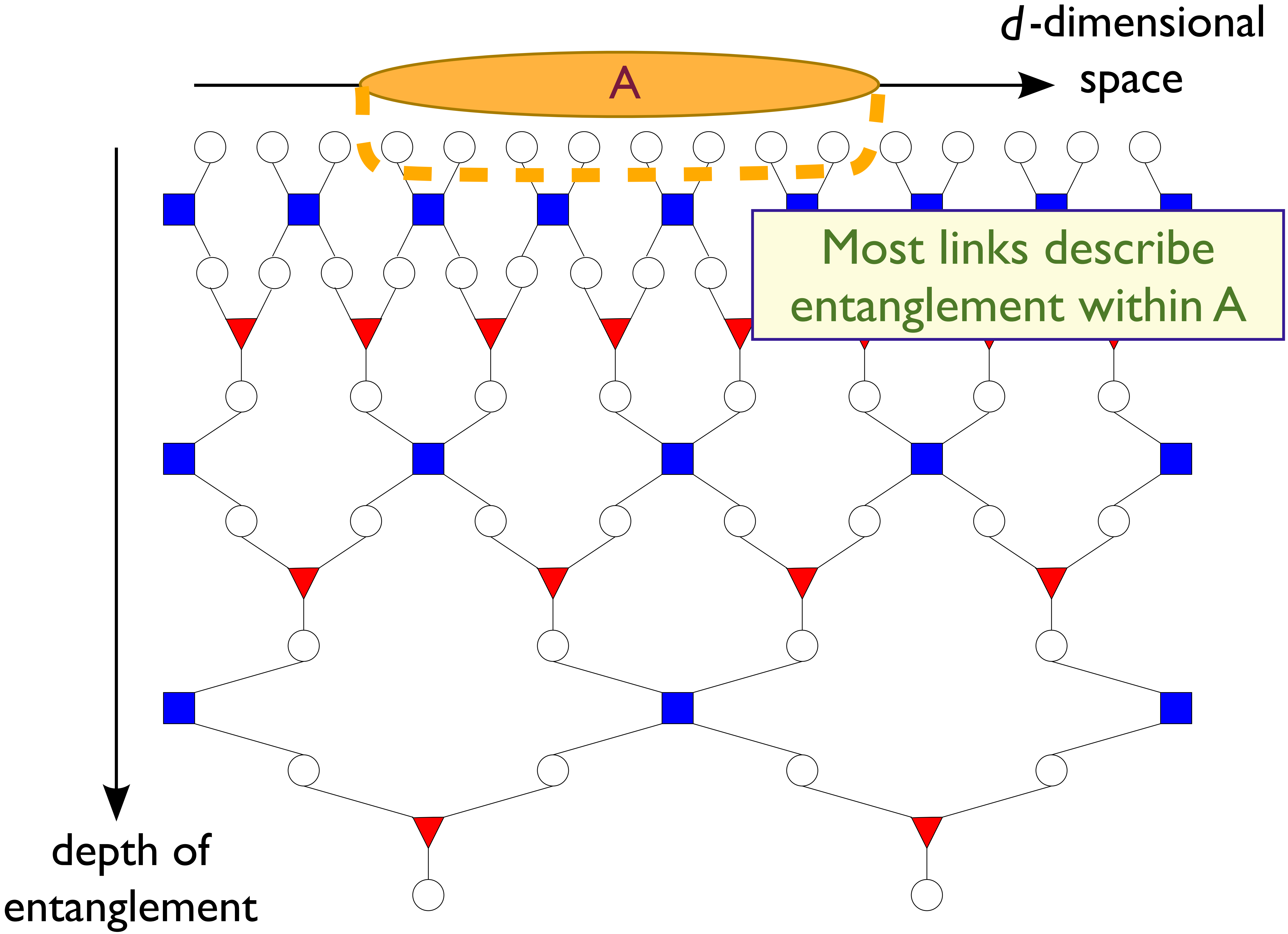
$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

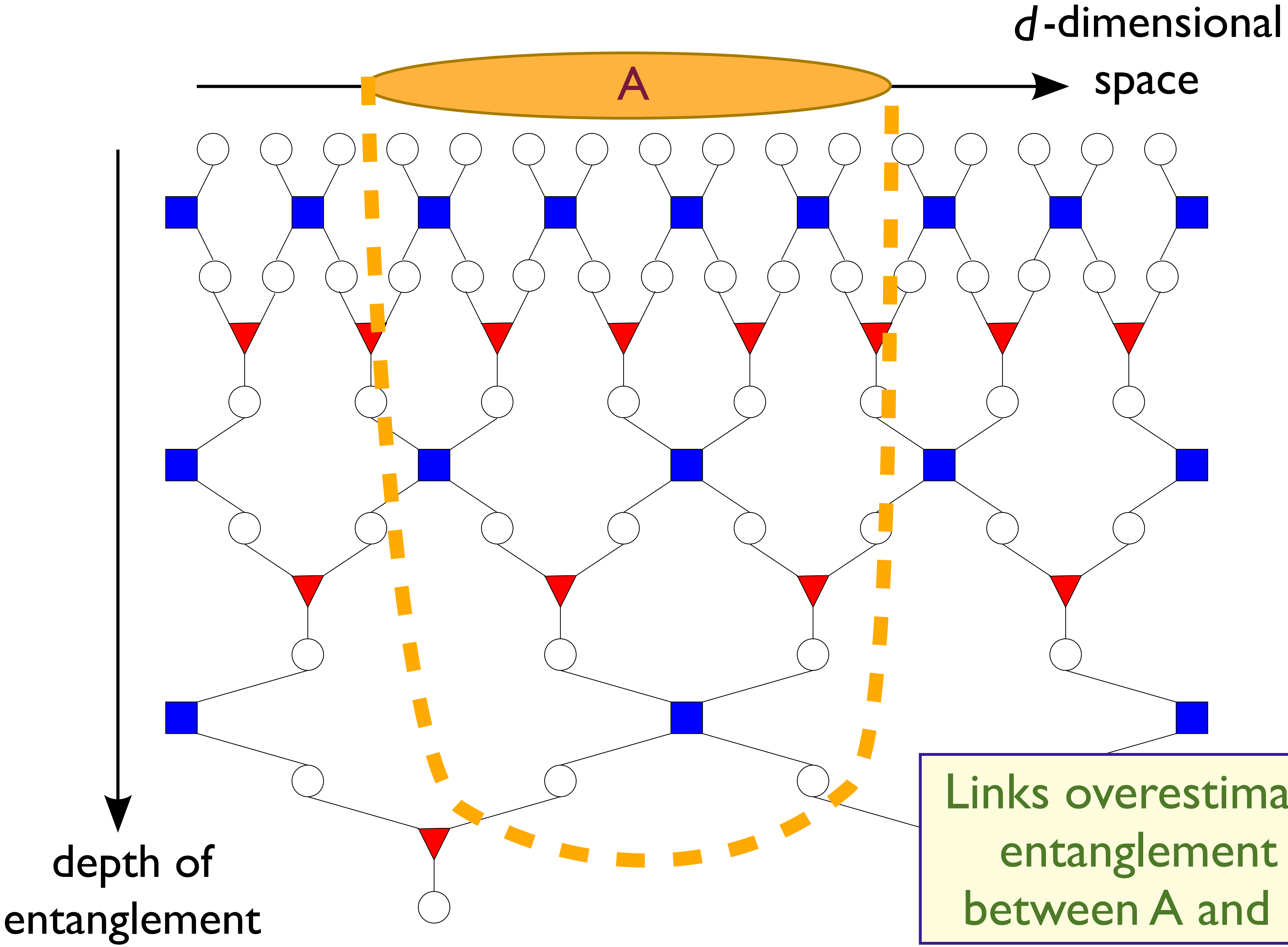
Entanglement entropy



Entanglement entropy

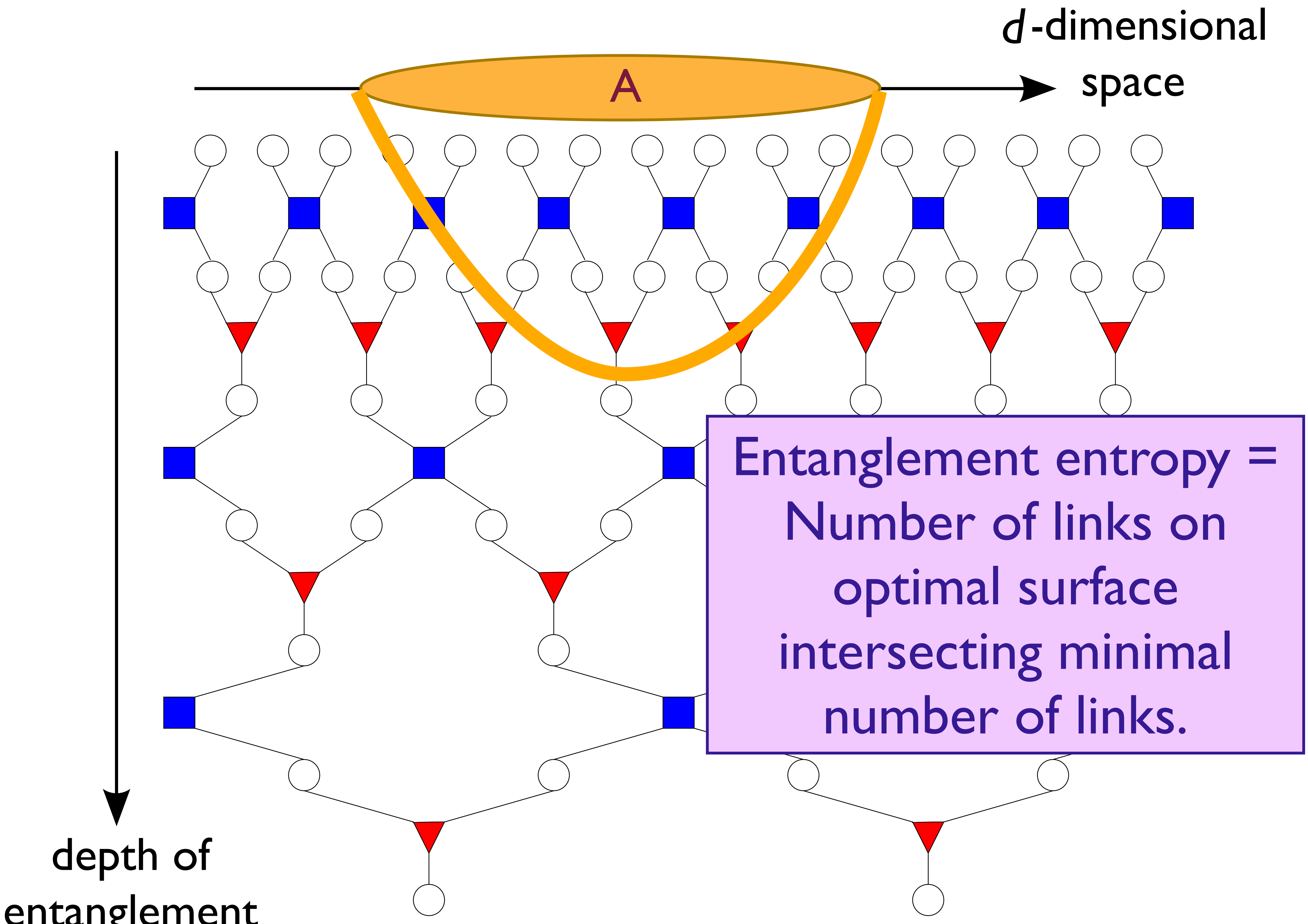


Entanglement entropy



Links overestimate entanglement between A and B

Entanglement entropy

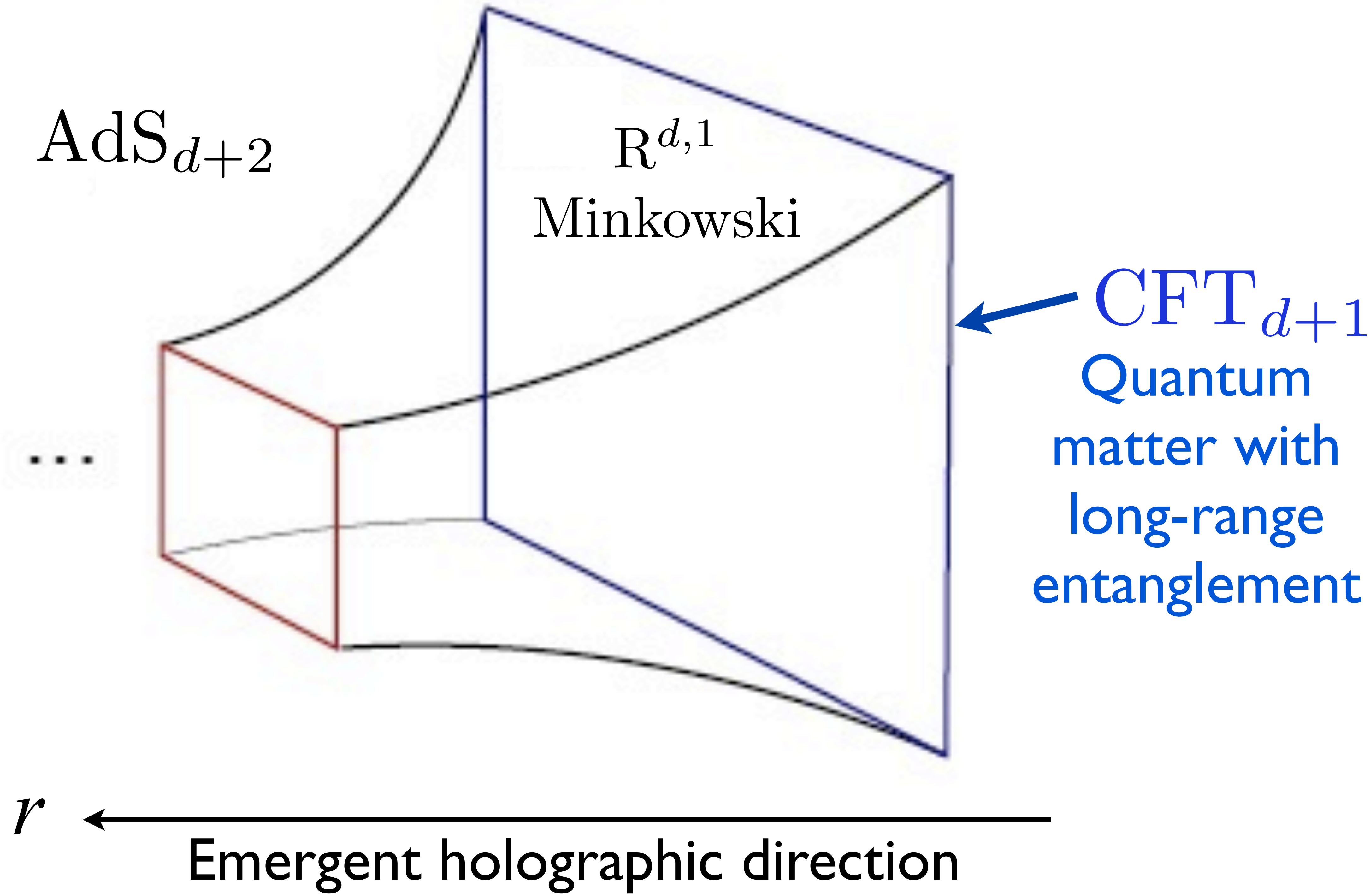


Entanglement entropy

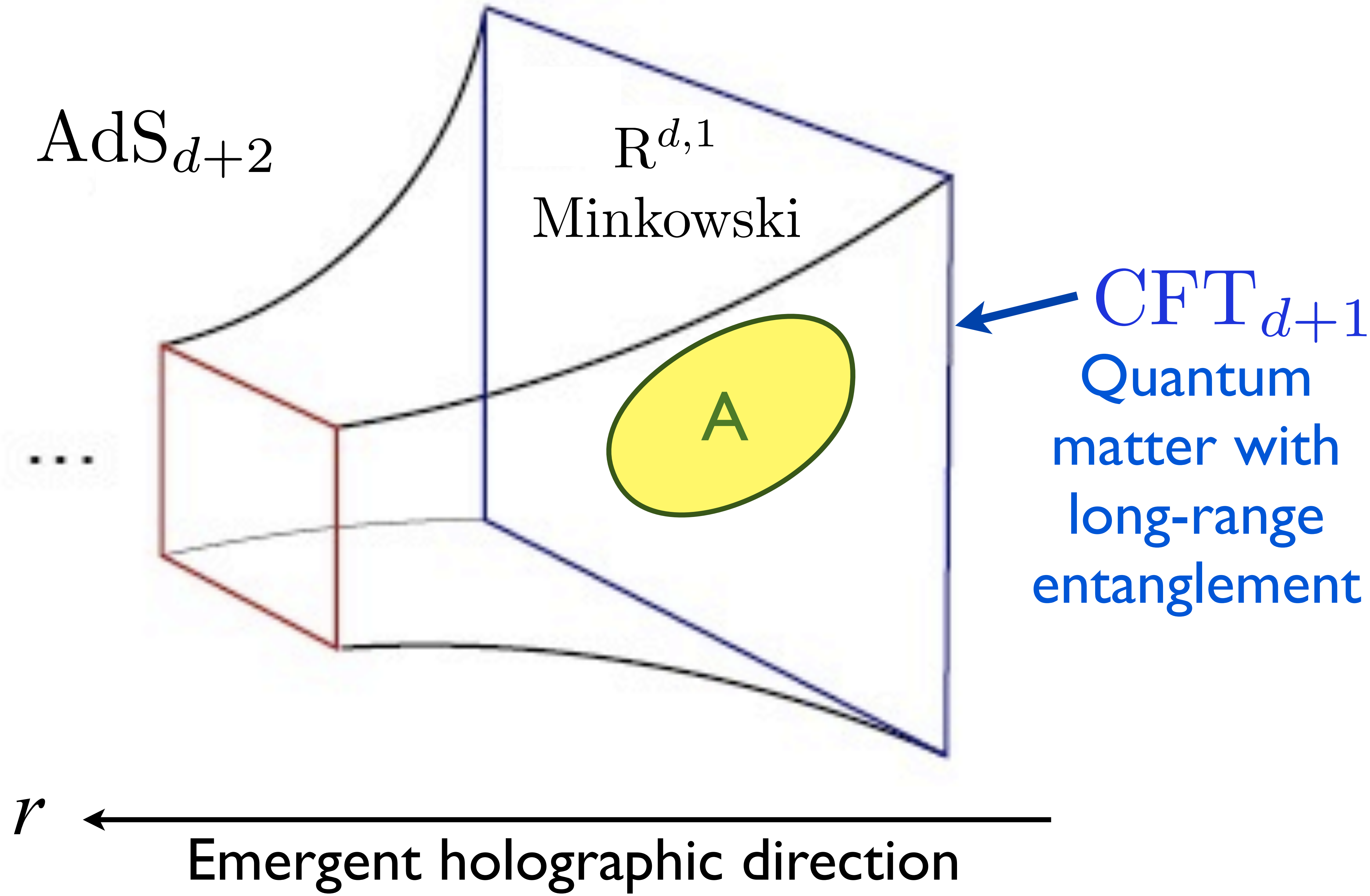
The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

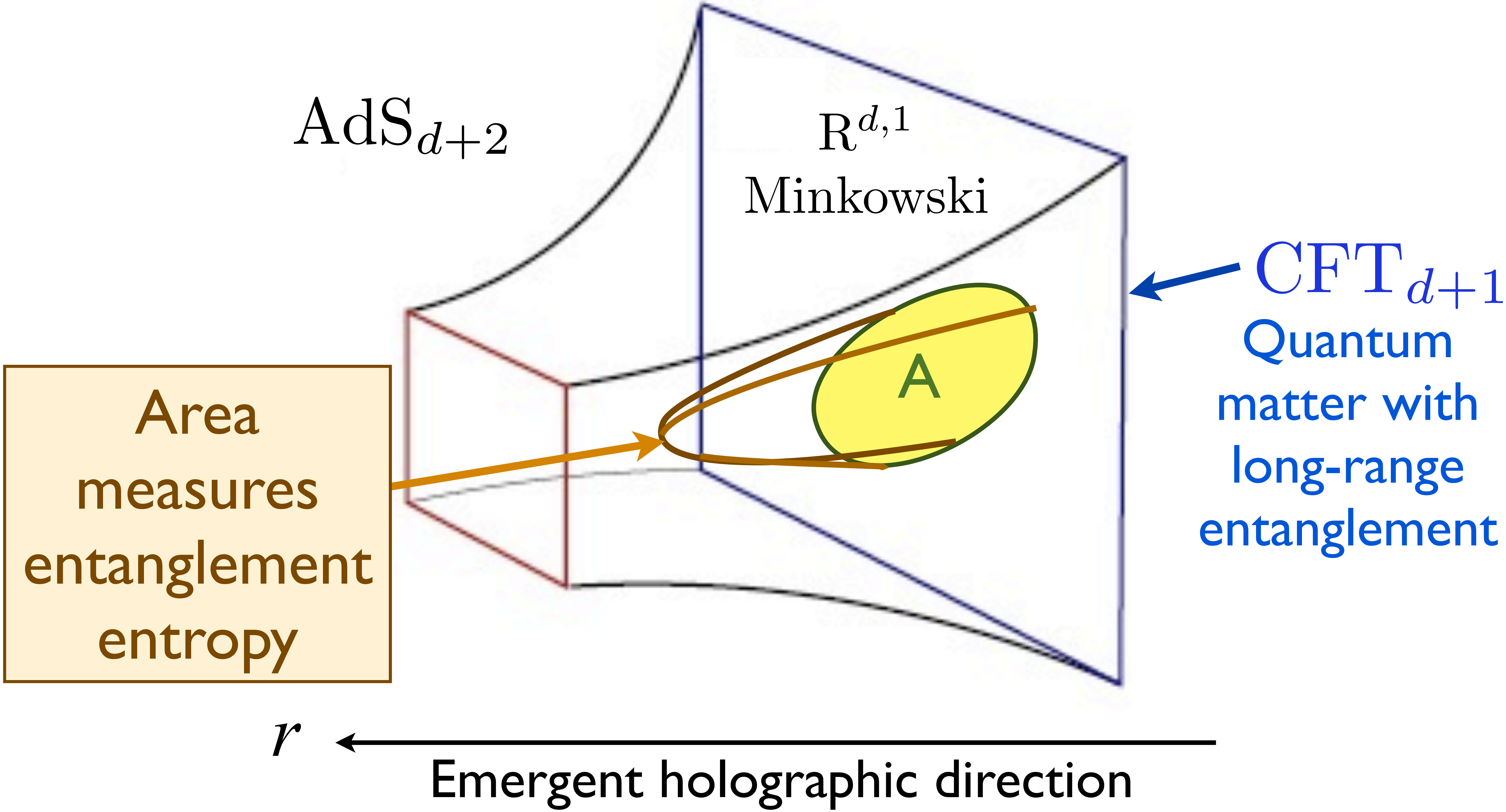
This can be seen both the string and tensor-network pictures

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317



J. McGreevy, arXiv0909.0518


AdS$_{d+2}$
R^{d,1}
Minkowski
A
CFT$_{d+1}$
Quantum matter with long-range entanglement
r
Emergent holographic direction



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Quantum superposition and entanglement

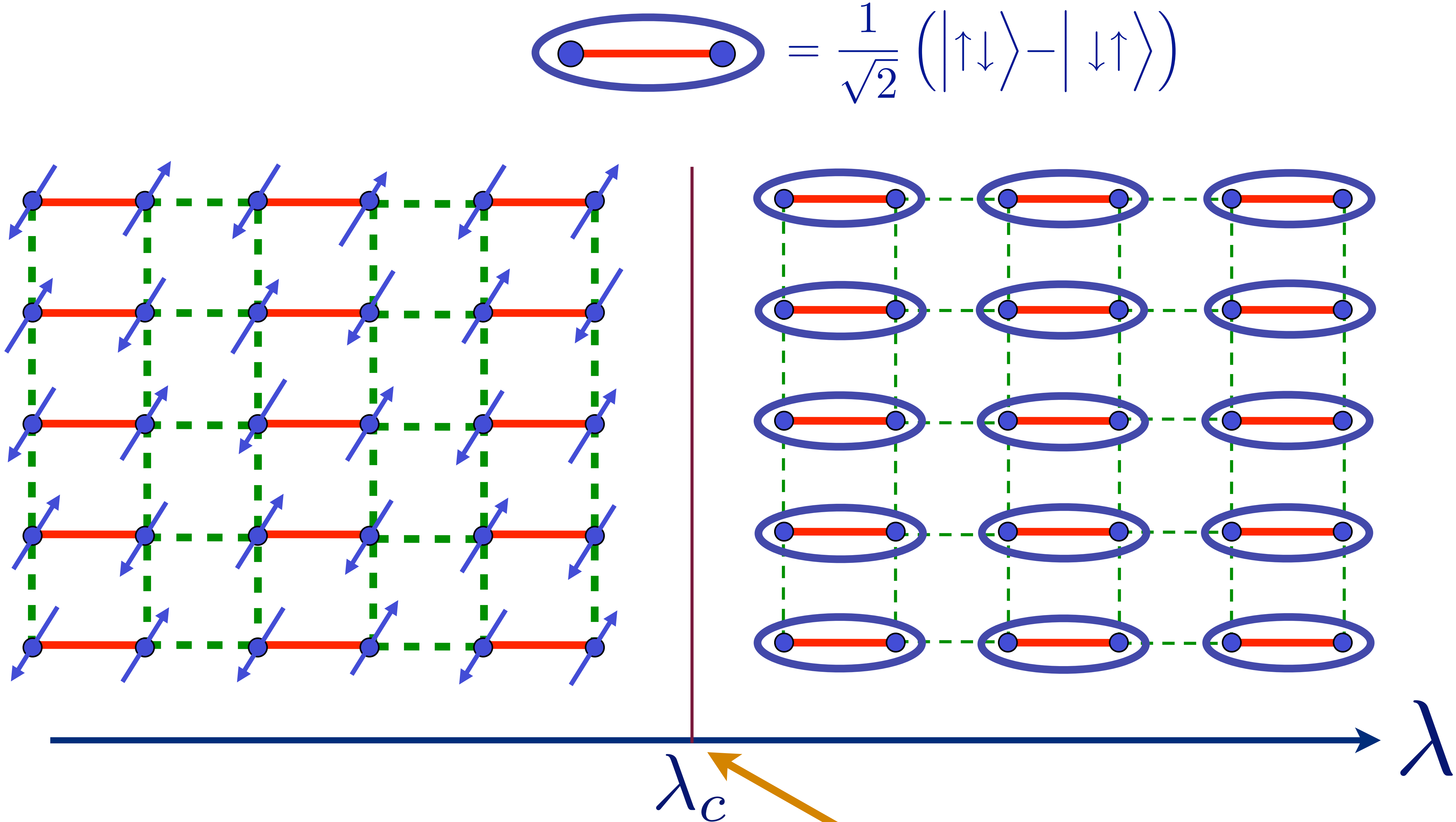
Quantum critical points of electrons in crystals

String theory and black holes

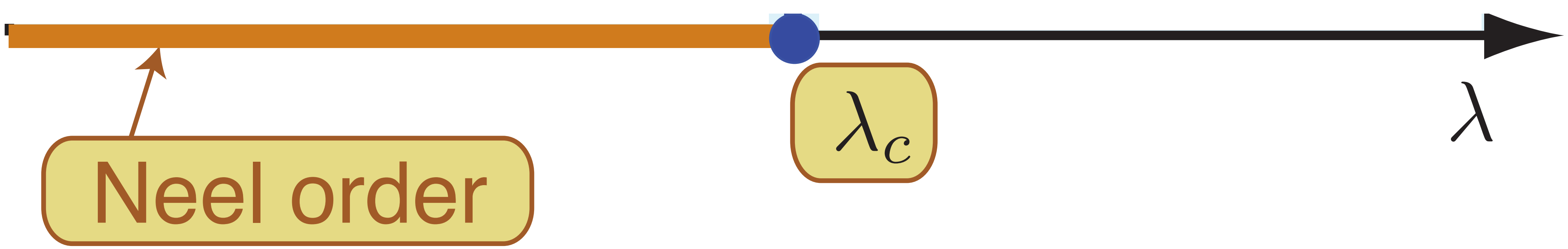
Quantum superposition and entanglement

Quantum critical points of electrons in crystals

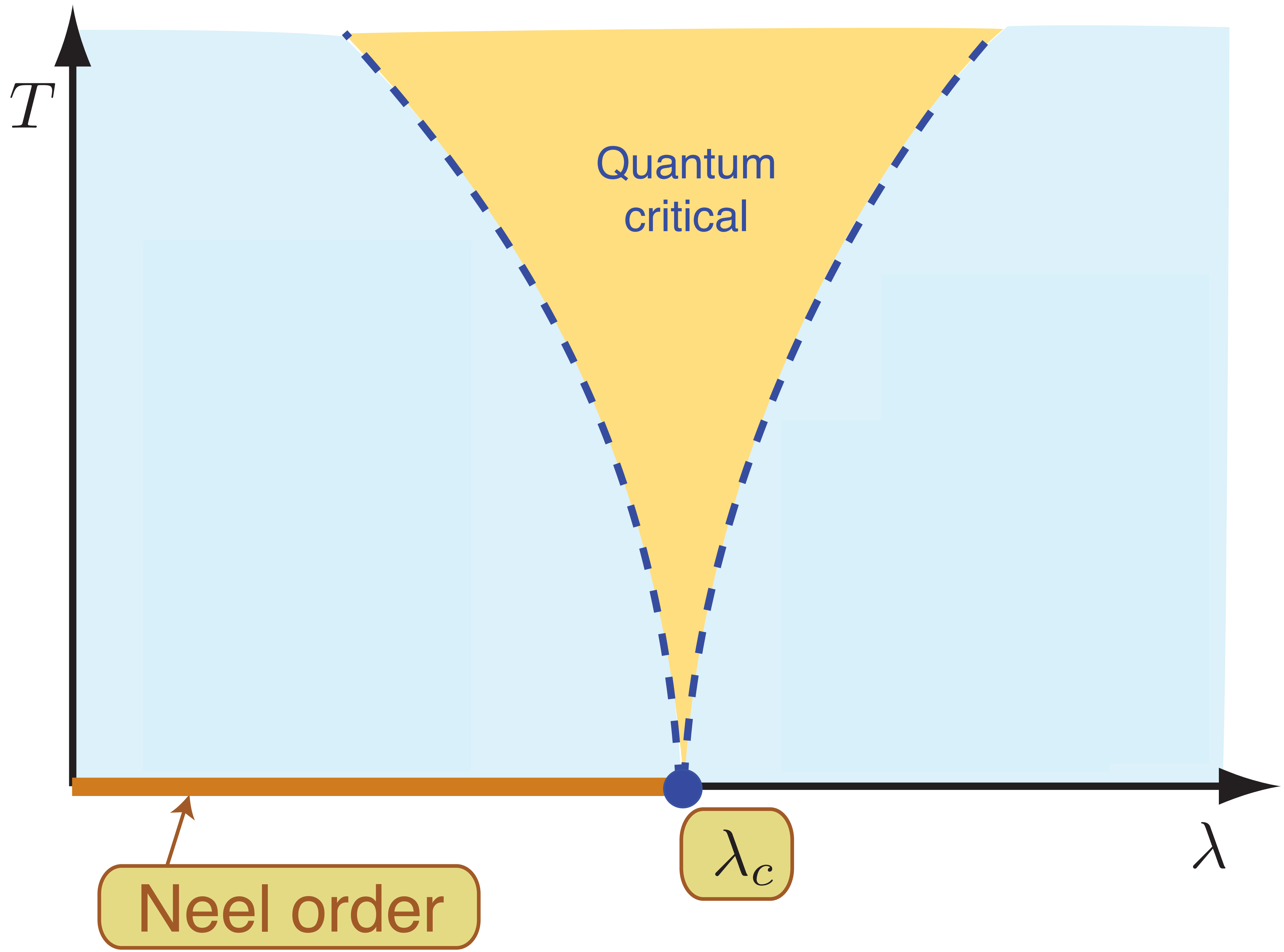
String theory and black holes



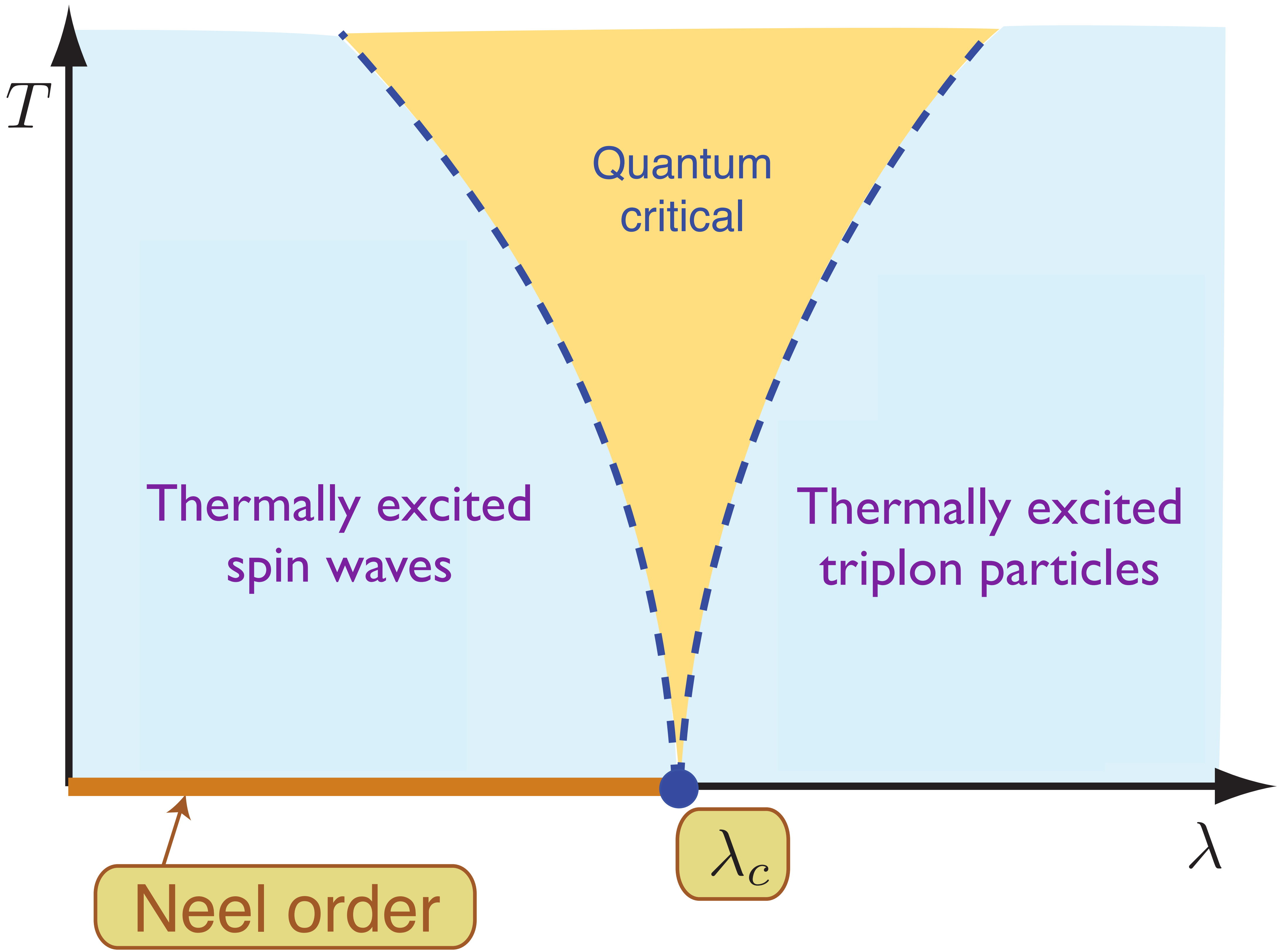
Quantum critical point with non-local entanglement in spin wavefunction



Neel order
λ_c
λ


T
Quantum
critical
Neel order
λ_c
λ

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
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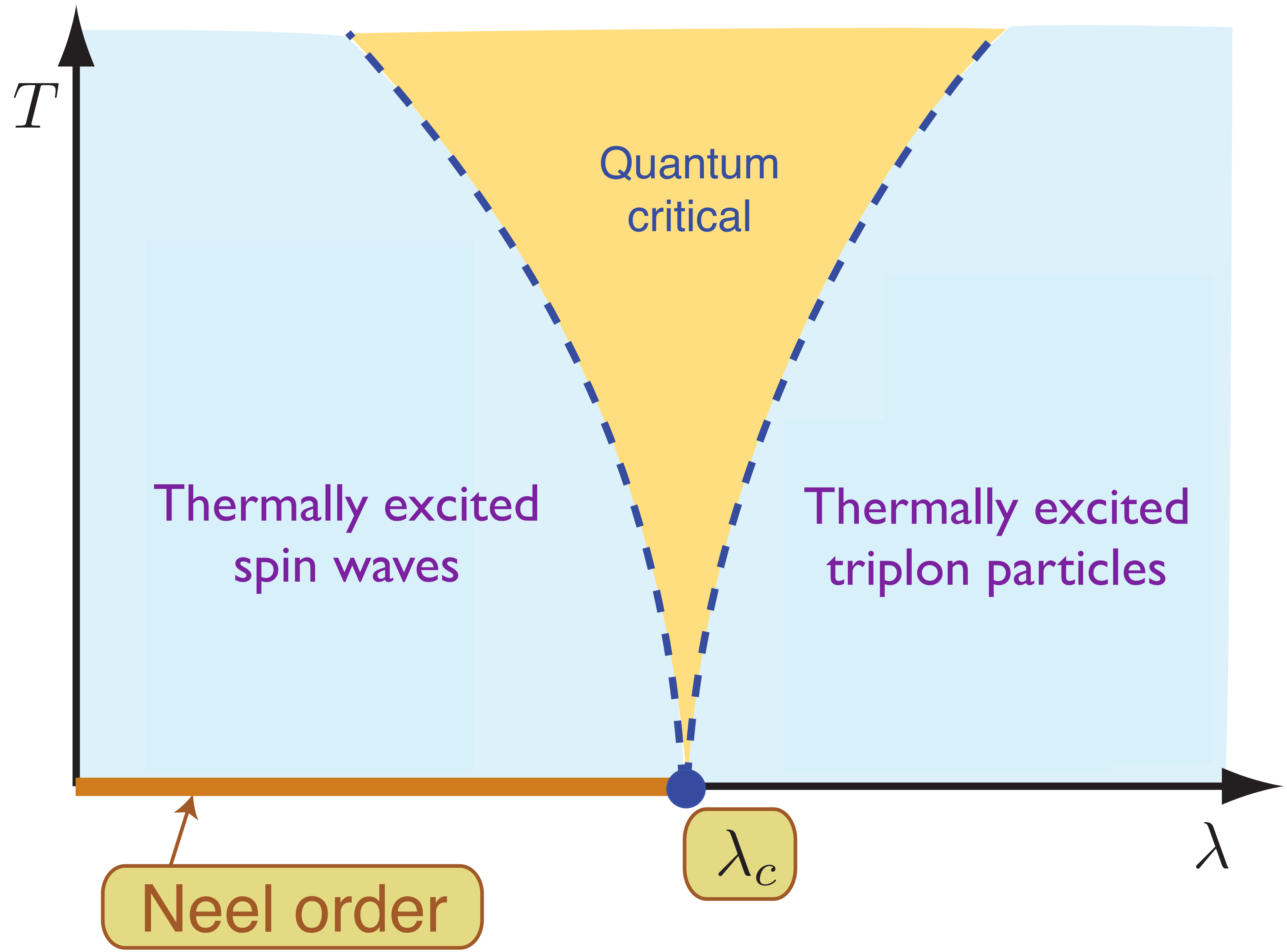

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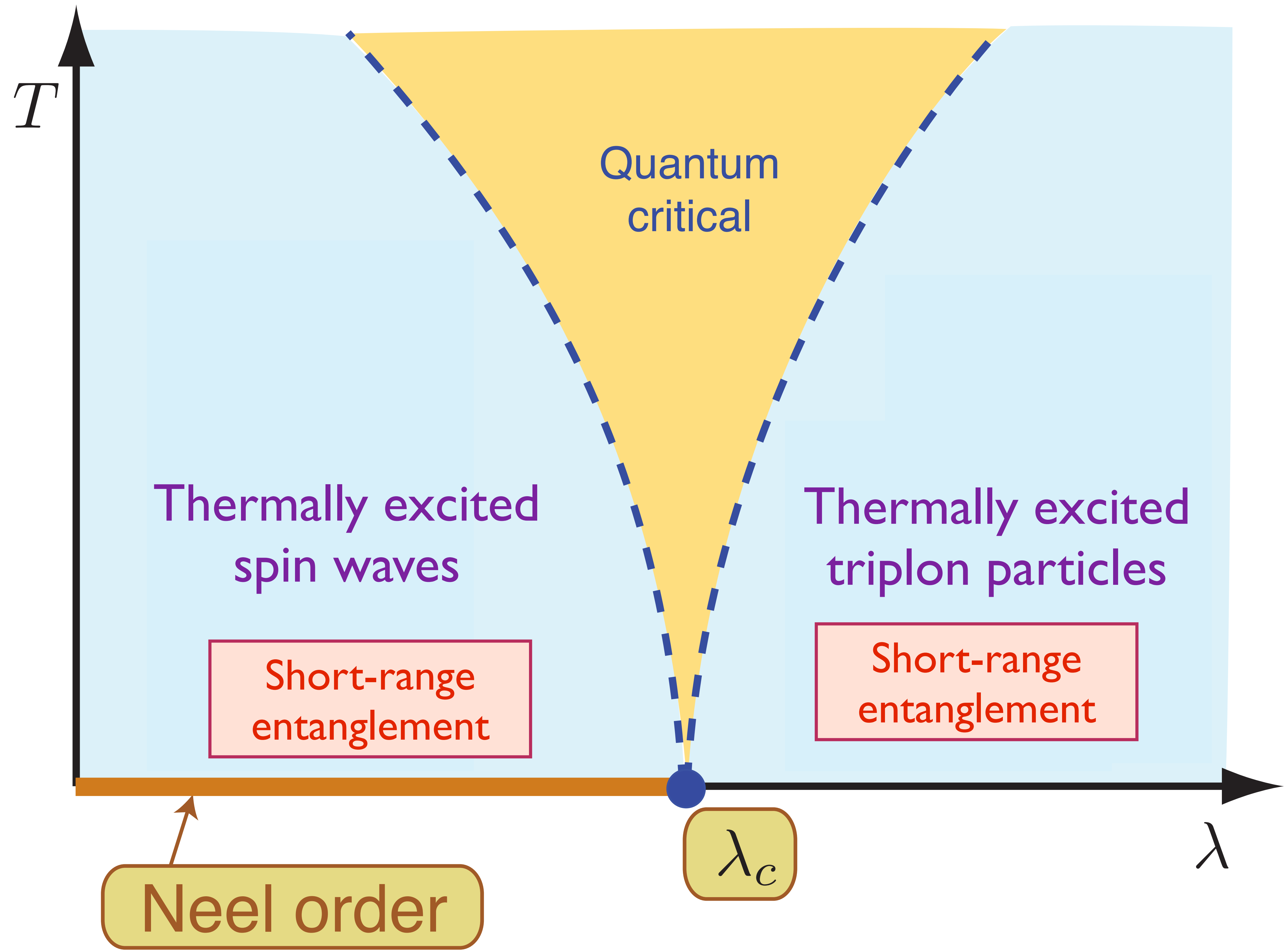

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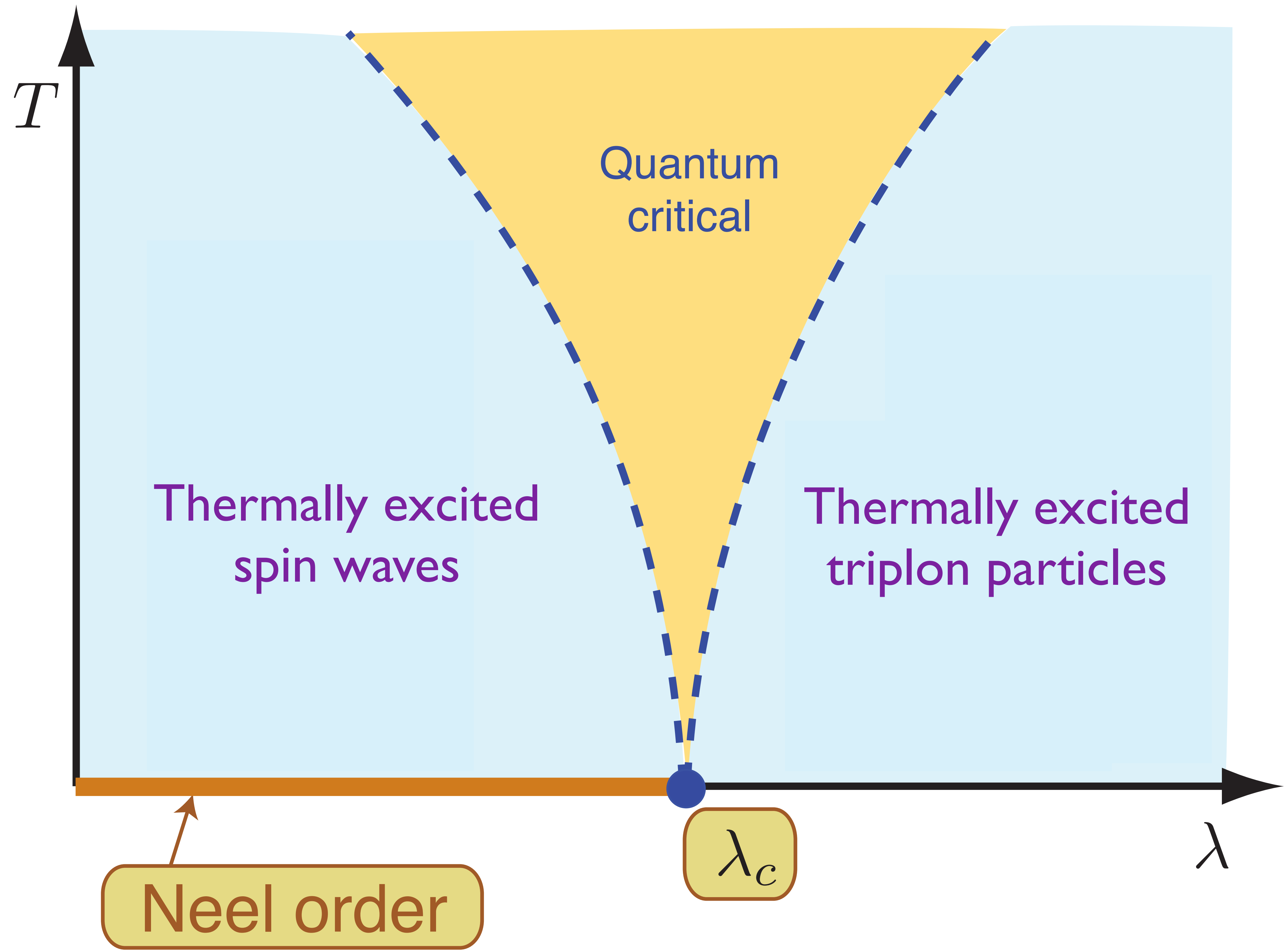
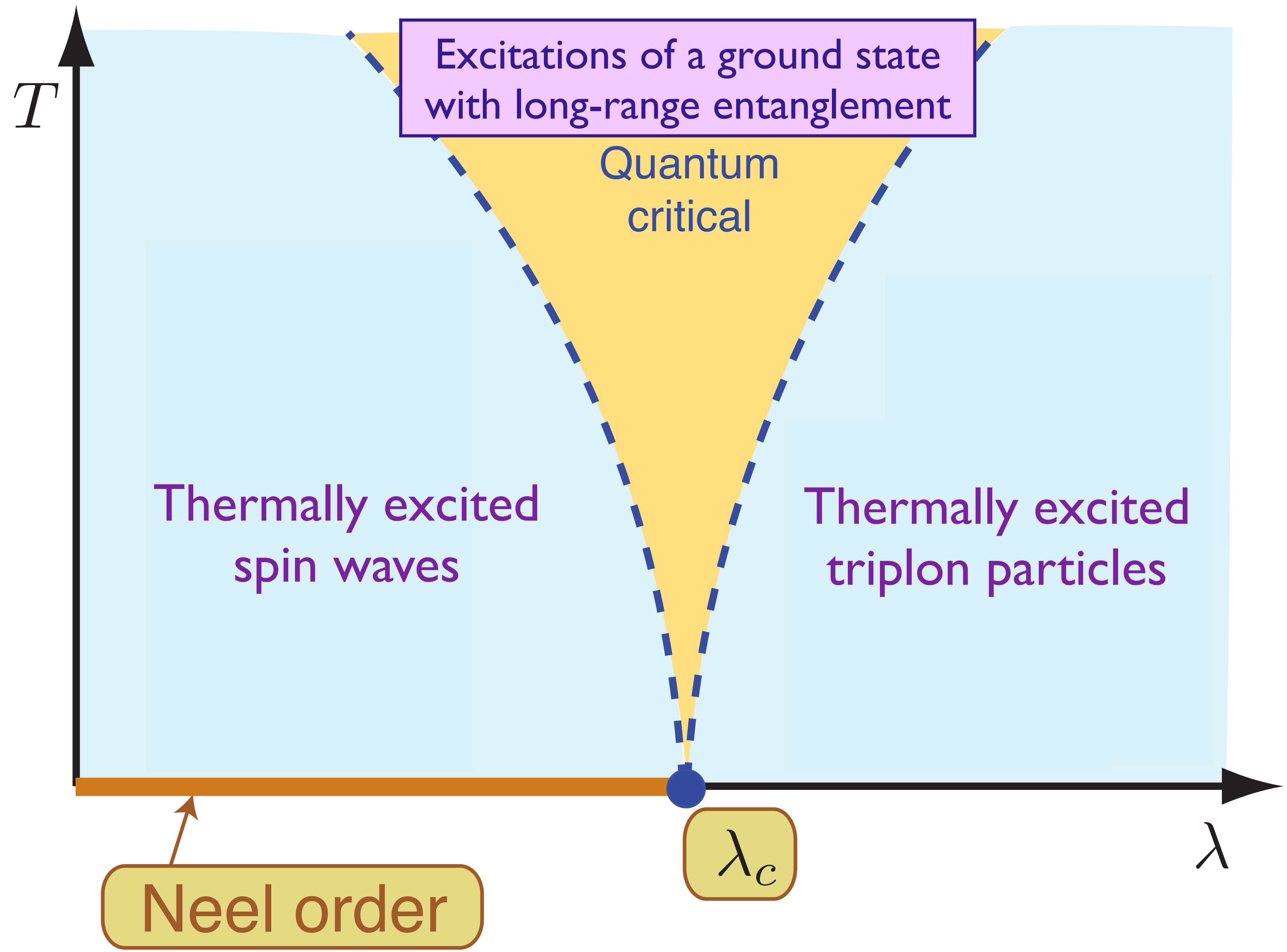
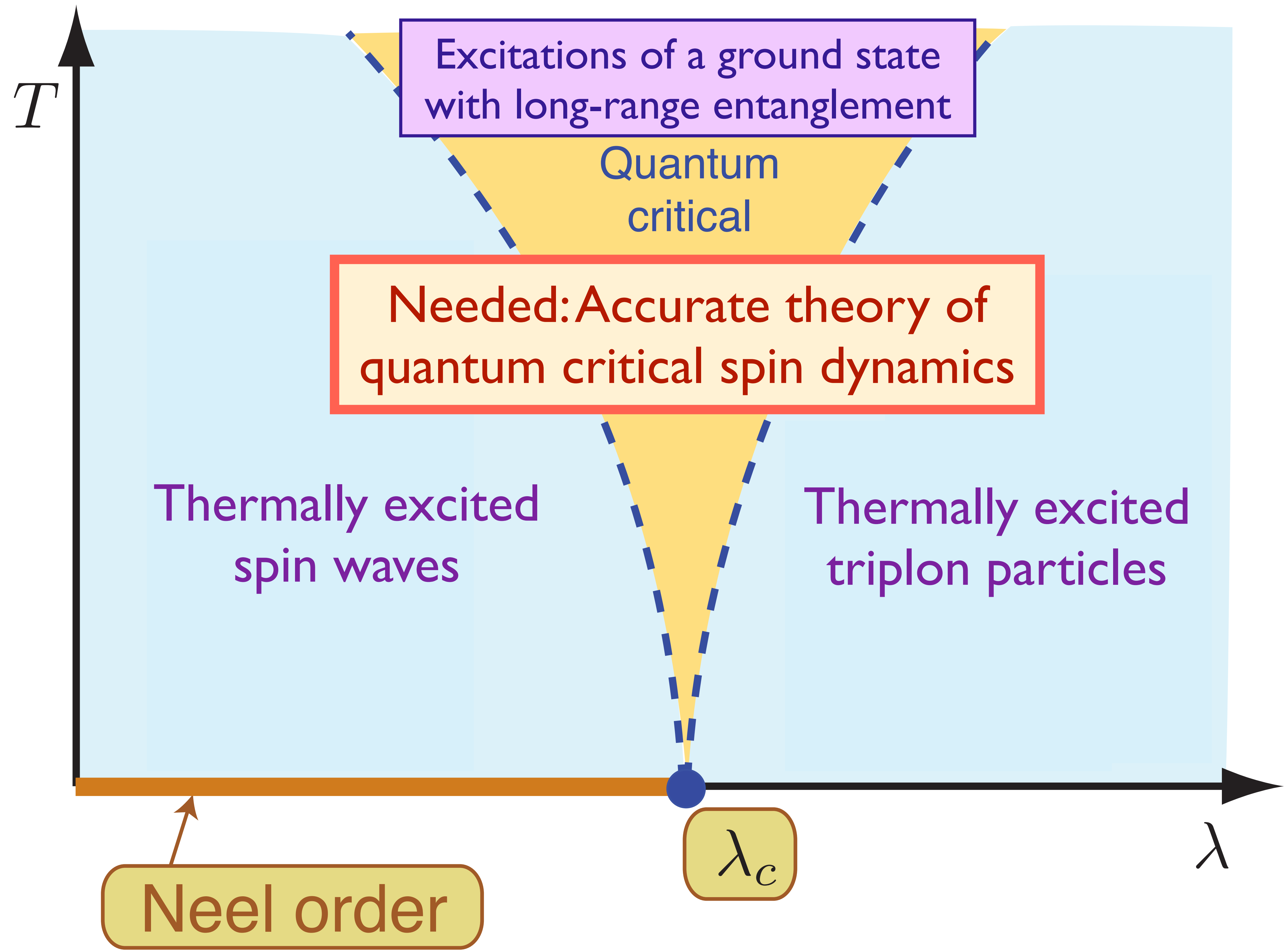


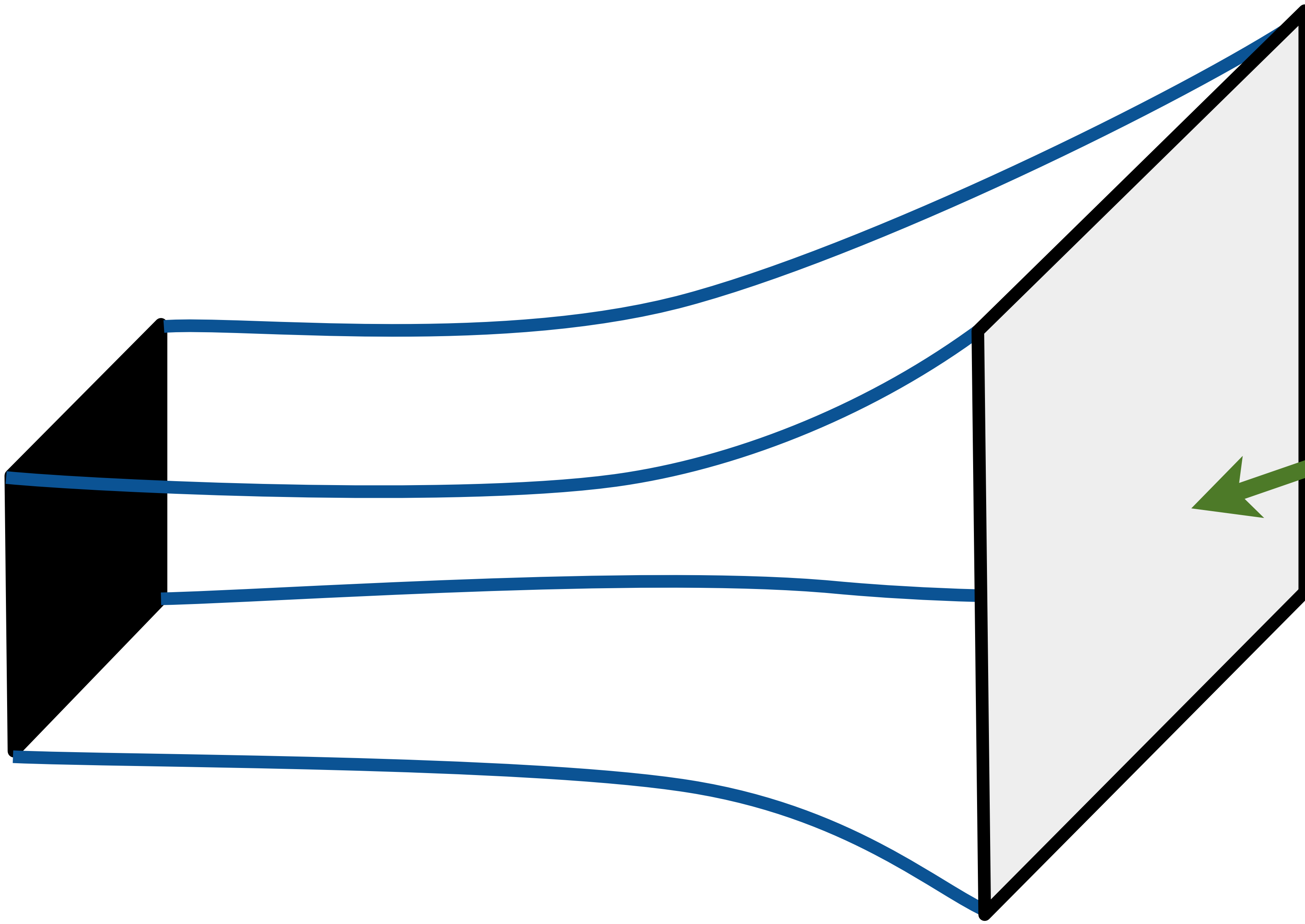
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

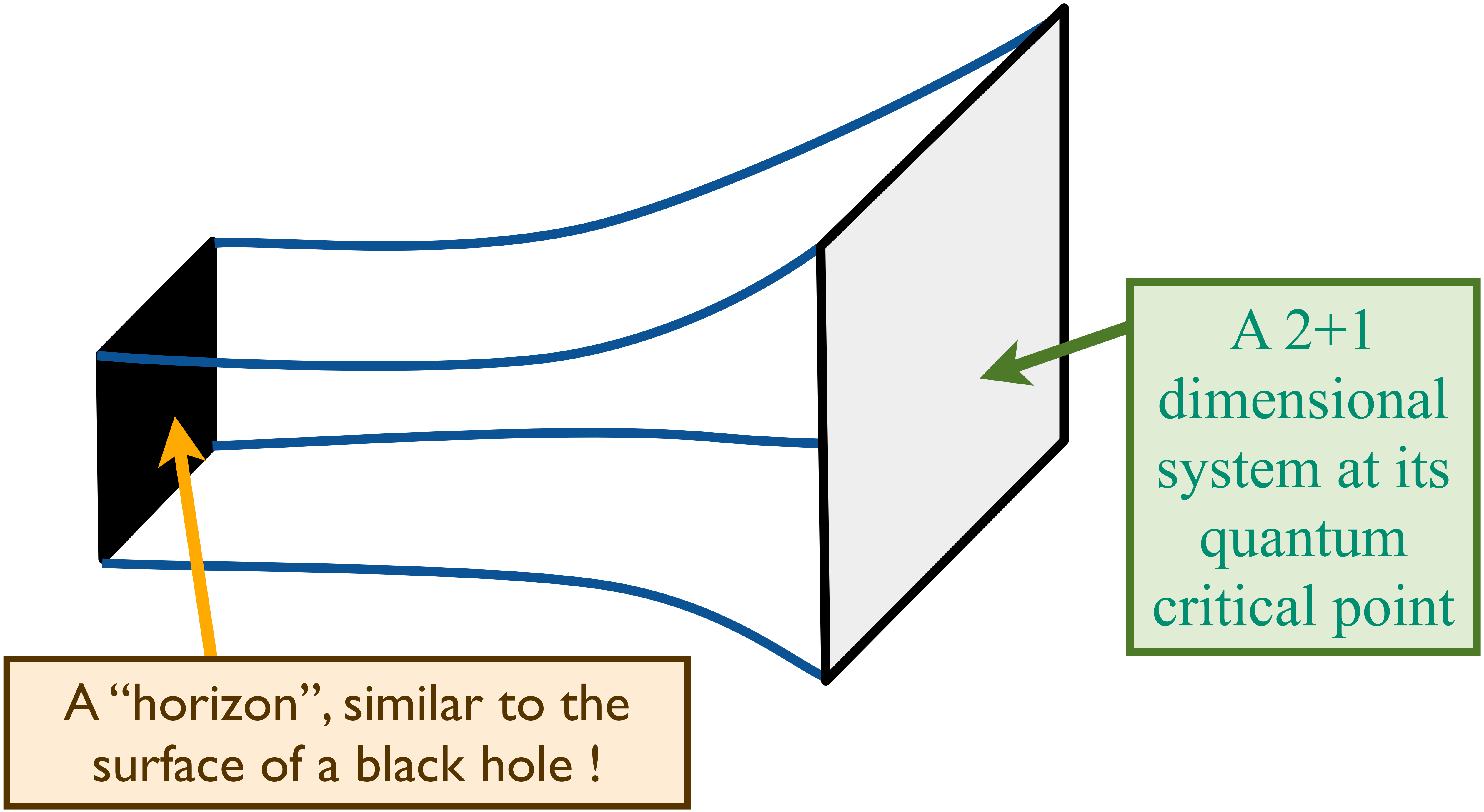


String theory at non-zero temperatures



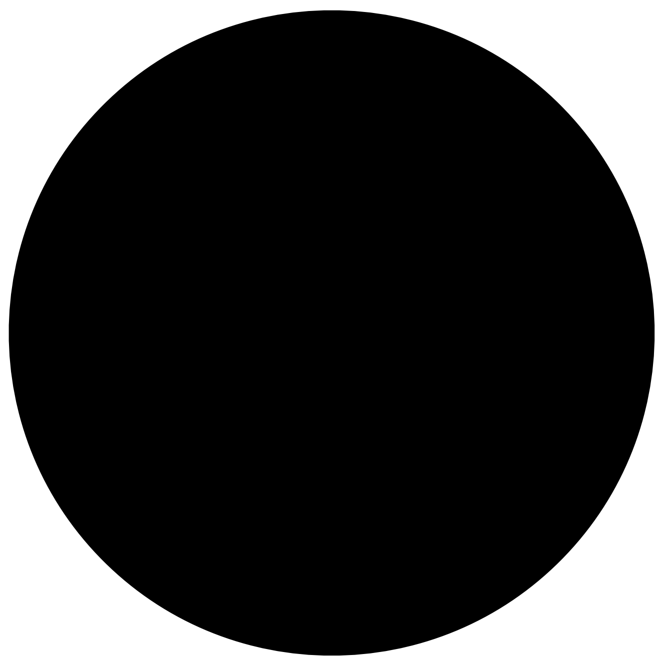
A 2+1 dimensional system at its quantum critical point

String theory at non-zero temperatures



Black Holes

Objects so massive that light is gravitationally bound to them.

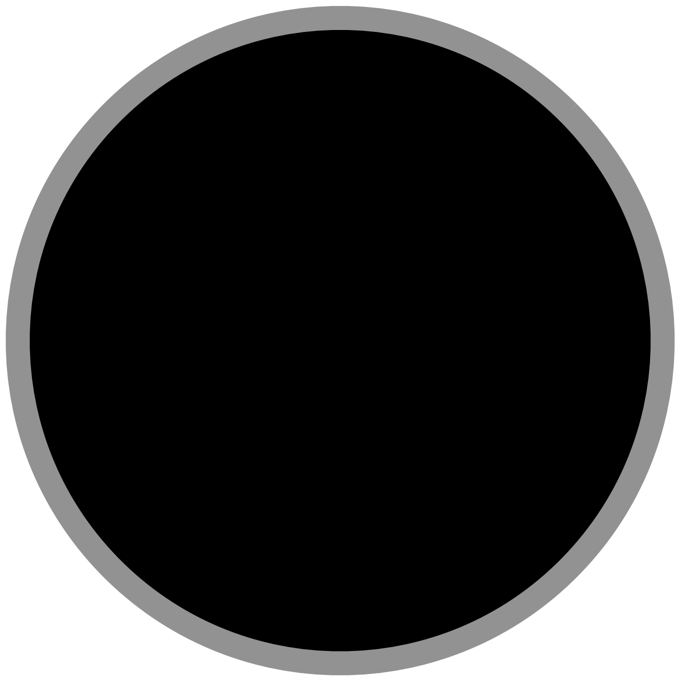


Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

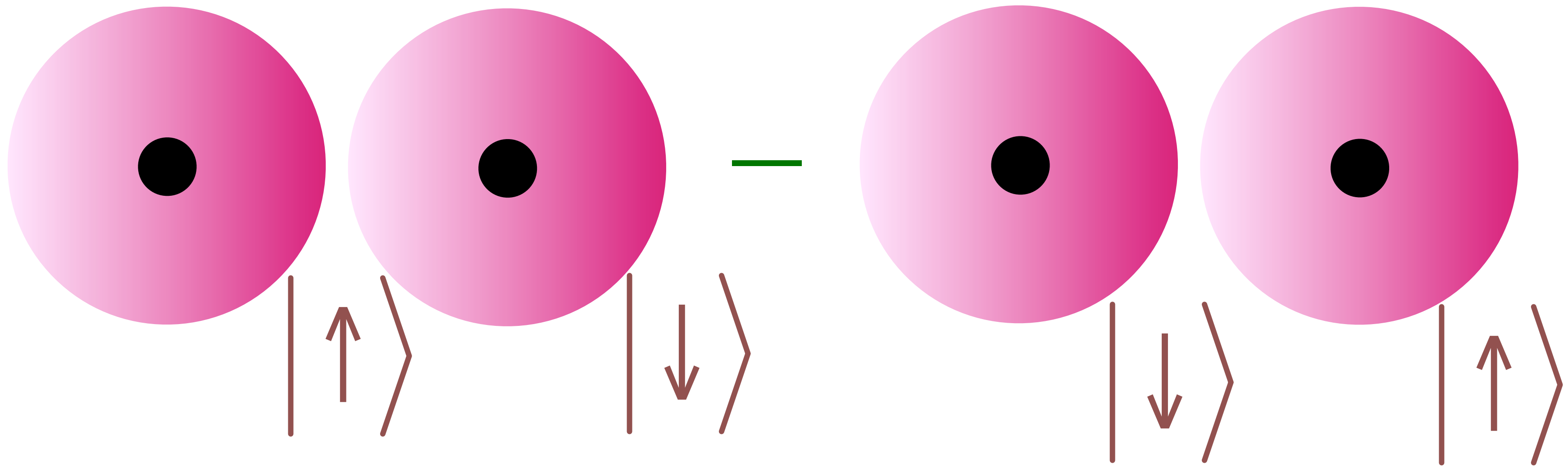
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



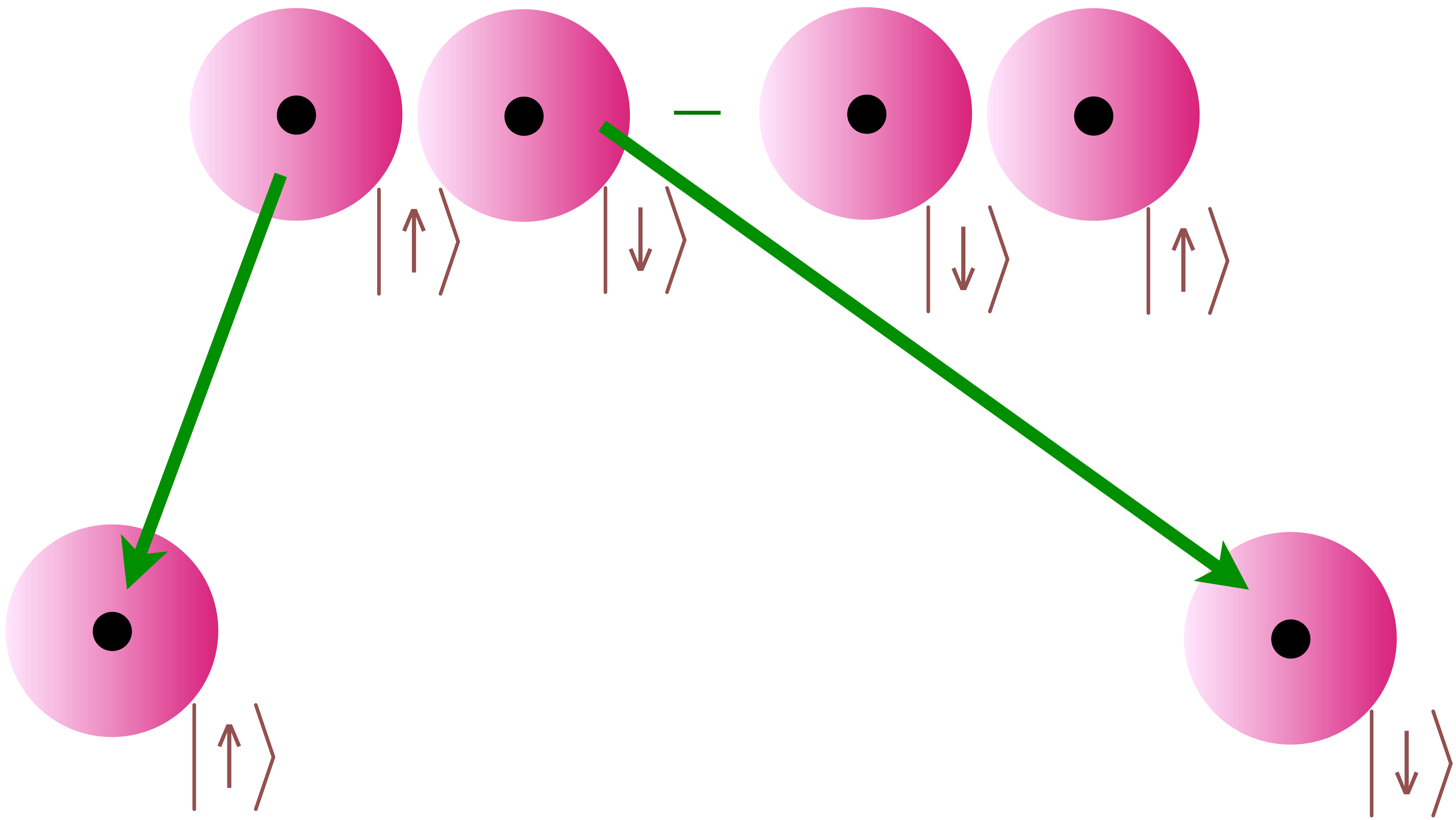
Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

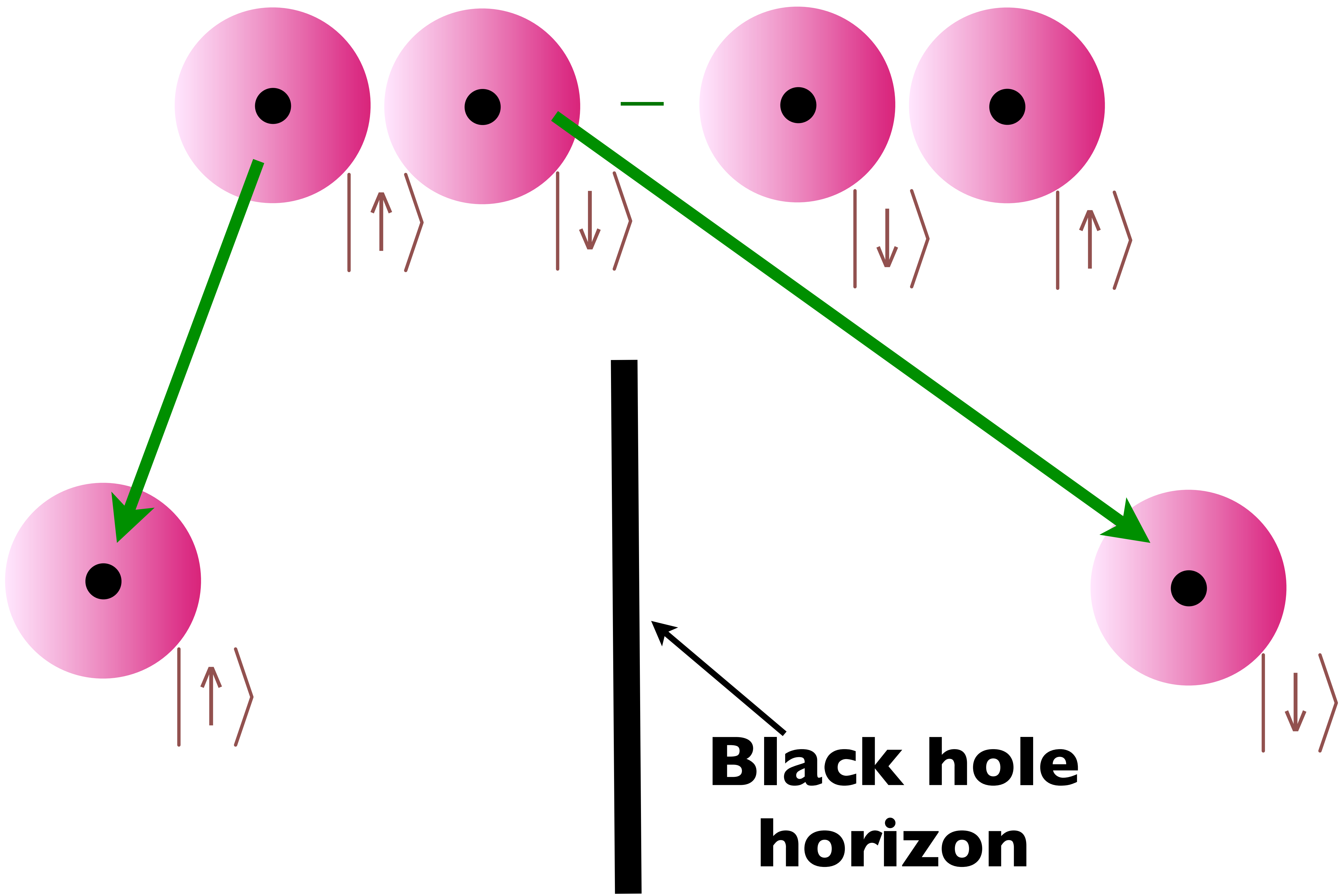
Quantum Entanglement across a black hole horizon



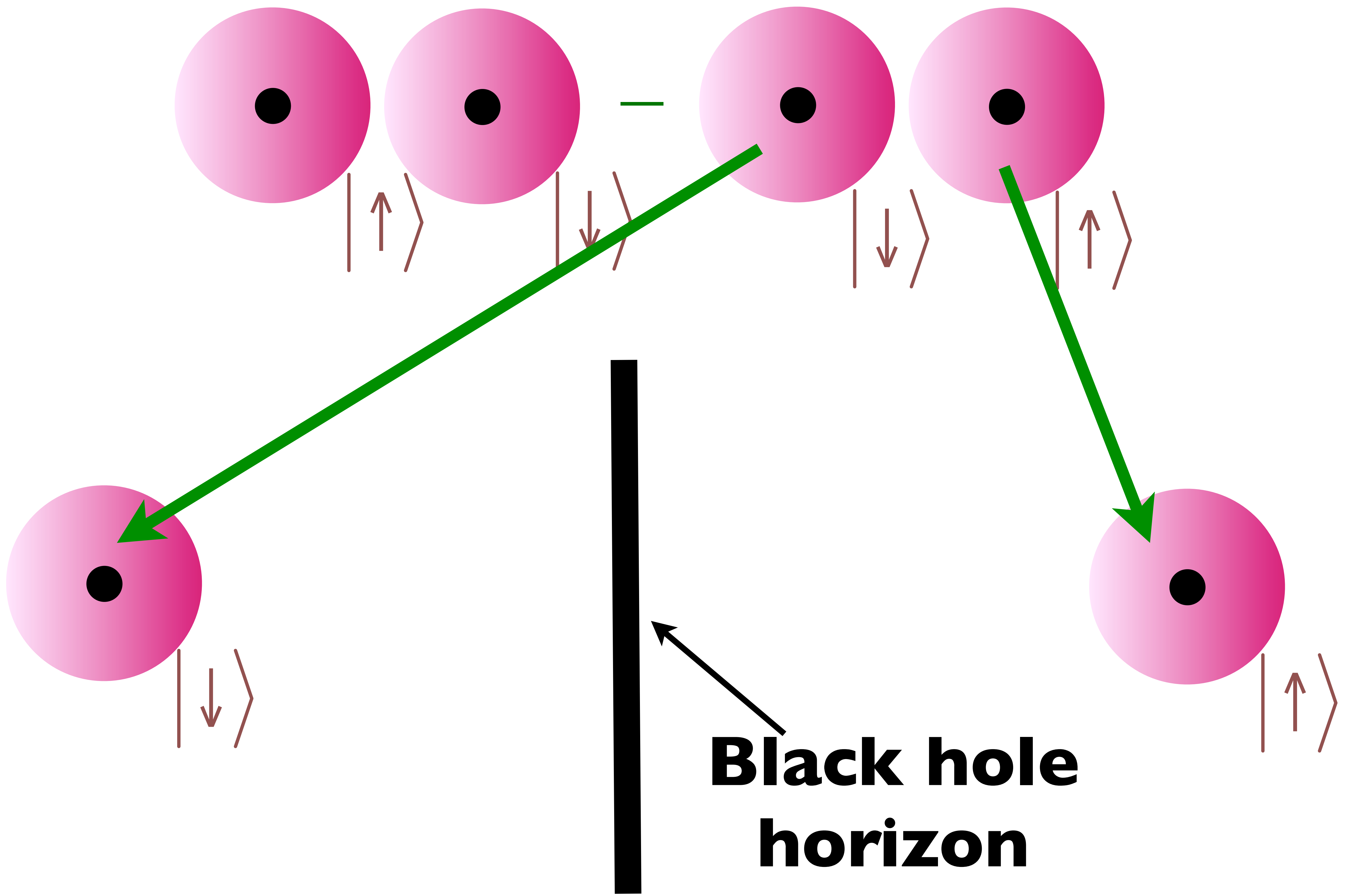
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

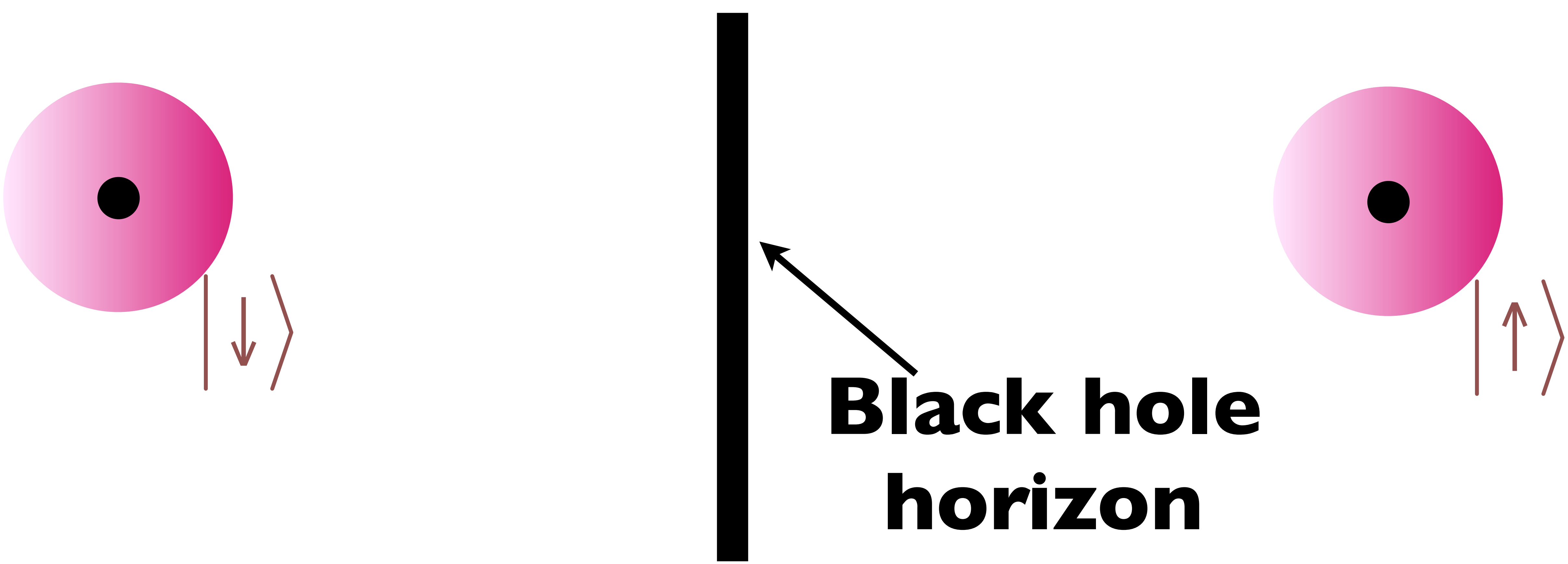


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

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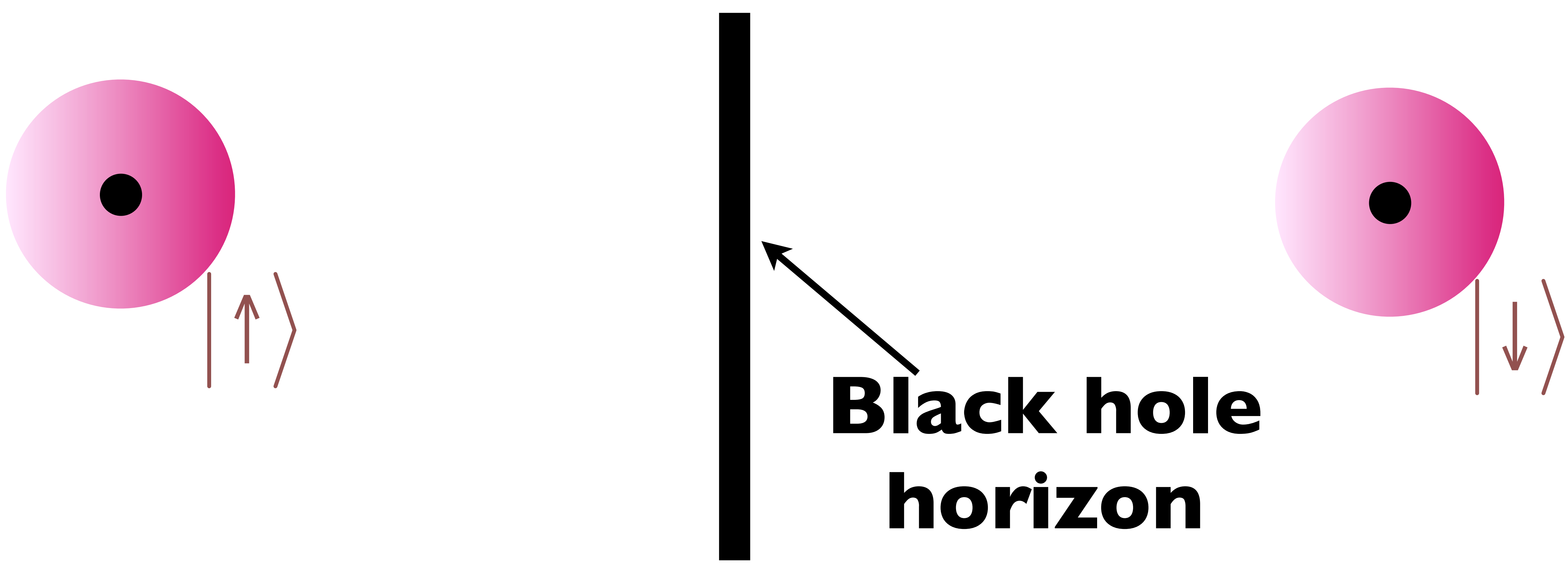
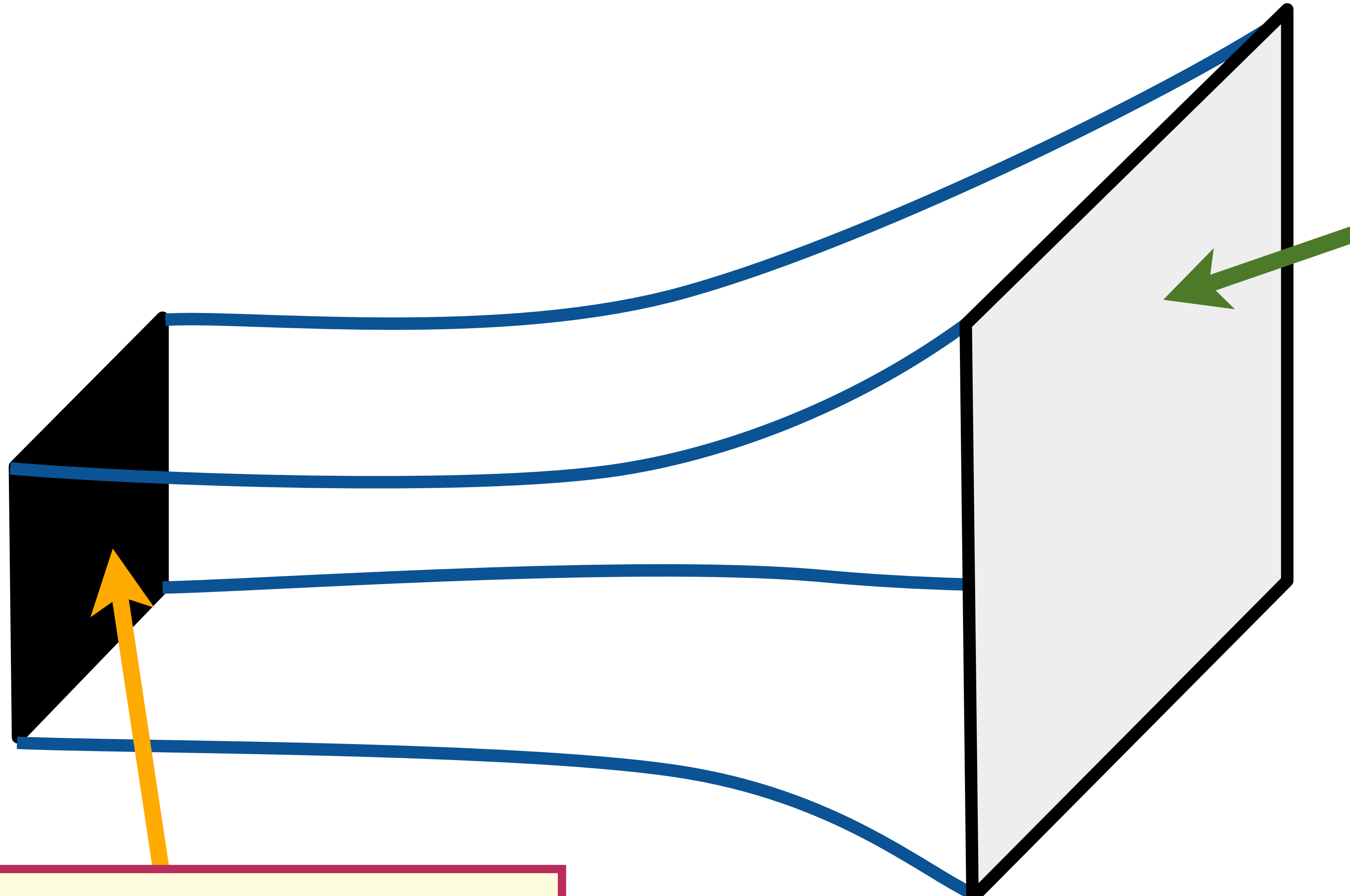


Quantum Entanglement across a black hole horizon

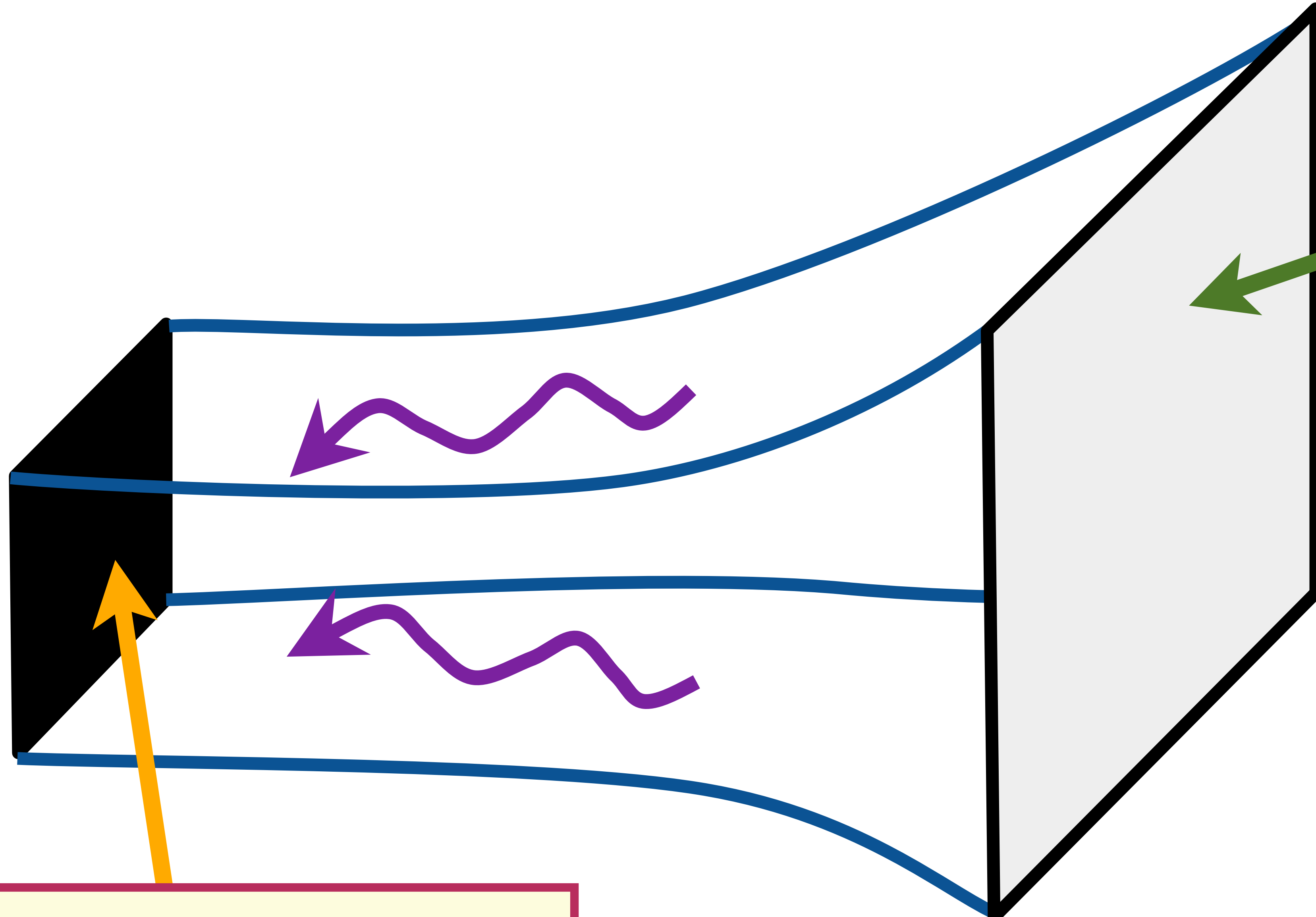
There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

String theory at non-zero temperatures



String theory at non-zero temperatures

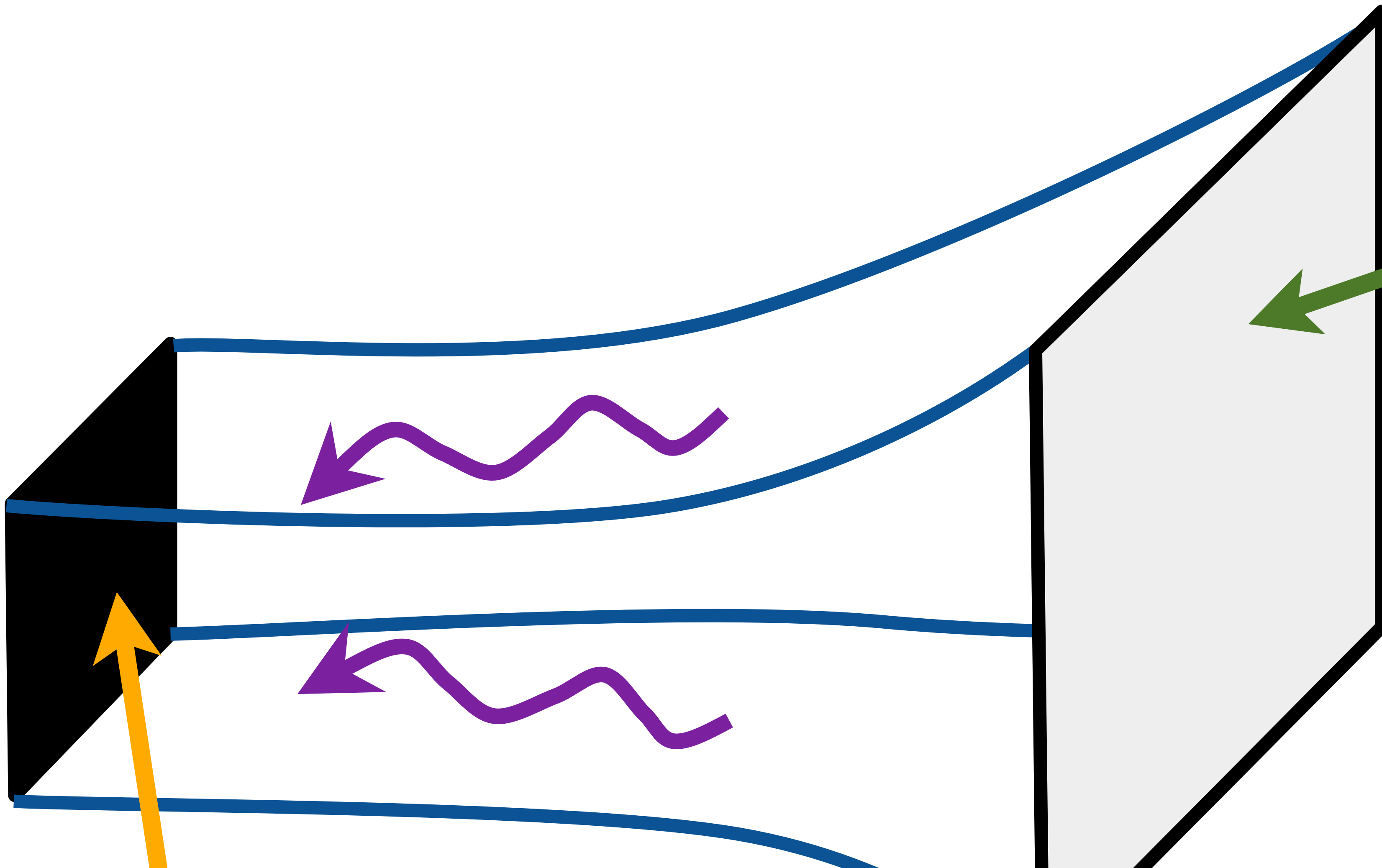


A 2+1 dimensional system at its quantum critical point

A "horizon", whose temperature and entropy equal those of the quantum critical point

Friction of quantum criticality = waves falling into black brane

String theory at non-zero temperatures



A 2+1 dimensional system at its quantum critical point

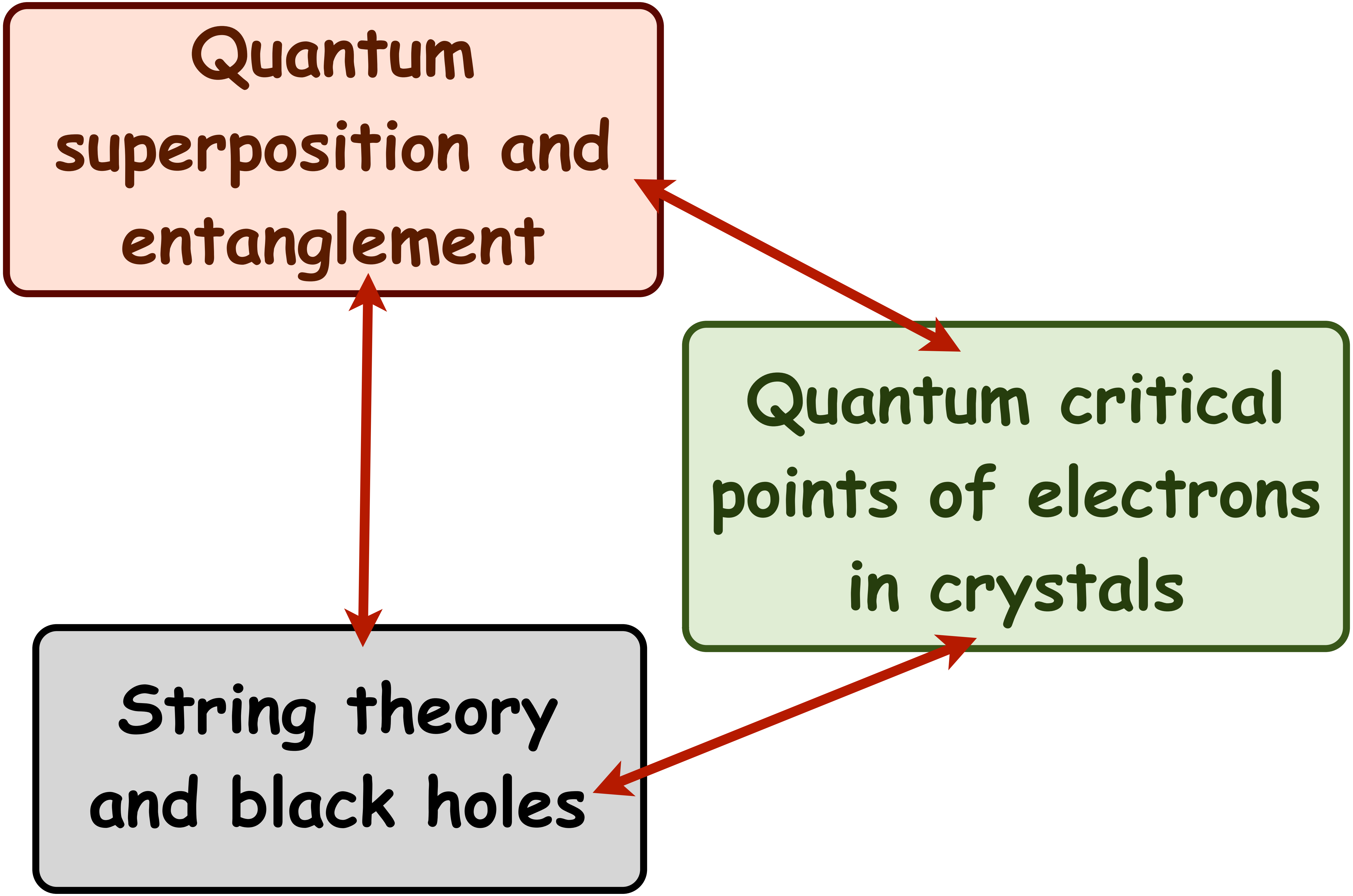
A “horizon”, whose temperature and entropy equal those of the quantum critical point

An (extended) Einstein-Maxwell provides successful description of dynamics of quantum critical points at non-zero temperatures (where no other methods apply)

Quantum superposition and entanglement

Quantum critical points of electrons in crystals

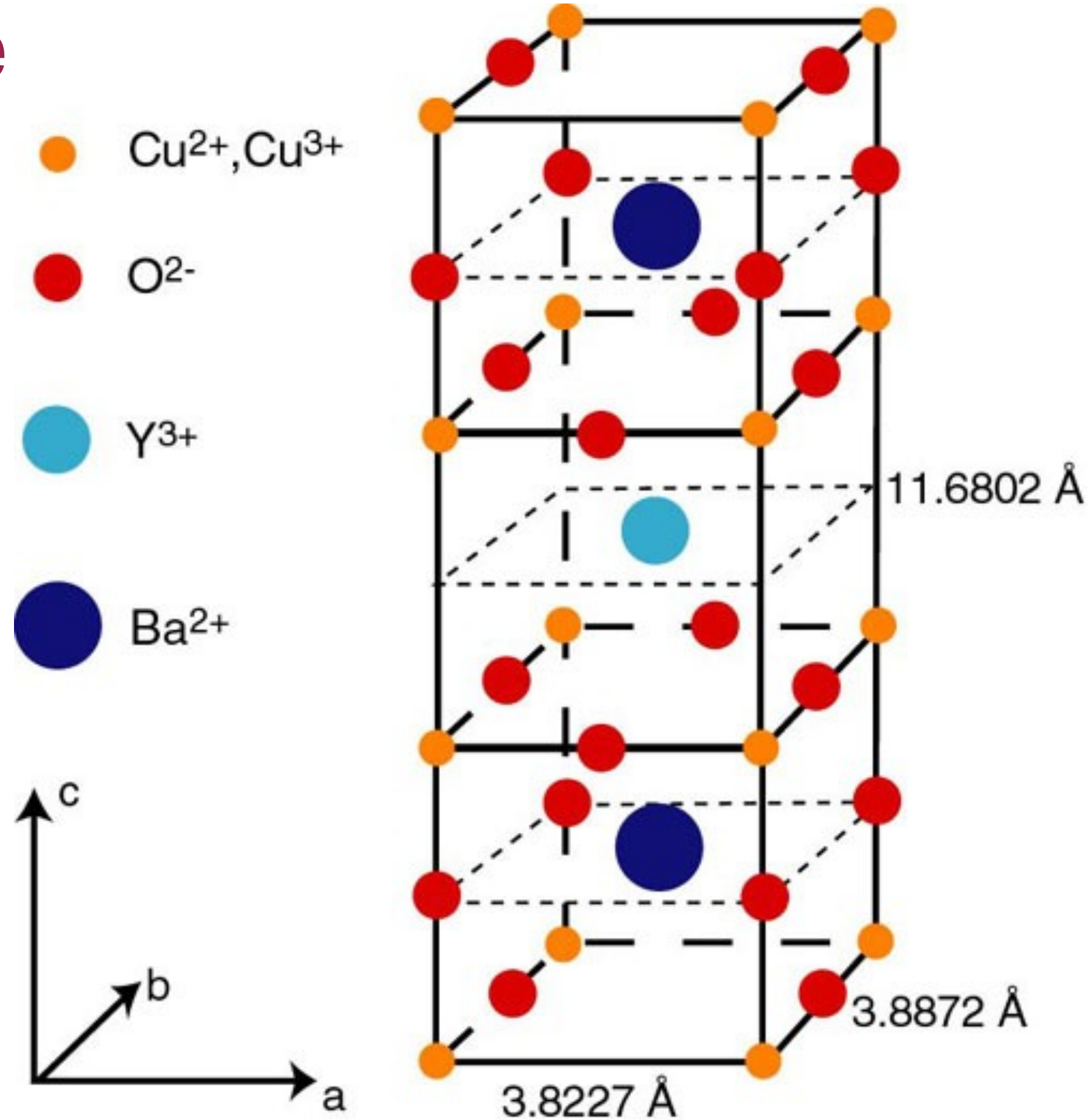
String theory and black holes


Quantum superposition and entanglement
Quantum critical points of electrons in crystals
String theory and black holes

Metals, “strange metals”, and high temperature superconductors

Insights from gravitational “duals”

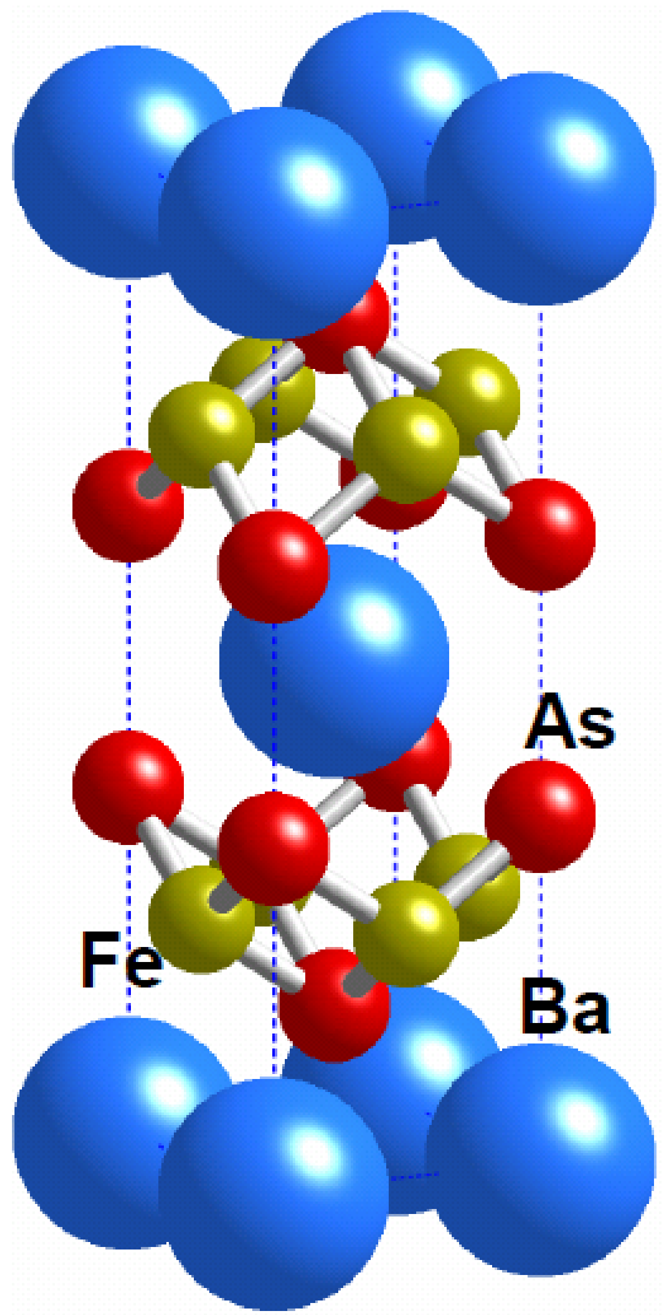
High temperature superconductors

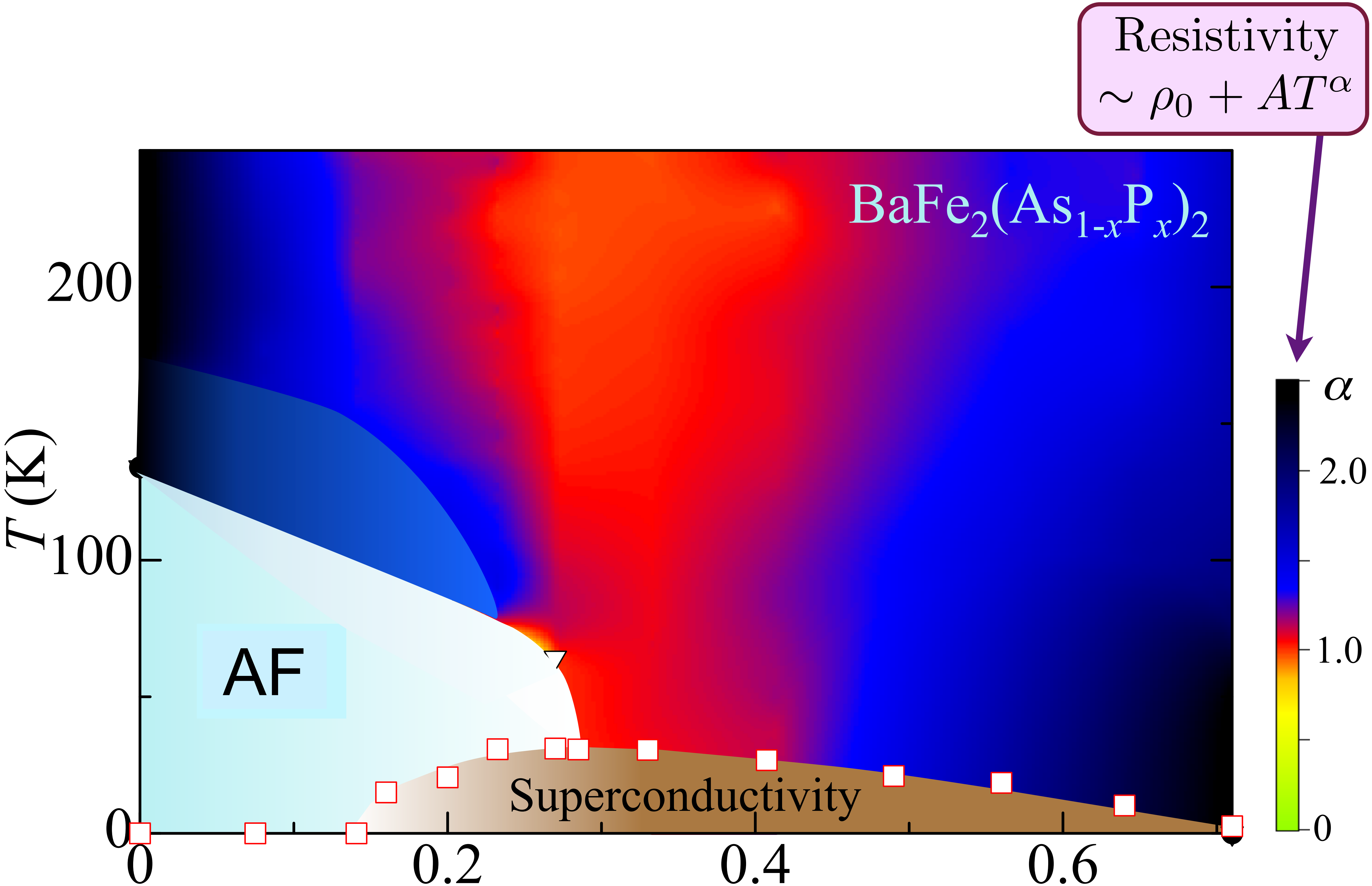


$$\mathrm{YBa_2Cu_3O}_{6+x}$$

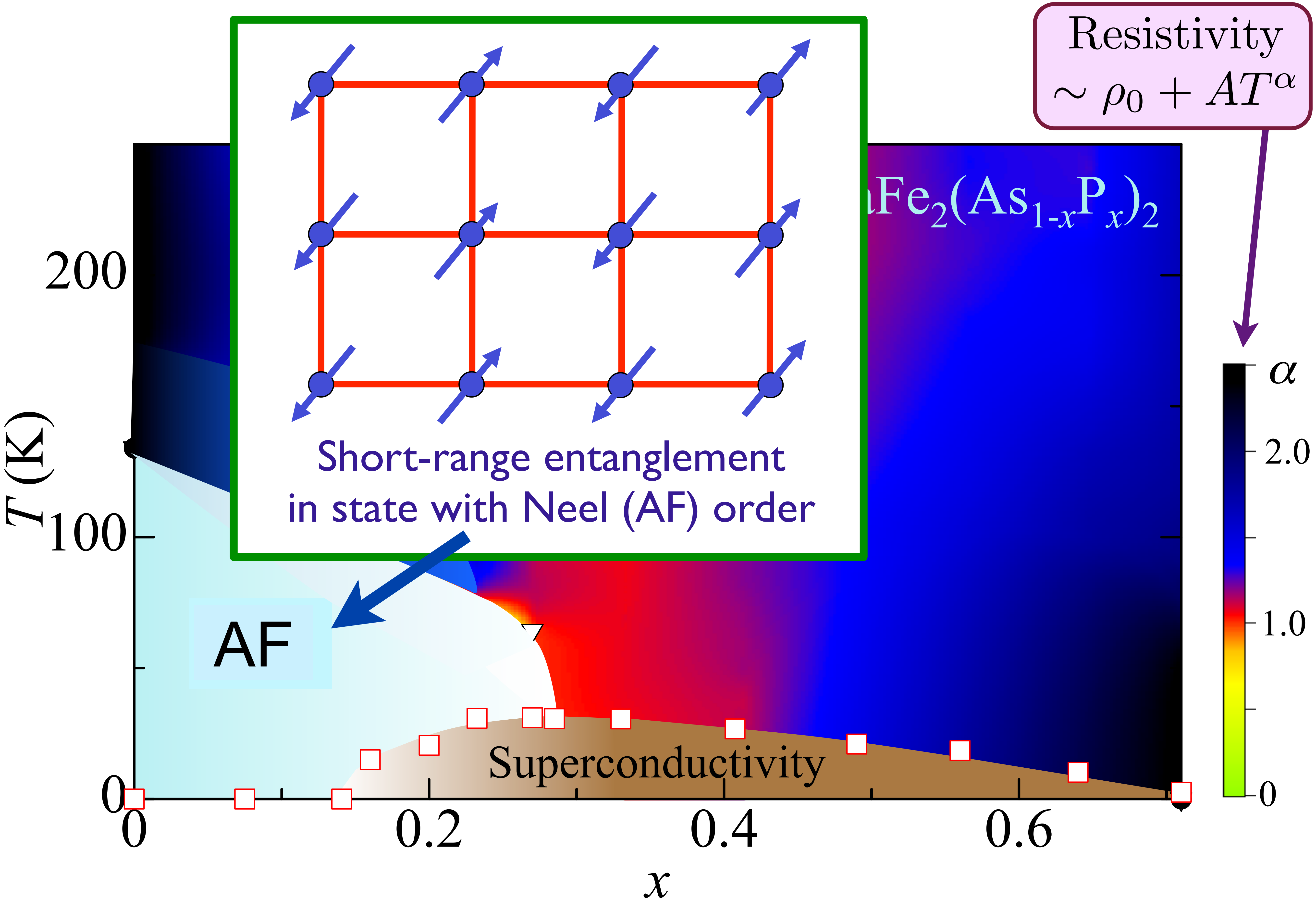
Iron pnictides:

a new class of high temperature superconductors



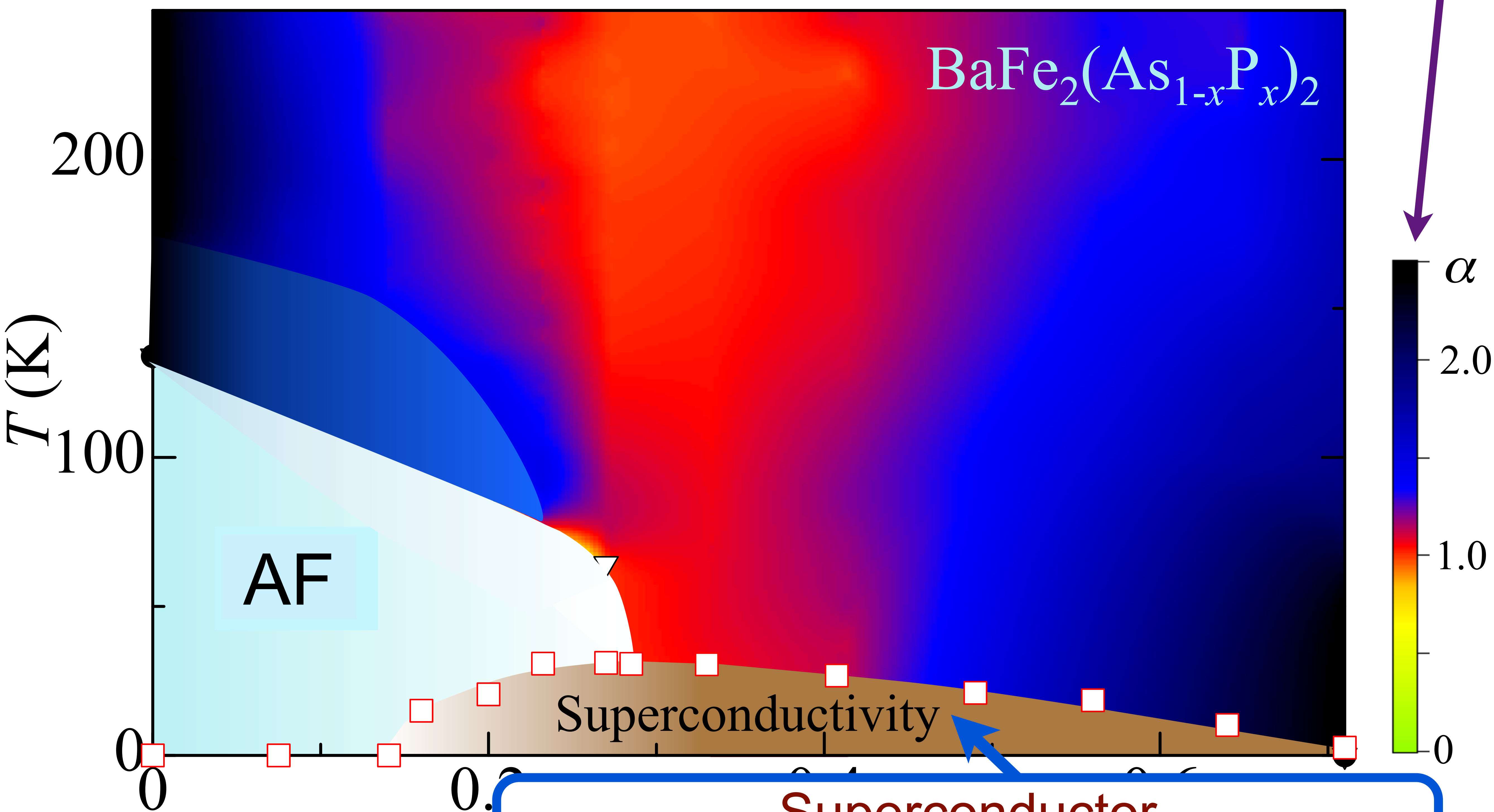


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



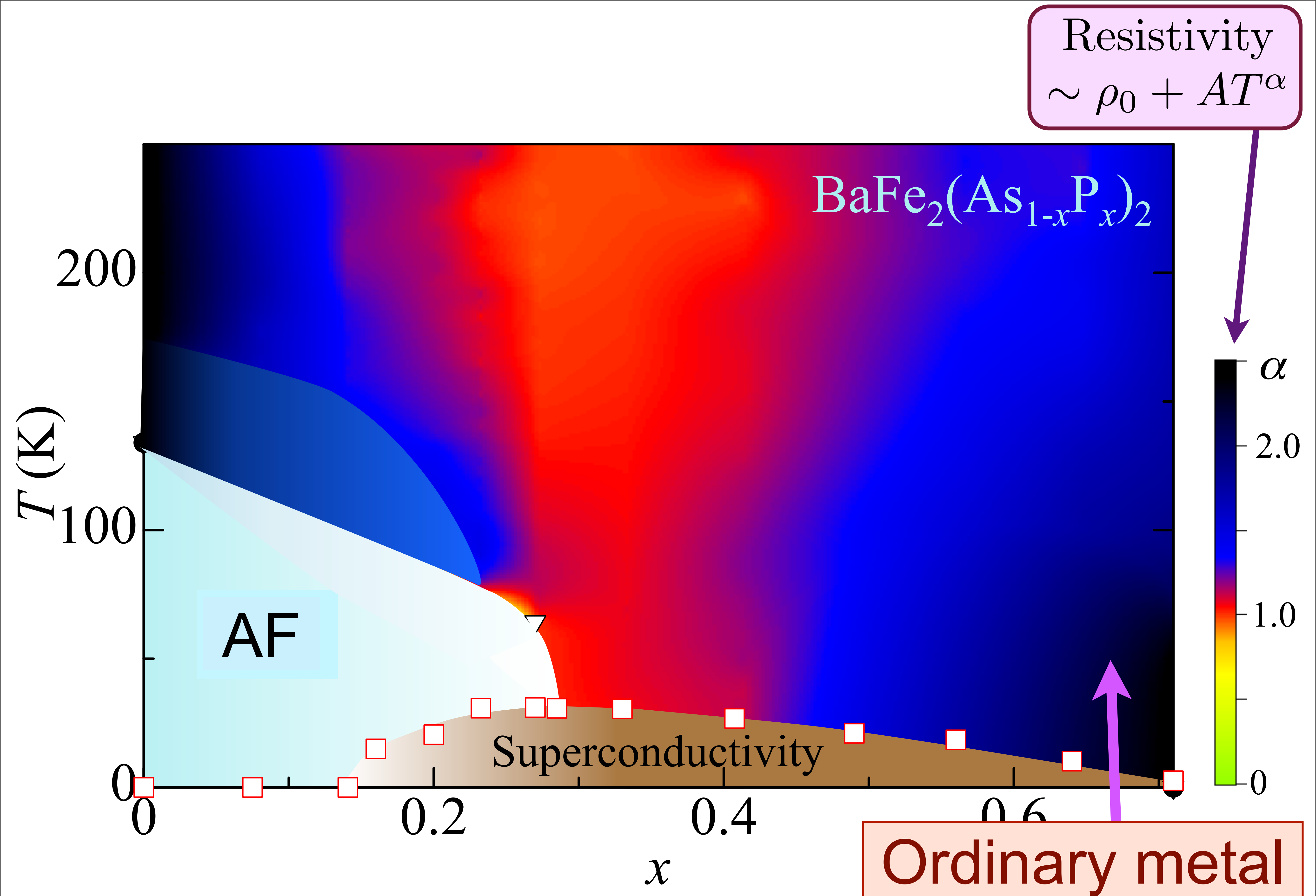
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)

Resistivity
$\sim \rho_0 + AT^\alpha$



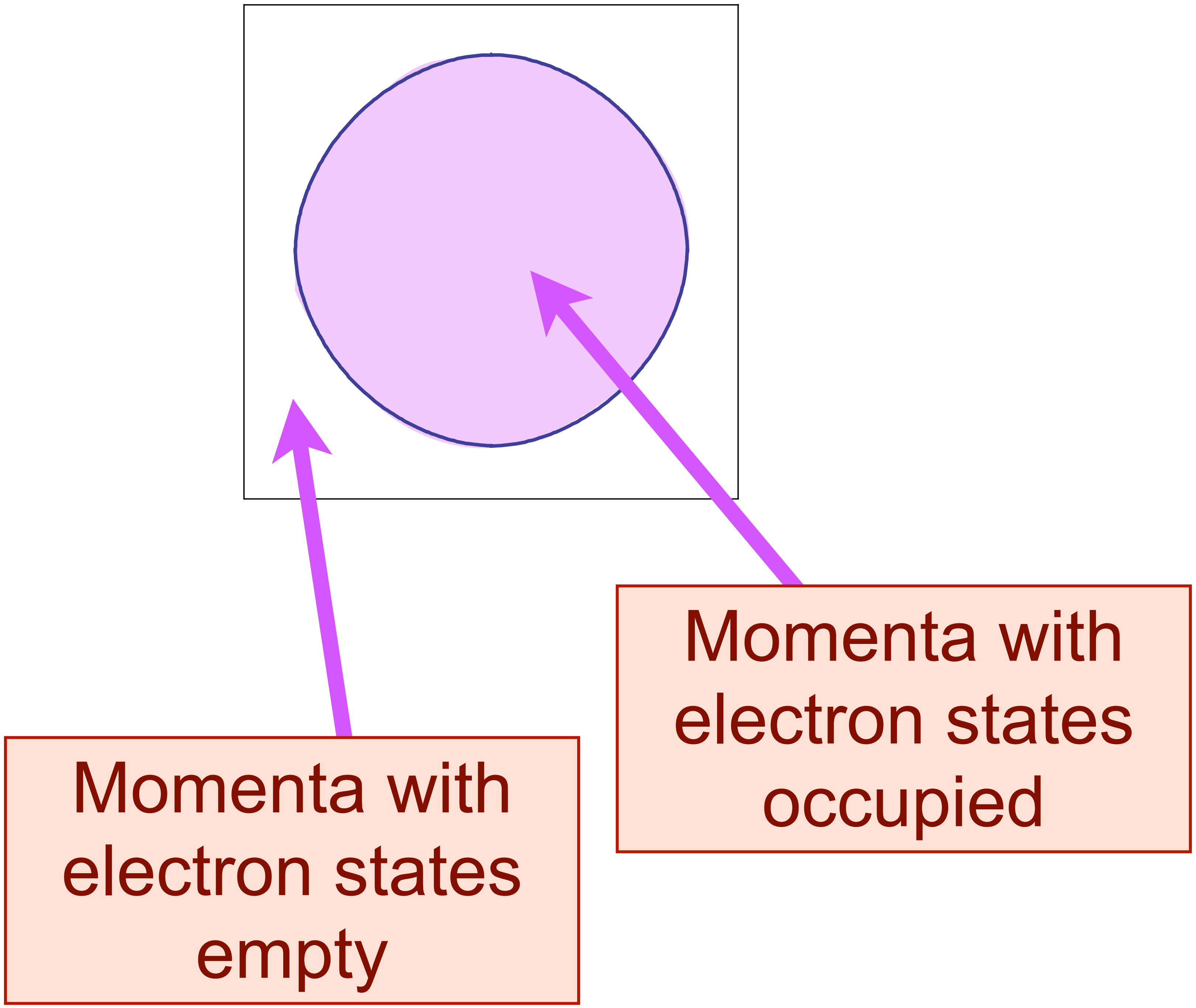
Superconductor
Bose condensate of pairs of electrons
Short-range entanglement

S. Kasahara, T. Shiba
H. Ike

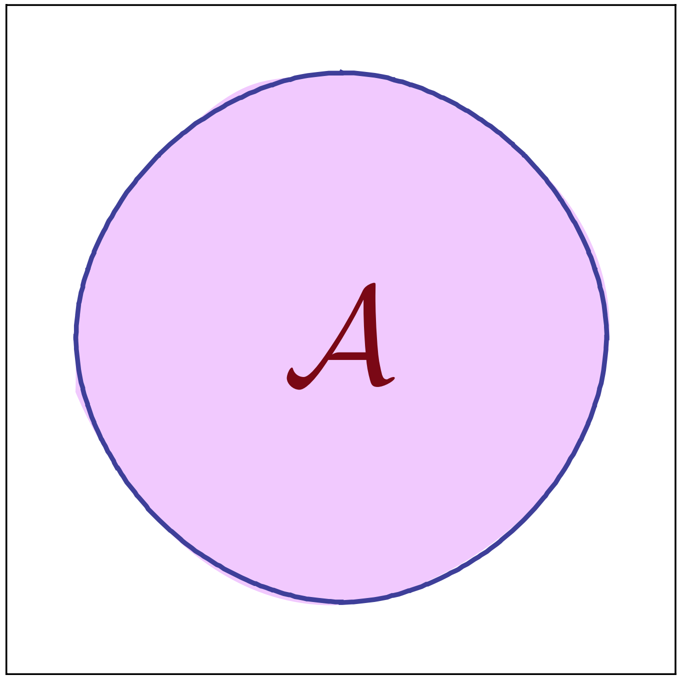


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Mat
Physical Review B **81**, 184519 (2010)

Sommerfeld-Bloch theory of ordinary metals

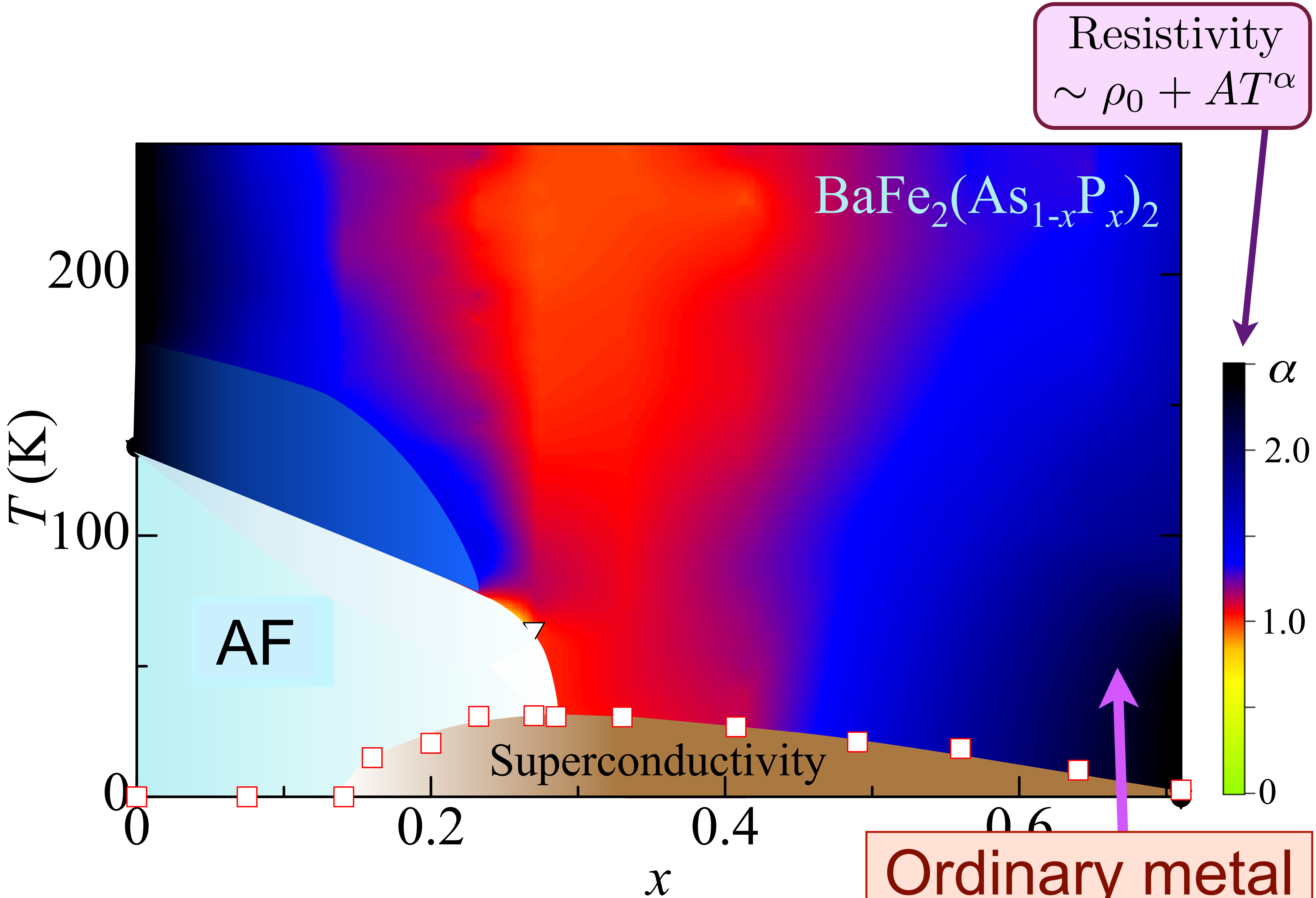


Sommerfeld-Bloch theory of ordinary metals



Key feature of the theory:
the **Fermi surface**

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the electron density
- Excitations near the Fermi surface are responsible for the familiar properties of ordinary metals, such as resistivity $\sim T^2$.



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O
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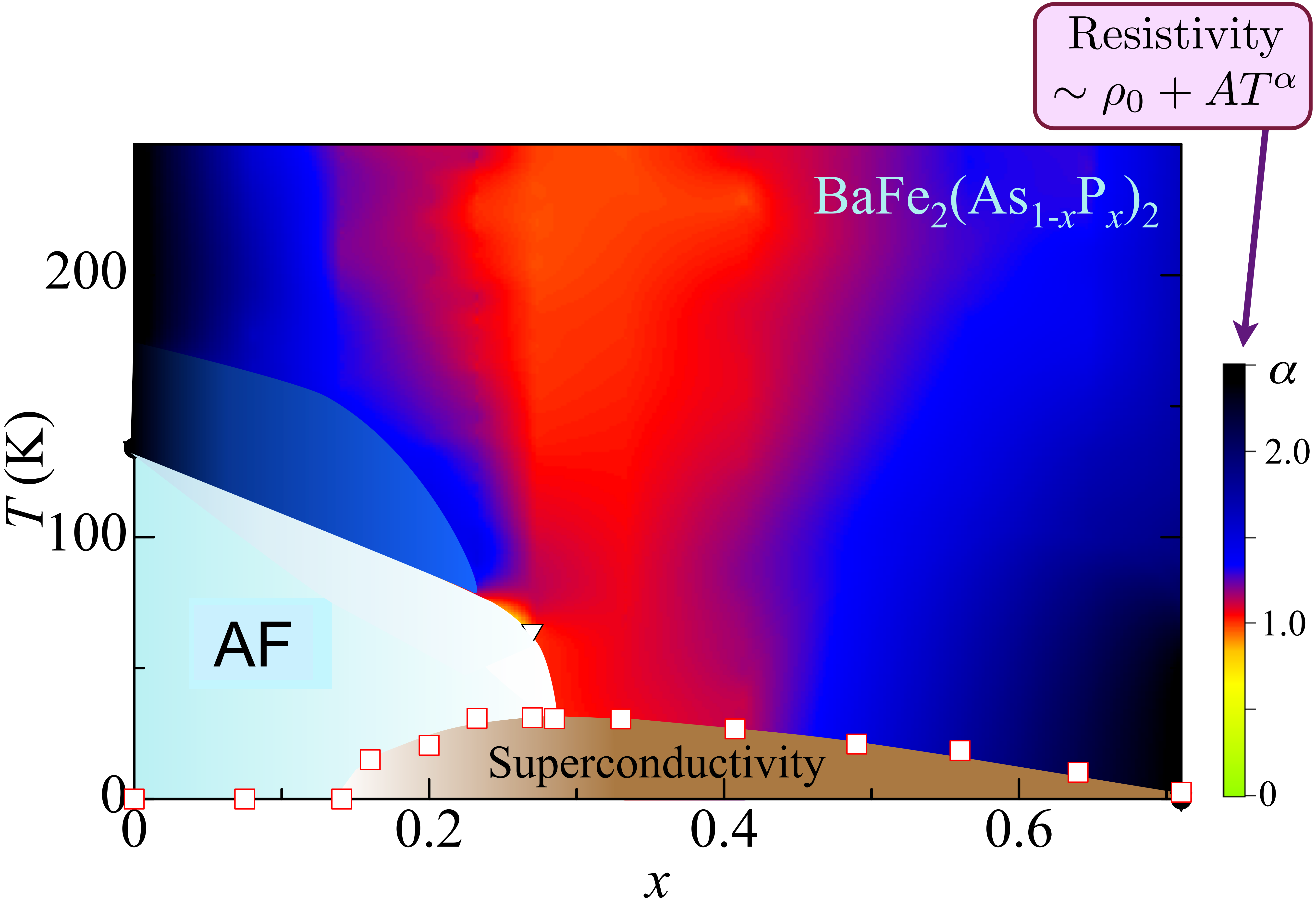
Physical Review B **81**, 184519 (2010)



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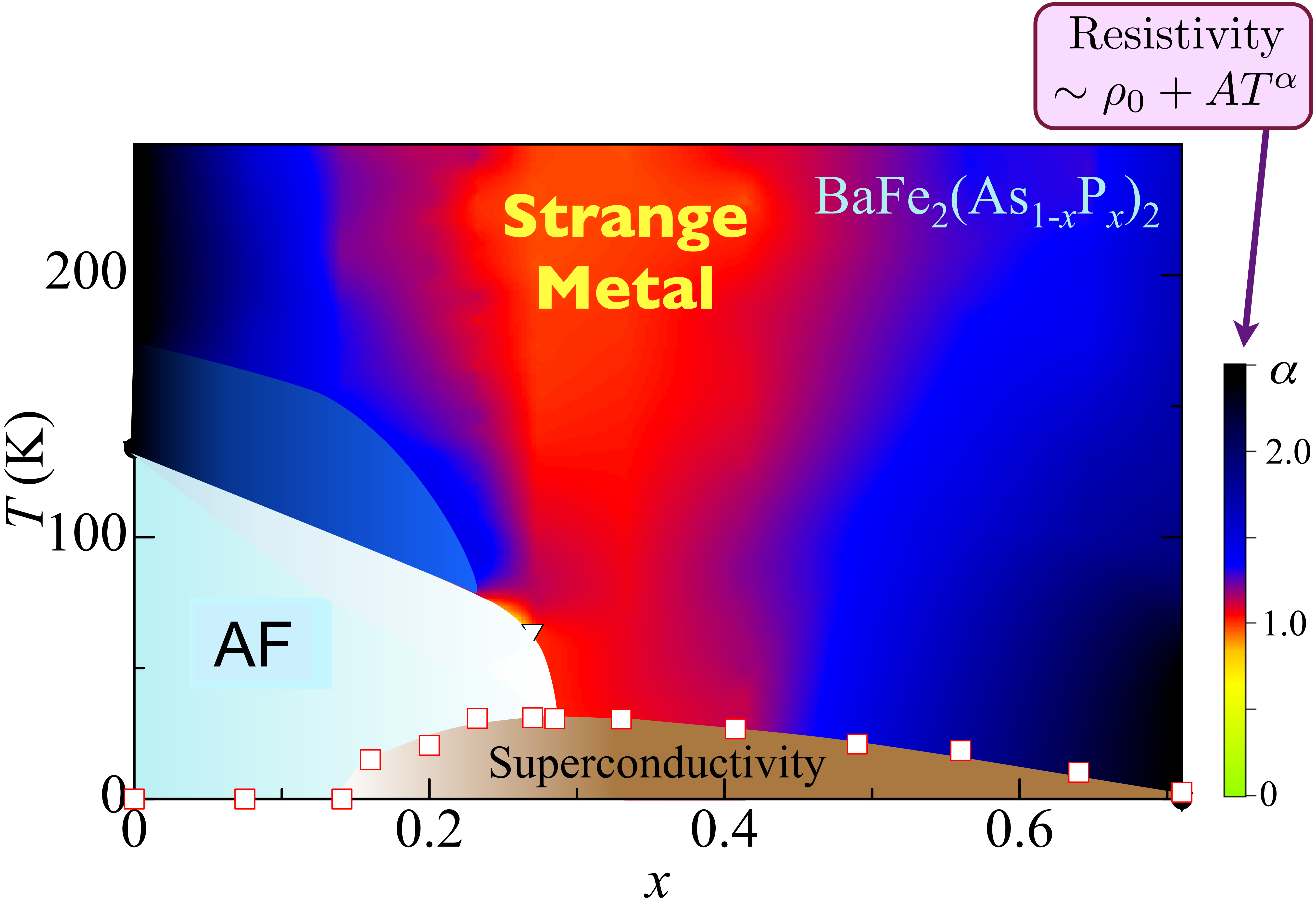

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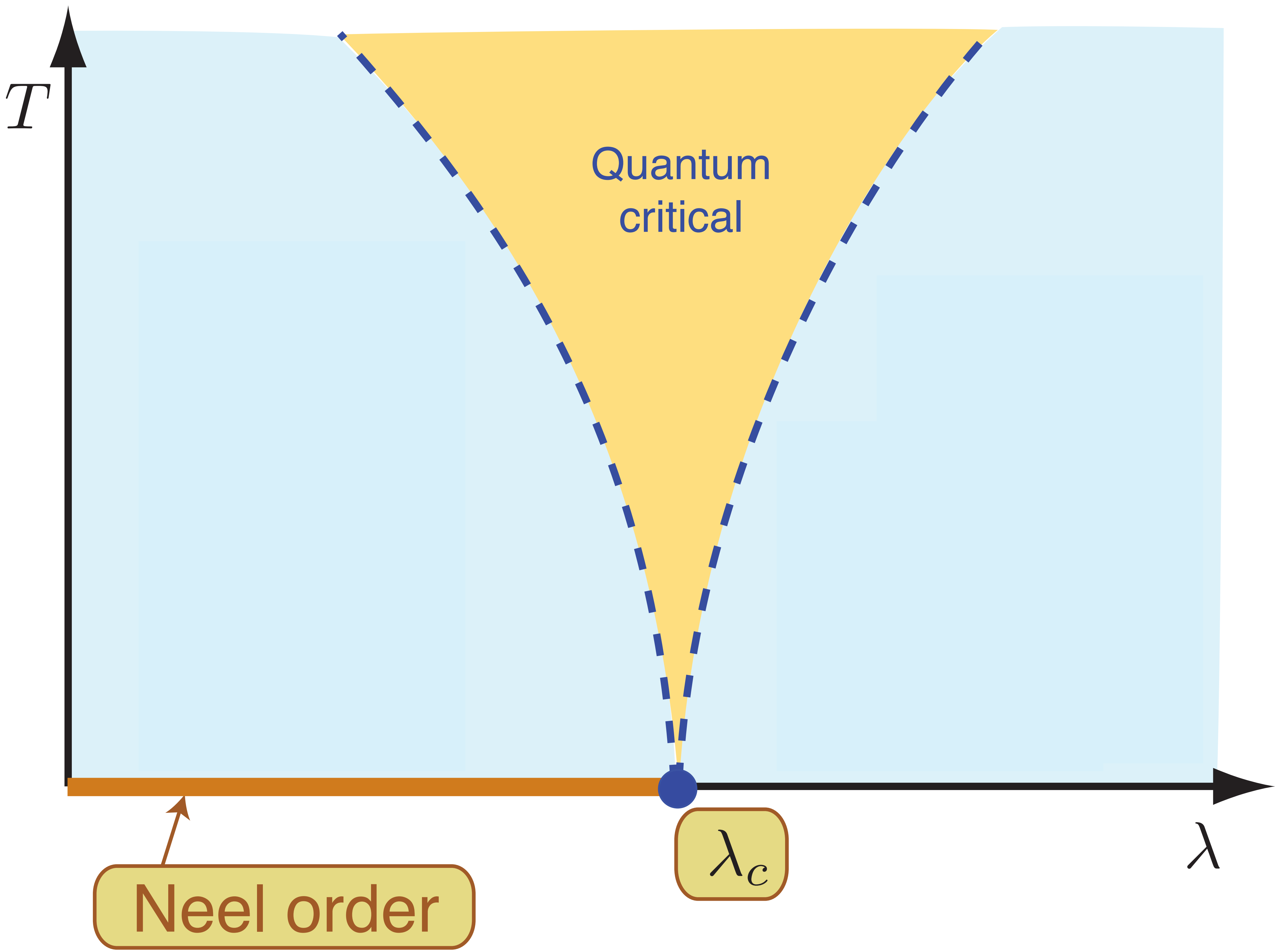
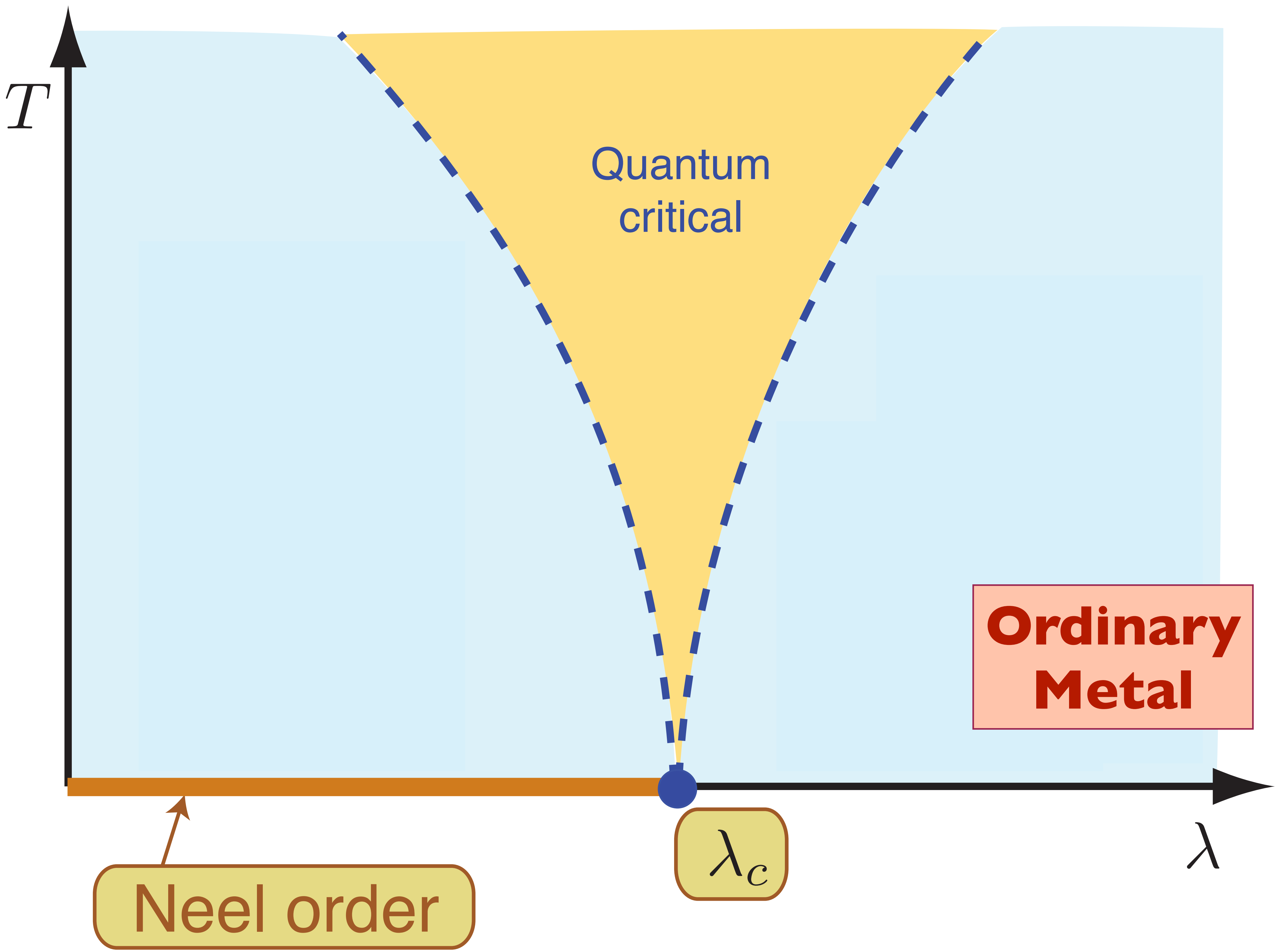
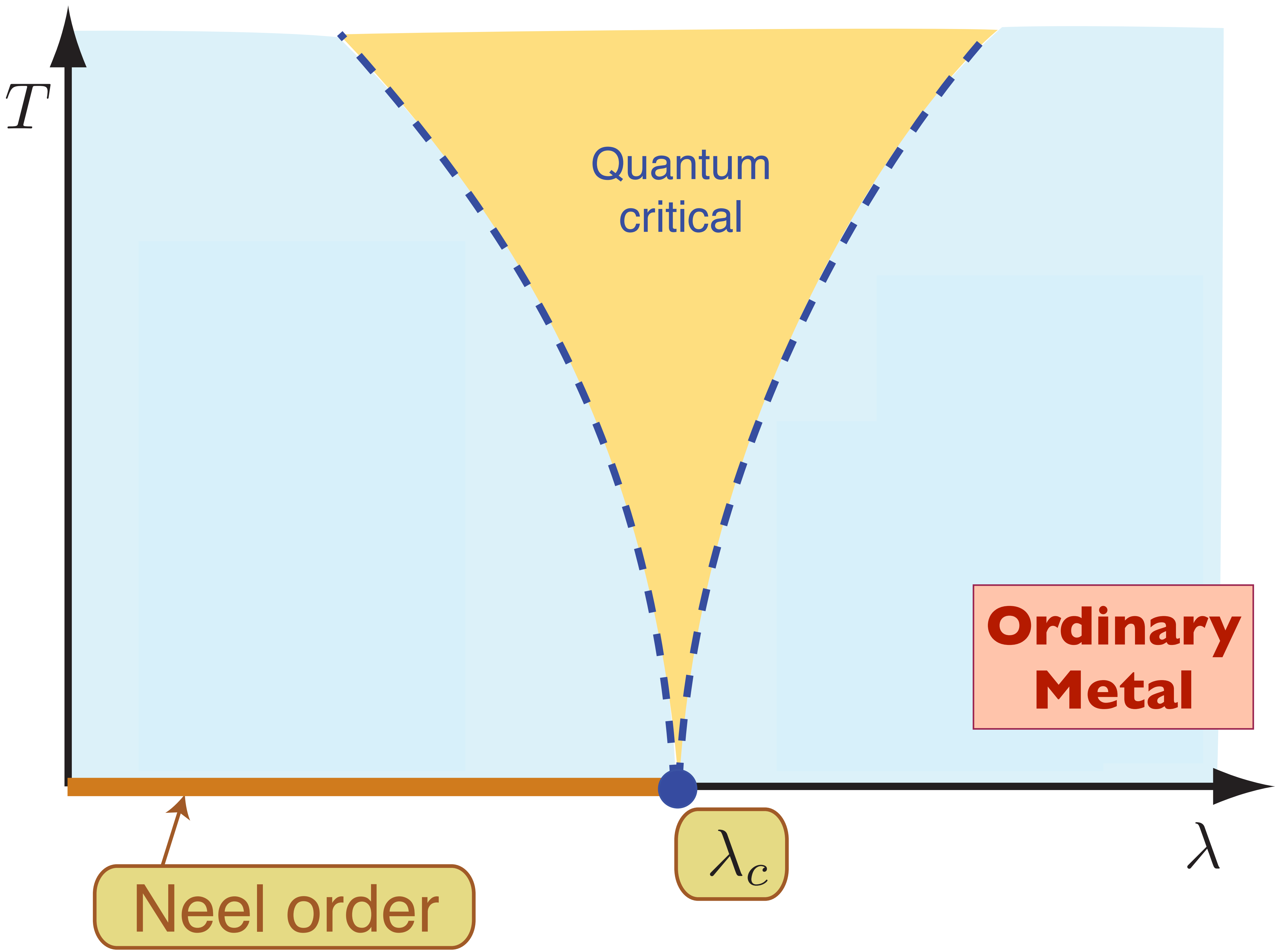
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

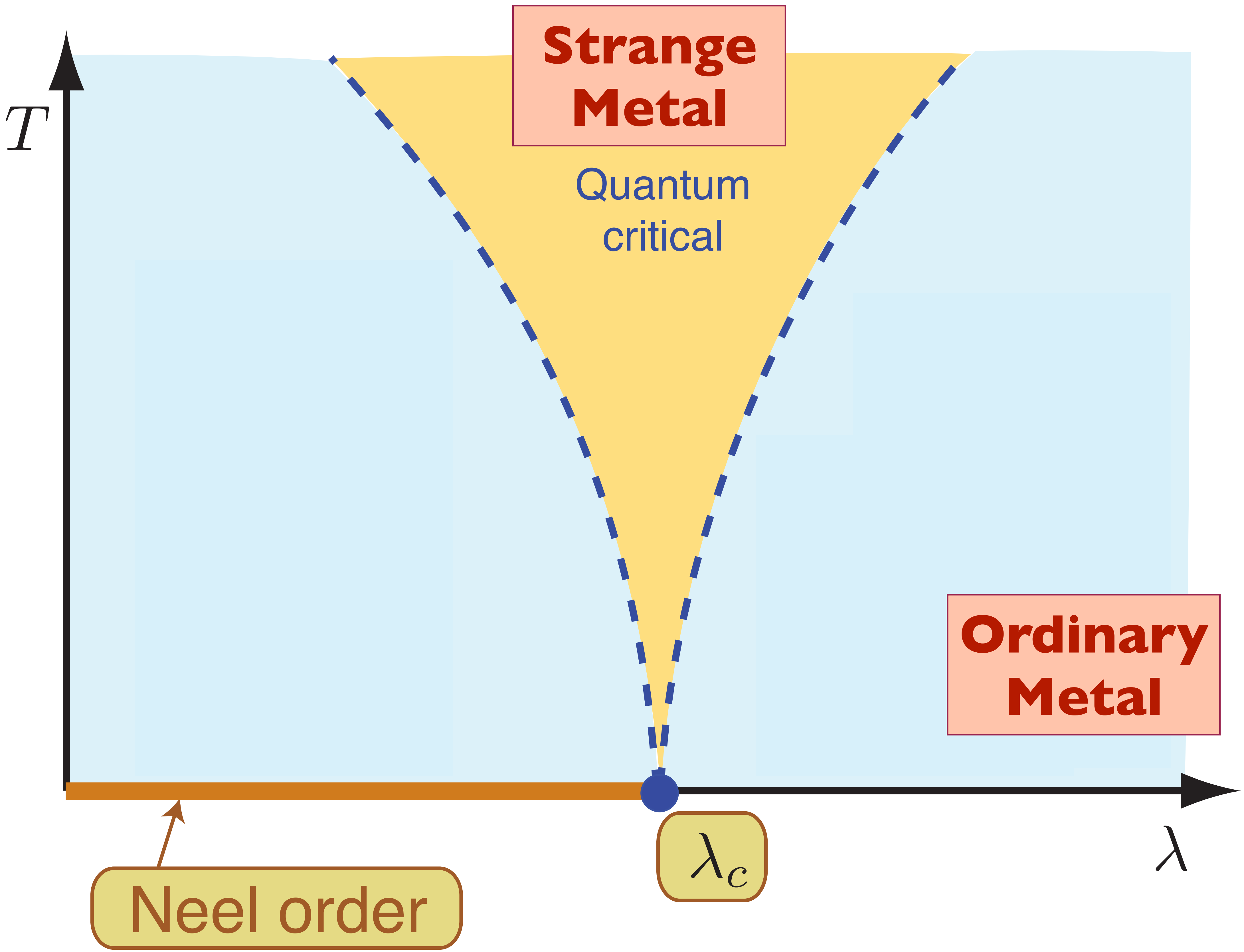


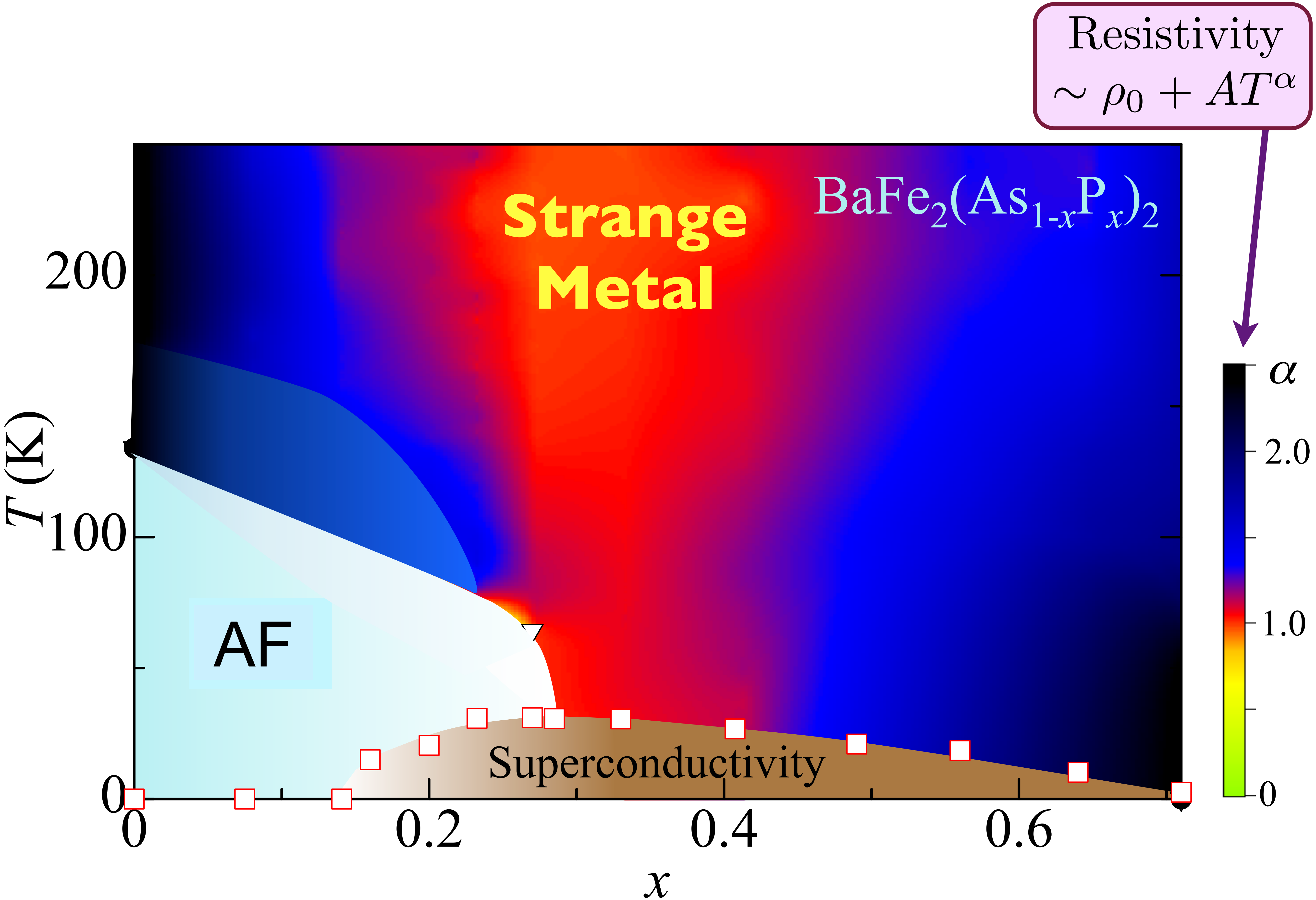


T
Quantum critical
Ordinary Metal
Neel order
λ_c
λ

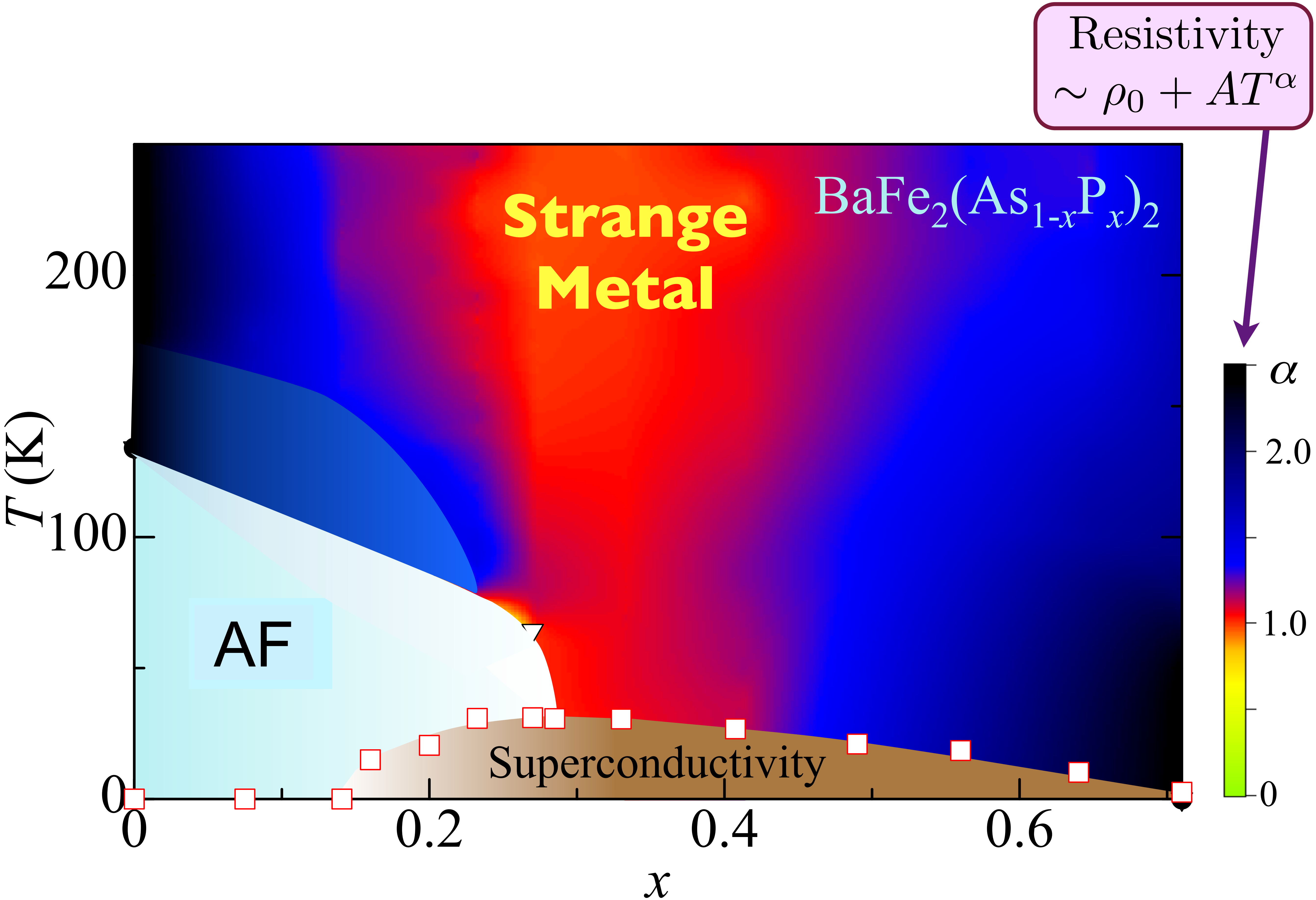


T
Quantum
critical
Ordinary
Metal
λ
λ_c
Neel order


Strange Metal
Quantum critical
Ordinary Metal
T
λ
λ_c
Neel order



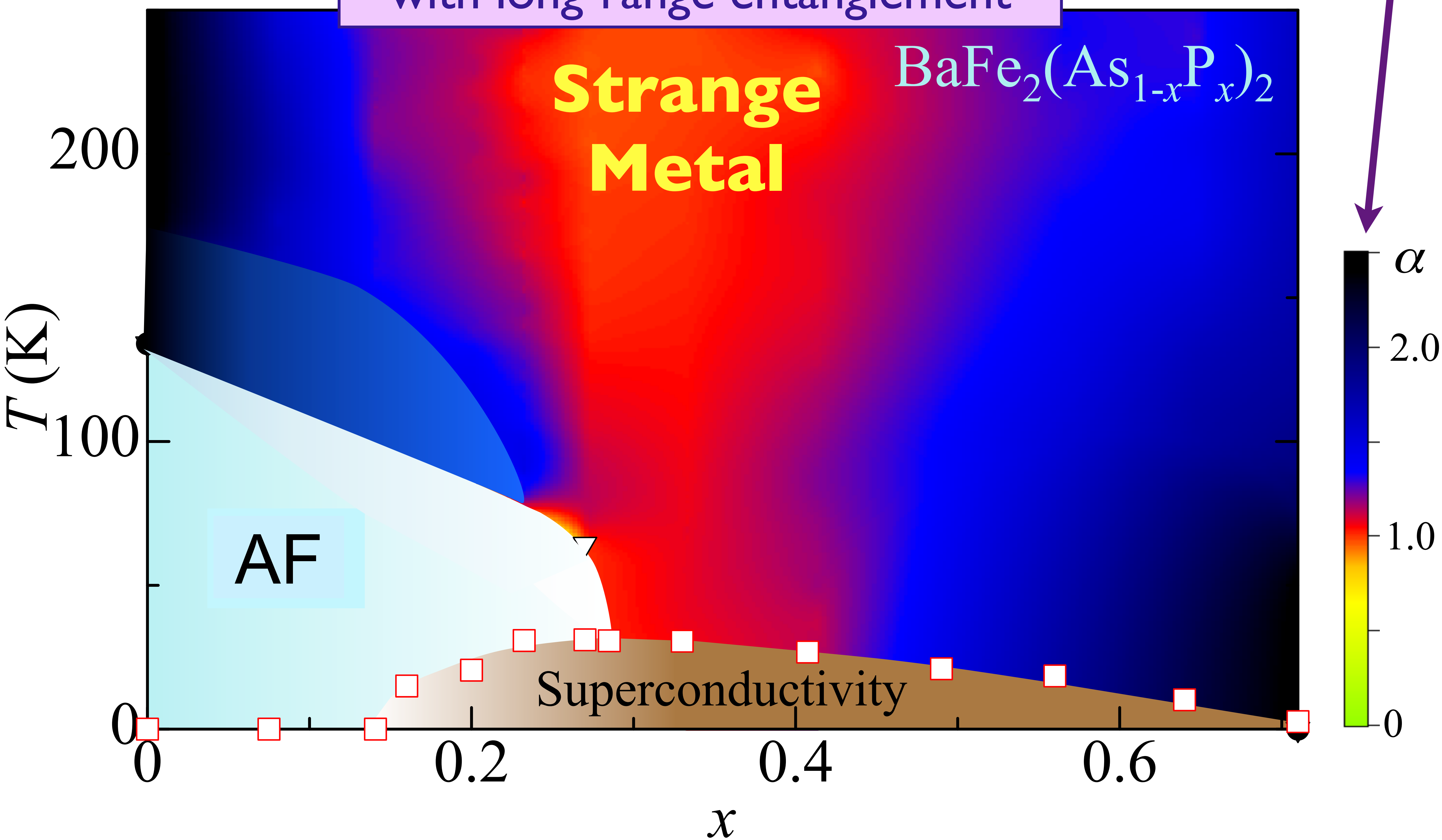
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Excitations of a ground state with long-range entanglement

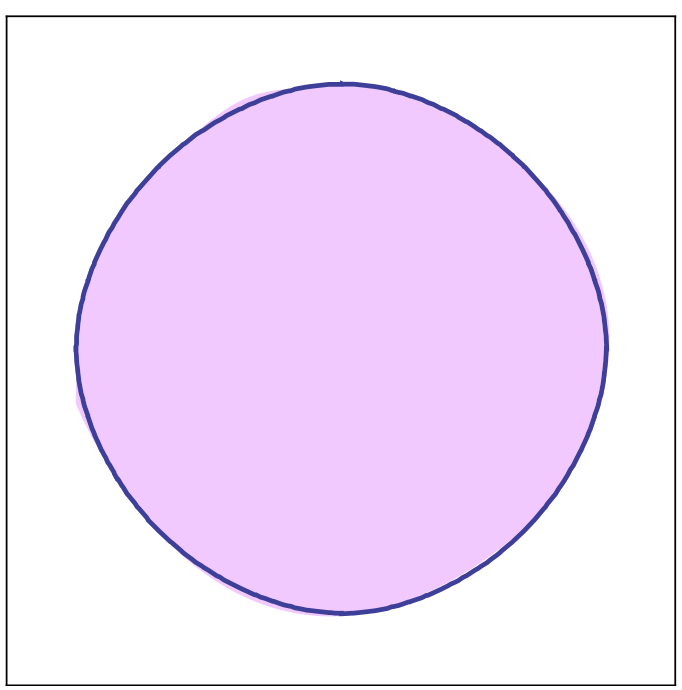
Resistivity $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

<u>Key (difficult) problem:</u>

Describe quantum critical points and phases of systems with Fermi surfaces leading to metals with novel types of long-range entanglement



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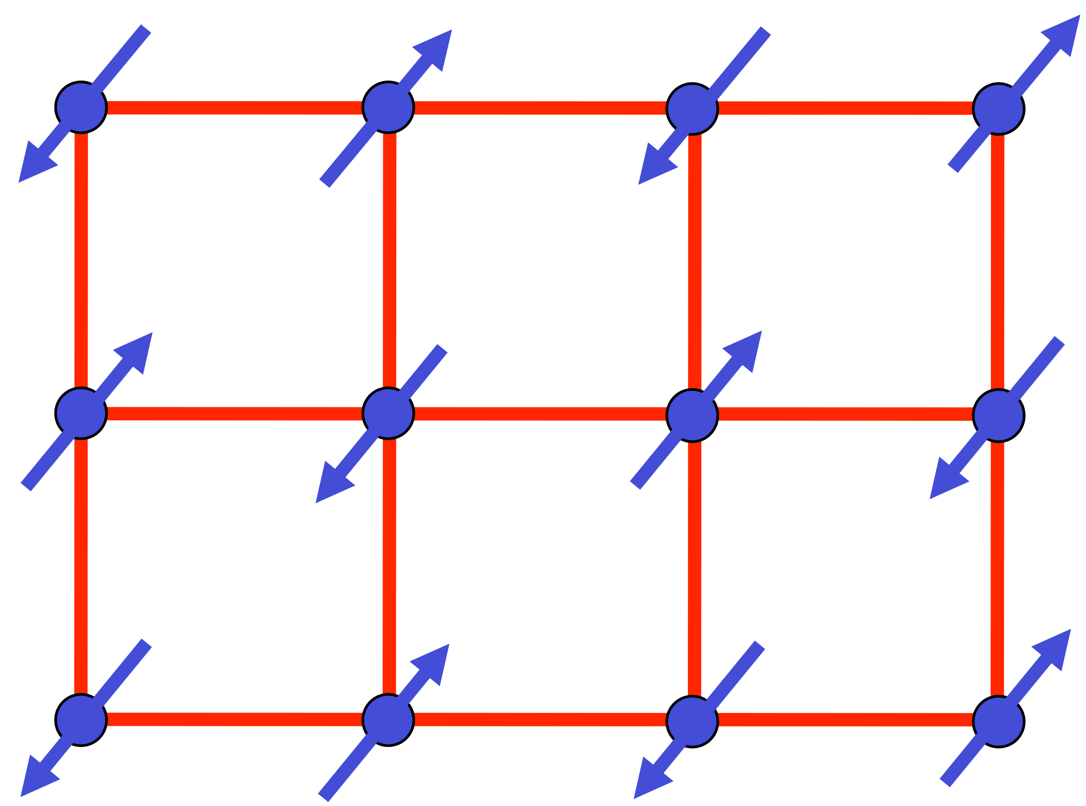


Challenge to string theory:

Describe quantum critical points and phases of metals

Challenge to string theory:

Describe quantum critical points and phases of metals

Can we obtain gravitational theories of superconductors and ordinary Sommerfeld-Bloch metals ?

Challenge to string theory:

Describe quantum critical points and phases of metals

Can we obtain gravitational theories of superconductors and ordinary Sommerfeld-Bloch metals ?

Yes

T. Nishioka, S. Ryu, and T. Takayanagi, JHEP **1003**, 131 (2010)
G. T. Horowitz and B. Way, JHEP **1011**, 011 (2010)
S. Sachdev, Physical Review D **84**, 066009 (2011)

Challenge to string theory:

Describe quantum critical points and phases of metals

Do the “holographic” gravitational theories also yield metals distinct from ordinary Sommerfeld-Bloch metals ?

Challenge to string theory:

Describe quantum critical points and phases of metals

Do the "holographic" gravitational theories also yield metals distinct from ordinary Sommerfeld-Bloch metals ?

Yes, lots of them, with many "strange" properties !

Challenge to string theory:

Describe quantum critical points
and phases of metals

How do we discard artifacts, and choose the
holographic theories applicable to condensed matter physics ?

Challenge to string theory:

Describe quantum critical points and phases of metals

How do we discard artifacts, and choose the holographic theories applicable to condensed matter physics ?

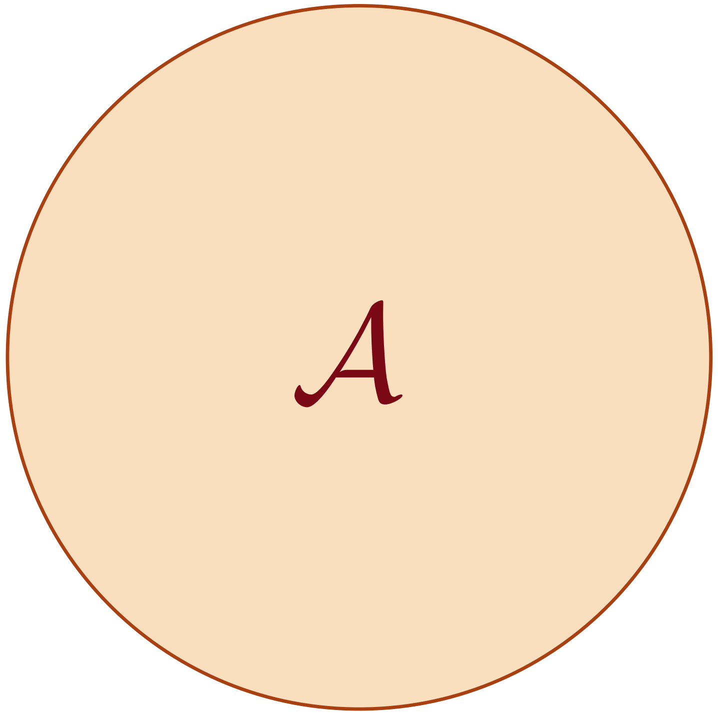
Choose the theories with the proper entropy density

Checks: these theories also have the proper entanglement entropy and Fermi surface size !

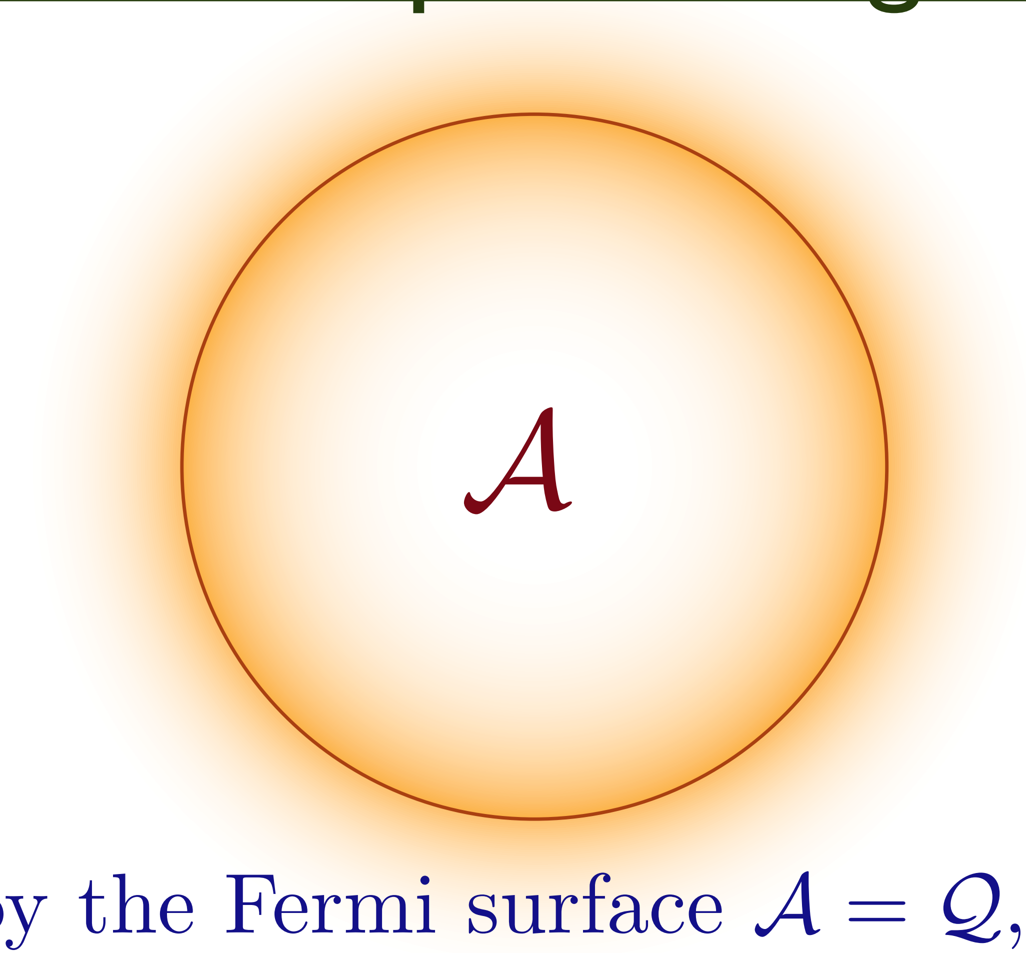
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

The simplest example of
a “strange metal”
is realized by
fermions with a Fermi surface
coupled to an Abelian
or non-Abelian gauge field.

Fermi surface of an ordinary metal



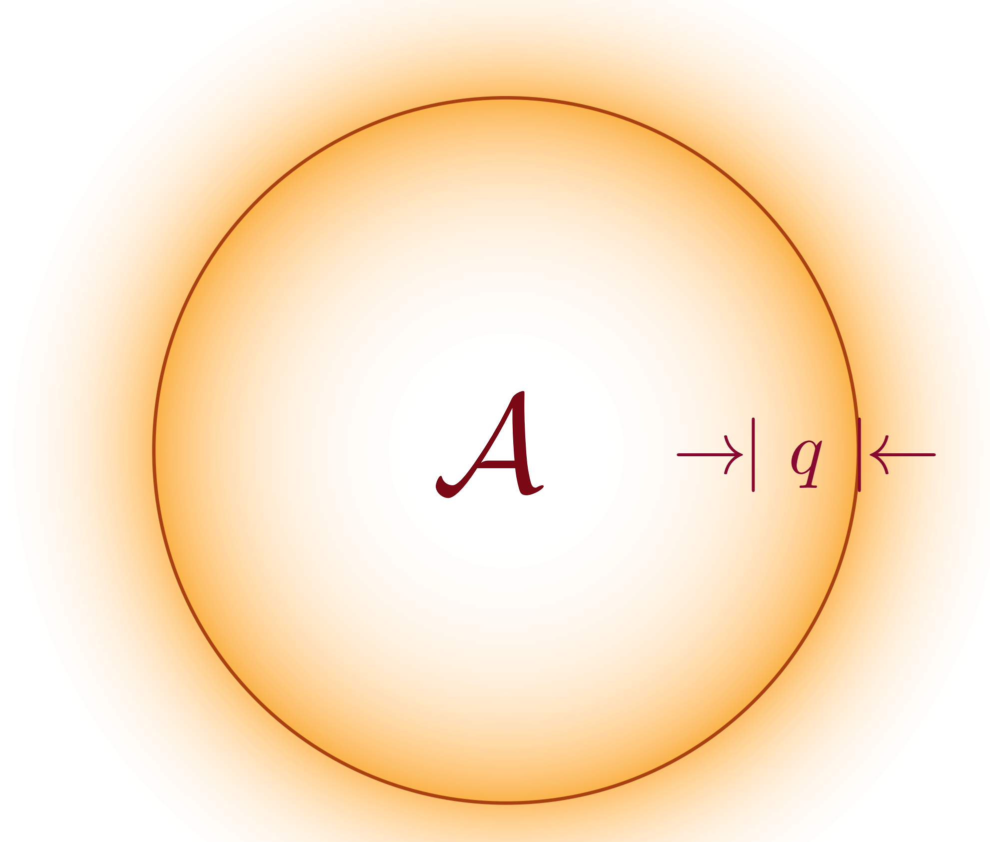
Fermions coupled to a gauge field



- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

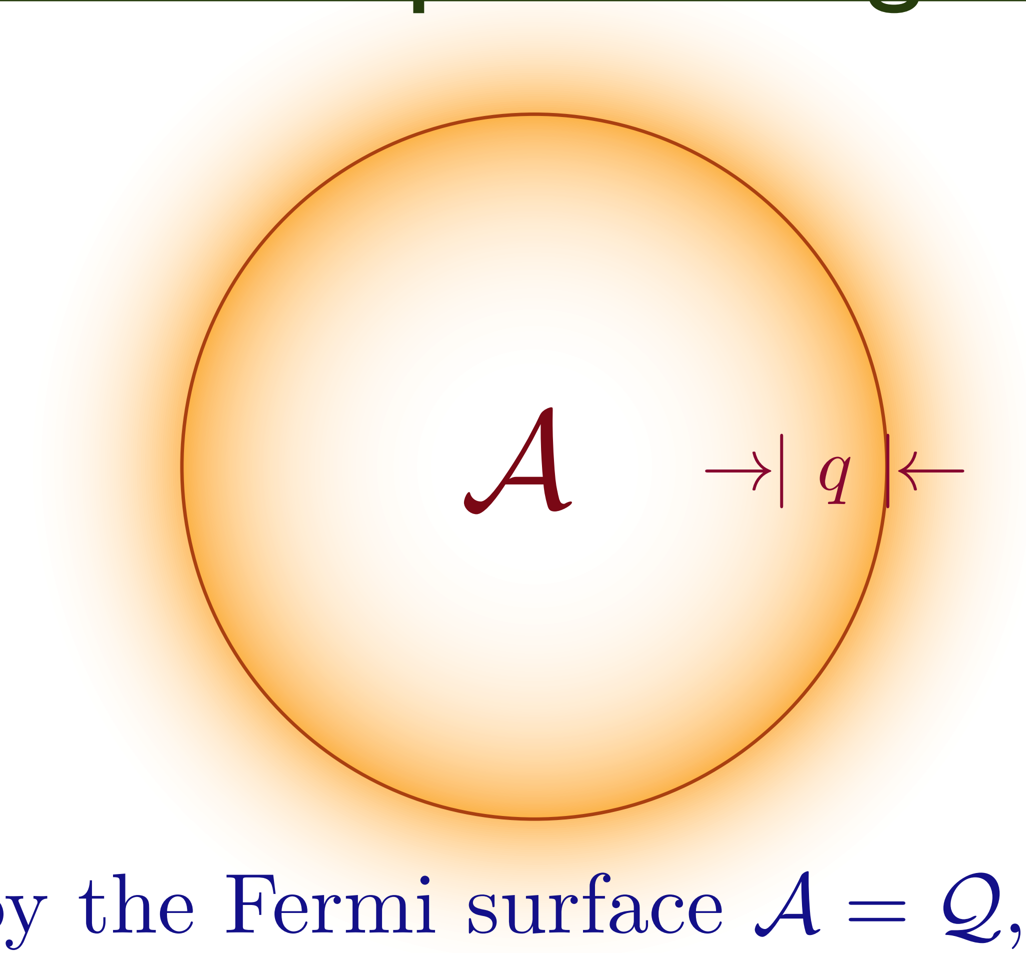
Fermions coupled to a gauge field



- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density
- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

Fermions coupled to a gauge field



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- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\rm eff}/z}$ with $d_{\rm eff} = 1$.

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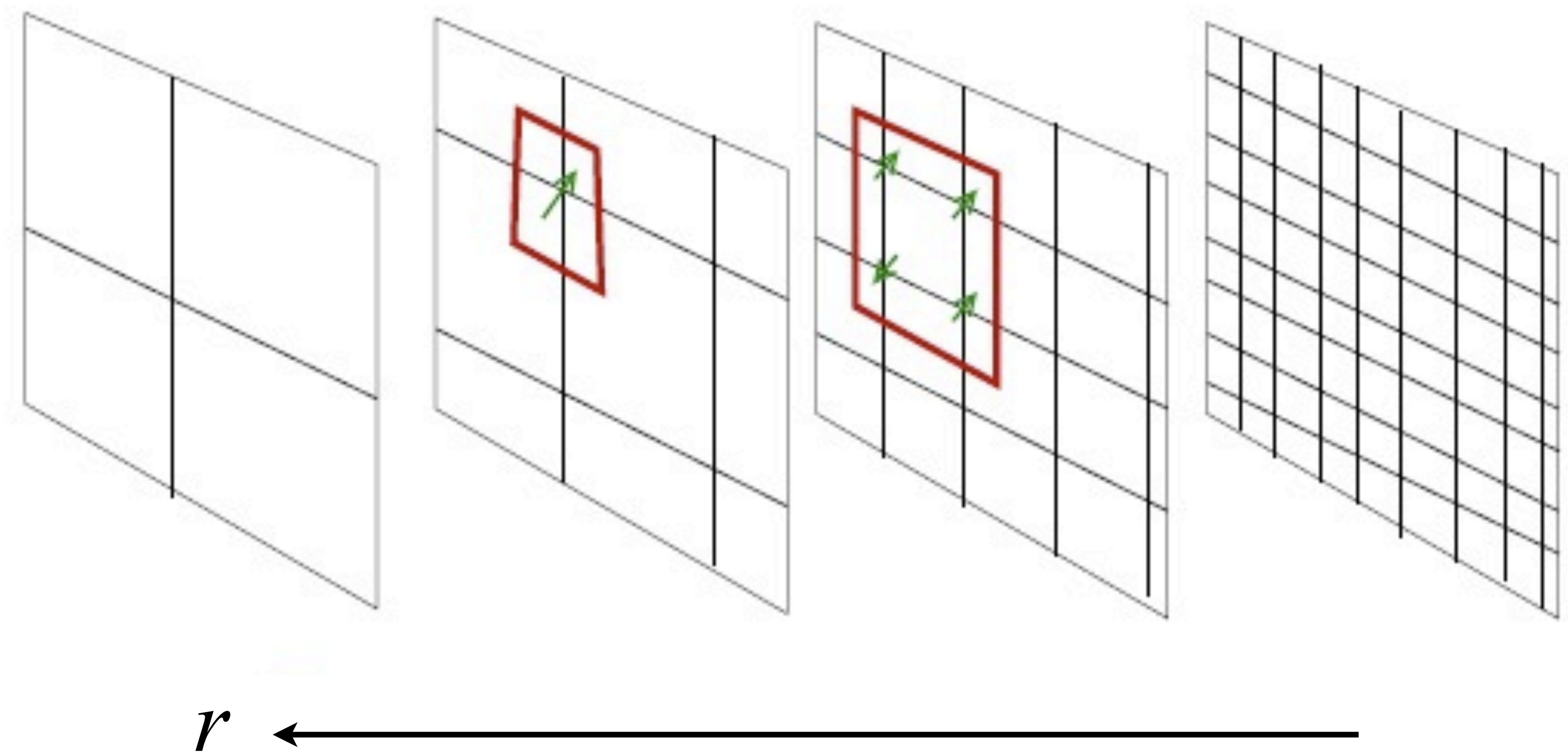
Holography of "strange metals"



J. McGreevy, arXiv0909.0518

Holography of "strange metals"

Consider the following (most) general metric for the holographic theory

$$\boxed{ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)}$$

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This metric transforms under rescaling as

$$\begin{aligned} x_i &\to \zeta\, x_i \\ t &\to \zeta^z\, t \\ ds &\to \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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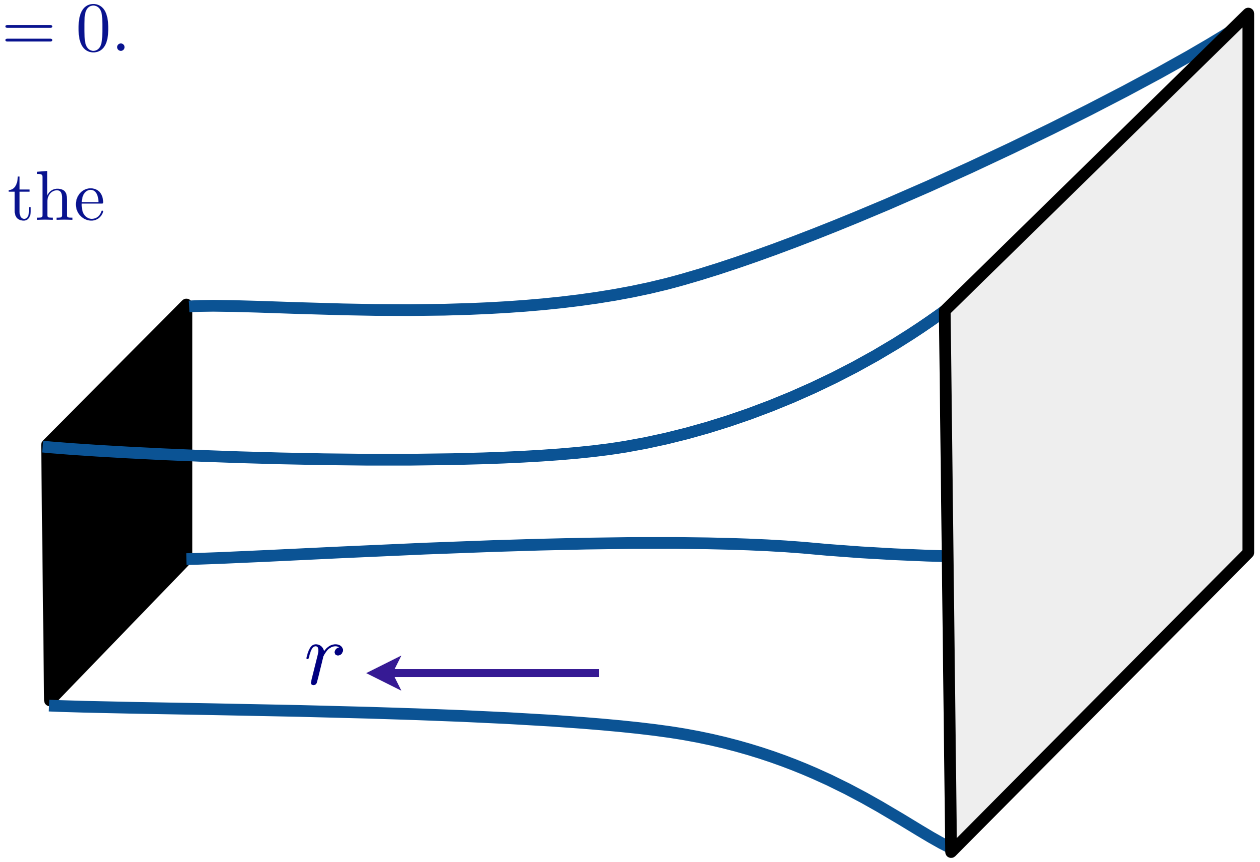
What is θ ? ($\theta = 0$ for "relativistic" quantum critical points).

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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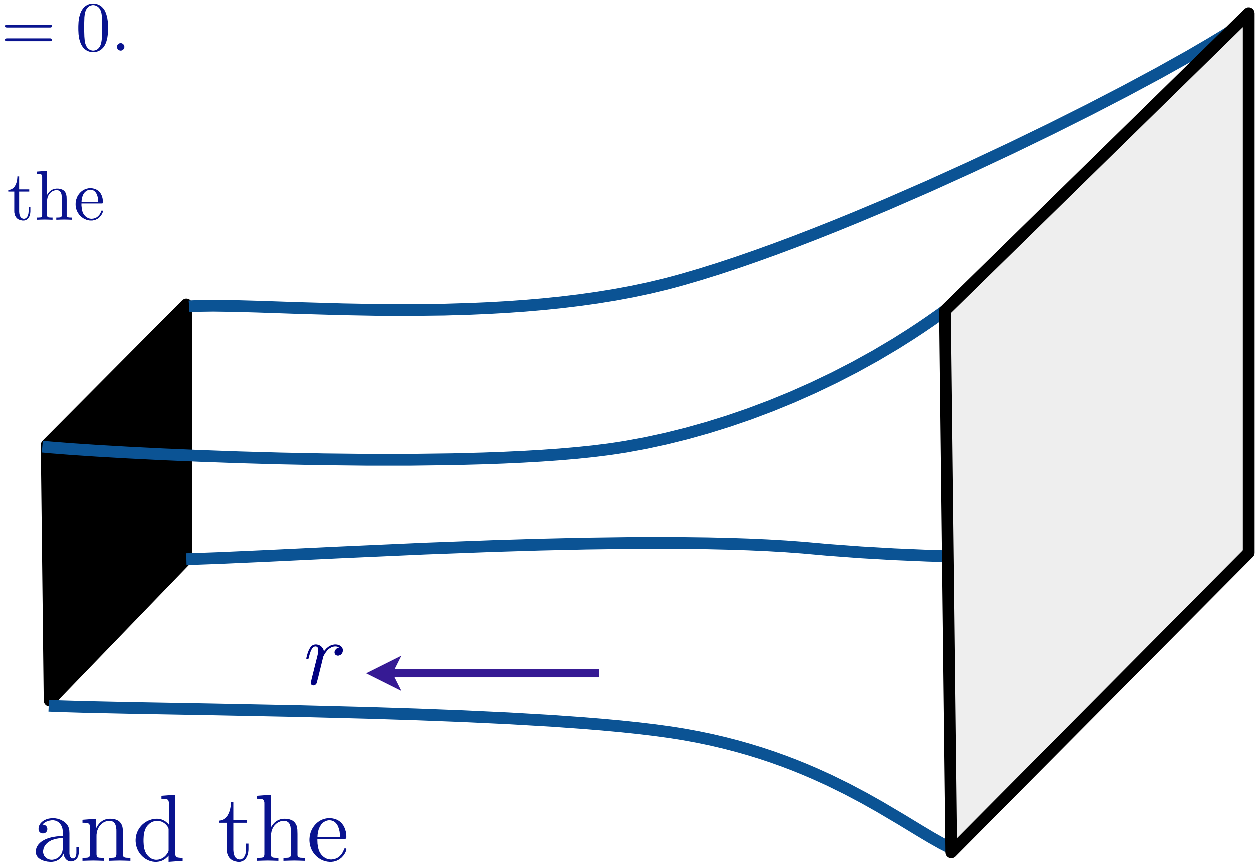

Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\mathrm{eff}}/z}$$

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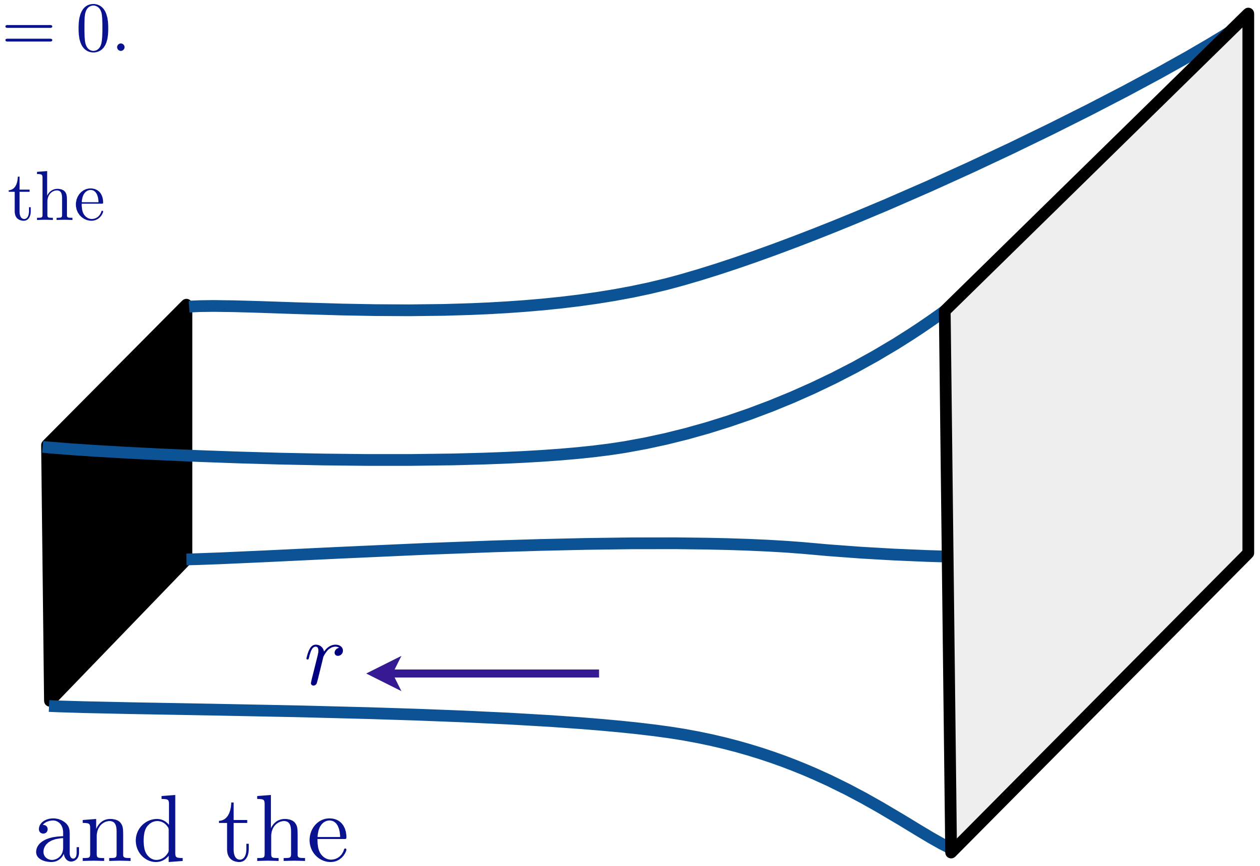

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For a strange metal should choose $\theta = d - 1$.

Holography of "strange metals"

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- The entanglement entropy exhibits logarithmic violation of the area law, expected for systems with Fermi surfaces, only for this value of θ !

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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- The entanglement entropy exhibits logarithmic violation of the area law, expected for systems with Fermi surfaces, only for this value of θ !
- The co-efficient of the logarithmic term is consistent with the Fermi surface size expected from $\mathcal{A} = \mathcal{Q}$.
- Many other features of the holographic theory are consistent with a boundary theory which has "hidden" Fermi surfaces of gauge-charged fermions.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.

Conclusions

Simplest examples of long-range entanglement are at quantum-critical points of insulating antiferromagnets

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

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String theory and gravity in
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Much recent progress offers hope of a
holographic description of "strange metals"