Supersolid matter, or How do bosons resolve their frustration?

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Colloquium UofT (October 2005)

Superfluid

Bose condensate, delocalized atoms (bosons), persistent flow, broken gauge symmetry, zero viscosity,…

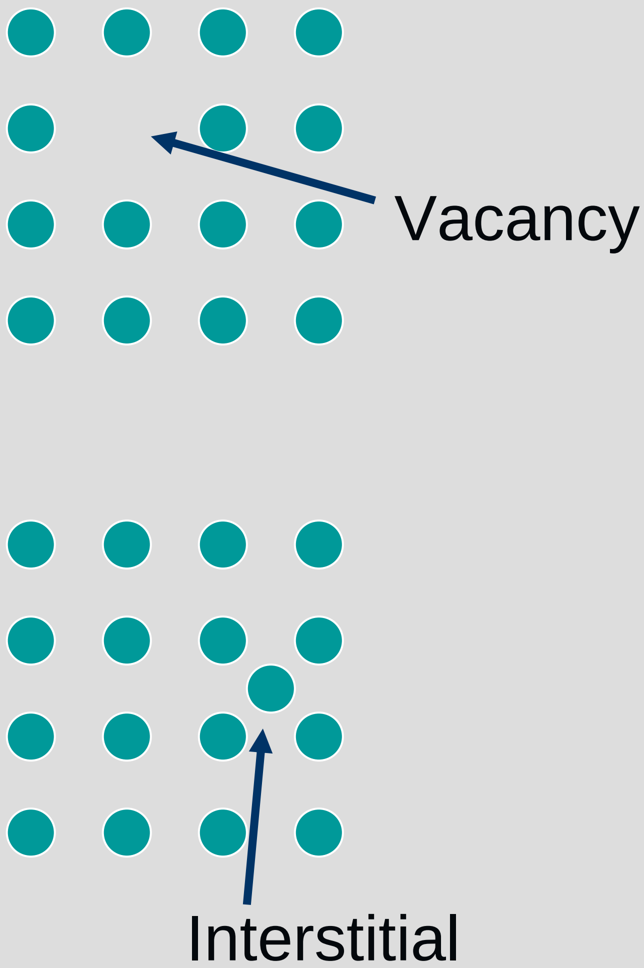
Crystal

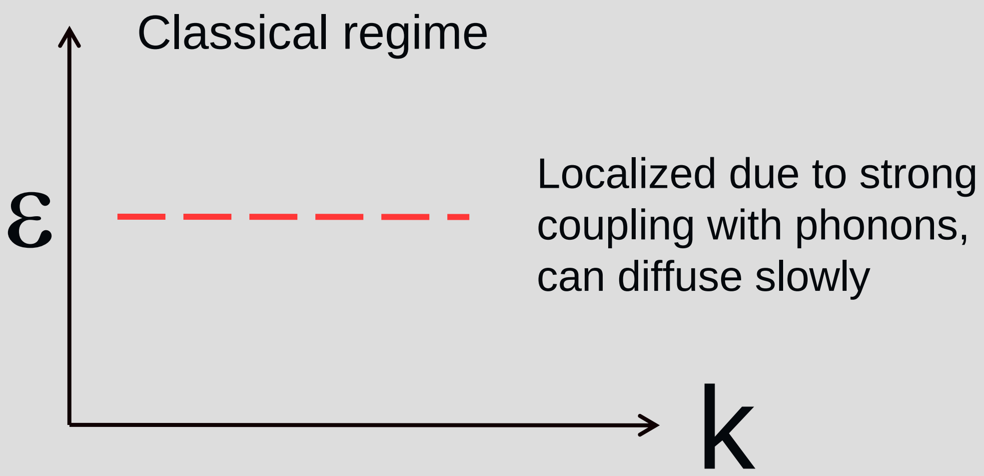
Density order, localized atoms (bosons), shear modulus, broken translational symmetry,…

Can we hope to realize both sets of properties in a quantum phase?

Bose condensation (superflow) *and* ***periodic arrangement of atoms (crystallinity)***

Crystals are not perfect: Quantum defects and a mechanism for supersolidity (Andreev & Lifshitz, 1969)





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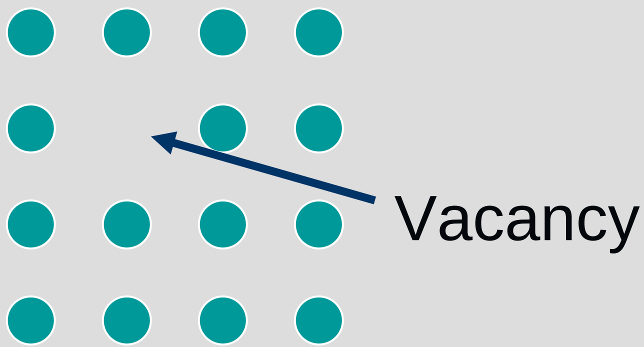
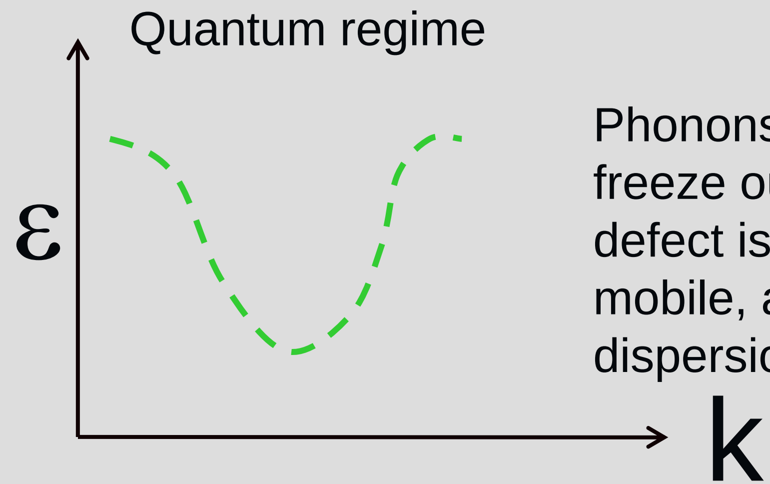
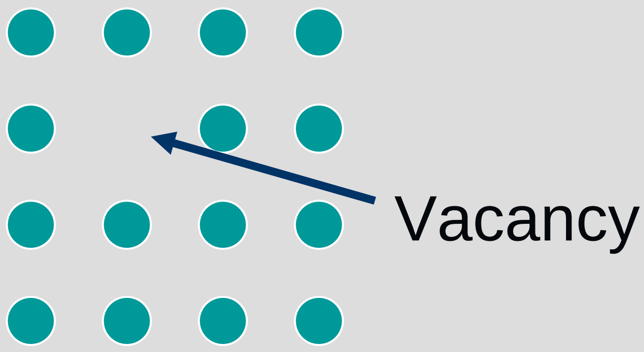
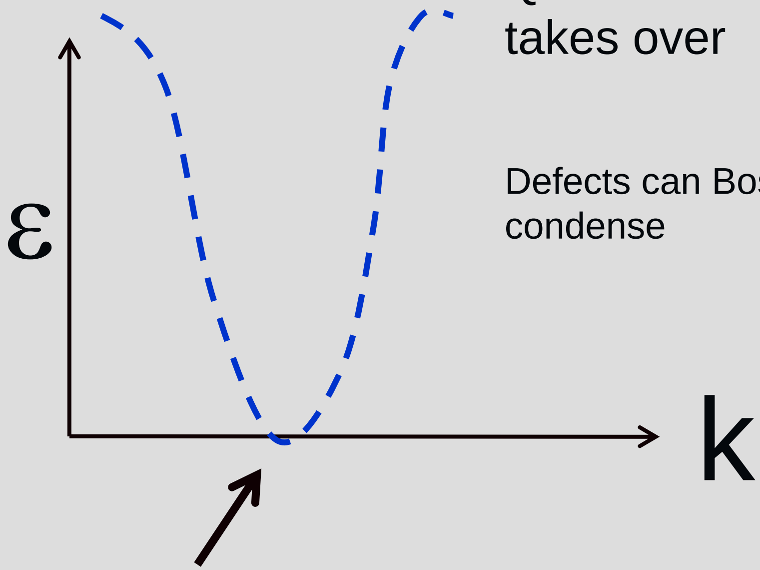




Phonons start to freeze out, and defect is more mobile, acquires dispersion

Crystals are not perfect: Quantum defects and a mechanism for supersolidity (Andreev & Lifshitz, 1969)



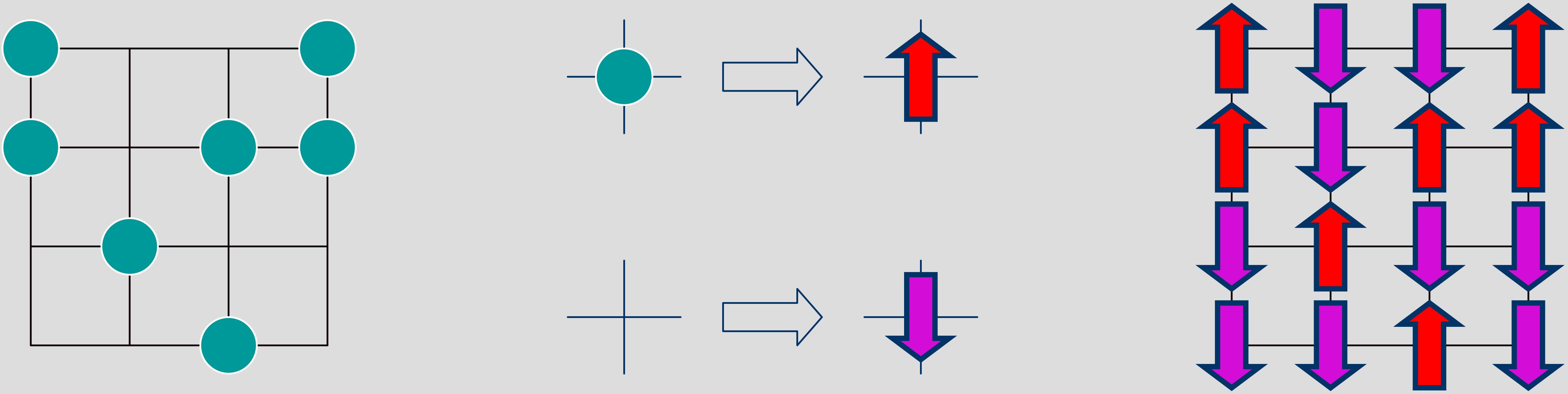


Perhaps condensation of a tiny density of quantum defects can give superfluidity while preserving crystalline order!

Andreev-Lifshitz (1969), Chester (1970)

Background crystal + Defect superflow = Supersolid

Lattice models of supersolids: Connection to quantum magnets

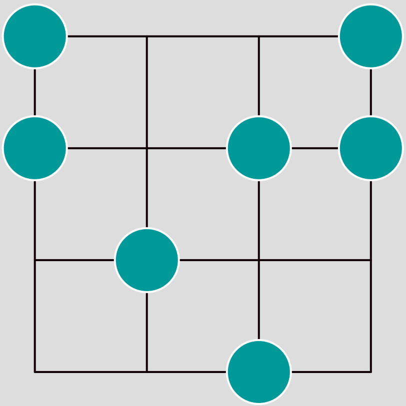


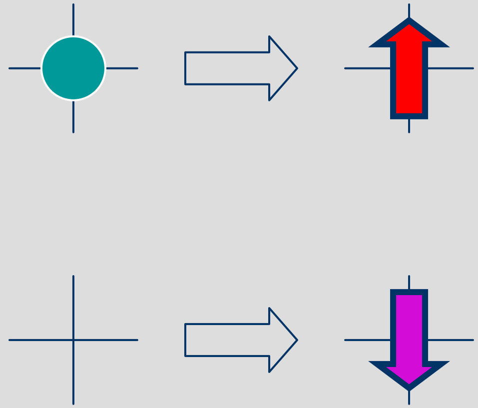
Classical Lattice Gas:

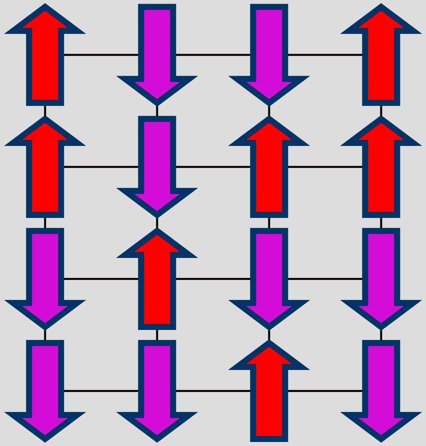
*1. Analogy between **classical fluids/crystals** and **magnetic systems***
*2. Keep track of **configurations** for thermodynamic properties*
*3. Define **"crystal"** as **breaking of lattice symmetries***
*4. Useful for understanding **liquid, gas, crystal phases** and **phase transitions***

Quantum Lattice Gas: *Extend to keep track of **quantum nature** and **quantum dynamics** (Matsubara & Matsuda, 1956)*

Lattice models of supersolids: Connection to quantum magnets

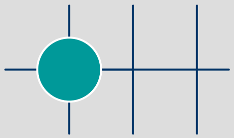


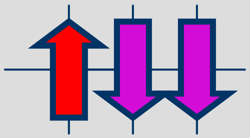




<u>Classical Lattice Gas:</u> *Useful analogy between classical statistical mechanics of fluids and magnetic systems, keep track of **configurations***

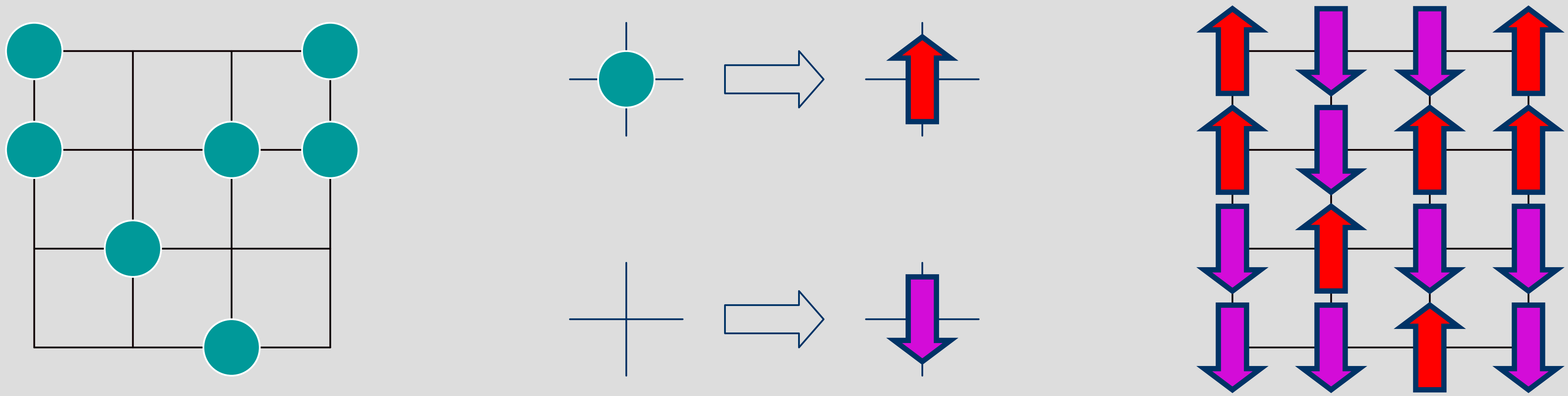
<u>Quantum Lattice Gas:</u> *Extend to keep track of **quantum** nature*





$$n(r) = S_Z(r)\ ;\ b^+(r) = S^+(r)$$

Lattice models of supersolids: Connection to quantum magnets



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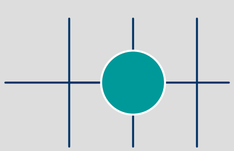
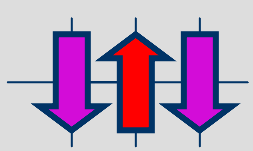


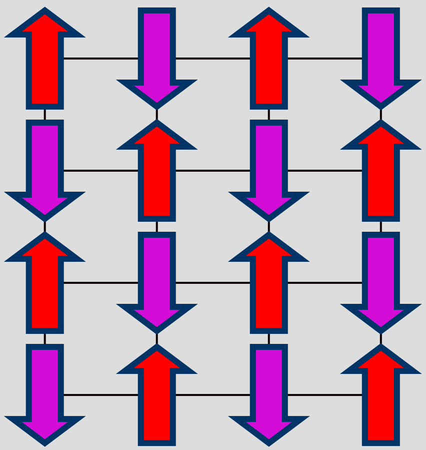


$$n(r) = S_z(r)\ ;\ b^+(r) = S^+(r)$$

Lattice models of supersolids: Connection to quantum magnets

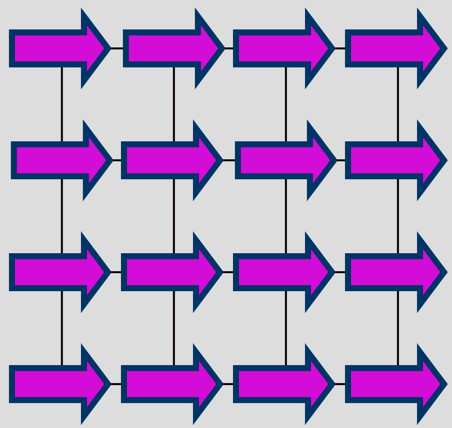
1. Borrow **calculational tools** from magnetism studies: *e.g.*, mean field theory, spin waves and semiclassics

2. **Visualize “nonclassical” states**: *e.g.*, superfluids and supersolids



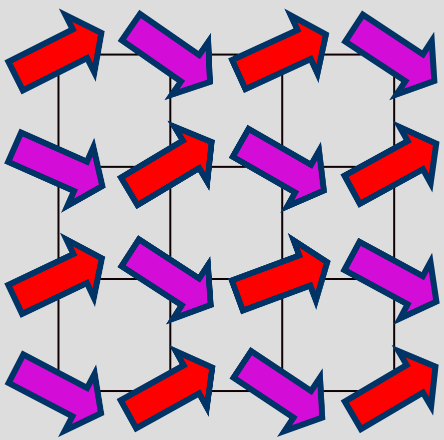
*Crystal: $\mathbf{S_z}$,**n** order*

Breaks lattice symmetries



*Superfluid : $\mathbf{S_x}$,<**b**> order*

Breaks spin rotation (phase rotation) symmetry



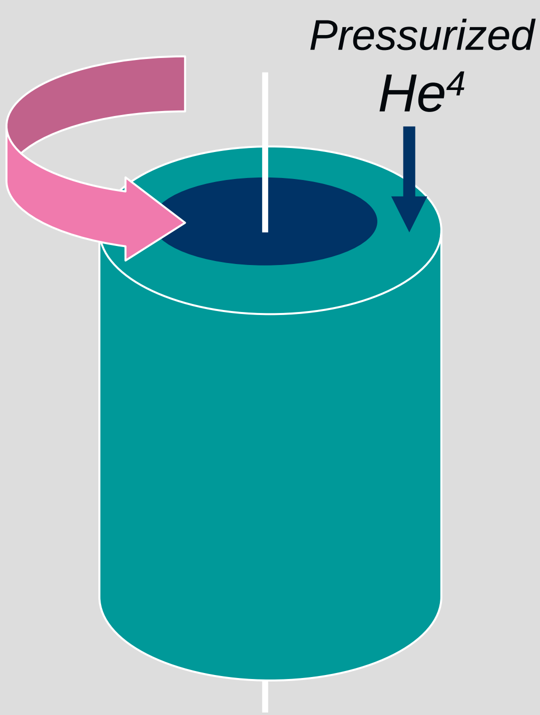
*Supersolid: **Both** order*

Breaks both symmetries

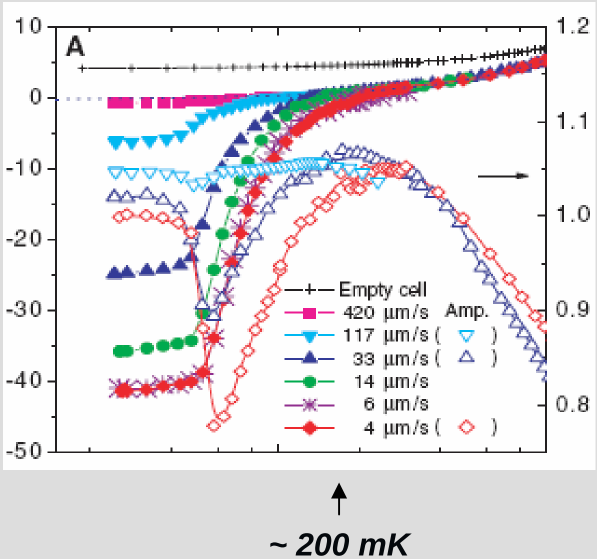
Lattice models of supersolids *: Matsubara & Matsuda (1956), Liu & Fisher (1973)*

Why are we interested **now?**

Superfluidity in He^4 in high pressure crystalline phase?



Supersolid should show **nonclassical rotational inertia** *due to superfluid component remaining at rest (Leggett, 1970)*

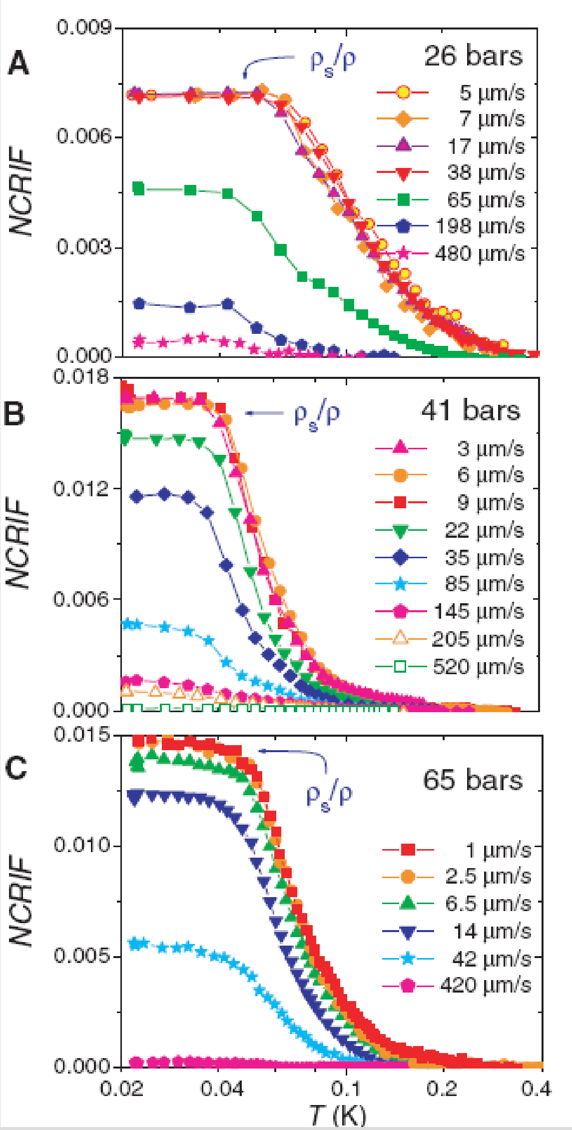


Reduced moment of inertia

*E. Kim and M.Chan (**Science**, 2004)*

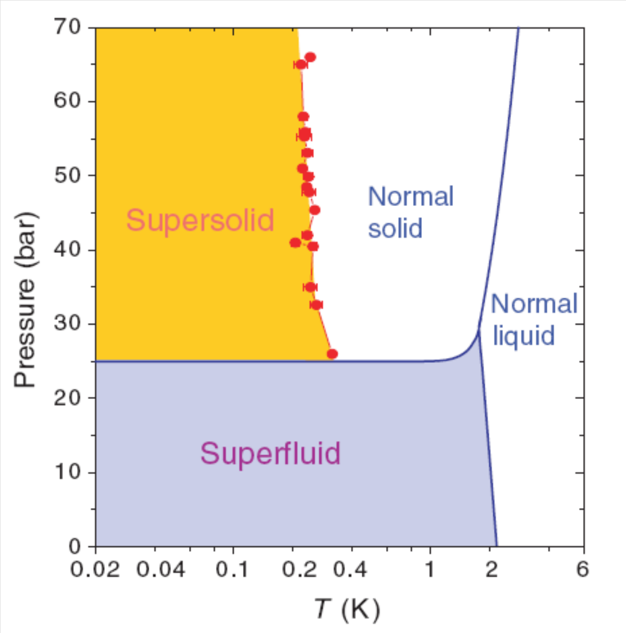
Earlier work (J.M. Goodkind & coworkers, 1992-2002) gave very indirect evidence of delocalized quantum defects in very pure solid He^4

Superfluidity in He^4 in high pressure crystalline phase?



Reduced moment of inertia = Supersolid?

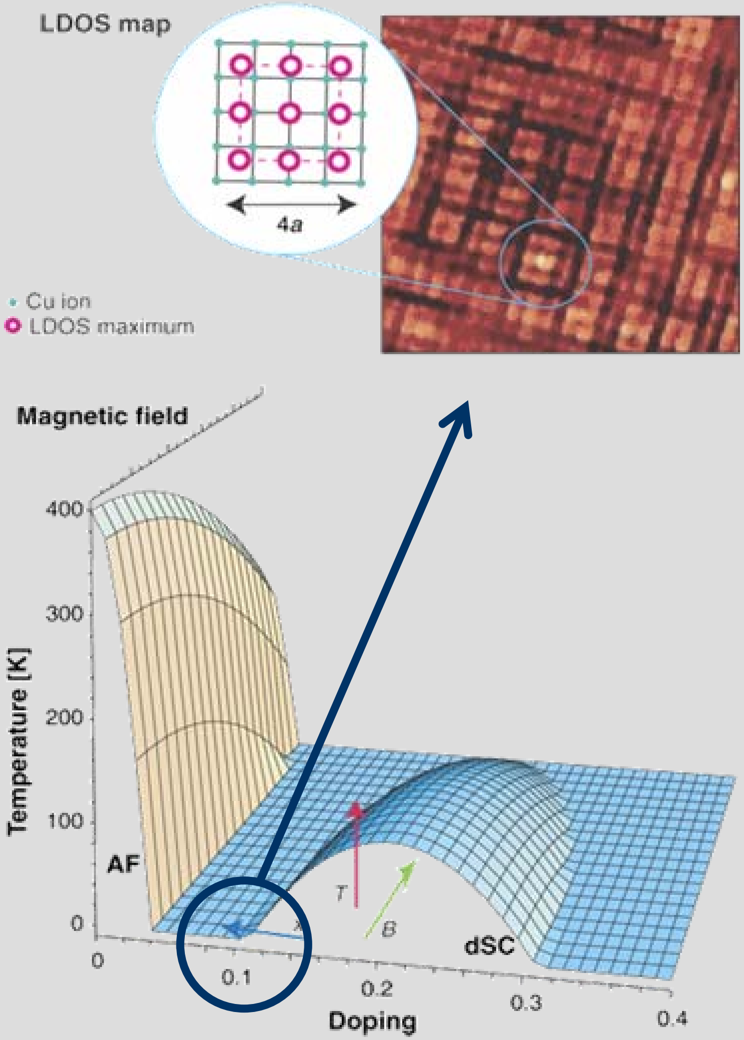
*E. Kim and M.Chan (**Science**, 2004)*



Bulk physics or not?

Microcrystallites? N.Prokofiev & coworkers (2005)

STM images of $Ca_{(2-x)}Na_xCuO_2Cl_2$



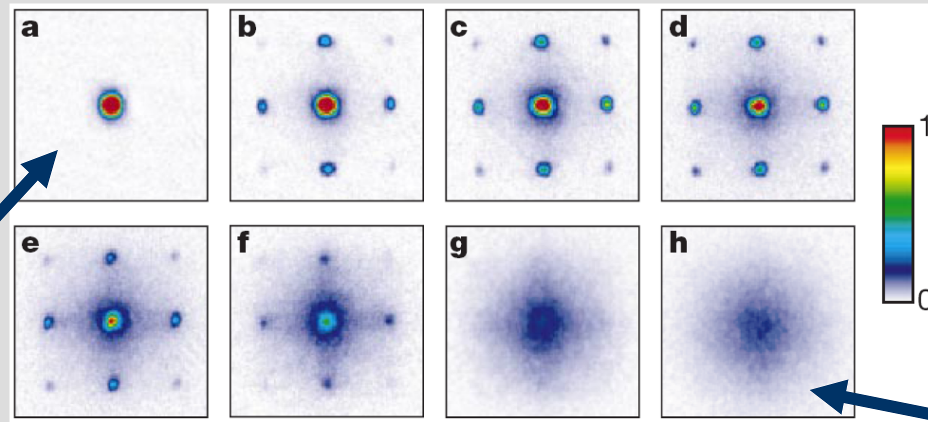
Nondispersive pattern over 10-100 meV range

*Evidence for a **$4a_0$ x $4a_0$ unit-cell solid** from tunneling spectroscopy **in underdoped superconducting samples (Tc=15K, 20K)***

*T. Hanaguri, et al (**Nature**, 2004)*

*M. Franz (**Nature N&V**, 2004)*

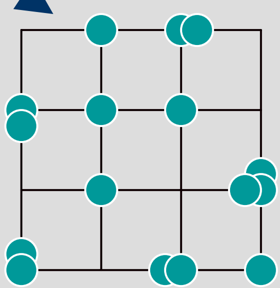
Engineering quantum Hamiltonians: Cold atoms in optical lattices

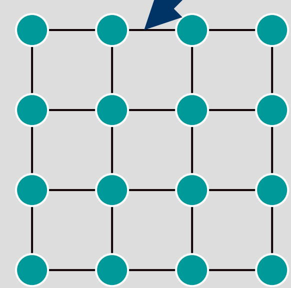


Coherent Superfluid

"Incoherent" Mott insulator

Decreasing kinetic energy





*M.Greiner, et al (**Nature** 2002)*

*Can one realize and study **new** quantum phases?*

Revisit lattice models for supersolids

1. ***Is the Andreev-Lifshitz mechanism realized in lattice models of bosons?***

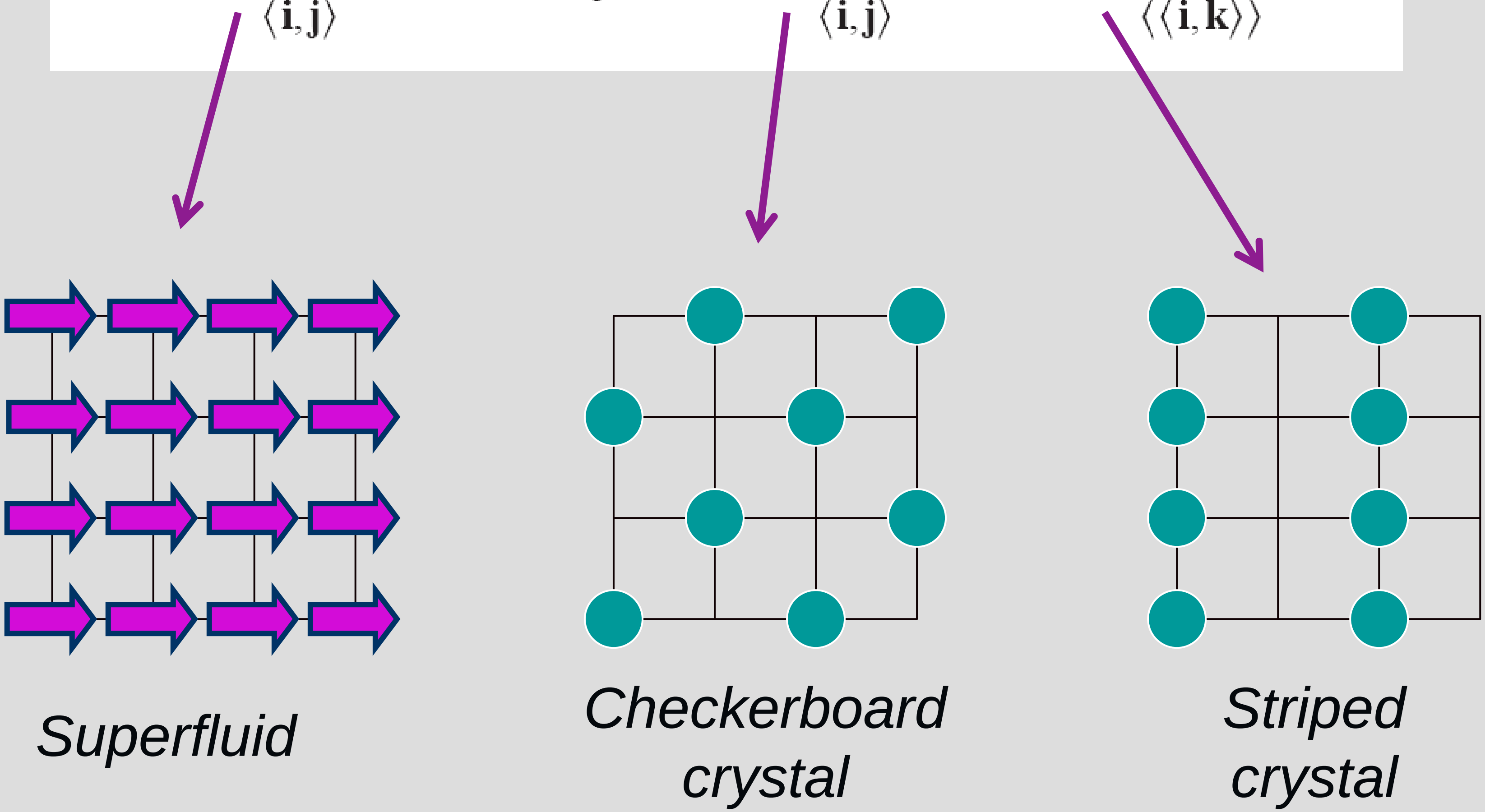
2. ***Are there other routes to supersolid formation?***

3. ***Is it useful to try and approach from the superfluid rather than from the crystal?***

4. ***Can we concoct very simple models using which the cold atom experiments can realize a supersolid phase?***

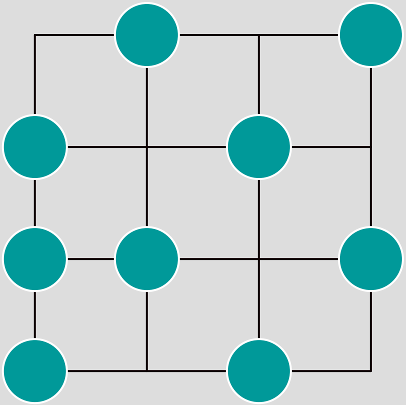
Bosons on the Square Lattice: Superfluid and Crystals

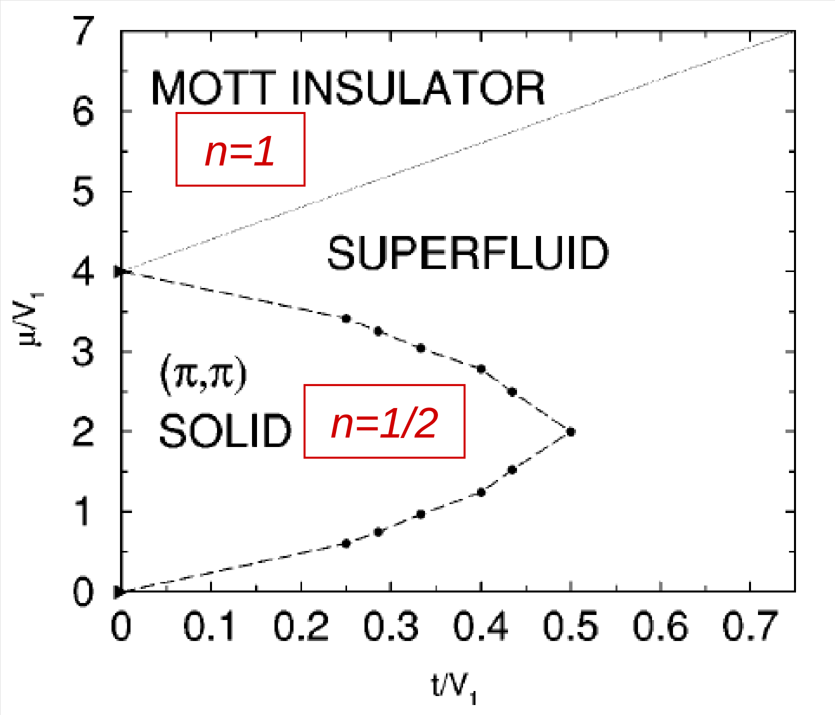
$$H = -t \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \left(a_{\mathbf{i}}^{\dagger} a_{\mathbf{j}} + a_{\mathbf{j}}^{\dagger} a_{\mathbf{i}} \right) + V_1 \sum_{\langle \mathbf{i},\mathbf{j} \rangle} n_{\mathbf{i}} n_{\mathbf{j}} + V_2 \sum_{\langle\langle \mathbf{i},\mathbf{k} \rangle\rangle} n_{\mathbf{i}} n_{\mathbf{k}}$$



Bosons on the Square Lattice: Is there a supersolid?

$$H = -t\sum_{\langle \mathbf{i},\mathbf{j}\rangle}(a_{\mathbf{i}}^{\dagger}a_{\mathbf{j}} + a_{\mathbf{j}}^{\dagger}a_{\mathbf{i}}) + V_1\sum_{\langle \mathbf{i},\mathbf{j}\rangle} n_{\mathbf{i}}n_{\mathbf{j}} + V_2\sum_{\langle\langle \mathbf{i},\mathbf{k}\rangle\rangle} n_{\mathbf{i}}n_{\mathbf{k}}$$





F. Hebert, et al (PRB 2002)

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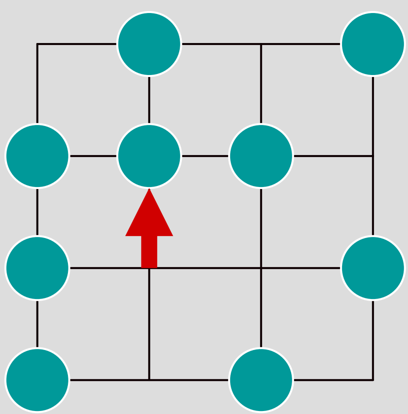

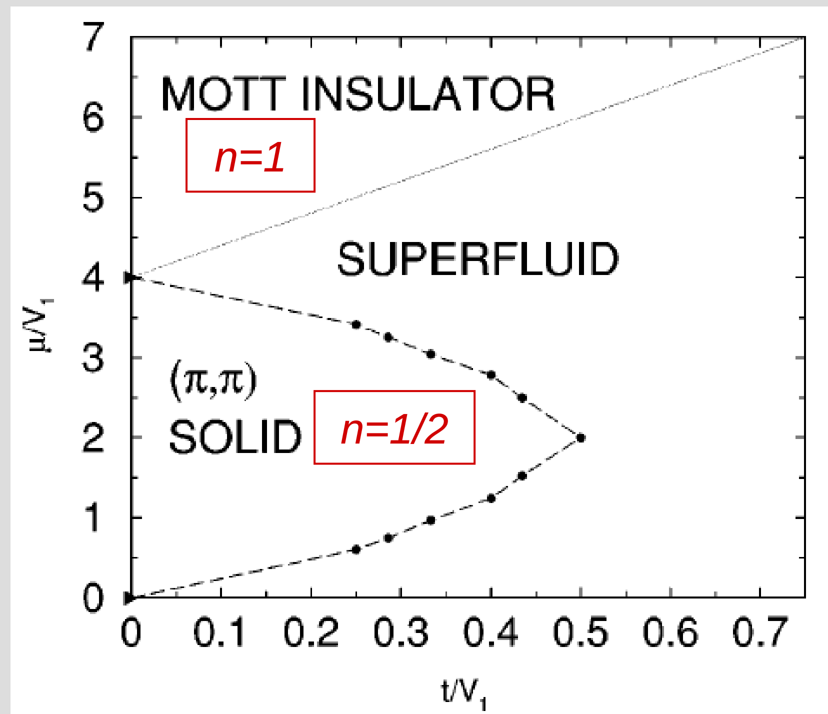

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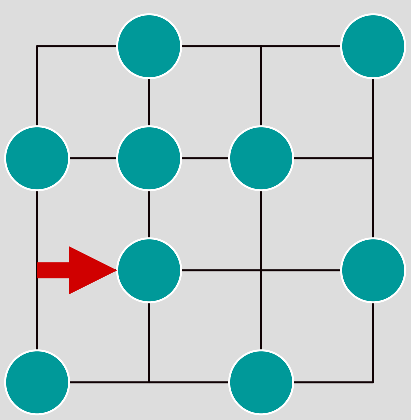

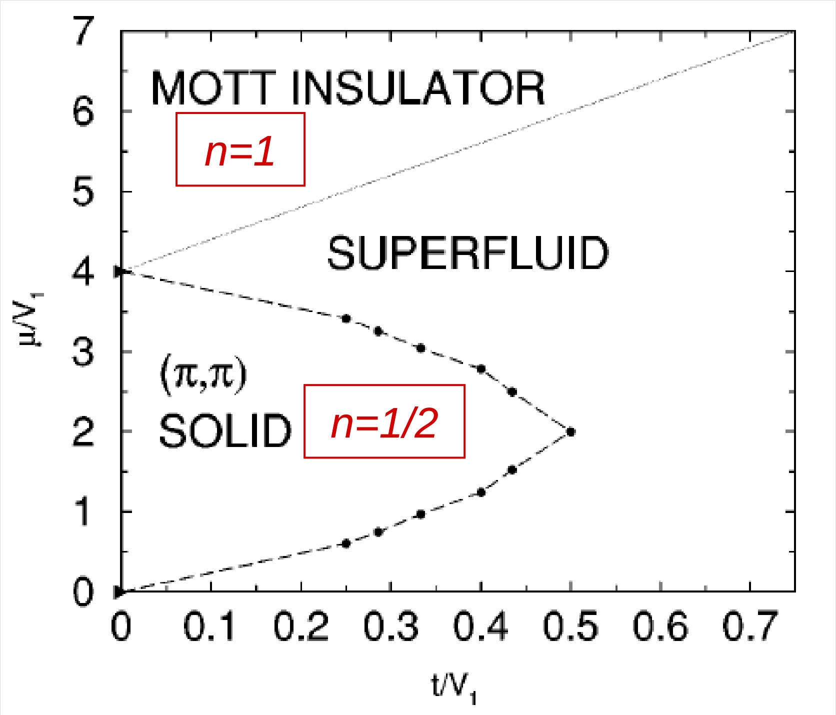

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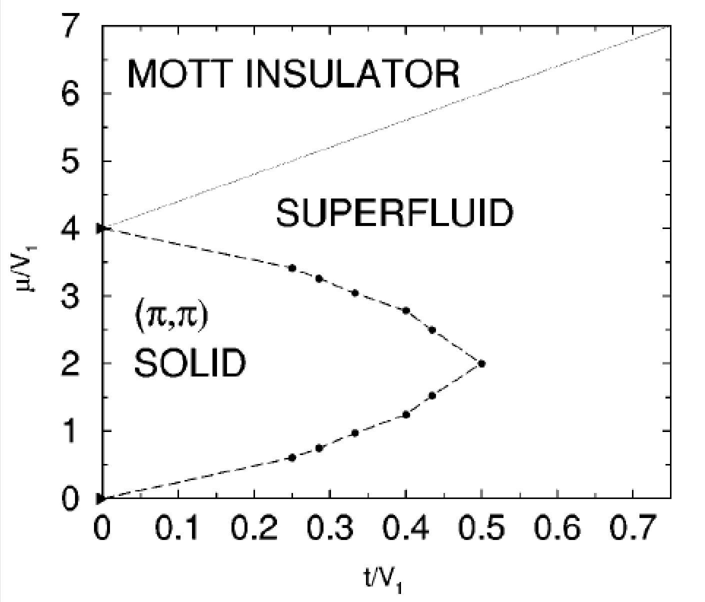
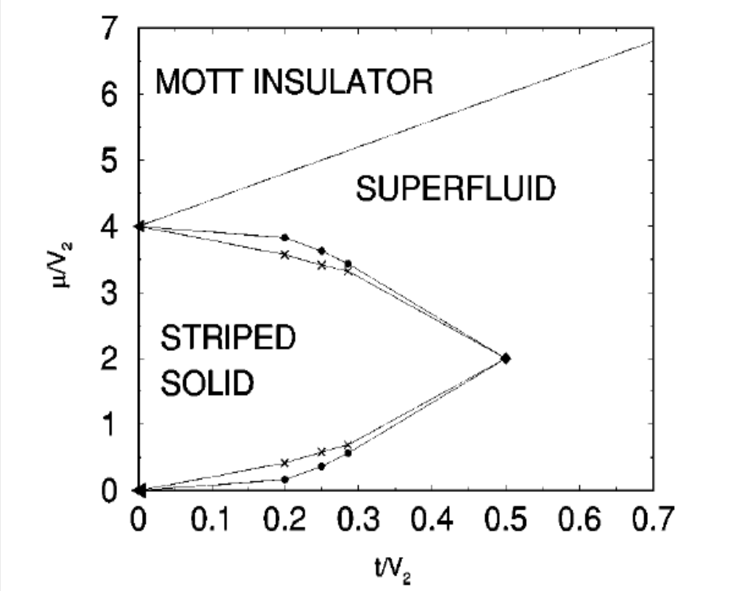


Andreev-Lifshitz supersolid could possibly exist with t'



Andreev-Lifshitz supersolid

Bosons on the Triangular Lattice

Superfluid, Crystal and Frustrated Solid

Boson model

$$H = -t\sum_{\langle ij\rangle}(b_i^\dagger b_j + b_j^\dagger b_i) + \sum_{\langle ij\rangle} V\left(n_i - \frac{1}{2}\right)\left(n_j - \frac{1}{2}\right)$$

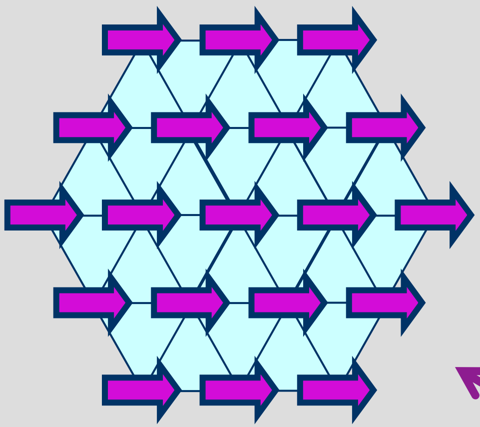
Quantum spin model

$$H = \sum_{\langle ij\rangle}[-J_\perp(S_i^x S_j^x + S_i^y S_j^y) + J_z S_i^z S_j^z]$$

Bosons on the Triangular Lattice

Superfluid

$$H = -t\sum_{\langle ij\rangle}(b_i^\dagger b_j + b_j^\dagger b_i) + \sum_{\langle ij\rangle} V\left(n_i - \frac{1}{2}\right)\left(n_j - \frac{1}{2}\right)$$



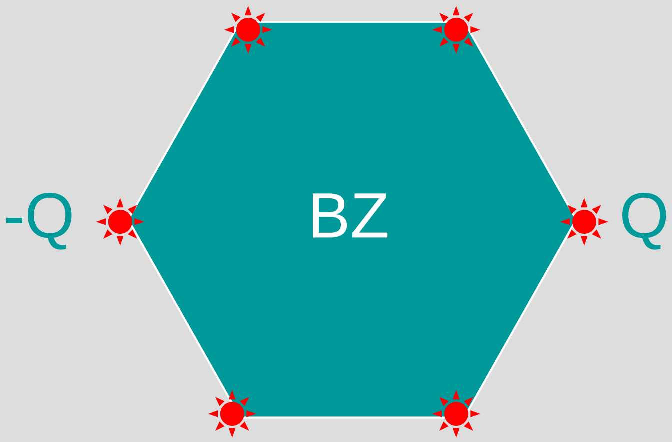
Superfluid

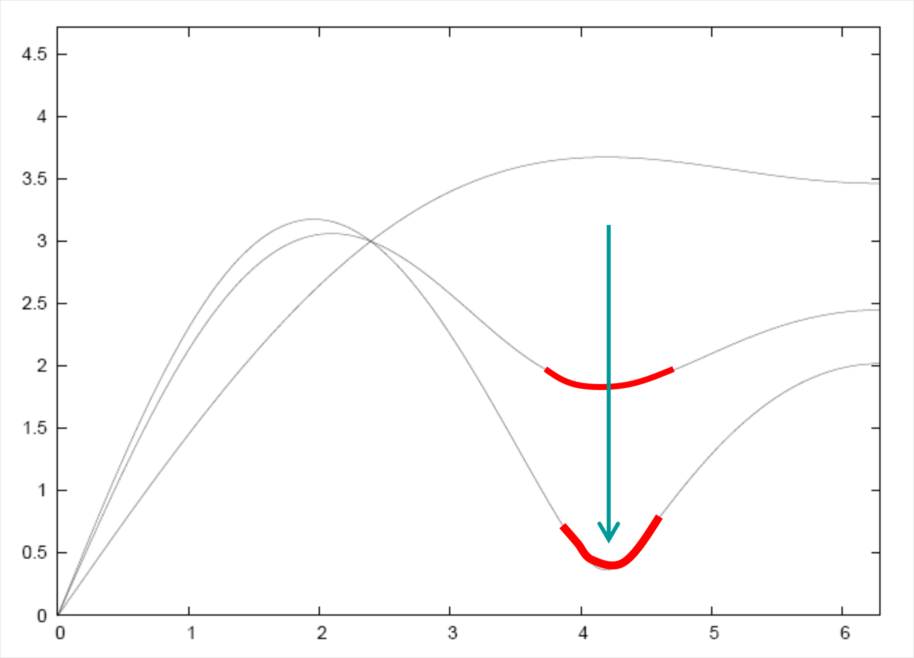
$$H = \sum_{\langle ij\rangle}\left[-J_\perp(S_i^x S_j^x + S_i^y S_j^y) + J_z S_i^z S_j^z\right]$$

Bosons on the Triangular Lattice

Spin wave theory in the superfluid & an instability at half-filling

How do interactions affect the excitation spectrum in the superfluid?



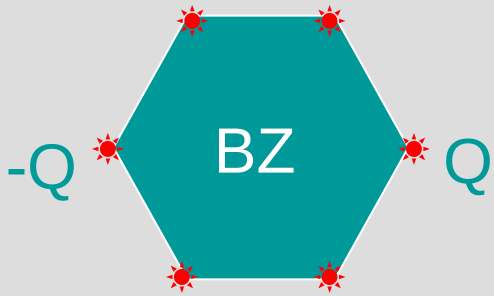


Roton minimum hits zero energy, signalling instability of superfluid

G. Murthy, et al (1997)
R. Melko, et al (2005)

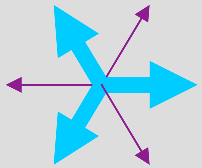
Bosons on the Triangular Lattice

Landau theory of the transition & what lies beyond



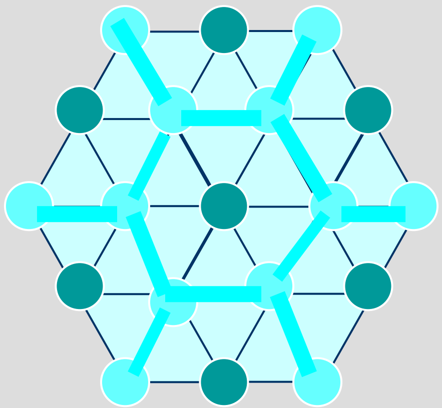
- *Focus on low energy modes: +Q,-Q,0*
- *Construct Landau theory*

$$S = \int d^2x \int_0^\beta d\tau [|\partial_\tau \psi|^2 + c^2 |\nabla \psi|^2 + r|\psi|^2 + u|\psi|^4 + v|\psi|^6 + w\mathrm{Re}(\psi^6) + M^2/(2\chi) - \lambda M \mathrm{Re}(\psi^3)]$$



w < 0

[2m’,-m,-m]

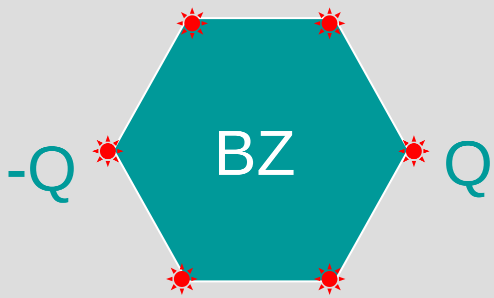


Supersolid #1

R. Melko, et al (2005)

Bosons on the Triangular Lattice

Landau theory of the transition & what lies beyond



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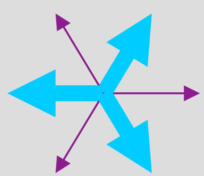
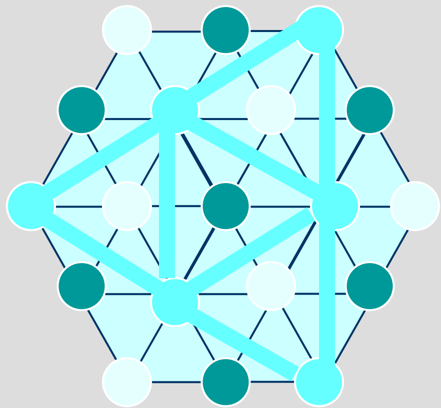


w > 0

[m,0,-m]



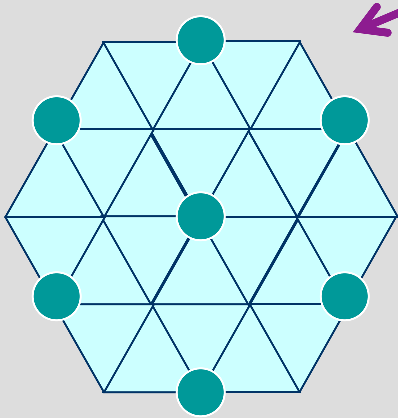
Supersolid #2

R. Melko, et al (2005)

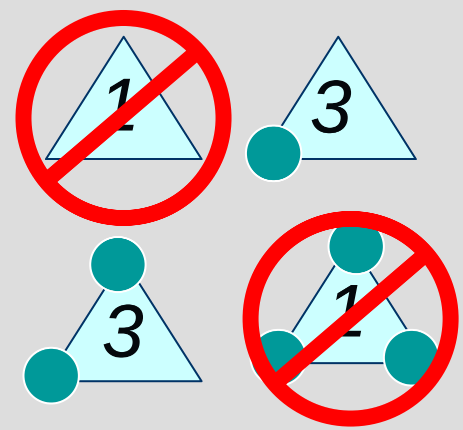
Bosons on the Triangular Lattice

Crystal and Frustrated Solid

$$H = -t\sum_{\langle i,j\rangle}(a_i^\dagger a_j + a_j^\dagger a_i) + V\sum_{\langle i,j\rangle} n_i n_j - \mu \sum_i n_i$$



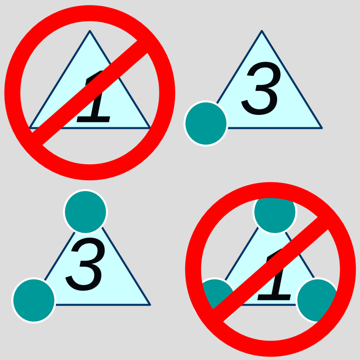
Crystal at n=1/3



Frustrated at n=1/2

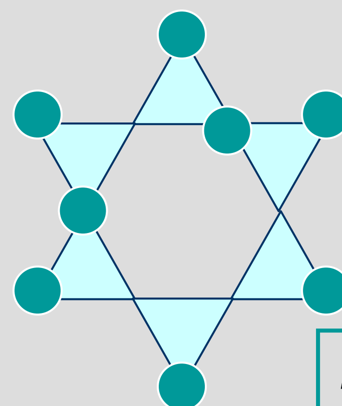
Quantifying “frustration”

Triangular Ising Antiferromagnet



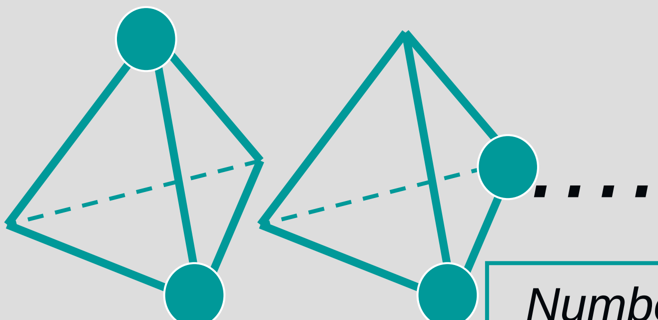
Number of Ising ground states ~ exp(0.332 N)

Kagome Ising Antiferromagnet



Number of Ising ground states ~ exp(0.502 N)

Pyrochlore “spin-ice”



Number of “spin ice” ground states ~ exp(0.203 N)

“Order-by-disorder”: Ordering by fluctuations

- *Even if the set of classical ground states does not each possess order, thermal states may possess order due to entropic lowering of free energy (states with maximum accessible nearby configurations)*

$$F = E - T\,S$$

- *Quantum fluctuations can split the classical degeneracy and select ordered ground states*

Many contributors (partial list)

- *J. Villain and coworkers (1980)*
- *E.F. Shender (1982)*
- *P. Chandra, P. Coleman, A.I.Larkin (1989): Discrete Z(4) transition in a Heisenberg model*
- *A.B.Harris,A.J.Berlinsky,C.Bruder (1991), C.Henley, O.Tchernyshyov: Pyrochlore AFM*
- *R. Moessner, S. Sondhi, P. Chandra (2001): Transverse field Ising models*

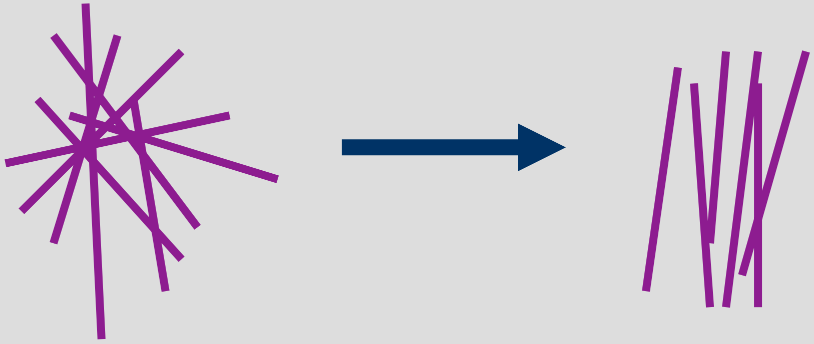
"Order-by-disorder": Ordering by fluctuations

- *Even if the set of classical ground states does not each possess order, thermal states may possess order due to entropic lowering of free energy (states with maximum accessible nearby configurations)*

$$F = E - T\,S$$

- *Quantum fluctuations can split the classical degeneracy and select ordered ground states*

- *L. Onsager (1949): Isotropic to nematic transition in hard-rod molecules*



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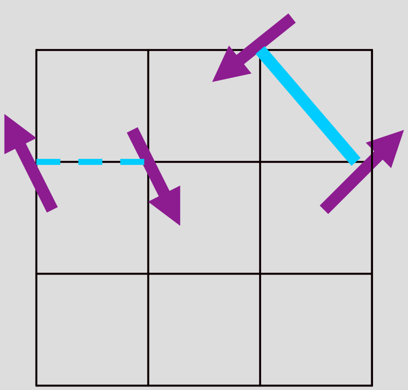

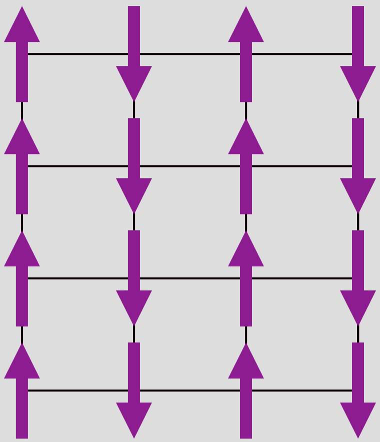

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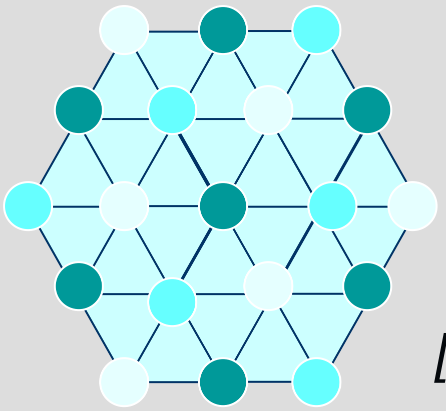
- ***R. Moessner, S. Sondhi, P. Chandra (2001): Triangular Ising antiferromagnet in a transverse field – related to quantum dimer model on the honeycomb lattice***

$$H = J_z \sum_{\langle ij \rangle} S_i^z S_j^z - h_{\text{eff}} \sum_i S_i^x$$



[m,0,-m]

Supersolid order from disorder

$$H = \sum_{\langle ij \rangle} [-J_{\perp}(S_i^x S_j^x + S_i^y S_j^y) + J_z S_i^z S_j^z]$$

Quantum fluctuations (exchange term, $J_{\perp}$) can split the classical degeneracy and select an ordered ground state

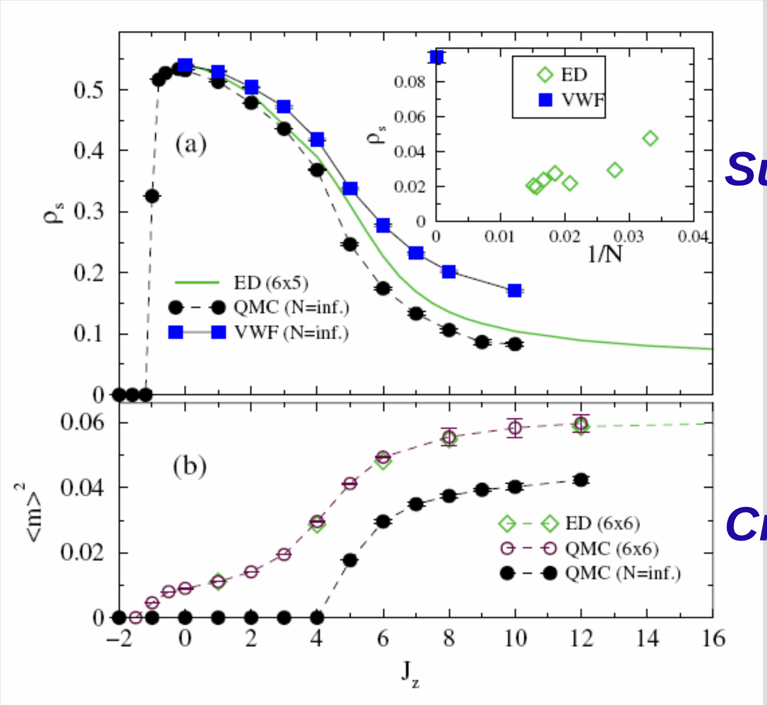
Variational arguments show that superfluidity persists to infinite J_z, hence "map" on to the transverse field Ising model (in a mean field approximation)

$$h_{\text{eff}} = J_{\perp} \langle S_i^x \rangle$$

Superfluid + Broken lattice symmetries = ***Supersolid***

Bosons on the Triangular Lattice

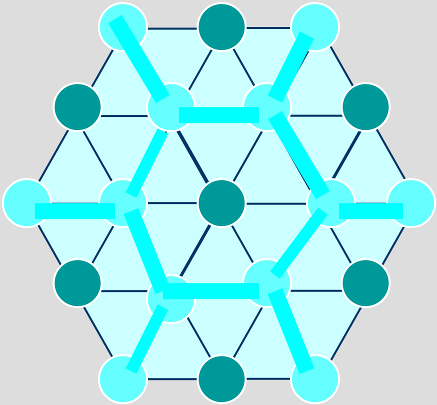
Phase Diagram



Superfluid order

\+

Crystal order



R. Melko et al (2005)

D. Heidarian, K. Damle (2005)

Bosons on the Triangular Lattice

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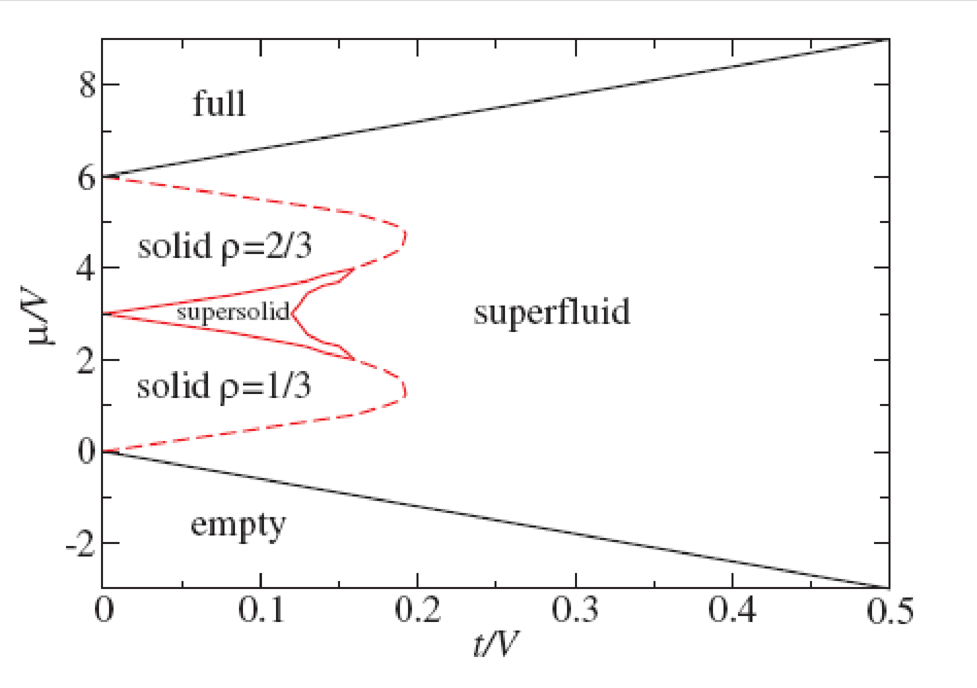

S. Wessel, M. Troyer (2005)

M. Boninsegni, N. Prokofiev (2005)

Summary

- *Is the Andreev-Lifshitz mechanism realized in lattice models of bosons?*

  Yes, in square lattice boson models

- *Are there other routes to supersolid formation?*

  Order-by-disorder in certain classically frustrated systems

  Continuous superfluid-supersolid transition from roton condensation

- *Can we concoct very simple models using which the cold atom experiments can realize a supersolid phase?*

  Possible to realize triangular lattice model with dipolar bosons in optical lattices

Open issues

- *What is the low temperature and high pressure crystal structure of solid He^4?*
- *How does a supersolid flow?*
  How do pressure differences induce flow in a supersolid? (J. Beamish, Oct 31)
- *Extension to 3D boson models? Is frustration useful in obtaining a 3D supersolid?*
- *Excitations in supersolid? Structure of vortices?*
- *Implications for theories of the high temperature superconductors?*