PHY385 Optics, Fall 2010.

Term Test 1 Solutions
Jason Harlow.

1. $\vec{E}_o = 2\left(\frac{V}{m}\right)\hat{j}$ $\qquad \lambda = 3\,m$

$\tau = 10^{-8}\,s$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3}$ in S.I. units.

$\omega = \frac{2\pi}{\tau} = \frac{2\pi}{10^{-8}} = 2\pi \times 10^8$ in S.I. units

$\vec{E} = \vec{E}_o \cos\left(kx + \omega t + \phi_o\right)$ for a wave traveling in the $+x$ direction.

$\vec{E} = 0$ when $x = t = 0 \Rightarrow \phi_o = \pm \pi/2$.

when t is slightly positive $\vec{E}$ is in $\hat{j}$ direction $\Rightarrow \left[\frac{d}{dt}\cos(kx + \omega t + \phi_o)\right]_{x=0,\ t=0} > 0$

$\Rightarrow -\sin\phi_o > 0$
$\sin\phi_o < 0 \Rightarrow \phi_o = -\pi/2$.

a) $$\boxed{\vec{E} = 2\left(\frac{V}{m}\right)\hat{j}\cos\left(\frac{2\pi}{3}x + (2\pi\times 10^8)t - \frac{\pi}{2}\right)}$$

b) In complex exponential form:
$$\boxed{\vec{E} = 2\left(\frac{V}{m}\right)\hat{j}\, e^{i\left[\frac{2\pi}{3}x + (2\pi\times10^8)t - \pi/2\right]}}$$

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2. Use the dispersion equation:

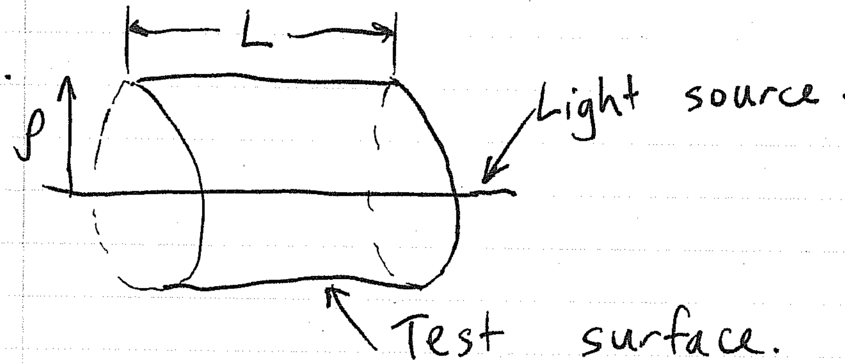
$$n^2 = 1 + \frac{Nq_e^2}{\epsilon_0 m_e}\left(\frac{1}{\omega_0^2 - \omega^2}\right)$$

$$\omega_0 = 2\pi f_0 = 2\pi(2.9\times10^{15}) = 1.822\times10^{16}\ \frac{\text{rad}}{\text{s}}$$

$$\omega = 2\pi f = 2\pi(4.1\times10^{14}) = 2.576\times10^{15}\ \frac{\text{rad}}{\text{s}}$$

$$n = \left[1 + \frac{1.4\times10^{29}(1.6\times10^{-19})^2}{8.85\times10^{-12}(9.11\times10^{-31})}\left(\frac{1}{(1.822\times10^{16})^2-(2.576\times10^{15})^2}\right)\right]$$

$n = 1.54$

3. 

Consider a test surface of radius ρ and length L, centred on light source.

→ By conservation of energy, the total power passing through this surface should not depend on ρ.

$$\text{Power} = I\cdot A \quad \text{, where } I = \text{irradiance}$$

$$\text{Power} = I(2\pi\rho L) = \text{constant}. \Rightarrow I \propto \frac{1}{\rho}$$

Eq. 3.44: $I = \frac{c\epsilon_0}{2}E_0^2 \Rightarrow E_0 \propto \sqrt{I} \propto \frac{1}{\sqrt{\rho}} = [\rho]^{-1/2}$

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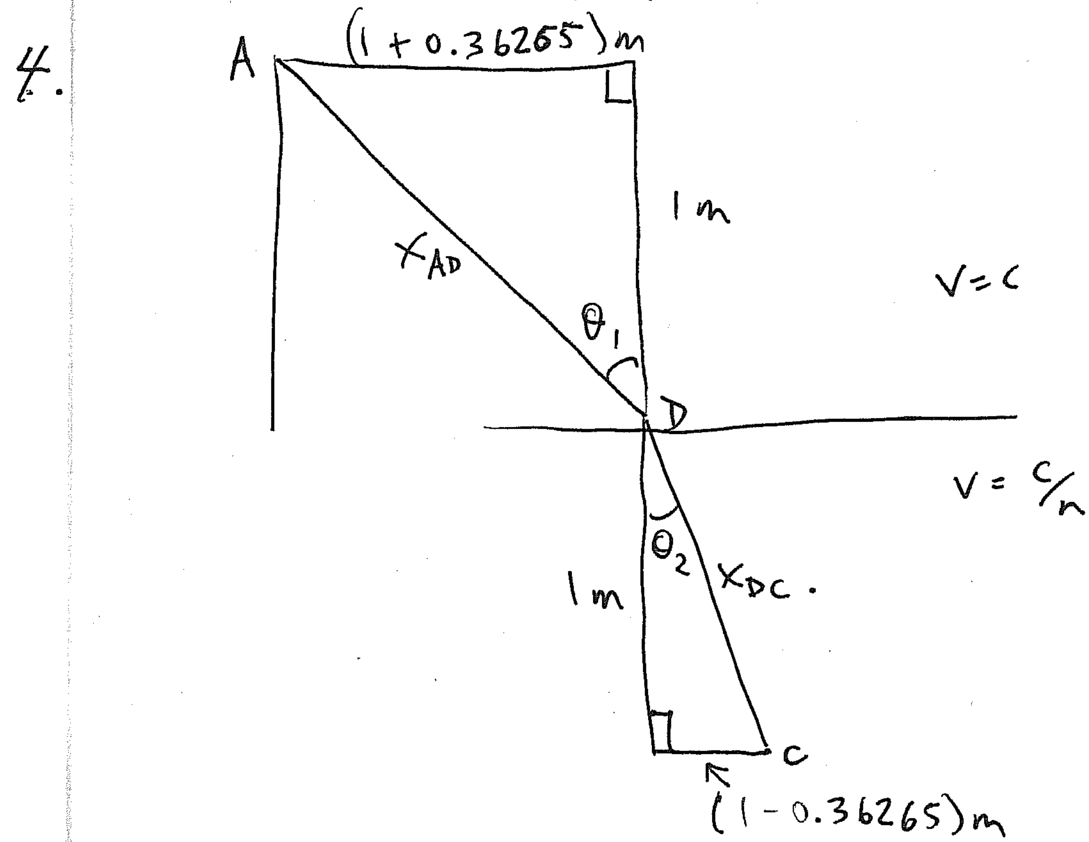
4.



a) $$t = \frac{x_{AD}}{c} + \frac{1.5\, x_{DC}}{c}$$

$$= \frac{1}{3\times 10^8}\left[\sqrt{1^2 + (1+0.36265)^2} + 1.5\sqrt{1^2+(1-0.36265)^2}\right]$$

$$t = 11.563 \text{ ns}$$

b). Snell's Law $\Leftrightarrow$ Path of least time.

$$\theta_1 = \tan^{-1}\left(\frac{1+0.36265}{1}\right) = 53.73^\circ$$

$$\theta_2 = \tan^{-1}\left(\frac{1-0.36265}{1}\right) = 32.51^\circ$$

Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin 53.73^\circ = 1.5 \sin 32.51^\circ$$

$$0.806 = 0.806$$ ✓

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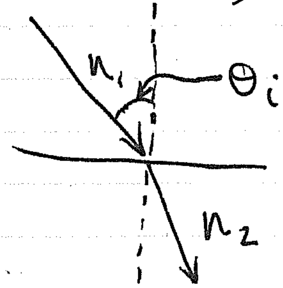
5. a) Polarizing angle



$$\theta_i = \theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.458}{1}\right)$$

$$\theta_p = 55.55^\circ$$

b) Incident Power $= P_i = P_{i\perp} + P_{i\parallel}$

Unpolarized $\Rightarrow P_{i\perp} = P_{i\parallel} = P_i/2$

Reflected Power

TE Mode: $\frac{P_{r\perp}}{P_{i\perp}} = R_\perp = r_\perp^2$

TM Mode: $\frac{P_{r\parallel}}{P_{i\parallel}} = R_\parallel = r_\parallel^2$

Ratio $\frac{P_{TE}}{P_{TM}} = \frac{P_{r\perp}}{P_{r\parallel}} = \frac{R_\perp}{R_\parallel} = \left(\frac{r_\perp^2}{r_\parallel^2}\right)$

Fresnel's eq's:

4.34: $$r_\perp = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}$$

where $\theta_i = 45^\circ$, $n_1 = 1$, $n_2 = 1.458$,

and θ_t is given by Snell's Law:

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\theta_t = \sin^{-1}\left[\frac{1}{1.458}\sin 45^\circ\right]$$

$$\theta_t = 29.011^\circ$$

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5 b) continued:

$$r_\perp = \frac{\cos 45^\circ - 1.458 \cos 29.011^\circ}{\cos 45^\circ + 1.458 \cos 29.011^\circ}$$

$$r_\perp = -0.28653$$

$$\Rightarrow R_\perp = r_\perp^2 = 8.210\%$$ power reflected.

TM-mode:
Eq. 4.40

$$r_\parallel = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

$$r_\parallel = \frac{1.458 \cos 45^\circ - \cos 29.011^\circ}{\cos 29.011^\circ + 1.458 \cos 45^\circ}$$

$$r_\parallel = 0.082097$$

$$\Rightarrow R_\parallel = r_\parallel^2 = 0.670\%$$ power reflected.

Ratio $\frac{P_{TE}}{P_{TM}} = \frac{R_\perp}{R_\parallel} = \frac{8.210\%}{0.670\%} = \boxed{12.2}$