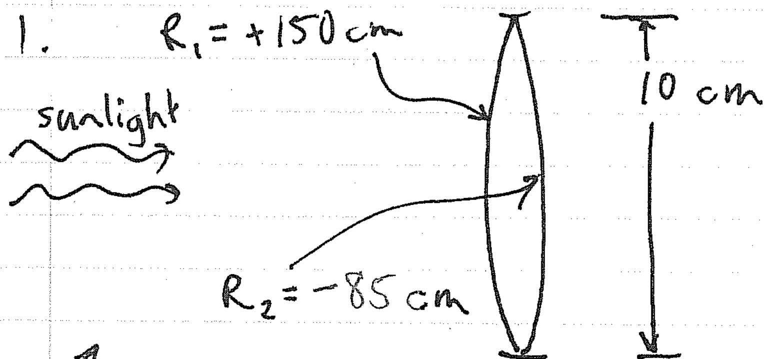
PHY 385
Optics.
Fall 2010

Term Test 2 Solutions.
Jason Harlow.

1.



$n = 1.52$

Using sign conventions of Fig. 5.12 on pg. 156.

(a) Eq. 5.15: $$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$f = \left[0.52\left(\frac{1}{150} + \frac{1}{85}\right)\right]^{-1}$$

$$\boxed{f = 104 \text{ cm}}$$

(b) $s_o \gg (s_i, f) \Rightarrow s_i \approx f = 104$ cm.

$$|M| = \frac{h_i}{h_o} = \frac{s_i}{s_o} \approx \frac{f}{s_o}$$

$$\Rightarrow h_i = \frac{h_o}{s_o} \cdot f = \frac{D_\odot}{1 \text{ A.U.}} \cdot f$$

$$h_i = \frac{1.4 \times 10^6 \text{ km}}{1.5 \times 10^8 \text{ km}} \cdot 104 \text{ cm} = \boxed{0.97 \text{ cm}}$$

[Note: limit of diffraction is $h_{min} \approx \Delta\theta \cdot f = \left(\frac{1.22\lambda}{D}\right) \cdot f$

$$h_{min} \simeq \frac{1.22\,(550 \times 10^{-9} \text{ m})}{0.1 \text{ m}} \cdot 104 \text{ cm} = 7 \times 10^{-4} \text{ cm}$$

$\rightarrow$ no problem].

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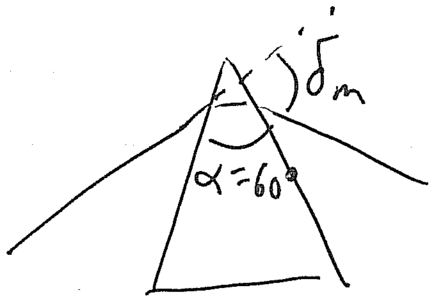
2.



Eq. 5.54: $n = \dfrac{\sin[(\delta_m + \alpha)/2]}{\sin(\alpha/2)}$, solve for δ_m.

$$\sin\left[\frac{\delta_m + \alpha}{2}\right] = n \sin(\alpha/2)$$

$$\frac{\delta_m + \alpha}{2} = \sin^{-1}[n \sin(\alpha/2)]$$

$$\delta_m = 2\sin^{-1}[n\sin(\alpha/2)] - \alpha$$

For $\alpha = 60°$, $n_{red} = 1.525$

$$\delta_m = 2\sin^{-1}[1.525 \sin 30°] - 60°$$

$$\delta_m = 39.37018°$$

For $\alpha = 60°$, $n_{blue} = 1.535$

$$\delta_m = 40.25984°$$

Difference: $\boxed{\Delta\delta_m = 0.89°}$

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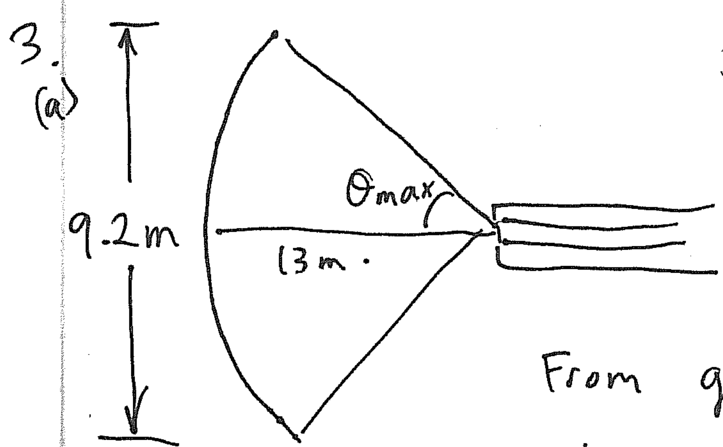
3.
(a)



$f = \frac{R}{2} = 13\,m$

core: $n_f = 1.620$
cladding $n_c = ?$

From geometry

$$\tan\theta_{max} = \frac{(9.2/2)}{13}$$

$$\theta_{max} = 19.49°.$$

From Eq. 5.64 : $NA = (n_f^2 - n_c^2)^{1/2}$

and $NA = n_o \sin\theta_{max}$, $n_o = 1$ in air.

$$\Rightarrow \sqrt{n_f^2 - n_c^2} = \sin\theta_{max}$$

$$n_f^2 - n_c^2 = \sin^2\theta_{max}$$

$$n_c = \sqrt{n_f^2 - \sin^2\theta_{max}}$$

$$= \sqrt{1.620^2 - (\sin 19.49)^2}$$

$\boxed{n_c = 1.585}$ ← maximum cladding index.

(b) Eq. 5.65 :

$$\frac{P_o}{P_i} = 10^{-\alpha L/10}$$

$\alpha = 0.85$ dB/km
$L = 50\,m\left(\frac{1\,km}{1000\,m}\right) = 0.05$ km.

$$\frac{P_o}{P_i} = 0.9903$$

$\boxed{99.03\%}$

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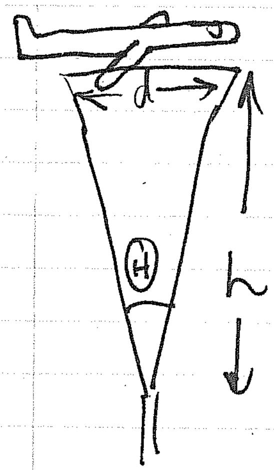
4. (a) From the equation above Eq. 13.19, the full angular width of a beam is



$$\Theta = \frac{2.44\,\lambda}{D} \quad *$$

$$\Theta \approx \frac{d}{h} \quad \Rightarrow \quad d = \Theta \cdot h$$

$$d = \frac{2.44\,\lambda}{D} \cdot h$$

$$d = \frac{2.44\,(532\times10^{-9}\ \mathrm{m})}{4\times10^{-3}\ \mathrm{m}} \cdot 500$$

$$d = 0.16\ \mathrm{m} = \boxed{16\ \mathrm{cm\ diameter}}$$

(b)
$$I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{P}{\pi (D/2)^2}$$

$$= \frac{0.7\times10^{-3}\ \mathrm{W}}{\pi\,(0.16/2)^2} = \boxed{0.034\ \frac{\mathrm{W}}{\mathrm{m}^2}}$$

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* If you use $\Theta = 0.637\,\dfrac{\lambda}{w_0}$ from pg. 595

this is $\Theta = 1.27\,\dfrac{\lambda}{D}$ $\Rightarrow$ (a) 8.4 cm diameter
(b) 0.13 W/m^2

Eq. 13.19.