UNIVERSITY OF TORONTO
Faculty of Arts and Science
DECEMBER 2010 EXAMINATION
PHY385H1F
Duration – 3 hours

Instructions:
Please complete the following problems in the examination booklet(s) provided. Completely fill out your identifying information on each booklet you use, and number your booklets. On each page clearly indicate at the top which question you are answering or continuing. Show all your work. You will be graded on correct method as well as correct answer. If you use an equation from the Hecht text, please give the equation number and page number.

Aids allowed:

- A pocket calculator with no communication ability.
- The full text of *Optics* 4th Edition (Copyright 2002) by Eugene Hecht.

Possibly helpful information:

Speed of light in a vacuum: $c = 3.00 \times 10^{8}$ m/s
Planck's constant: $h = 6.63 \times 10^{-34}$ J s
Boltzmann's constant: $k = 1.38 \times 10^{-23}$ J/K
Permittivity constant: $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
Permeability constant: $\mu_0 = 1.26 \times 10^{-6}$ H/m
1 electron-Volt = 1 eV = 1.60×10^{-19} J
Atomic mass number of the most common isotope of Rubidium = 85
Atomic mass unit 1 u = 1.66×10^{-27} kg

Most probable speed for a gas which obeys the Maxwell-Boltzmann distribution: $v = \sqrt{\frac{2kT}{m}}$

Doppler shift in frequency, f, for light when speed $v << c$: $\left|\frac{\Delta f}{f_0}\right| = \frac{v}{c}$

Ordinary and extraordinary indices of refraction for calcite: $n_o = 1.6584$, $n_e = 1.4864$.

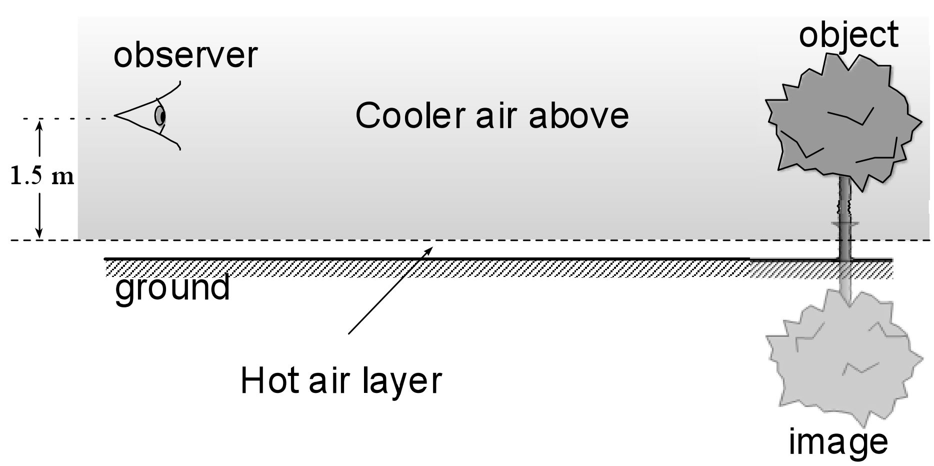
[100 points total for the exam]

1. Professor Thywissen uses an infrared laser to optically cool a sample of Rubidium atoms which are at a temperature of 3.0 mK. The tabulated transition wavelength is $\lambda_0 = 775$ nm.
   a. **[1 point]** If Professor Thywissen wishes to cool the gas, should he tune the frequency of his laser slightly above or slightly below the tabulated transition frequency, $f_0 = c/\lambda_0$, for this transition?
   b. **[5 points]** Estimate the optimal difference in frequency, Δf, between the laser and the Rubidium transition for Professor Thywissen to cool this sample of atoms.

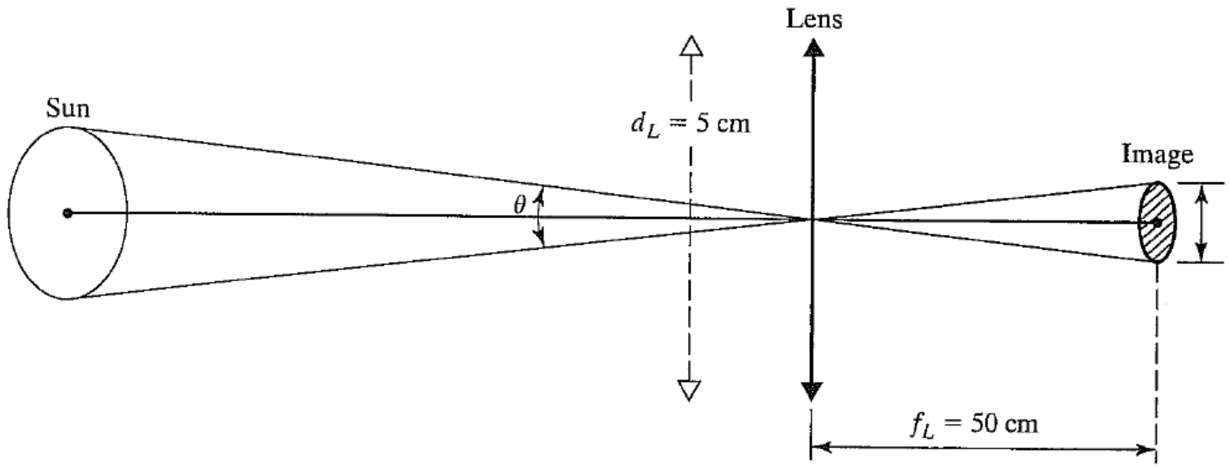
2. **[6 points]** One of the metal-cutting lasers in Professor Miller's lab provides pulses lasting 10^{-12} s, with a wavelength of 532 nm, and a peak irradiance of 10^{20} W/m^2. What is the maximum amplitude of the electric field of the EM-wave oscillations?

3. **[12 points]** Calculate the percent reflectance and transmittance for both (a) TE and (b) TM modes of light in air incident at 50° on a glass surface of index of refraction 1.60. [Recall that TE waves are linear-polarized, and have electric fields that oscillate perpendicular to the plane of incidence and therefore are parallel to the boundary plane separating the two media. TM waves are the orthogonal linear polarization state, and have electric fields that oscillate in the plane of incidence.]

4. **[8 points]** When the sun shines on a flat, dark surface, such as that of a highway, it creates a thin layer of hot air just above the surface. An observer whose eyes are 1.5 m above the hot air layer, looks forward, as shown in the sketch below. Assume the index of refraction of the cooler air at the position of the observer has a constant value of $n = 1.00029$, and the index of refraction of the hot air layer is $n = 1.00003$. If the observer looks at the surface of the road, how far along the road ahead does she need to look to see total internal reflection from the hot air layer? (This will form an image, which we call a mirage.)



5. **[8 points]** The Sun subtends an angle of 0.50° at the Earth's surface, where the irradiance is 1300 W/m^2 at normal incidence. What is the irradiance of an image of the Sun formed by a lens with diameter 5.0 cm and focal length 50.0 cm?



6. **[8 points]** A bi-convex thin lens with index of refraction 1.50 has a focal length of 30.0 cm in air. When immersed in a certain transparent liquid, it becomes a negative lens with a focal length of magnitude 188 cm. Determine the index of refraction of the liquid.

7. A system of 2-level atoms and photons is in thermal equilibrium at a temperature, *T*. The spectral energy density of the photons is given by Equation 13.10 of Hecht:

$$u_\nu = \frac{A_{21}/B_{21}}{(B_{12}/B_{21})e^{h\nu/kT} - 1}$$

where ν is the transition frequency between levels 1 and 2, in Hz.

   a. **[4 points]** Show that the ratio of the transition rates of stimulated to spontaneous emission from level 2 to level 1 is given by:
   $$\left[\frac{1}{e^{h\nu/kT} - 1}\right]$$
   b. **[2 points]** If $E_2 - E_1 = 2.0$ eV, determine the ratio of the transition rates of stimulated emission to spontaneous emission at a temperature of 300 K.
   c. **[4 points]** Redo the calculation from the previous part for a temperature of 30,000 K, and compare the results of both calculations.

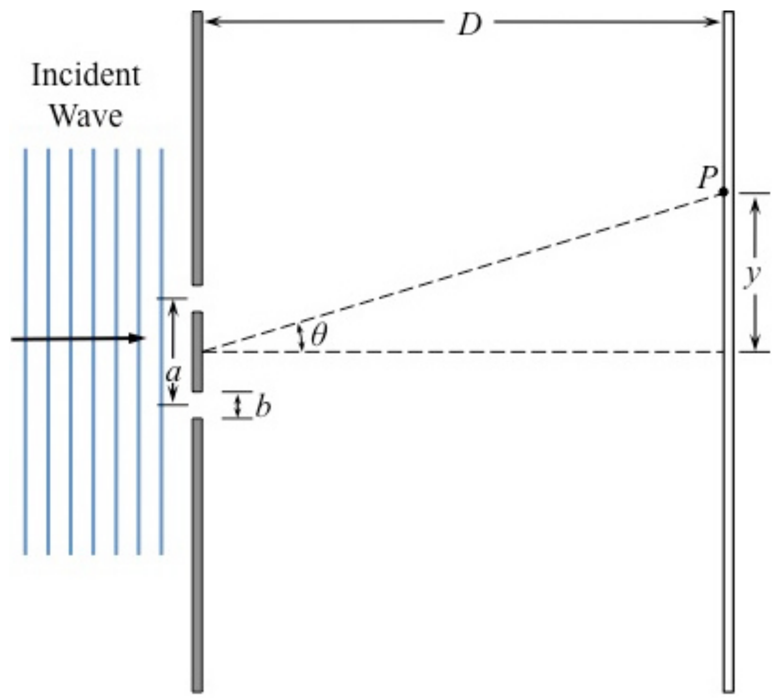
8. **[8 points]** Each eyepiece of a set of RealD 3-D glasses popular in movie theatres in 2010 uses a linear polarizer and a quarter wave-plate. Consider initially unpolarized light travelling horizontally toward the observer in the +*z*-direction that first falls on the linear polarizer, which is aligned at +45° relative to the horizontal *x*-axis. Light emerges from the linear polarizer with normalized Jones vector $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$. Suppose the light next falls onto the quarter wave-plate, whose fast axis is aligned with the vertical *y*-axis. What will be the Jones vector of the light which emerges from the quarter wave-plate, and what will be its polarization state?

9. **[4 points]** A thin plate of calcite is cut with its Optical Axis (OA) parallel to the plane of the plate. What minimum thickness is required to produce a quarter-wave path difference for sodium light whose wavelength in a vacuum is 589 nm?

10. **[8 points]** Two 1.0 MHz radio antennas emitting in-phase are separated by 600 m along a North-South line. A radio receiver placed 2.0 km East is equidistant from both transmitting antennas and picks up a fairly strong signal. How far North should that receiver be moved if it is again to detect a signal nearly as strong?

11. In a double-slit experiment as shown in the figure, the viewing screen is at a distance D = 4.00 m, and point P lies at a distance y = 20.5 cm from the centre of the pattern. Each slit has a width of b = 1.0 µm, the centre-to-centre separation of the slits is 4.0 µm, and the wavelength of the light is 580 nm.

    a. **[4 points]** Can the interference pattern be accurately modelled using Fraunhofer diffraction? Justify your answer.
    b. **[8 points]** Assuming Fraunhofer diffraction, what is the ratio of the irradiance I_P at point P to the irradiance I_{cen} at the centre of the pattern?



12. **[10 points]** Sunlight impinges on a transmission grating that is formed with 3000 lines per centimetre. Does the third-order visible spectrum overlap the second-order visible spectrum? Justify your answer. Take the limits of the visible spectrum to be 400 nm to 700 nm.