Optics 2010
Final Exam
Solutions

1. (a) Below 1

(b) 5: Most probable speed: $v = \sqrt{\frac{2kT}{m}}$ 1

Doppler Shift: $\frac{v}{c} = \frac{|\Delta f|}{f_0}$ 1

$$\Delta f = \frac{f_0 v}{c} = \frac{f_0}{c}\sqrt{\frac{2kT}{m_{Rb}}}$$

$c = f_0 \lambda \Rightarrow f_0 = \frac{c}{\lambda}$ 1 $\quad m_{Rb} = 85\,u$. 1

$$|\Delta f| = \frac{c\,v}{\lambda\,c} = \frac{1}{\lambda}\sqrt{\frac{2kT}{m_{Rb}}}$$

$$|\Delta f| = \frac{1}{775\times10^{-9}}\sqrt{\frac{2(1.38\times10^{-23})(3\times10^{-3})}{85(1.661\times10^{-27})}}$$

$|\Delta f| =$

9.88×10^{5} Hz 1 below

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2. $I = \frac{c\epsilon_0}{2} E_0^2$ (Eq. 3.44)

$$E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(10^{20})}{(3\times10^8)(8.85\times10^{-12})}}$$

$E_0 = 2.7 \times 10^{11}\ \frac{V}{m}$ or $2.7\times10^{11}\ \frac{N}{C}$

(wavelength and pulse duration do not matter!)

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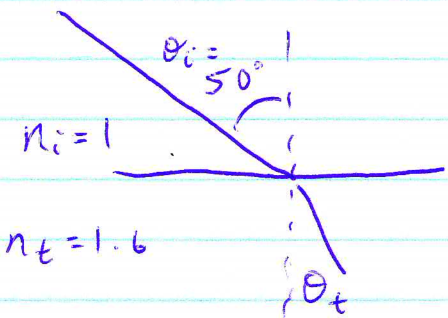
3.

a) TE : $r_\perp$ use eq. 4.34:

$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$



$$\theta_t = \sin^{-1}\left(\frac{n_i}{n_t} \sin\theta_i\right)$$

$$= \sin^{-1}\left(\frac{\sin 50}{1.6}\right)$$

$$\theta_t = 28.606^\circ$$

$$r_\perp = \frac{\cos 50 - 1.6 \cos(28.606)}{\cos 50 + 1.6 \cos(28.606)}$$

$$r_\perp = -0.372118$$

$$R_\perp = r_\perp^2 = 13.8\%$$ 3

$$T_\perp = 100\% - R_\perp = 86.2\%$$ 3

b). $$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$ [Eq. 4.40]

$$= \frac{1.6 \cos 50 - \cos(28.606)}{\cos(28.606) + 1.6 \cos 50}$$

$$r_\parallel = +0.078959$$

$$R_\parallel = r_\parallel^2 = 0.62\%$$ 3

$$\Rightarrow T_\parallel = 99.38\%$$ 3

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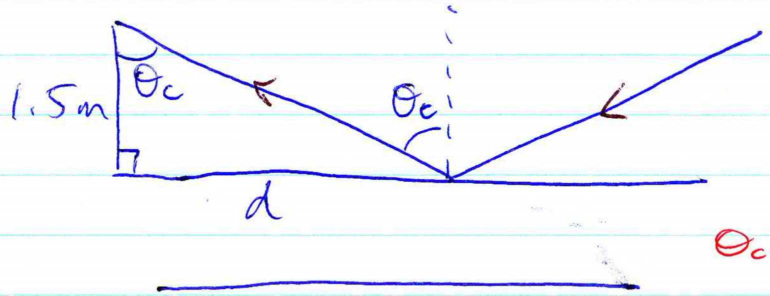
4.



$\theta_c = 88.7^\circ$

$$\tan\theta_c = \frac{d}{1.5}$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$d = 1.5\tan\left[\sin^{-1}\left(\frac{1.00003}{1.00029}\right)\right]$$

$$d = 65.8\ m$$

$$d = 66\ m$$

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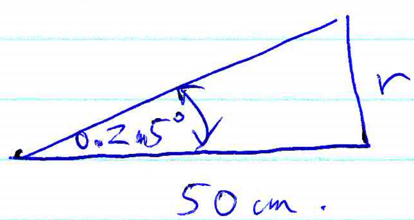
5. 2 steps: (1) Find Power that is transmitted by a 5 cm -diameter lens.

(2) Find area of the image of the sun → divide Power by this area to find irradiance.

Step (1): $P = I_{sun} \cdot A_{lens} = 1300 \frac{W}{m^2} \pi \left(\frac{0.05m}{2}\right)^2$

$P = 2.55254$ W

Step (2): image ~~diameter~~ radius $= r$



$\tan 2.5 = \frac{r}{0.5m}$

$r = 0.5 \tan(2.5)$

image area $A_{im} = \pi r^2$

$I = \frac{P}{A_{im}} = \frac{P}{\pi r^2} = \frac{2.55254}{\pi(0.5\tan(2.5))^2}$

$I = \cancel{1705 \, W/m^2} \; 1.707 \times 10^5 \, W/m^2$

$I = 1.7 \times 10^5 \, W/m^2$

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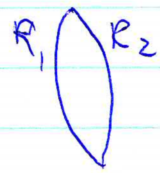
6. Air:



$$\left(\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}\right)$$

Adaptation of Eq. 5.14 gives:

$$\frac{n_m}{f} = (n_\ell - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where n_m = index of surrounding medium.

n_ℓ = index of lens.

$n_\ell = 1.5$

$n_{air} = 1$

$f_{air} = 30$ cm.

$$\Rightarrow \frac{1}{f_{air}} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f_{air}} = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2}{30\text{ cm}}$$

$\downarrow$

in liquid: $f_\ell = -188$ cm

$$\frac{n_m}{f_\ell} = (n_\ell - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n_\ell - n_m)\frac{2}{30\text{ cm}}$$

$$\frac{n_m}{f_\ell} = \frac{2n_\ell}{30\text{cm}} - \frac{2}{30\text{cm}}n_m$$

$$n_m\left(\frac{1}{f_\ell} + \frac{2}{30\text{cm}}\right) = \frac{2n_\ell}{30\text{cm}}$$

$$n_m = \frac{2(1.5)}{30\text{cm}}\left[\frac{1}{-188} + \frac{2}{30}\right]^{-1}$$

$$n_m = \boxed{1.63}$$

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7. (a) $R_{stim} = u_\nu B_{21}$

$R_{spon} = A_{21}$

$$\frac{R_{stim}}{R_{spon}} = \frac{u_\nu B_{21}}{A_{21}} = \frac{\cancel{B_{21}}}{\cancel{A_{21}}}\left(\frac{\cancel{A_{21}}}{\cancel{B_{21}}}\right) \frac{1}{(B_{12}/B_{21})\, e^{h\nu/kT} - 1}$$ 2

and $B_{12} = B_{21} \Rightarrow (B_{12}/B_{21}) = 1.$ 2

$$\Rightarrow \frac{R_{stim}}{R_{spon}} = \frac{1}{e^{h\nu/kT} - 1}$$ $= 4$

(b). $h\nu = E_2 - E_1 = 2\,eV = 2 \times 1.6 \times 10^{-19}\ J.$

$kT = 1.38 \times 10^{-23} \cdot (300)$

$$\frac{1}{\exp(h\nu/kT) - 1} = 2.7 \times 10^{-34}$$ 2

(c) $= 0.86$ 2

wow, a lot more! 2

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8. linear polarizer at 45°

QWP. fast axis vertical.

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$\frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix}1 & 0\\0 & -i\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$$

Jones vector.

R-circular polarized.

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9. Eq. 8.32:

$$\Delta\varphi = \frac{2\pi}{\lambda_0} d \left| n_o - n_e \right|$$

$$QWP \Rightarrow \Delta\varphi = \frac{2\pi}{4} = \frac{\pi}{2}$$ ← minimum $\Delta\varphi$.

solve for d:

$$\frac{\pi}{2} = \frac{2\pi}{\lambda_0} d (n_o - n_e)$$

$$\frac{\lambda_0}{4} = d (n_o - n_e)$$

$$d = \frac{\lambda_0}{4(n_o - n_e)} = \frac{589 \times 10^{-9}}{4(1.6584 - 1.4864)}$$

$$d = 8.56 \times 10^{-7} \text{ m}$$

$$\boxed{d = 0.86\ \mu m}$$

856 nm

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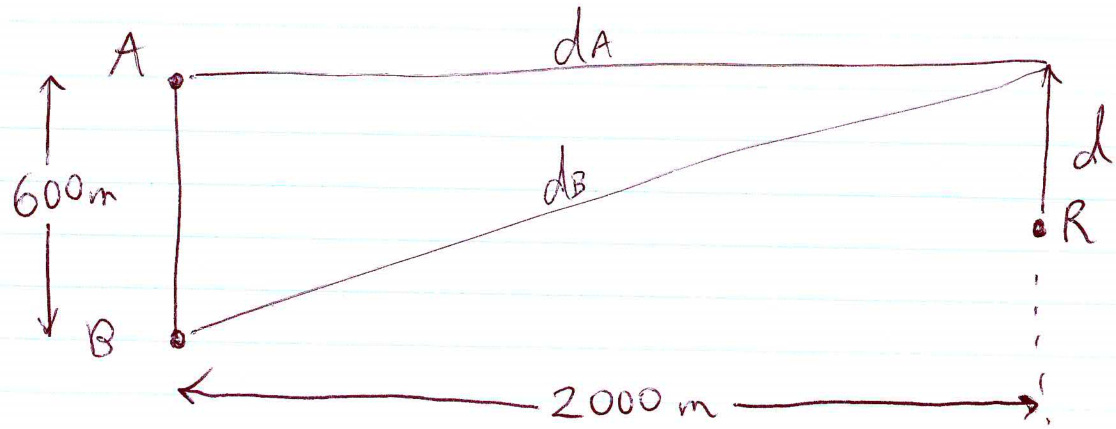
10.



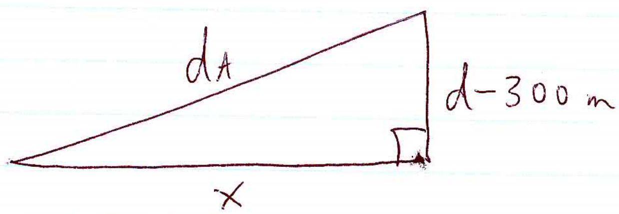
Constructive interference $\Rightarrow$ $d_B - d_A = \lambda$

$f = \frac{c}{\lambda} \Rightarrow \boxed{\lambda = \frac{c}{f} = \frac{3\times10^8\ m/s}{1.0\times10^6\ s^{-1}} = 300m}$

$\Rightarrow d_B = d_A + \lambda$

- Assuming $d > 300$ m, the triangle for source A is:

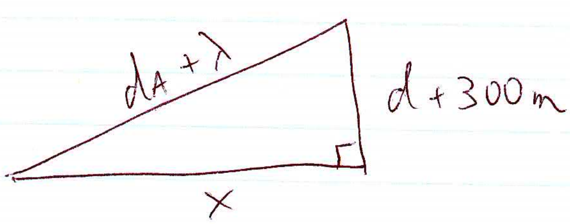
+4



where $x = 2000$m

Pythagorous $\Rightarrow$ $\boxed{d_A^2 = x^2 + (d-300m)^2}$ ⊗

- The triangle for source B is:



Pythagorous $\Rightarrow (d_A+\lambda)^2 = x^2 + (d+300m)^2$

$d_A^2 + 2d_A\lambda + \lambda^2 = x^2 + d^2 + d(600m) + (300m)^2$

10. continued.

Use eq. ⊛ to eliminate d_A :

$$\cancel{x^2} + \cancel{d^2} - d(600m) + \cancel{(300m)^2} + 2\lambda\sqrt{x^2 + (d-300m)^2} + \lambda^2$$

$$= \cancel{x^2} + \cancel{d^2} + d(600m) + \cancel{(300m)^2}$$

$$2\lambda\sqrt{x^2 + (d-300m)^2} = d(1200m) - \lambda^2$$

$$x^2 + (d-300m)^2 = \left[\frac{d(600m)}{\lambda} - \frac{\lambda}{2}\right]^2$$

$$x^2 + d^2 - \cancel{d(600m)} + (300m)^2 = d^2\left(\frac{600m}{\lambda}\right)^2 - \cancel{d(600m)} + \frac{\lambda^2}{4}$$

$$x^2 + (300m)^2 - \lambda^2/4 = d^2\left[\left(\frac{600m}{\lambda}\right)^2 - 1\right]$$

$$d = \sqrt{\frac{x^2 + (300m)^2 - \lambda^2/4}{(600m/\lambda)^2 - 1}} \quad , \; x = 2000m, \; \lambda = 300m$$

$$d = \sqrt{\frac{2000^2 + 300^2 - 300^2/4}{2^2 - 1}} = \boxed{1164.4\ m}$$

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11. (a) $R \sim D = 4\ m.$

$$\frac{a^2}{\lambda} = 2.76\times10^{-5}\ m \ll 4\ m,$$

so, yes, Fraunhofer is well justified.

(b). $$\theta = \tan^{-1}\left(\frac{20.5\ cm}{400\ cm}\right) = 0.0512\ rad$$
$$= 2.93^\circ.$$

$$\beta = \frac{\pi b}{\lambda}\sin\theta = 0.277\ rad.$$

$$\alpha = \frac{\pi a}{\lambda}\sin\theta = 1.11\ rad.$$

$$\left(\frac{\sin\beta}{\beta}\right)^2 = 0.975\ , \quad \cos^2\alpha = 0.199$$

$$\frac{I}{I_0} = 0.774$$

77.4%

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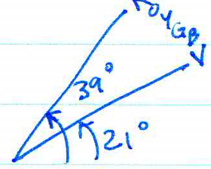
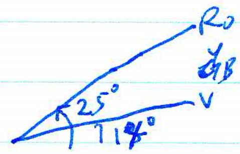
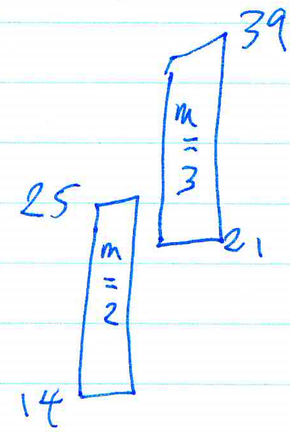
12. $\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right)$

m = 3.



m = 2.





Yes, they overlap a bit

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