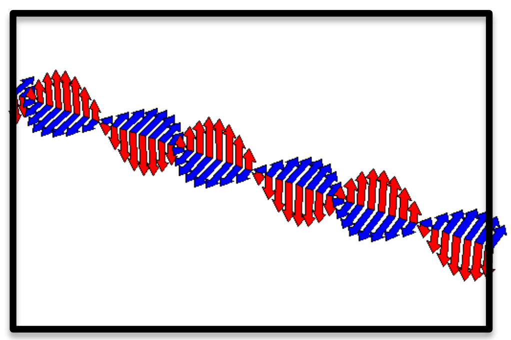
PHY385-H1F Introductory Optics

Class 3 – Outline: Sec. 3.1, 3.2

- Finishing Spherical, Cylindrical Waves
- Maxwell's Equations in a Vacuum
- Magnetic Field: $\vec{B}$ and Magnetizing Field: $\vec{H}$
- Intuitive Look at EM waves (powerpoint)
- Electromagnetic Waves: speed, general complex-exponential form
- Constraints on the EM-wave imposed by Maxwell's Equations



In-Class Task from last time

- Consider the function: $\psi(x,t) = f(x - vt)$
- Where: $f(y) = \dfrac{1}{y^2 + 1}$

1. Consider the time $t = 0$. At what value of x is $\psi(x,0)$ a minimum? At what value of x is $\psi(x,0)$ a maximum?

ANSWER: *ψ is minimum at $x = \pm\infty$, ψ is maximum at $x = 0$*

2. Consider the time $t = +1$ second. At what value of x is $\psi(x,1)$ a maximum?

ANSWER: *ψ is maximum when $y = 0$. $y = x - vt$. $x = y + vt$, so ψ is a maximum when $x = v$. [I probably should have specified the speed!]*

Electromagnetic Waves: An intuitive look…

[These following slides have been used in class with the gracious permission of their author, Dr. Lawrence P. Staunton, of Drake University
http://www.drake.edu/artsci/physics/
I have made a couple of small changes to adapt the slides for my PHY385 course – any errors that result are entirely my fault.]

Maxwell's Equations: Partial Differential Form

Faraday's Induction Law in partial differential form [B is magnetic field in Tesla]:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Gauss's Law – Electric:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Gauss's Law – Magnetic:

$$\vec{\nabla} \cdot \vec{B} = 0$$

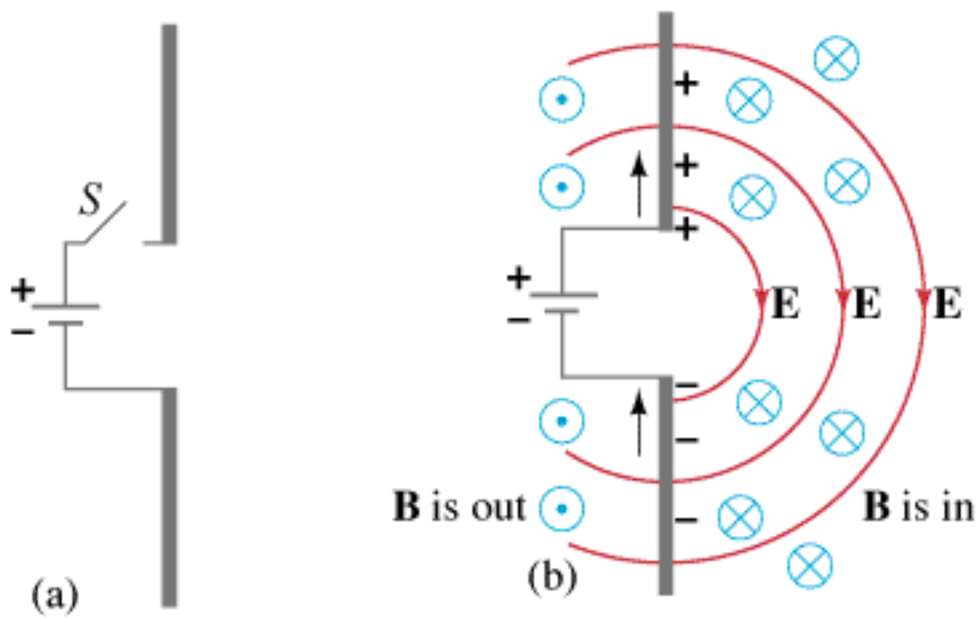
Ampère's Circuital Law [B is magnetic field in Tesla]:

$$\vec{\nabla} \times \vec{B} = \mu \vec{J} + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$$

Electromagnetic Waves

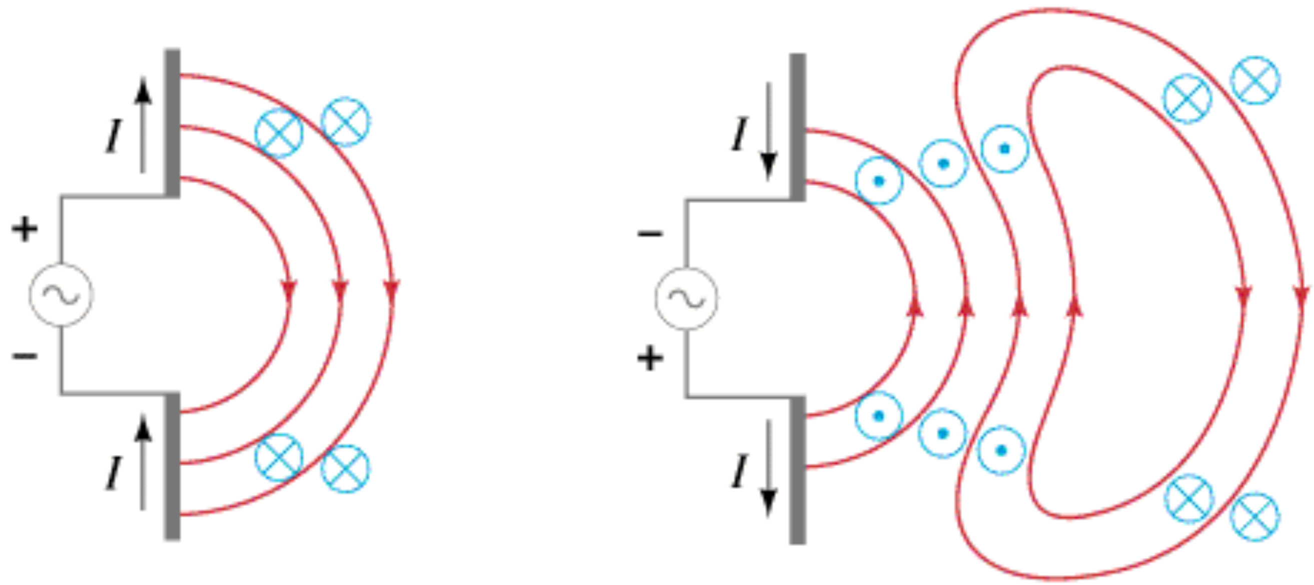
- So, a magnetic field will be produced in space if there is a changing electric field [Ampere's Law, using "displacement current" $d\Phi_E/dt$]
- But, this magnetic field is changing since the electric field is changing [derivative of cosine is sine and vice-versa]
- A changing magnetic field produces an electric field that is also changing [Faraday's Law]
- We have a self-perpetuating system!

Electromagnetic Waves



- Close switch and current flows briefly
- Sets up electric field
- Current flow sets up magnetic field as little circles around the wires
- Fields not instantaneous, but form in time
- Energy is stored in fields and cannot move infinitely fast

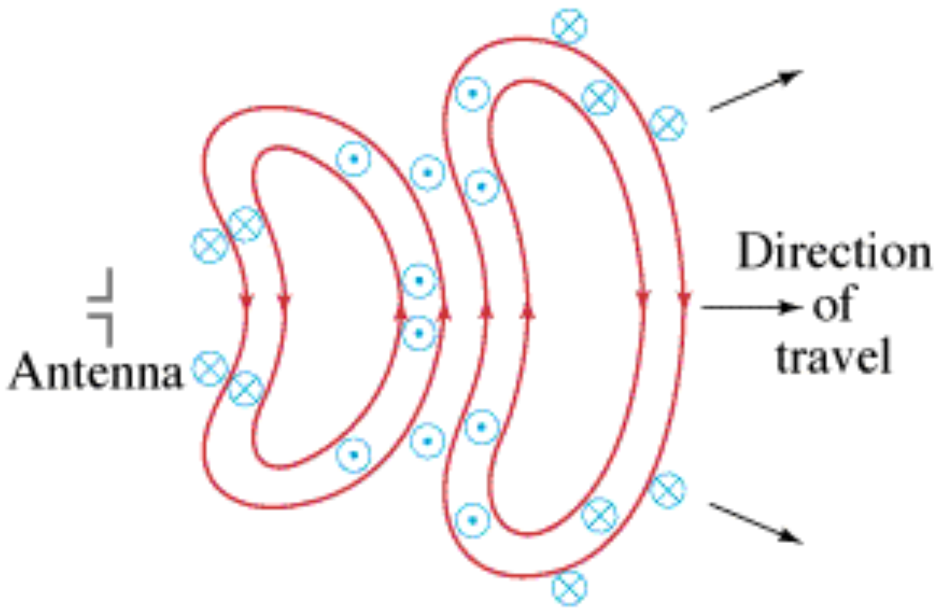
Electromagnetic Waves



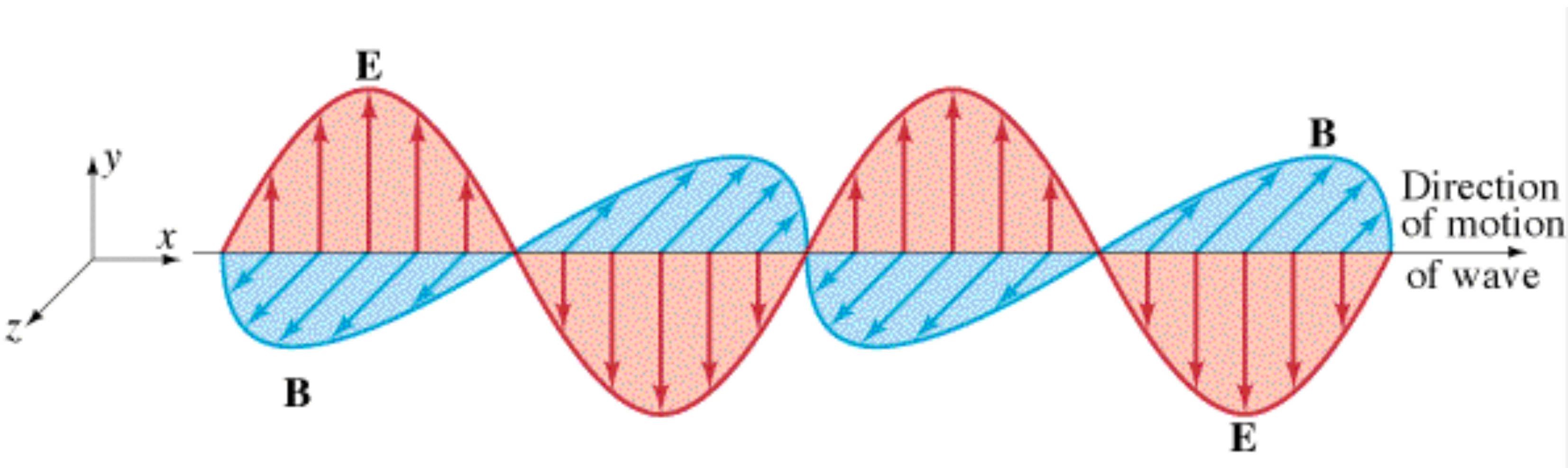
(a) (b)

- Picture (a) shows first half cycle
- When current reverses in picture (b), the fields reverse
- See the first disturbance moving outward: These are the electromagnetic waves.

Electromagnetic Waves



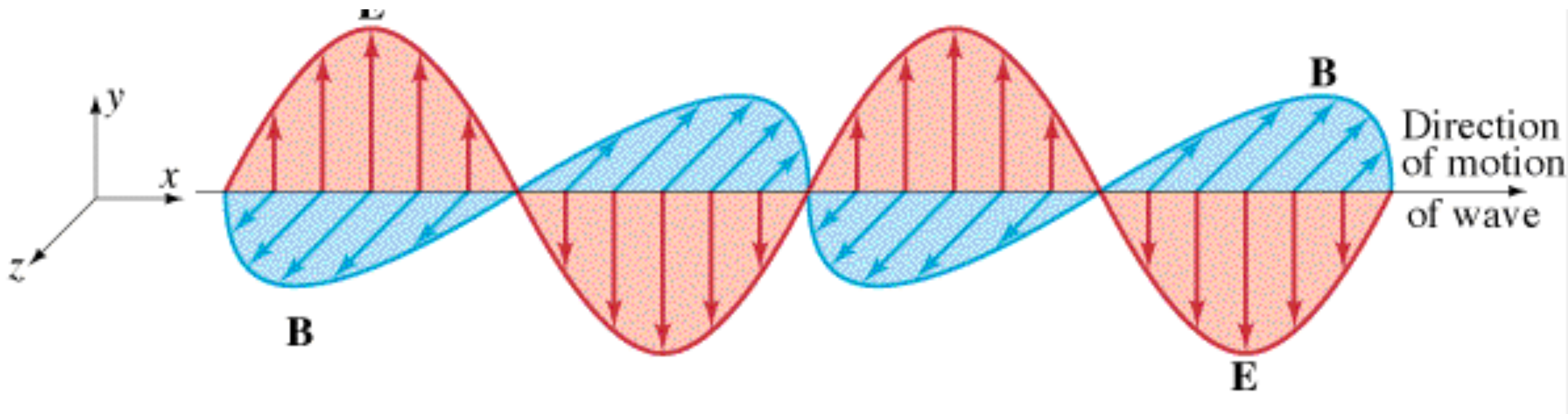
- Notice that the electric and magnetic fields are at right angles to one another
- They are also perpendicular to the direction of motion of the wave



Speed of EM Waves

- Now that we have shown how the waves are formed from oscillating charges, we need to see if we can predict how fast they move
- We move far away from the source so that the wave fronts are essentially flat
- Just like dropping a rock in a pond and looking at the waves a few metres away from the impact point

Speed of EM Waves

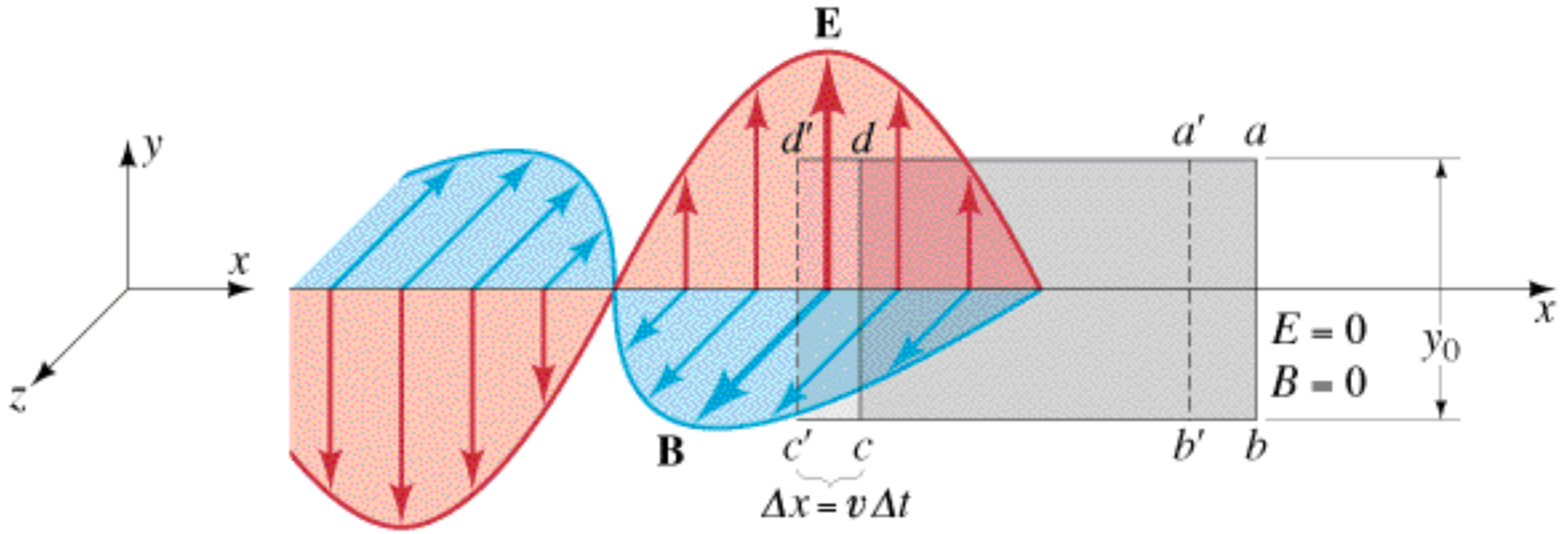


- This picture defines the coordinate system used in Hecht

- Wave propagates along the *x*-axis

- The electric field varies in the *y*-direction and the magnetic field in the *z*-direction.

Speed of EM Waves



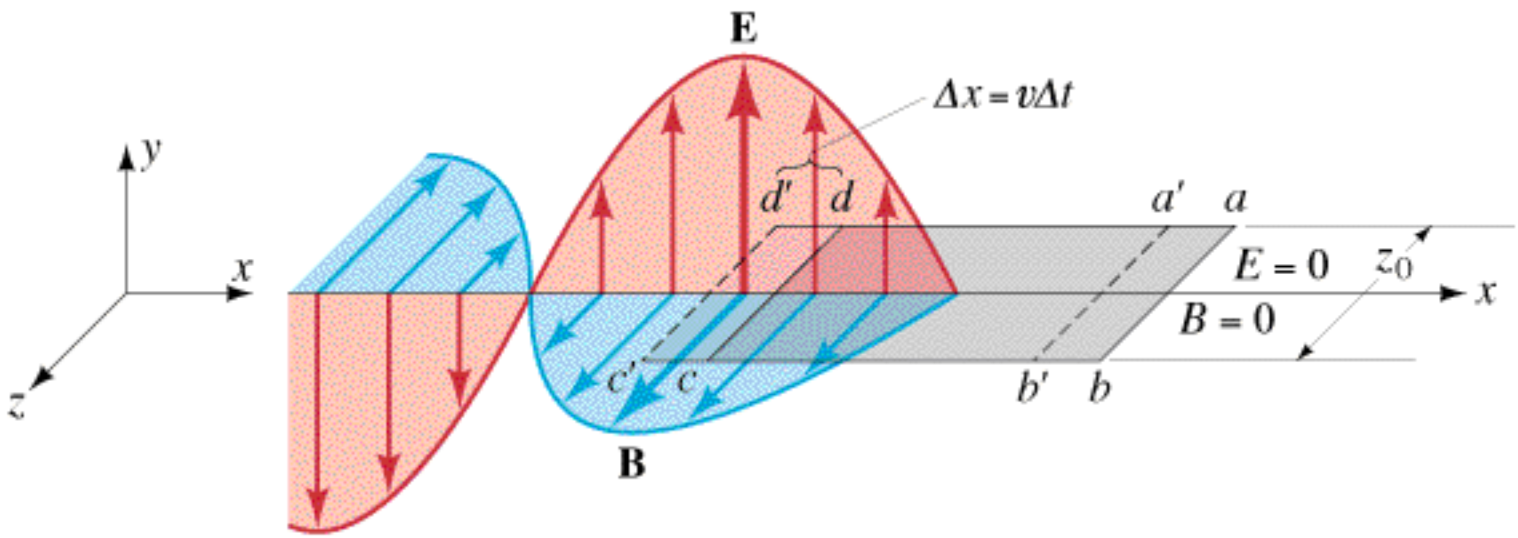
We are going to apply Faraday's Law to the imaginary moving rectangle *abcd*. Compute the magnetic flux change:

$$emf = \frac{\Delta\Phi_B}{\Delta t} = \frac{B\Delta A}{\Delta t} = \frac{By_0 v\Delta t}{\Delta t} = By_0 v$$

Speed of EM Waves

- We can say the emf around the loop is the sum of the individual emfs going along each straight line segment in the loop
- We look at the work done in moving a test charge around the loop
- $emf = W/q = Fd/q = Ed$
- $emf = Ey_0 = By_0v$
- $E = Bv$

Speed of EM Waves



Now we are going to look at the change in electric flux. Set a new imaginary rectangle and play the same game as before:

$$\sum B_{\parallel}\Delta l = \mu_0\varepsilon_0 \frac{\Delta\Phi_E}{\Delta t} = \mu_0\varepsilon_0 \frac{E z_0 v\Delta t}{\Delta t} = \mu_0\varepsilon_0 E z_0 v$$

Speed of EM Waves

$$Bz_0 = \mu_0 \varepsilon_0 E z_0 v$$

$$B = \mu_0 \varepsilon_0 E v$$

From before: → $E = Bv$

$$B = \mu_0 \varepsilon_0 (Bv) v$$

$$1 = \mu_0 \varepsilon_0 v^2$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$v = \frac{1}{\sqrt{8.85 \times 10^{-12} \times 4\pi \times 10^{-7}}}$$

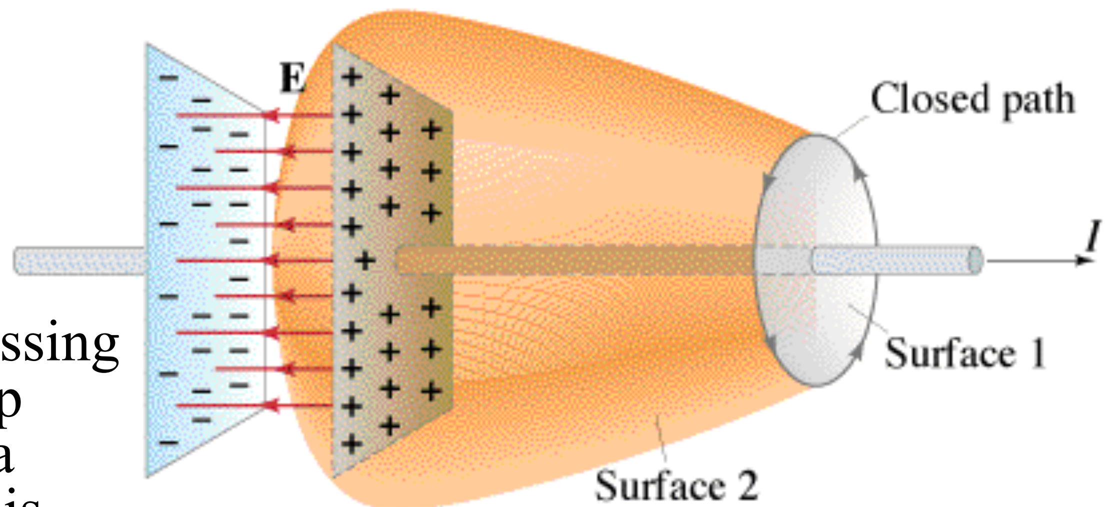
$$v = 3 \times 10^8$$

5-minute In-Class Task

- Please take out a piece of paper that you don't mind handing to me at the end
- WRITE YOUR NAME at the top of the piece of paper
- Discussion with your friends or me during this task is ***encouraged!***
- Ampère's Law is:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$

Where I_{through} is the current passing through a surface, and the loop integral is carried out around a closed path which encircles this surface.



1. Consider the surfaces shown near a discharging capacitor. What is the current through surface 1? What is the current through surface 2? Should the loop integral of B around the closed path be the SAME or DIFFERENT for the two surfaces?
2. How can Ampère's Law be applied to this situation?