Term Test 1 **PHY385H1F** – "Introductory Optics"
October 11, 2011 Duration: 50 minutes

NAME: Jason Harlow **Student Number:** Answer Key

Aids allowed: A pocket calculator with no communication ability. "Optics" 4th Edition (Copyright 2002) by Eugene Hecht.

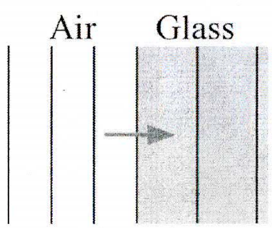
Possibly helpful information:
The speed of light in a vacuum is $c = 3.00 \times 10^8$ m s^{-1}

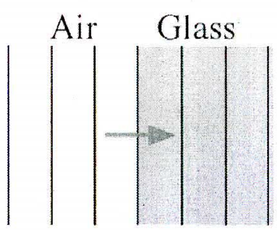
You may not communicate with anyone other than the invigilator during the test.

Multiple Choice Part (6 points)

Circle the letter of the best answer for each question.

1. A light wave travels, as a plane wave, from air ($n = 1.0$) into glass ($n = 1.5$). Which diagram shows the correct wave fronts?

A.


B.


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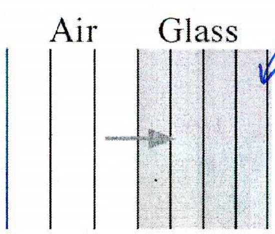
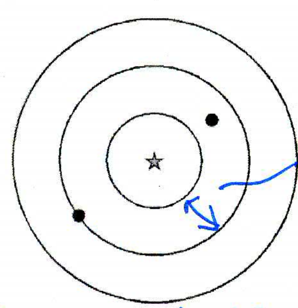


$\lambda_{glass} = \frac{\lambda_0}{n}$

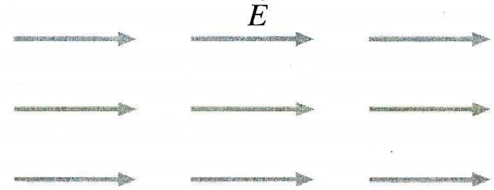
2. A spherical wave travels outward from a point source. What is the phase difference between the two points on the wave marked with dots?
   A. $\pi/4$ radians
   B. $\pi/2$ radians
   C. π radians
   D. $7\pi/2$ radians
   E. 7π radians



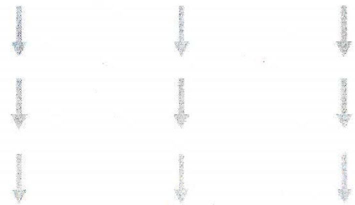
2π phase diff. between wave fronts.

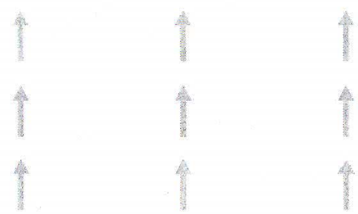
$\Delta\phi = \pi$ half-way

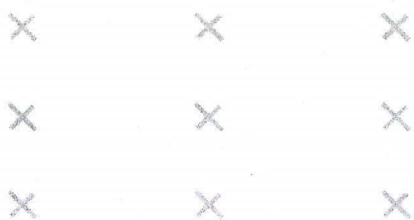
3. An electromagnetic plane wave is coming toward you, out of the screen. At one instant, the electric field looks as shown. Which is the wave's magnetic field at this instant?



$\vec{E} \times \vec{B} = \odot$ = dir. of wave

A. 

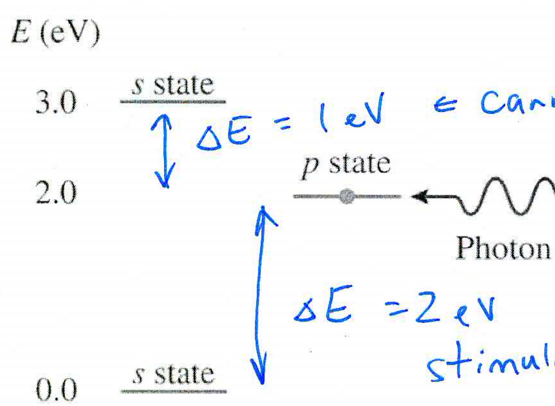
B. 

C. 

D. 

E. The magnetic field is instantaneously zero.

4. A photon with energy 2.0 eV is incident on an atom that is in the p state. Which can happen?
   A. Absorption
   B. Stimulated emission
   C. Both
   D. Neither
   E. Not enough info to tell

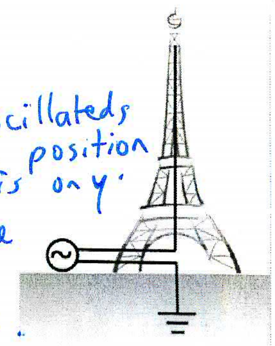


$\Delta E = 1$ eV ← cannot absorb.

$\Delta E = 2$ eV can cause stimulated emission.

I suppose there could be a higher state not shown or the atom could be ionized by the photon, so I accept C as a correct answer.

5. Consider a big transmitting tower, which is aligned with the vertical $+z$ axis. Choose coordinates so that $+z$ is up, $+x$ is East, and $+y$ is North.
An AC generator is connected to the tower, sending a current up and down its length.
If you are in a car 1 km North of the tower:



vertical current ⇒ $\vec{B}$ oscillateds along x when position is on y.
Dipole field in x-y plane is vertical, along z.

A. The $\vec{E}$-field will oscillate in the $\pm x$ direction, and the $\vec{B}$-field will oscillate in the $\pm z$ direction
B. The $\vec{E}$-field will oscillate in the $\pm z$ direction, and the $\vec{B}$-field will oscillate in the $\pm y$ direction
C. The $\vec{E}$-field will oscillate in the $\pm y$ direction, and the $\vec{B}$-field will oscillate in the $\pm x$ direction
D. The $\vec{E}$-field will oscillate in the $\pm z$ direction, and the $\vec{B}$-field will oscillate in the $\pm x$ direction
E. The $\vec{E}$-field will oscillate in the $\pm y$ direction, and the $\vec{B}$-field will oscillate in the $\pm z$ direction

6. Which of the following is true: (I) A reflected beam always has the same irradiance as the incident beam; (II) a reflected beam lies in the same plane as the incident beam; (III) a reflected beam always makes an angle $\theta = \sin^{-1}(n_t/n_i)$ with the normal to the interface.
A. I only
B. I and II
C. II and III
D. II only
E. I, II and III

Long Answer Part (14 points)

Please complete the following problems in the examination booklet provided. Show all your work. You will be graded more on correct method than correct answer. If you take an equation from the Hecht text, please give the equation number and page number.

1. Consider the following mathematical expressions, where x, y and z are positions in metres, and t is time in seconds:
1. $\psi(z,t) = A\sin^2[4\pi(t+z)]$
2. $\psi(x,t) = A(x-t)^2$
3. $\psi(x,t) = \dfrac{A}{Bx^2 - t}$

(a) Which qualify as travelling waves? Prove your conclusion.
(b) If they qualify, give the magnitude and direction of the wave velocity.

2. A plane, harmonic, linearly polarized light wave has an electric field intensity given by:

$$\vec{E}(x,t) = E_0\hat{k}e^{i\pi\cdot 10^{15}\left(t - \frac{x}{0.65c}\right)}$$

while travelling in a piece of glass. Find
(a) The frequency of the light in Hz.
(b) The index of refraction of the glass.

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LA1. (a). 1. qualifies because it satisfies the wave equation: $\frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$

Also, if $w = z - vt$, it is a function of w:

$$\psi = A \sin^2(4\pi w)$$

where $v = -1 \ m/s$

2. qualifies because it satisfies the wave eq.

Again, if we set $w = x - vt$, then

$$\psi = A w^2 \quad , \text{ and } \quad v = +1 \ m/s$$

3. does not qualify.

$$\frac{\partial \psi}{\partial x} = \frac{-2ABx}{(Bx^2 - t)^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{AB}{(Bx^2-t)^2}\left[\frac{8Bx^2}{Bx^2-t} - 2\right]$$

$$\frac{\partial \psi}{\partial t} = \frac{A}{(Bx^2-t)^2}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{2A}{(Bx^2-t)^3}$$

$\frac{\partial^2 \psi}{\partial x^2}$ and $\frac{\partial^2 \psi}{\partial t^2}$ are not proportional.

$\Rightarrow$ This is not a travelling wave.

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LA1 (b) 1. $v = 1 \, m/s$, $-z$-direction ✓

2. $v = 1 \, m/s$, $+x$-direction ✓

LA2 Compare with the form:

$$\vec{E} = \vec{E}_0 \, e^{i(kx - \omega t)}$$

Then $k = \dfrac{\pi \cdot 10^{15}}{0.65\,c}$

$$\omega = \pi \cdot 10^{15}$$

(a) $f = \dfrac{\omega}{2\pi} = \dfrac{\cancel{\pi} \cdot 10^{15}}{2\cancel{\pi}} = 0.5 \times 10^{15}$

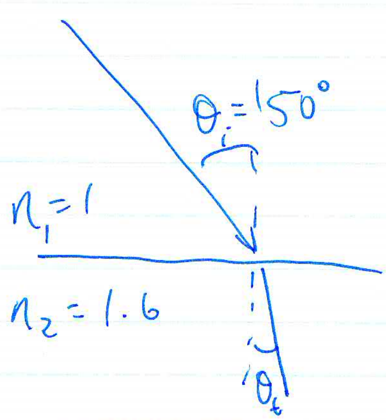
$$\boxed{f = 5 \times 10^{14} \text{ Hz}}$$

−0.5 for missing units.
2

(b) $\dfrac{\omega}{k} = \dfrac{c}{n} \Rightarrow n = \dfrac{ck}{\omega} = c\left(\dfrac{\pi \cdot 10^{15}}{0.65\,c}\right)\dfrac{1}{\pi \cdot 10^{15}}$

$$\boxed{n = \frac{1}{0.65} = 1.54}$$

LA3.



Equal mix of $\parallel$ and $\perp$

Snell's Law:

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\theta_t = \sin^{-1}\left[\frac{n_1}{n_2}\sin\theta_i\right]$$

$$\theta_t = 28.606°$$

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LA3 continued.

Eq. 4.34

$$r_\perp = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{\cos 50^\circ - 1.6\cos(28.606)}{\cos 50^\circ + 1.6\cos(28.606)}$$

$$r_\perp \approx -0.37212$$

Eq. 4.61 $R_\perp = r_\perp^2 = 0.138$ of $\perp$ component.

Eq. 4.40

$$r_\parallel = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_1 \cos\theta_t + n_2 \cos\theta_i}$$

$$= \frac{1.6\cos 50 - \cos(28.606)}{\cos(28.606) + 1.6\cos 50}$$

$$r_\parallel = 0.078959$$

Eq. 4.62 $R_\parallel = r_\parallel^2 = 0.006$ of $\parallel$ component

$$R = 0.5 R_\perp + 0.5 R_\parallel$$

$$= 0.5(0.138) + 0.5(0.006)$$

$$= 0.07235$$

Reflected ~~A~~ irradiance is $\boxed{0.0724 \frac{W}{m^2}}$

Did not ÷ by 2: 0.145 W/m² ... +4

Used only $R_\perp$: 0.138 W/m² ... +2.5