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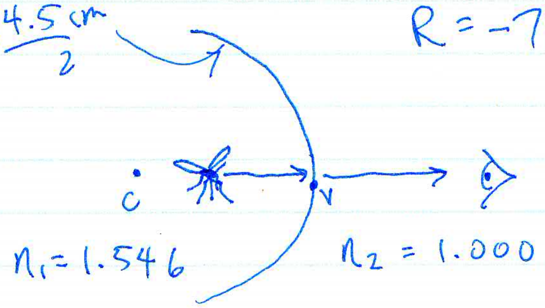
Optics
Test 2
Fall 2011

1. $R = \frac{14.5\text{ cm}}{2}$ $R = -7.25$ cm



Sign convention from Table 5.1 pg. 154: R is negative if C is left of V

Known: $s_i = -2.5$ cm (virtual image, to the left of V)
need: s_o

Use eq. 5.8: $\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$

Solve for s_o: $\frac{s_o}{n_1} = \left[\frac{n_2 - n_1}{R} - \frac{n_2}{s_i}\right]^{-1}$

$$s_o = n_1\left[\frac{n_2 - n_1}{R} - \frac{n_2}{s_i}\right]^{-1}$$

$$= 1.546\left[\frac{-0.546}{-7.25} - \frac{1}{-2.5}\right]^{-1}$$

$$= 1.546\,[0.075310 + 0.4]^{-1}$$

(a) $\boxed{s_o = +3.25\text{ cm}}$ 2 points.

(b) Use eq. 5.25: $M_T = -\frac{s_i}{s_o} = \frac{y_i}{y_o}$

known: $y_i = 0.8$ cm
need y_o

$$y_o = -\frac{y_i s_o}{s_i} = \frac{-0.8\,(3.25)}{-2.5} = \boxed{1.04\text{ cm}}$$

2 points.

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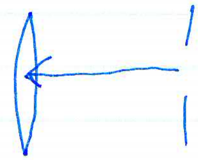
2. (a) → if the diaphragm is the aperture stop, then the entrance pupil is the image of the diaphragm as seen from the object:

$f = +20.0\,cm$



$S_o = +5.0\,cm \qquad y_o = +2.0\,cm.$

Thin Lens eq.:

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

$$s_i = \left[\frac{1}{f} - \frac{1}{S_o}\right]^{-1} = \left[\frac{1}{20} - \frac{1}{5}\right]^{-1}$$

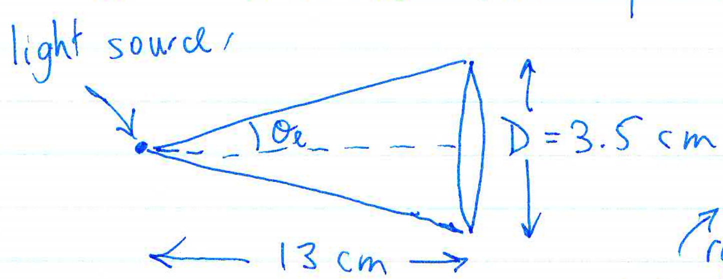
$$s_i = -\frac{20}{3}\ cm = -6.7\ cm$$

$$M_T \equiv \frac{y_i}{y_o} = -\frac{S_i}{S_o} \Rightarrow y_i = y_o\left(\frac{-S_i}{S_o}\right) = 2\left(\frac{-6.7}{5}\right)$$

$$y_i = 2.7\ cm.$$

1 point Entrance pupil is $\boxed{6.7\ cm \text{ behind lens}}$,
1 point and it has a $\boxed{\text{diameter of } 2.7\ cm.}$

(b) If the lens is the aperture stop:



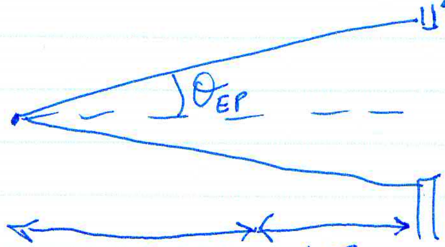
$$\tan\theta_e = \frac{3.5/2}{13}$$

$$\Rightarrow \theta_e = 7.7°$$

radius of light cone that passes through lens.

2 (b) continued

If the diaphragm is the aperture stop.



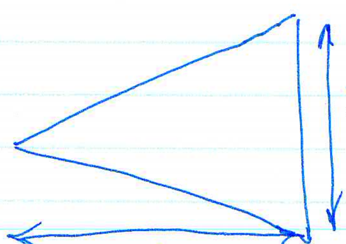
$$\tan\theta_{EP} = \frac{2.7/2}{(13+6.7)}$$

$$\theta_{EP} = 3.9^\circ$$ ← much smaller

2 points.

$\boxed{\text{So the effective aperture stop is the diaphragm}}$

(c) Use the entrance pupil to find how much power goes through the system:



$d = 13 + 6.7 = 19.7$ cm $= 0.197$ m

Irradiance at entrance pupil: $I = \frac{P}{4\pi d^2} = \frac{60}{4\pi(0.197)^2}$

$I = 123.4$ W/m^2

Area of entrance pupil $A_{EP} = \pi(D/2)^2 = \pi\left(\frac{0.027}{2}\right)^2$

$A_{EP} = 5.59 \times 10^{-4}$ m^2

Power through system: $P = I \cdot A_{EP} = \boxed{69 \text{ mW}}$ 2 points

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3. (a) $NA = (n_f^2 - n_c^2)^{1/2}$ [Eq. 5.64, pg. 195]

$= \sqrt{1.58^2 - 1.52^2}$

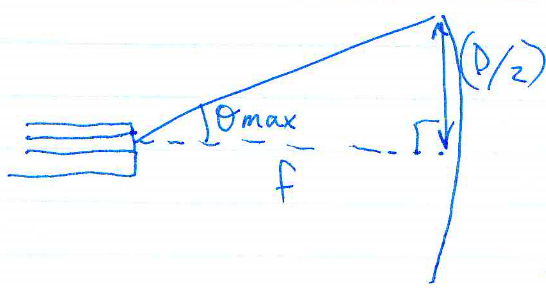
$\boxed{NA = 0.43}$ 1 point

(b) If $n_i = 1.000$, $NA = \sin\theta_{max}$

$\Rightarrow \theta_{max} = \sin^{-1}(NA)$

$\boxed{\theta_{max} = 25.5°}$ 1 point

(c) $f = -\frac{R}{2}$ for spherical mirror, for parallel rays the fibre should be a distance f from the mirror.



$\tan\theta_{max} = \frac{D/2}{f}$

$D = 2f\tan\theta_{max}$

$D = 2\left(-\frac{R}{2}\right)\tan\theta_{max}$

$D = -R\tan\theta_{max} = 0.60\ m\ \tan 25.5°$

$\boxed{D = 0.29\ m}$ 3 points

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4.(a) Use Wien's Law eq. 13.3

$$\lambda_{max} = \frac{0.002898}{T}$$

$$\boxed{\lambda_{max} = 483\ nm}$$ 1 point

(b) Use Eq. 13.4:

$$I_\lambda = \frac{2\pi h c^2}{\lambda^5}\left[\frac{1}{\exp[hc/(\lambda k_B T)] - 1}\right]$$

At $\lambda = 550.5 \times 10^{-9}$ m

$$I_\lambda = \frac{2\pi(6.626\times10^{-34})(3\times10^8)^2}{(550.5\times10^{-9})^5}\left[\frac{1}{\exp[6.626\times10^{-34}\cdot 3\times10^8/(550.5\times10^{-9}\cdot 1.38\times10^{-23}\cdot 6000)] - 1}\right]$$

$$= 7.38\times10^{15}\left[\frac{1}{e^{4.36} - 1}\right]$$

$$I_\lambda = 9.56\times10^{13}\ \frac{W}{m^3}$$ ← Power per m^2 cross-section area per m of wavelength.

2 points

Over 1 nm $= 10^{-9}$ m of wavelength, the irradiance is:

$$I = I_\lambda(\Delta\lambda) = 95{,}550\ \frac{W}{m^2}$$

Total power emerging from a circular hole of Area $= \pi(D/2)^2$:

$$P = I A = I\pi\left(\frac{D}{2}\right)^2 = 95550\cdot\pi\cdot\left(\frac{0.001}{2}\right)^2$$

$$\boxed{P = 75.6\ mW}$$ 2 points