

## PHY131H1S - Class 15

- Today:
- Stable and Unstable Equilibrium
- Hooke's Law
- Elastic Potential Energy
- Energy in Collisions
- Elastic, Inelastic Collisions



### Ch.10 Reading Quiz 1:

- Two objects collide. All external forces on the objects are negligible.
- If the collision is “perfectly elastic”, that means it conserves
  - A. Momentum  $p=mv$
  - B. Kinetic energy  $E = \frac{1}{2} mv^2$
  - C. Both
  - D. Neither

### Ch.10 Reading Quiz 2:

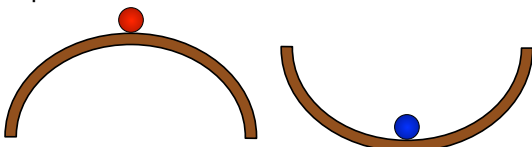
- Two objects collide. All external forces on the objects are negligible.
- If the collision is “inelastic”, that means it conserves
  - A. Momentum  $p=mv$
  - B. Kinetic energy  $E = \frac{1}{2} mv^2$
  - C. Both
  - D. Neither
- All collisions conserve momentum.
- Energy can sometimes be lost if the objects that collide are deformed or create heat in some way.

### Ch.10 Reading Quiz 3:

- Two objects collide. All external forces on the objects are negligible.
- If the collision is “perfectly inelastic”, that means
  - A. momentum is not conserved.
  - B. the final kinetic energy is zero.
  - C. the objects stick together.
  - D. one of the objects ends with zero velocity.

Last day I asked at the end of class:

- A red marble is balanced on the top of a smooth hill. A blue marble sits at the bottom of a smooth valley. Which marble is in equilibrium?
- ANSWER: They both are!
- What is the difference between these two situations?
- ANSWER: The red marble is in *unstable* equilibrium. The blue marble is in *stable* equilibrium.



### Dominoes

- A domino is a rectangular solid which can be balanced on its edge
- When standing upright, its gravitational potential energy is a maximum
- This is a state of **unstable equilibrium**: a small perturbation can cause the domino to fall, transforming its gravitational potential energy into kinetic energy
- As it is falling, it can perturb its neighbor, which then releases its potential energy: a **chain reaction** can ensue!



### Hooke's Law

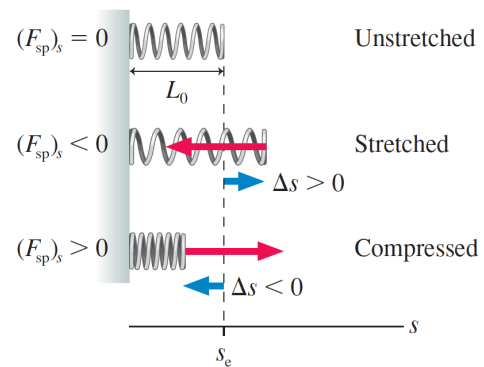
- If you stretch a rubber band, bend a ruler or other solid object, a force appears that tries to pull the object back to its equilibrium, or unstretched, state.
- A force that restores a system to an equilibrium position is called a **restoring force**.
- If  $s$  is the position, and  $s_e$  is the equilibrium position, we define  $\Delta s = s - s_e$ .

$$(F_{sp})_s = -k \Delta s \quad (\text{Hooke's law})$$

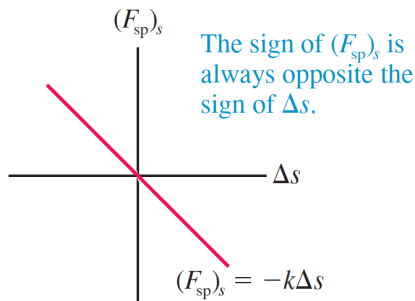
where  $(F_{sp})_s$  is the  $s$ -component of the restoring force, and  $k$  is the spring constant of the spring.

- The minus sign reminds you that it is a *restoring force*.

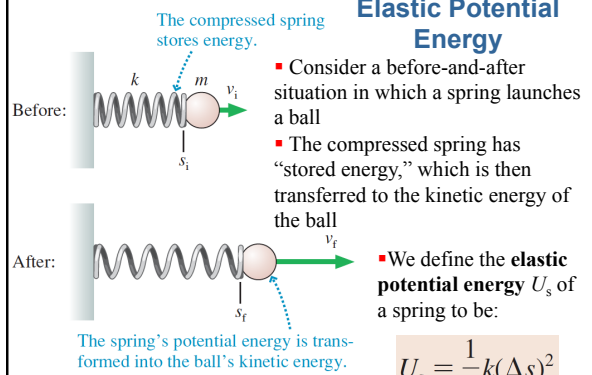
### Hooke's Law



### Hooke's Law



### Elastic Potential Energy



- Consider a before-and-after situation in which a spring launches a ball
- The compressed spring has "stored energy," which is then transferred to the kinetic energy of the ball

- We define the **elastic potential energy**  $U_s$  of a spring to be:

$$U_s = \frac{1}{2} k (\Delta s)^2$$



A spring-loaded gun shoots a plastic ball with a speed of 4 m/s. If the spring is compressed twice as far, the ball's speed will be

- A. 1 m/s.
- B. 2 m/s.
- C. 4 m/s.
- D. 8 m/s.
- E. 16 m/s.

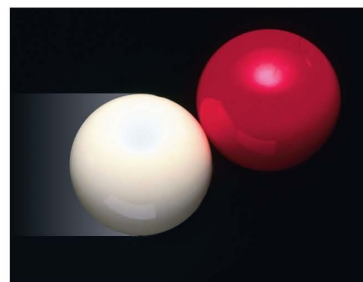
$$E_f = E_i$$

$$\frac{1}{2} m v_f^2 + \frac{1}{2} k x_f^2 = \frac{1}{2} m v_i^2 + \frac{1}{2} k x_i^2$$

$$\frac{1}{2} m v_f^2 + 0 = 0 + \frac{1}{2} k x_i^2$$

$$v_f = \sqrt{\frac{k}{m}} x_i$$

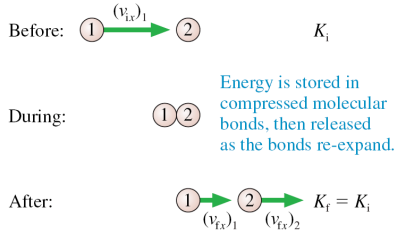
### Elastic Collisions



A perfectly elastic collision conserves both momentum and mechanical energy.

### 1D Elastic Collision when ball 2 is initially at rest.

Consider a head-on, perfectly elastic collision of a ball of mass  $m_1$  having initial velocity  $(v_{ix})_1$ , with a ball of mass  $m_2$  that is initially at rest.



The balls' velocities after the collision are  $(v_{fx})_1$  and  $(v_{fx})_2$ .

### 1D Elastic Collision when ball 2 is initially at rest.

momentum conservation:  $m_1(v_{ix})_1 + m_2(v_{ix})_2 = m_1(v_{ix})_1$

energy conservation:  $\frac{1}{2}m_1(v_{ix})_1^2 + \frac{1}{2}m_2(v_{ix})_2^2 = \frac{1}{2}m_1(v_{ix})_1^2$

There are two equations, and two unknowns:  $v_{fx1}$  and  $v_{fx2}$ . You can solve this. The solution is:

Eq. 10.43:

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \quad (\text{perfectly elastic collision with ball 2 initially at rest})$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

### Elastic Collision when ball 2 is initially at rest.

Eq. 10.43:

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \quad (\text{perfectly elastic collision with ball 2 initially at rest})$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

These equations come in especially handy, because you can always switch into an inertial reference frame in which ball 2 is initially at rest!

### Demonstration and Example

- A 0.50 kg basketball and a 0.05 kg tennis ball are stacked on top of each other, and then dropped from a height of 0.82 m above the floor.
- How high does the tennis ball bounce?
- Assume all perfectly elastic collisions.



### Demonstration and Example

- Divide motion into segments.
- Segment 1: free-fall of both balls from a height of  $h = 0.82$  m. Use conservation of energy:  $U_f + K_f = U_i + K_i$
- $0 + \frac{1}{2} m v_f^2 = mgh + 0$
- $v_f = [2gh]^{1/2} = -4.0$  m/s, for both balls.
- Segment 2: basketball bounces elastically, so its new velocity is +4.0 m/s.



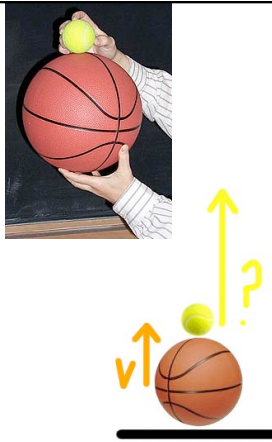
### Demonstration and Example

- Segment 3: A 0.50 kg basketball moving upward at 4.0 m/s strikes a 0.05 kg tennis ball, initially moving downward at 4.0 m/s.
- Their collision is perfectly elastic.
- What is the speed of each ball immediately after the collision?



### Demonstration and Example

- Segment 4: freefall of tennis ball on the way up.  $v_i = +10.5$  m/s.
- Use conservation of energy:  $U_f + K_f = U_i + K_i$
- $mgh + 0 = 0 + \frac{1}{2} m v_i^2$
- $h = v_i^2 / (2g) = 5.6$  m.
- So the balls were dropped from 0.82 m, but the tennis ball rebounds up to 5.6 m!  
(Assuming no energy losses.)



### Before Class 16 next Wednesday

- Note that U of T is closed on Monday and Tuesday for "Fall Reading Break"
- Remember MasteringPhysics.com Problem Set 7 on Chapter 10 is due Friday
- The Pre-Class Quiz on Chapter 11 is due by 10am on **Wednesday** of next week
- Something to think about:
- If one object does work on another object, does energy always get transferred from one object to the other?
- Have a great break!!

