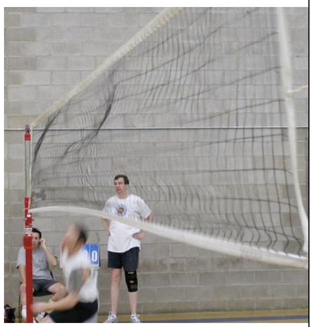
PHY131H1S - Class 20

Today:

- Oscillations, Repeating Motion
- Simple Harmonic Motion
- Oscillations follow a sine or cosine function



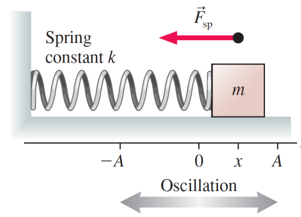
A little pre-class reading quiz on Ch.14

A mass on a spring forms an **oscillator**. Which of the following quantities in the description of **simple harmonic motion** is *not* determined by the initial position and velocity of the mass?

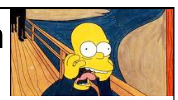
A. the amplitude, A

B. the phase constant, ϕ_0

C. the angular frequency, ω

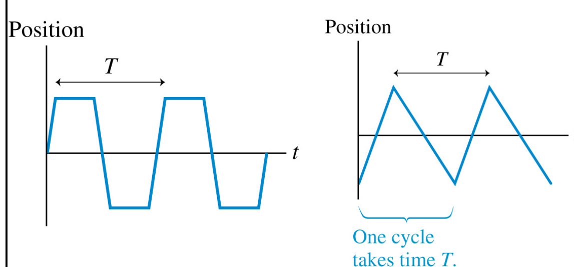


The Final Exam



- The final exam will be Dec. 15 2-4pm.
- The room depends on the first letters of your last name: A-CAR in EX310, CAS-FR in EX200, FU-O in EX200 and P-Z in EX100.
- If you have a time conflict you must contact the Examinations Office in SS1006
- As with your term tests, allowed aids include a calculator and **one** double-sided hand-written aid sheet prepared by you
- The 2 hour exam is comprehensive covering all aspects of the course evenly from Sep.12 to Dec. 7. Mainly it is chapters 1-15 of Knight (with some exclusions) and Error Analysis.

Some oscillations are ***not*** sinusoidal:



Period, frequency, angular frequency

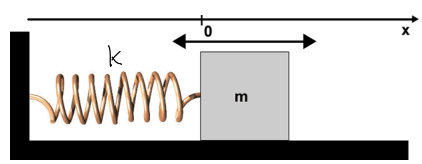
- For any kind of oscillation, the time to complete one full cycle, and return to the same condition, is called the **period**, T.
- The **frequency**, f, is the number of cycles per second. Frequency and period are related by:

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

- The frequency f is measured in cycles per second, or Hertz.
- We may also define an **angular frequency** ω in radians per second, to describe the oscillation:

$$\omega \text{ (in rad/s)} = \frac{2\pi}{T} = 2\pi f \text{ (in Hz)}$$

The Spring-Mass System



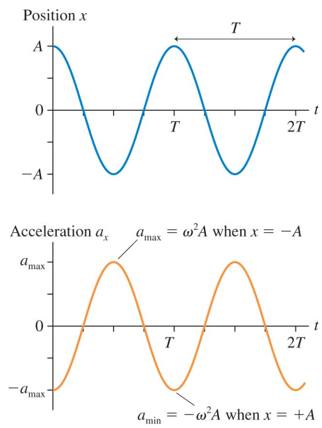
The force exerted on the mass by the spring:

$F = -k\,x$ (Hooke's Law)

$F = m\,a$ (Newton's Second Law)

Combine to form a differential equation:

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$



$$F_s = -kx$$

$$a = \frac{F_{Net}}{m}$$

$$a_x = -\frac{k}{m}x$$

Mass on a Spring: Summary of solution

Acceleration depends on x: $a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$

"Trial Solution" for x (guess): $x = A\cos(\omega t)$

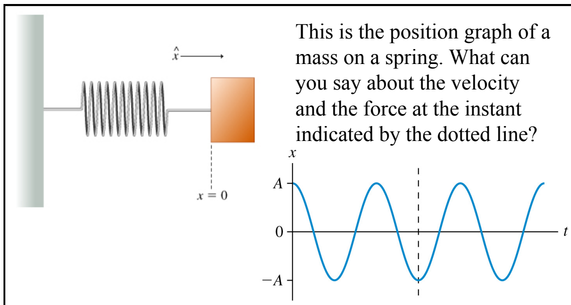
where A and ω are constants.

Velocity is dx/dt: $v = A[-\sin(\omega t)]\omega = -A\omega\sin(\omega t)$

Acceleration is dv/dt: $a = -A\omega\cos(\omega t)\omega = -A\omega^2\cos(\omega t)$

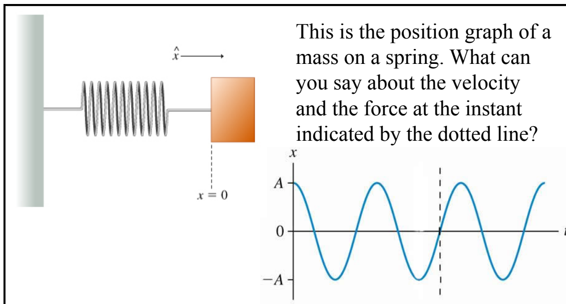
This is just $a = -\omega^2 x$, which is our original equation for acceleration if: $\omega^2 = \frac{k}{m}$

A is arbitrary: in practice it is set by initial conditions.



This is the position graph of a mass on a spring. What can you say about the velocity and the force at the instant indicated by the dotted line?

A. Velocity is positive; force is zero.
B. Velocity is negative; force is zero.
C. Velocity is negative; force is to the right.
D. Velocity is zero; force is to the right.
E. Velocity is zero; force is to the left.



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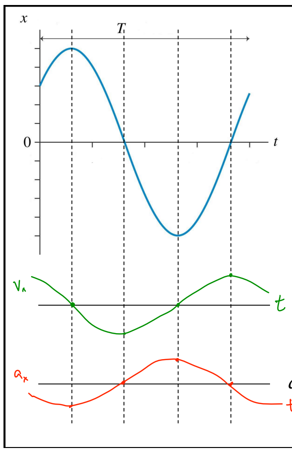
Simple Harmonic Motion with a Phase Constant

- If the initial position of an object in S.H.M. is not A, then we may still use the cosine function by introducing a **phase constant** ϕ_0 measured in radians.
- In this case, the primary kinematic equations of S.H.M. are:

$x = A\cos(\omega t + \phi_0)$	$x_{max} = A$
$v_x = -v_{max}\sin(\omega t + \phi_0)$	$v_{max} = A\omega$
$a_x = -a_{max}\cos(\omega t + \phi_0)$	$a_{max} = A\omega^2$

Simple Harmonic Motion (SHM)



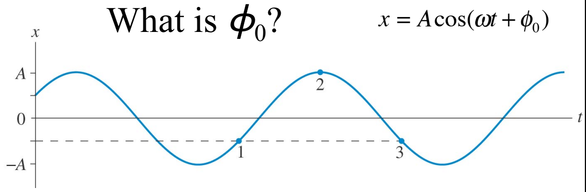
$$x = A\cos(\omega t + \phi_0)$$

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi_0)$$

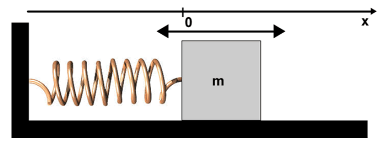
$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi_0)$$

What is ϕ_0?

$x = A\cos(\omega t + \phi_0)$



A. $-\pi/3$
B. $+\pi/3$
C. $-2\pi/3$
D. $+2\pi/3$
E. 0



A mass is oscillating on a spring in S.H.M. When it passes through its equilibrium point, an external "kick" suddenly decreases its speed, but then it continues to oscillate. As a result of this slowing, the frequency of the oscillation

A. goes up
B. goes down
C. stays the same

S.H.M. notes.

- The frequency, f, is set by the properties of the system. In the case of a mass m attached to a spring of spring-constant k, the frequency is always

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- A and ϕ_0 are set by the initial conditions: x_0 (initial position) and v_0 (initial velocity).
- A turns out to be related to the total energy of the spring oscillator system: $E = \frac{1}{2} k A^2$.

Before Class 21 on Monday

- Please finish reading Knight Chapter 14

- Something to think about: A mass hanging from a string is swinging back and forth with a period of 2 seconds.
- What is the period if the mass is doubled?
- What is the period if the length of the string is doubled?