

Last Time I asked:

- The two identical beakers shown are filled to the same height with water. Beaker $B$ has a plastic sphere floating in it.
A. Beaker A with all its contents weighs more than Beaker B with all its contents
B. Beaker $B$ with all its contents weighs more than Beaker A with all its contents
C. Beaker A with all its contents weighs the same as Beaker B with all its contents


Ch. 15 Reading Quiz
The buoyant force on an object submerged in a liquid depends on:
A. the object's mass.
B. the object's volume.
C. the density of the liquid.
D. both A and B .
E. both $B$ and $C$.
C. Beaker $A$ with all its contents weighs the same as Beaker $B$ with all its contents

- Beaker B will have $V_{\mathrm{f}}$ less water than beaker A , where $V_{\mathrm{f}}$ is the amount of water displaced by the ball. So the water will weigh $V_{\mathrm{f}} \rho_{\mathrm{w}} g$ less.
- The ball is in equilibrium, so the buoyancy force equals its weight. The buoyancy force is $V_{\mathrm{f}} \rho_{\mathrm{w}} g$.
- The ball weighs exactly the same as the displaced water, so both beakers + contents weigh the same.



## Pascal's Law for liquids

- Consider a small element of fluid in a beaker.
- Pressure acts inward on all surfaces of the small element.
- Gravity pulls it downward.
- To balance the force of gravity, the upward pressure on the bottom surface must be greater than the downward pressure on the top surface: "buoyancy"
- $\quad p_{2}-p_{1}=-\rho g\left(y_{2}-y_{1}\right)$
- This is the equation for the pressure of an incompressible fluid in hydrodynamic equilibrium in a gravitational field.
- Pressure increases with depth! Scuba divers know this!


## Buoyancy: Archimedes Principle

- Let's do a "thought experiment" (Gedanken).
- Imagine a beaker with a fluid and a block, $B$, hanging near it.
- There is a fluid element $F$ with the same shape and volume as the block $B$.
- The fluid element $F$ is in mechanical equilibrium:

$$
\text { - } F_{\text {up }}-F_{\text {down }}-W_{F}=0
$$

- where $F_{\text {up }}$ is the pressure force on the
 bottom surface, $F_{\text {down }}$ is the pressure force on the top surface, and $W_{F}$ is the weight of fluid $F$.


## Buoyancy: Archimedes Principle

- Step 1: Remove F from the beaker and place it in a small container, leaving an empty bubble of the same size in the beaker.
- The bubble is not in mechanical equilibrium, since its weight is much less than that of the removed fluid, but the pressure forces are the same.:

$$
\text { - } F_{\mathrm{up}}-F_{\mathrm{down}}=W_{F}>0
$$

- where $F_{\text {up }}$ is the pressure force on the bottom surface, $F_{\text {down }}$ is the pressure force on the top surface, and $W_{F}$ is the
 weight of the removed fluid F .


## Buoyancy: Archimedes Principle

- Step 2: Block B, with weight $W_{B}$, is placed in the bubble.
- There is a net force on Block B:
- $F_{\text {net }}=F_{\text {up }}-F_{\text {down }}-W_{B}=W_{F}-W_{B}$
- where $W_{F}$ is the weight of the removed fluid F , and $W_{B}$ is the weight of the block B.
- This is equal to the force of gravity, $-W_{B}$, plus a new force called "Buoyancy", which is due to the pressure gradient in the
 fluid.

Archimedes' principle: When an object is immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid displaced by the object.

If a very large floating iceberg were to melt, what, in principle, would happen to the overall sealevel?
A. sea-level would rise very slightly
B. sea-level would fall very slightly. C. sea-level would stay the same.


## Iceberg Reasoning:

- When an object is immersed in a fluid, the fluid exerts an upward force on the object equal to the weight of the fluid displaced by the object.
- Iceberg not melted has weight $W$ and volume $V_{1}$. It displaces a weight of water $W$ with volume $V_{w}$ which is less than $V_{1}$. So it floats: some of the iceberg sticks up above the water.
- Iceberg melted has the same weight $W$ and less volume. But it still displaces the same amount of water. It displaces a weight of water $W$.
- So melting an iceberg which is floating does not change sea level.
- If the iceberg were not floating, but sitting on a land-mass, and it melted and added water to the ocean, this would increase the sea-level.


## The Ideal Fluid Model

- The fluid is incompressible. It does not change its volume with pressure.
- The fluid is nonviscous. It slips along the walls of the container with zero friction.
- The flow is steady. The flow follows smooth lines which do not cross or twist.

- A tube widens from a cross-sectional area $A_{1}$ to a cross sectional area $A_{2}=4 A_{1}$. As a result the speed of an ideal dynamic fluid in the tube changes from $v_{1}$ to
A. $v_{2}=v_{1} / 16$
B. $v_{2}=v_{1} / 4$
C. $v_{2}=v_{1}$
D. $v_{2}=4 v_{1}$
E. $v_{2}=16 v_{1}$



## Bernoulli's Law

- Consider an ideal fluid, flowing through a tube which narrows.

- It increases its velocity. This means the kinetic energy per volume of the fluid will increase.
- How can this be? There must be a force which does work on the fluid to speed it up.
- The force must come from a pressure difference.
- Pressure must be lower in the region of increased fluid velocity.


## Fluid Dynamics

Comparing two points in a flow tube of cross section $A_{1}$ and $A_{2}$, we may use the equation of continuity:

$$
v_{1} A_{1}=v_{2} A_{2}
$$

where $v_{1}$ and $v_{2}$ are the fluid speeds at the two points.
The flow is faster in narrower parts of a flow tube, slower in wider parts.
This is because the volume flow rate $Q$, in $\mathrm{m}^{3} / \mathrm{s}$, is constant.

$$
Q=\frac{\Delta V}{\Delta t}=A \cdot|v|
$$

- We study the steady flow of water from a water tap, e.g., in your kitchen sink. The jet of water
A. broadens as it falls.
B. narrows as it falls.
C. does not change its cross-sectional shape.
D. slows before hitting the bottom of the sink.



## Bernoulli's Law

- Bernoulli's law is an expression of the conservation of energy for a closed system.
- It states that an increase in the speed of an ideal dynamic fluid is accompanied by a drop in its pressure.
- When combined with Pascal's law for pressure drop with height in a fluid, it is written:

$$
p_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=p_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2}
$$

or:

$$
p+\frac{1}{2} \rho v^{2}+\rho g y=\text { constant }
$$

## Bernoulli's Example

- A cylinder with water of height $\mathbf{h}$ has a small hole cut in the side at height $\mathbf{z}$. The water strikes the ground at $\mathbf{x}$. The figure shows the streamline from the top of the water at $\mathbf{A}$ to just outside the hole $\mathbf{B}$.
- If the hole is small, it is reasonable to approximate that the speed of the water at $\mathbf{A}$ is zero. Since point $A$ and $B$ are in contact with the outside air, it is reasonable to approximate that the pressure is the same at point $A$ and $B$, that of atmospheric pressure in the room.
- Find $\mathbf{x}$.

that's all the new material for PHY131...
- Final Exam is Thursday, Dec. 15, 2:00 PM in EX building
- The 2 hour final exam will cover the entire course, including all of the assigned reading, error analysis assignment, plus Practicals materials and what was discussed in class
- Approximately even spread over Knight Chs. 1-15 minus the exclusions
- I recommend you are familiar with all Masteringphysics homework and Practicals work you did.
- Please email me ( jharlow @ physics.utoronto.ca ) with any questions or review suggestions for Wednesday.

