## PHY131H1F

Class 3
Today: Error Analysis

- Significant Figures
- Unit Conversion
- Normal Distribution
- Standard Deviation
- Reading Error
- Propagation of Errors
- Error in the Mean



## Clicker Question 1

What is the approximate density of water?
A. $10^{-5} \mathrm{~kg} / \mathrm{m}^{3}$
B. $0.01 \mathrm{~kg} / \mathrm{m}^{3}$
C. $0.1 \mathrm{~kg} / \mathrm{m}^{3}$
D. $1 \mathrm{~kg} / \mathrm{m}^{3}$
E. $1000 \mathrm{~kg} / \mathrm{m}^{3}$

## Class 3 Preclass Quiz on MasteringPhysics

- This was due this morning at 8:00am
- 864 students submitted the quiz on time
- $69 \%$ correct: If a digital thermometer reads $12.8^{\circ} \mathrm{C}$ and the error specified in the manual is "half the last digit", in this case it would be $\pm 0.05^{\circ} \mathrm{C}$.
" 78\% correct: In the physical sciences, an "error" is the number to the right of the $\pm$ symbol, also known as an uncertainty.
- $80 \%$ correct: In an equation for the normal distribution (Gaussian) sigma is related to the width of the distribution.
" $97 \%$ correct: "Mean" is the same as "average".


## Class 3 Preclass Quiz on MasteringPhysics

- Some common student comments/feedback:
- "Everything seems fine except for the terms and the formulas that gets me dizzy. I hope I wouldn't need to memorize complicated looking formulas in the near future."
- propagation of errors of precision
- Standard deviation
- "I want to know if there are specidic problem sets that i can practice from in the book without doing every single question at the end of each chapter?"
- "I am wondering if anyone would click the 'song by Taylor Swift' option for kicks, though."

From Knight Chapter 1:
FIGURE 1.25 Determining significant figures.


## Example Problem - Rounding

- A cart begins at rest, and accelerates down a ramp with acceleration: $a=0.518 \mathrm{~m} / \mathrm{s}^{2}$.

- After 3.2 s , how far has it traveled?
- Use $\underline{d}=1 / 2 a t^{2}$.
$d=0.5(0.518)(10.24)$
$d=2.65216 \mathrm{~m}$ $d=$
- A cart begins at rest, and accelerates down a $\zeta^{2} s y$ ramp with acceleration: $a=0.518 \mathrm{~m} / \mathrm{s}^{2}$. After 3.2 s . fig how far has it traveled? $d=\left(\frac{1}{2}\right) a t^{2}$ gives the result 2.65216 on your calculator. How should you best report this answer in your "Final Answer" box?
A. 2.65216 m
B. 2.65 m
C. 2.60 m
D. 2.6 m
E. 2.7 m


## Example Problem - Rounding Early

- A cart begins at rest, and accelerates down a ramp with acceleration: $a=0.518 \mathrm{~m} / \mathrm{s}^{2}$.
- After 3.2 s , how far has it traveled?

$$
t=3.2 \mathrm{~s}
$$

- Use $d=1 / 2 a t^{2}$.

$$
t^{2}=10.24 \mathrm{~s}^{2}
$$

$$
\begin{aligned}
d & =0.5(0.518)(10) \\
& =2.59 \mathrm{~m} \\
d & =2.6 \mathrm{~m} \longleftarrow \text { This is wrong }
\end{aligned}
$$

## An important skill: Unit Conversion

- Example
- Knight Ch. 1 Problem 24d (page 30):

Convert $14 \mathrm{in}^{2}$ to SI units.

- Known:
- 1 in $=2.54 \mathrm{~cm}$
- $100 \mathrm{~cm}=1 \mathrm{~m}$
$\rightarrow 1=\left(\frac{1 \mathrm{in}}{2.54 \mathrm{~cm}}\right)=\left(\frac{2.54 \mathrm{~cm}}{1 \mathrm{in}}\right)$
$1=\left(\frac{100 \mathrm{~cm}}{1 \mathrm{~m}}\right)=\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)$
- The SI unit of area is $\mathrm{m}^{2}$.


## When do I round?

- The final answer of a problem should be displayed to the correct number of significant figures
- Numbers in intermediate calculations should not be rounded off
- It's best to keep lots of digits in the calculations to avoid round-off error, which can compound if there are several steps

$$
\begin{aligned}
& 1=\left(\frac{1 \mathrm{in}}{2.54 \mathrm{~cm}}\right)=\left(\frac{2.54 \mathrm{~cm}}{1 \mathrm{in}}\right) \\
& \text { Convert } 14 \mathrm{in}^{2} \text { to } \mathrm{m}^{2} . \\
& 1=\left(\frac{100 \mathrm{~cm}}{1 \mathrm{~m}}\right)=\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right) \\
& A=14 \text { n }^{k}\left(\frac{2.54 \mathrm{~cm}}{1 \mathrm{k}}\right)^{2}\left(\frac{1 \mathrm{~m}}{100 \mathrm{ck}}\right)^{2} \\
&= \frac{14 \cdot 2.54^{2}}{100^{2}}=0.00903 \mathrm{~m}^{2} \\
& A=9.0 \times 10^{-3} \mathrm{~m}^{2}
\end{aligned}
$$



I have been informed that a clipboard was being passed around in Convocation Hall last week with a "sign-in sheet".

I will never authorize a clipboard to be passed around this room.

In the future, if you see something like this being passed around, please place it on the floor and refuse to pass it on.


## Real 2-hour Practicals begin this week!

- All you need is a calculator, your textbook and something to write with.
- Dr. Meyertholen has posted a good 6 minute video with answers to lots of FAQs about Practicals at:
- http://youtu.be/DVsLJgxJVgo


Demo and Example:

What is the Period of a Swinging Ball?
(Pendulum)

- Procedure: Measure the time for 5 oscillations, $t_{5}$.
- The period is calculated as $T=t_{5} / 5$.



## Last Wednesday I asked at the end of class:

- If your height is 150 cm , is there an error in that number?
- ANSWER: YES! Almost every measured number has an error, even if it is not stated. If you told me your height was 150 cm , I would guess the error is probably between 1 and 5 cm . [But there is no way to know this, unless you investigate how the 150 was measured.]

Here were Harlow's measurements of $\mathrm{t}_{5}$ : 7.53 s
7.38 s Which of the following might be a 7.47 s good estimate for the error in $7.43 \mathrm{~s} \quad$ Harlow's first measurement of 7.53 seconds?
A. 0.005 s
B. 0.05 s
C. 0.5 s
D. 5 s
E. Impossible to determine




The Gaussian: $\quad N(x)=A e^{\frac{-(x-\bar{x})^{2}}{2 \sigma^{2}}}$


Gaussian Distributions turn up everywhere!
Heights of some People
(London, 1886)


Propagation of Errors

- Rule \#1 (sum or difference rule):
- If $z=\underline{x}^{\text {ecrror }}+\underline{y}^{\leftarrow-\text { error }} \Delta x$ is $\Delta y$
- or $z=x-y$
- then $\Delta z=\sqrt{\Delta x^{2}+\Delta y^{2}}$

- Rule \#2 (product or division rule):
- If $z=x y$
- or $z=x / y$ then $\frac{\Delta z}{z}=\sqrt{\left(\frac{\Delta x}{x}\right)^{2}+\left(\frac{\Delta y}{y}\right)^{2}}$

Where does an object end up, if it takes N steps randomly left or right?

The final distribution is described by a Gaussian function!


- Rule \#3 (exponent rule):
- If $z=x^{\mathrm{n}}$
- then $\frac{\Delta z}{z}=n \frac{\Delta x}{x}$


## Propagation of Errors

> | - Rule \#2.1 (multiply by exact constant rule): |
| :--- |
| - If $z=x y$ |
| - and $x$ is an exact number, so that $\Delta x=0$ |
| - then $\Delta z=\|x\|(\Delta y) \leftarrow \begin{array}{l}\text { important in } \\ \text { unit conversion }\end{array}$ |

$\square$

## The $t_{5}$ data

- What is the error in the mean of $N$ measurements with the same error $\Delta x$ ?

$$
\begin{aligned}
& \text { Mean: } \bar{x}=\frac{\left(x_{1}+x_{2}+\cdots+x_{N}\right)}{N}=\frac{x_{\text {sum }}}{N} \\
& \text { Rule\#1 } \text { to find error in sum: } \\
& \Delta x_{\text {sum }}=\sqrt{\Delta x_{1}^{2}+\Delta x_{2}^{2}+\cdots+\Delta x_{N}^{2}}=\sqrt{\Delta x^{2}+\Delta x^{2}+\cdots+\Delta x^{2}} \\
& \Delta x_{\text {sum }}=\sqrt{N \cdot x^{2}}=\sqrt{N} \Delta x \\
& \text { Rule\#2.1 } \text {, divide by exact constant: } \\
& \Delta \bar{x}=\frac{\Delta x_{\text {sum }}}{N}=\frac{\sqrt{N} \Delta x}{N}\left(\frac{\sqrt{N}}{\sqrt{N}}\right)=\frac{\sqrt{N \sqrt{N} \Delta x}}{N \sqrt{N}}=\frac{\infty \Delta x}{x \sqrt{N}} \\
& \Delta \bar{x}=\frac{\Delta x}{\sqrt{N}} \leftarrow \text { Ersor in mean }
\end{aligned}
$$

## The Error in the Mean

- Many individual, independent measurements are repeated $N$ times
- Each individual measurement has the same error $\Delta x$
- Using error propagation you can show that the error in the estimated mean is:

$$
\Delta \bar{x}_{\text {est }}=\frac{\Delta x}{\sqrt{N}}
$$

## Significant Figures

- Discussed in Section 1.8 of Knight Ch. 1
- The rules for significant figures when errors are involved are:


## 1.Errors should be specified to one or two significant figures.

2. The most precise column in the number for the error should also be the most precise column in the number for the value.

You wish to know the time it takes to travel from Finch station to Yonge/Bloor by subway. You ask 10 people to take a stopwatch and time the trip. After analyzing all the data you find that it takes an average of 26 minutes and 40 seconds, with an error in this average of $\pm 100$ seconds.

- If you expand your survey and ask 1000 people to time the trip, when you analyze the data, what would you expect to be the error in the average time?
A. 100 seconds
B. 50 seconds
C. 10 seconds
D. 1 second
E. 0.05 seconds



## Clicker Question 6

- Example: If a calculated result is $\mathrm{d}=7.056$ +/- 0.705 m , how should you report this?
A. $7.1 \pm 0.7 \mathrm{~m}$
B. $7.06 \pm 0.71 \mathrm{~m}$
C. $7.056 \pm 0.705 \mathrm{~m}$
D. Any of the above
E. Either A or B, but not C


## Before Class 4 on Wednesday

- Please read Chapter 2, Sections 2.1-2.4 of Knight (or at least watch the pre-class video for Class 4)
- Please do the pre-class quiz before Wednesday at 8:00am.
- Don't forget to start the Problem Set due Sunday - there's no time limit, just a deadline
- Something to think about: Does constant velocity imply constant acceleration? Does constant acceleration imply constant velocity?

