Harlow- Solutions.

Possibly helpful information for this test:
$\pi = 3.14159$ is the ratio of the circumference to the diameter of a circle
$g = 9.80$ m/s^2 is the acceleration due to gravity near the Earth's surface.
1 minute = 60 seconds; 60 minutes = 1 hour

The quadratic equation: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Air resistance may be neglected in all questions, unless otherwise stated.

MULTIPLE CHOICE (16 points total)

1. You wish to find the floor area of a rectangular room. You measure the length to be 6.00 ± 0.50 m, and the width to be 3.50 ± 0.30 m. The area is closest to $A = l \times w = 6 \times 3.5 = 21\ m^2$
   A. 21.00 ± 0.58 m^2
   B. 21.0 ± 2.5 m^2
   C. 21.0 ± 1.8 m^2
   D. 21.0 ± 3.6 m^2
   E. 21 ± 12 m^2

Rule #2: $\Delta A = A\sqrt{\left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta w}{w}\right)^2}$

$$\Delta A = 21\sqrt{\left(\frac{0.5}{6}\right)^2 + \left(\frac{0.3}{3.5}\right)^2} = 2.5\ m^2$$

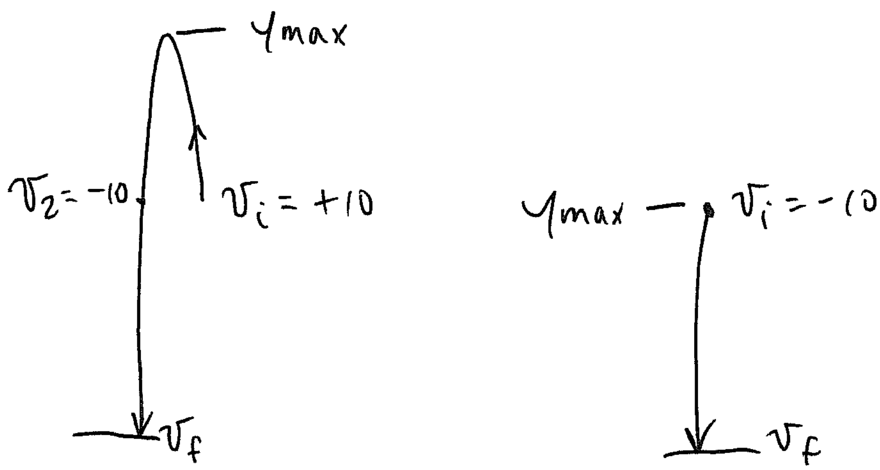
2. Home Depot sells carpet for \$1.12 per square foot. [Note that there are 12 inches in 1 foot, and 1 inch equals 2.54 cm.] You wish to cover the floor of a room with carpet. The room is 3.5 m wide and 6.0 m long. The cost will be closest to
   A. \$2.50
   B. \$25
   C. \$250
   D. \$2,500
   E. \$25,000

cost = Area × rate, rate = $1.12 \frac{\text{dollars}}{ft^2}$

$$\text{Cost} = (3.5 \times 6)\,m^2 \left(\frac{100\ cm}{1\ m}\right)^2 \left(\frac{1\ in}{2.54\ cm}\right)^2 \left(\frac{1\ ft}{12\ in}\right)^2 \cdot 1.12 \frac{\text{dollars}}{ft^2}$$

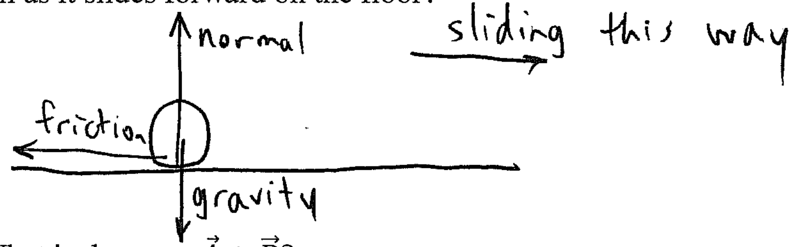
$$= \frac{3.5 \times 6 \times 100^2 \times 1.12}{2.54^2 \times 12^2} = 253 \text{ dollars}$$

3. You throw a ball vertically from the top of an 80 m tower with an initial upward velocity of 10 m/s. The ball reaches a maximum height y_{max}, and spends a total amount of time t_{flight} in the air. Just before it reaches the ground, its speed is v_f. Next, you throw the same ball downward with an initial velocity of 10 m/s. Compared to the previous throw, what will change or be the same?
   A. y_{max} will be less, t_{flight} will be less, and v_f will be greater
   B. y_{max} will be less, t_{flight} will be less, and v_f will be the same
   C. y_{max} will be less, t_{flight} will be the same, and v_f will be the same
   D. y_{max} will be the same, t_{flight} will be less, and v_f will be greater
   E. y_{max} will be less, t_{flight} will be the same, and v_f will be greater



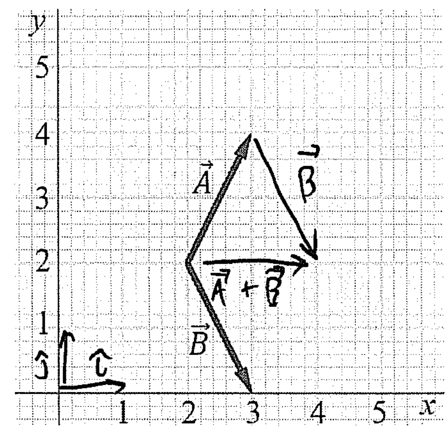
4. A bowler throws a bowling ball which slides in a forward direction along a flat, horizontal wooden floor. Consider the following forces which may or may not be acting on the ball as its slides forward on the floor:
   1. A downward force of gravity
   2. A forward force from the throw × No → no contact
   3. An upward force from the floor
   4. A backward frictional force from the floor

   Which of the choices below lists only forces acting on the ball as it slides forward on the floor, and no forces that are not acting on the ball as it slides forward on the floor?
   A. 1, 2
   B. 1, 2, 3
   C. 1, 2, 3, 4
   D. 1, 3
   E. 1, 3, 4



5. Two vectors lie in the x-y plane, as shown. What is the sum, $\vec{A} + \vec{B}$?

   A. $2\hat{\imath}$
   B. $2\sqrt{5}\hat{\imath}$
   C. $2\hat{\jmath}$
   D. $2\hat{\imath} + 4\hat{\jmath}$
   E. $4\hat{\imath} + 2\hat{\jmath}$



6. You measure the basal heart rate of five test-subjects to be: 65, 56, 73, 78, and 60, all in beats per minute (bpm). The error in the mean of these five measurements is closest to
   A. 1.8 bpm
   B. 3.0 bpm
   C. 4.1 bpm
   D. 9.1 bpm
   E. 66 bpm

Mean = 66.4 bpm

$\sigma = 9.07$ bpm

Error in Mean = $\frac{\sigma}{\sqrt{N}} = \frac{9.07}{\sqrt{5}} = 4.06$ bpm

7. A centrifuge used for separating plasma from blood operates at a maximum rotation speed of 5000 rotations per minute (rpm). It starts at rest, and has a constant angular acceleration for 2 seconds before it reaches the maximum rotation speed. The number of complete rotations the centrifuge makes during this time is closest to
   A. 80
   B. 300
   C. 3000
   D. 5000
   E. 300,000

Known: $\omega_0 = 0$

$\omega_f = 5000 \ \frac{\text{rot}}{\text{min}}$

$t = 2\,s$

Need $\Delta\theta$

Don't care about α.

Use: $\Delta\theta = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{5000+0}{2}\right)\left(\frac{2}{60}\right)\text{min}$

$\Delta\theta = 83$ rotations.

8. At time $t = 0$, small red marble is released from rest at the top of a smooth, frictionless incline that is at an angle θ relative to the horizontal. The red marble begins rolling down the incline. A short time later, when $t = T$, a blue marble is released from rest at the top of the same incline, and begins to roll in the same direction as the red marble. At a time $t = 2T$, what is the speed of the red marble relative to the blue marble?

A. $Tg\sin(\theta)$

B. $2Tg\sin(\theta)$

C. $\frac{1}{2}T^2g\sin(\theta)$

D. $\frac{3}{2}T^2g\sin(\theta)$

E. zero

$a = g\sin\theta$

red: $v_r = g\sin\theta\, t$

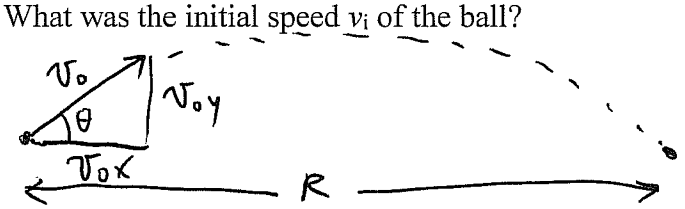
blue: $v_b = g\sin\theta\,(t - T)$

$v_r - v_b = g\sin\theta\,(t - t + T)$

$= Tg\sin\theta$

FREE-FORM PART (16 points total)
Clearly show your reasoning and work as some part marks may be awarded. Write your final answers in the boxes provided.

PART A [5 points]
Tennis ball launcher "A" fires a ball so that the initial velocity has an angle of 25° above the horizontal. The ball returns to the same height it was launched at after traveling a horizontal distance of 23 m. What was the initial speed v_i of the ball?



$$v_{0y} = v_0 \sin\theta$$
$$v_x = v_0 \cos\theta = \text{constant}$$
$$v_x = \frac{R}{t} \Rightarrow \boxed{R = v_x t = v_0 \cos\theta\, t}$$

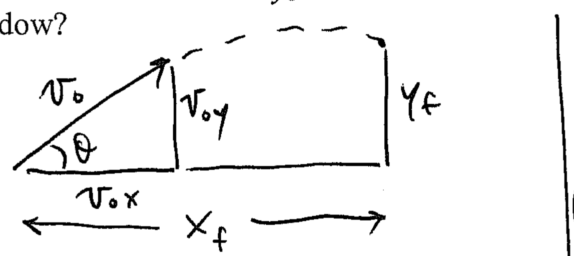
After a time t, the y-component of the velocity will be negative of its initial value (by symmetry)

$$v_{fy} = -v_0 \sin\theta$$
Use: $v_{fy} = v_{0y} - gt$
$$-v_0 \sin\theta = v_0 \sin\theta - gt$$
$$\Rightarrow t = \frac{2v_0}{g}\sin\theta$$
$$R = v_0 \cos\theta \left[\frac{2v_0}{g}\sin\theta\right]$$
$$\Rightarrow v_0 = \sqrt{\frac{gR}{2\sin\theta\cos\theta}} = \sqrt{\frac{9.8(23)}{2\sin 25 \cos 25}}$$
$$= 17.1 \text{ m/s}$$

$v_i =$ 17 m/s

← 2 sig figs

PART B [5 points]
Tennis ball launcher "B" fires a ball with an initial velocity of 25 m/s, at an angle of 35° above the horizontal. The ball travels a horizontal distance of 12 m, then passes through the window of a building. What is the vertical distance y_f of the ball above its initial height at the moment when it passes through the window?



$$v_x = v_0 \cos\theta = \text{constant} = \frac{x_f}{t}$$
$$\Rightarrow x_f = v_x t = v_0 \cos\theta\, t$$
$$\Rightarrow \boxed{t = \frac{x_f}{v_0 \cos\theta}}$$

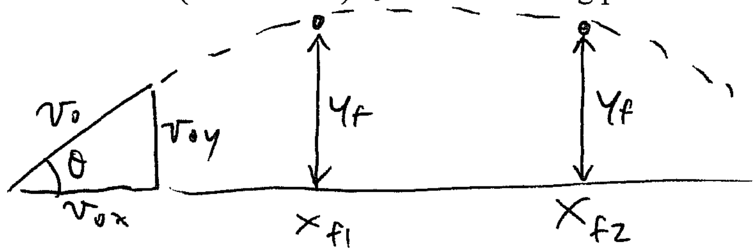
$$y_f = y_i + v_{0y} t + \tfrac{1}{2} a_y t^2$$
$$y_f = 0 + v_0 \sin\theta\, t - \tfrac{1}{2} g t^2$$
$$y_f = \cancel{v_0} \sin\theta \left(\frac{x_f}{\cancel{v_0} \cos\theta}\right) - \frac{1}{2} g \left(\frac{x_f}{v_0 \cos\theta}\right)^2$$
$$y_f = x_f \tan\theta - \frac{g x_f^2}{2 v_0^2 \cos^2\theta} \quad (*)$$
$$= 12 \tan 35 - \frac{9.8(12)^2}{2(25)^2(\cos 35)^2}$$
$$= 8.402 - 1.682 = 6.72 \text{ m}$$

$y_f =$ 6.7 m

← 2 sig figs

PART C [6 points]

Tennis ball launcher "B" fires a ball with an initial velocity of 25 m/s, at an angle of 35° above the horizontal. At what horizontal distance (or distances) x_f from its starting point will the ball be 10.0 m above its initial height?



In Part B I derived (*):

$$y_f = x_f \tan\theta - \frac{g\,x_f^2}{2 v_0^2 \cos^2\theta}$$

Rearrange this as a quadratic equation in x_f:

$$\underbrace{\frac{g}{2v_0^2\cos^2\theta}}_{\equiv a} x_f^2 \underbrace{- \tan\theta}_{\equiv b}\, x_f + \underbrace{y_f}_{\equiv c} = 0$$

Quadradic formula is $x_f = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x_f = \left[\tan\theta \pm \sqrt{\tan^2\theta - \frac{2 g y_f}{v_0^2 \cos^2\theta}}\right] \cdot \frac{v_0^2 \cos^2\theta}{g}$$

$$= \left[\tan 35 \pm \sqrt{(\tan 35)^2 - \frac{2(9.8)(10)}{25^2 (\cos 35)^2}}\right] \cdot \frac{25^2 (\cos 35)^2}{9.8}$$

$$= \left[0.70021 \pm 0.15144\right] 42.794$$

$x_{f1} = 23.48$ m , $x_{f2} = 36.45$ m

$x_f =$ 23 m or 36 m

2 sig figs