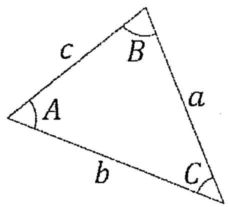
Solutions. by Jason Harlow PHY131

Test 1, Fall 2015

$\boxed{\text{Version A}}$

Possibly helpful information for this test:
$\pi = 3.14159$ is the ratio of the circumference to the diameter of a circle
$g = 9.80$ m/s^2 is the acceleration due to gravity near the Earth's surface.

Common Prefixes:
k = "kilo-" = 10^3 c = "centi-" = 10^{-2} m = "milli-" = 10^{-3} µ = "micro-" = 10^{-6}
n = "nano-" = 10^{-9}



Sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos(A)$

Air resistance may be neglected in all questions, unless otherwise stated.

MULTIPLE CHOICE PART (20 points total)
Choose the best answer for each question. 2 points per question, no penalty for guessing.

Question 1
A Canadian \$1 coin (called a "loonie") has a mass of 7 g, a diameter of 3 cm, and a thickness of 2 mm. Approximately how many loonies can you fit into an empty peanut butter jar? (Assume that the peanut butter jar originally held 1 kg of peanut butter, and that peanut butter has a density of 1000 kg/m^3. Please choose the closest answer.)

A. 50
B. 500
C. 5000
D. 50,000
E. 500,000

$\rho = \frac{m}{V}$

Volume of jar $= \frac{m}{\rho} = \frac{1\text{ kg}}{1000\text{ kg/m}^3} = 10^{-3}\text{ m}^3$

Volume of loonie $\approx (3\times10^{-2})(3\times10^{-2})(2\times10^{-3}) = 2\times10^{-6}\text{ m}^3$

Number of coins $\approx \frac{V_{jar}}{V_{coin}} = \frac{10^{-3}}{2\times10^{-6}} = 500$

Question 2
Two identical objects A and B fall from rest from different heights to the ground and feel no appreciable air resistance. If object B takes twice as long as object A to reach the ground, what is the ratio of the heights from which A and B fell?

A. $h_A/h_B = \sqrt{2}$
B. $h_A/h_B = 2$
C. $h_A/h_B = 1/2$
D. $h_A/h_B = 1/\sqrt{2}$
E. $h_A/h_B = 1/4$

$h_A = \frac{1}{2}g t_A^2$ $h_B = \frac{1}{2} g t_B^2$ $t_B = 2t_A$

$\frac{h_A}{h_B} = \frac{t_A^2}{t_B^2} = \frac{t_A^2}{(2t_A)^2} = \frac{1}{4}$

Question 3
Your friend is sitting directly above you on a tree branch. You wish to throw an apple up to her. The upward distance the apple must travel between your hand and your friend's hand is 3.5 m. What is the minimum initial speed that you must throw the apple so that it reaches her?

A. 5.9 m/s
B. 8.3 m/s
C. 11 m/s
D. 34 m/s
E. 69 m/s

$v = 0$ $\Delta y = 3.5\text{ m}$
$v_0 = ?$ $a_y = -9.8\text{ m/s}^2$

$v^2 = v_0^2 + 2a_y\Delta y$

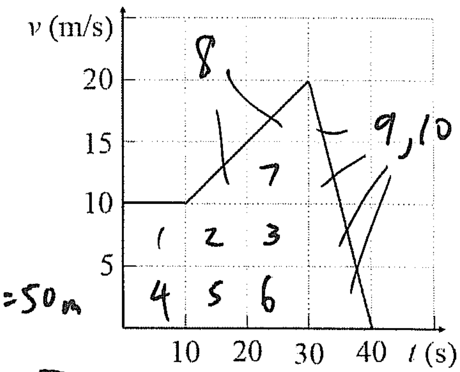
$v_0 = \pm\sqrt{-2(-9.8)(3.5)}$

$v_0 = \pm 8.2825\text{ m/s}$

choose positive.

Question 4

The velocity vs time graph represents the motion of a car along a straight line. What was the displacement of the car between $t = 0$ s and $t = 40$ s?



$\Delta x = \int_0^{40} v\,dt$

A. 350 m
B. 400 m
C. 500 m
D. 550 m
E. 800 m

Count 10 squares.
each square has area $5 \times 10 = 50$ m

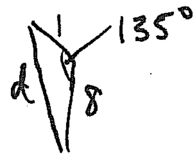
$\Delta x = 10 \times 50 = 500$ m

Question 5

A rabbit trying to escape a fox runs north for 8.0 m, then runs in a direction 45° west of north for 1.0 m, then drops 1.0 m down a hole into its burrow. What is the magnitude of the net displacement of the rabbit away from her starting point?

A. 7.1 m
B. 7.7 m
C. 8.1 m
D. 8.8 m
E. 10 m

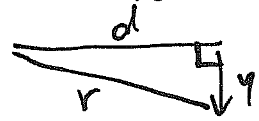
Top view of first two movements:



Cosine rule $d = [8^2 + 1^2 - 2(8)(1)\cos 135°]^{1/2}$

$d = 8.736$ m

Side view including last movement:



$r = \sqrt{d^2 + y^2} = \sqrt{8.736^2 + 1^2}$

$r = 8.793$ m

Question 6

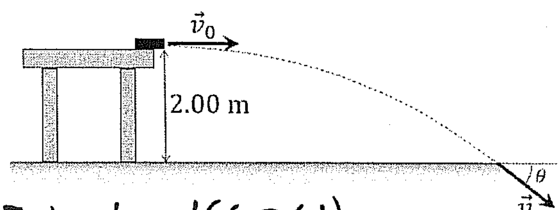
A hockey puck slides off the edge of a table with an initial velocity of $\vec{v}_0 = 28.0$ m/s, directly toward the right, as shown. The height of the tabletop above the flat floor is 2.00 m. What is the angle θ below the horizontal of the velocity of the puck just before it hits the ground?

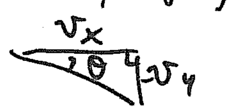
A. 12.6°
B. 12.8°
C. 31.8°
D. 72.6°
E. 77.2°

$v_y^2 = v_{oy}^2 + 2a_y \Delta y$

$v_y^2 = 0 - 2g(-2)$

$v_y = \pm\sqrt{4g} = -6.261$ m/s





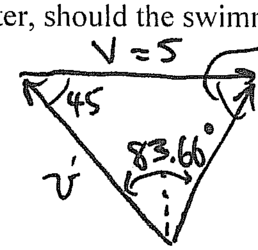
$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{6.261}{28}\right) = 12.604°$

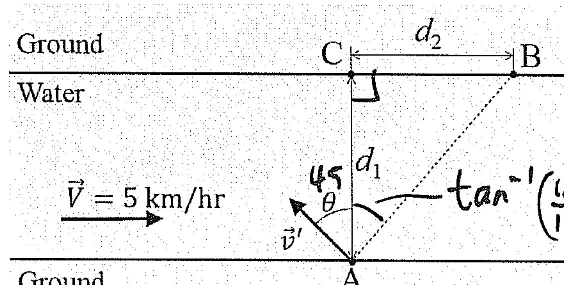
Question 7

A swimmer wants to cross a river, from point A to point B, as shown in the figure. The distance d_1 (from A to C) is 150 m, the distance d_2 (from C to B) is 120 m, and the speed V of the current in the river is 5 km/hr. Suppose that the swimmer's velocity relative to the water makes an angle of $\theta = 45°$ relative with the line from A to C, as indicated in the figure. To swim directly from A to B, what speed v', relative to the water, should the swimmer have?

$\vec{v} = \vec{v}' + \vec{V}$

A. 2.8 km/hr
B. 3.9 km/hr
C. 4.0 km/hr
D. 6.3 km/hr
E. 7.1 km/hr





$\tan^{-1}\left(\frac{120}{150}\right) = 38.66°$

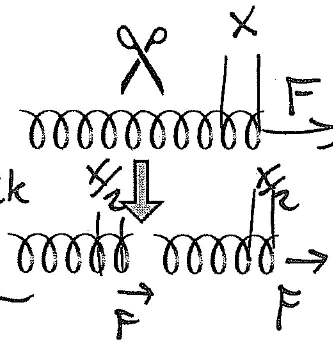
Sine rule: $\frac{v'}{\sin(51.34°)} = \frac{5}{\sin(83.66°)}$

$v' = 3.928$ m/s

Question 8

A metal spring has a spring constant of k. If you cut the spring in half, as shown, what is the spring constant of each new spring?

A. k
B. $2k$
C. $k/2$
D. $0.1k$
E. zero



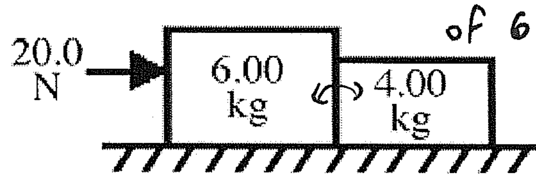
$k = \frac{F}{x}$

$k_2 = \frac{F}{x/2} = 2\left(\frac{F}{x}\right) = 2k$

Question 9

A 6.00-kg block is in contact with a 4.00-kg block on a horizontal frictionless surface as shown in the figure. The 6.00-kg block is being pushed by a horizontal 20.0-N force as shown. What is the magnitude of the force that the 4.00-kg block exerts on the 6.00-kg block? ← Newton 3 ⇒ same as F of 6 kg on 4 kg.

A. 6.00 N
B. 20.0 N
C. 8.00 N
D. 4.00 N
E. 10.0 N



fbd of 4 kg: $n\uparrow$ $F = 4a$ $mg\downarrow$

fbd of both: $n\uparrow$ $20\rightarrow$ $mg\downarrow$

$a = \frac{F_{net}}{M_{tot}} = \frac{20}{6+4} = \frac{20}{10} = 2\ m/s^2$

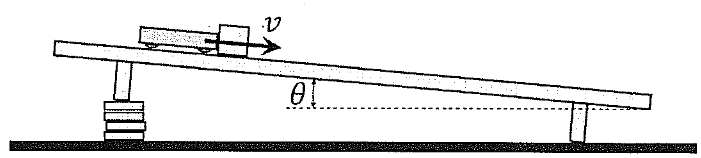
$F = 4(2)$

$F = 8$ N

Question 10

A cart with frictionless wheels is rolling down a straight metal track which is inclined at an angle of θ = 5° to the horizontal. As it rolls, it is pushing a sliding wooden block. Because of friction from the wooden block, the cart rolls at a constant speed. If you were to raise the left leg of the track more to increase θ, what would be the motion of the cart pushing the same block?

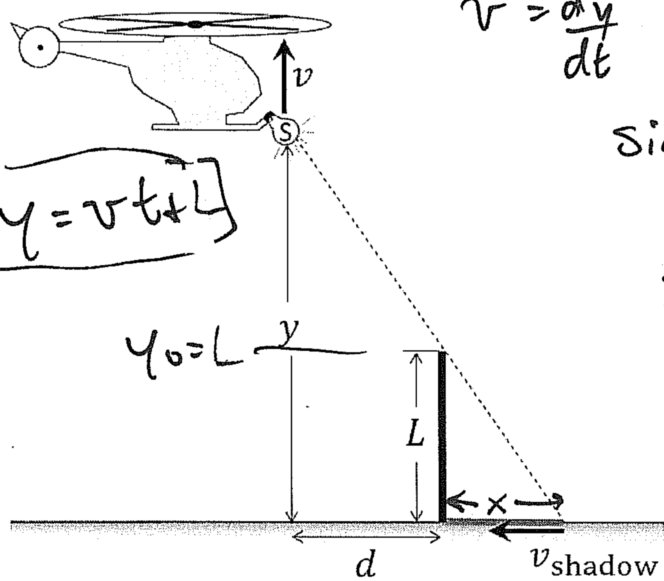
A. The cart would have negative acceleration down the incline, so that it would be slowing down.
B. The cart would have positive acceleration down the incline, so that it would be speeding up.
C. The cart would roll with the same constant speed down the incline as before.
D. The cart would roll with a constant speed, which is greater than before.
E. The cart would roll with a constant speed, which is less than before.



FREE-FORM PART (12 points total)
For full marks, you must clearly show all of your work and reasoning in the space provided. State any assumptions you make, and show all the steps of your calculations. Write your final answers in the boxes provided.

Problem A (6 points)
A helicopter flies upward at a speed, v, and holds a very bright light source, S, at an increasing height y above the flat ground. The light source is always above a point on the ground which is a distance d away from the bottom of a vertical post of height L. At time $t = 0$, the light source is at $y = L$. As the helicopter continues to fly upward at constant velocity, the shadow of the vertical post gets shorter, as shown. Find an expression for the speed of the tip of the shadow, v_{shadow}. Express your final result in terms of some or all of the following variables only: v, t, d and L.

$$v = \frac{dy}{dt} \qquad v_{shadow} = -\frac{dx}{dt}$$



$\boxed{y = vt + L}$

Similar triangles: $\frac{y}{d+x} = \frac{L}{x}$

Solve for x: $yx = L(d+x)$

$$yx = Ld + Lx$$

$$(y-L)x = Ld$$

$$x = \frac{Ld}{y-L}$$

Chain Rule: $\frac{dx}{dt} = \frac{dx}{dy} \cdot \frac{dy}{dt}$

$$\frac{dx}{dt} = \frac{d}{dy}\left[\frac{Ld}{y-L}\right] \cdot \frac{dy}{dt} = Ld \frac{d}{dy}(y-L)^{-1} v$$

$$-v_{shadow} = Ld\left(-1(y-L)^{-2}\right)v = \frac{-Ldv}{(y-L)^2}$$

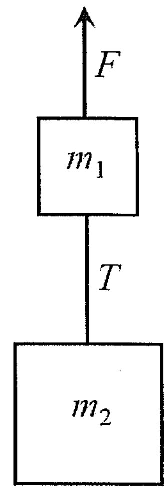
$$-v_{shadow} = \frac{-Ldv}{(vt+L-L)^2} = \frac{-Ldv}{v^2t^2}$$

$$v_{shadow} = \frac{Ld}{vt^2}$$

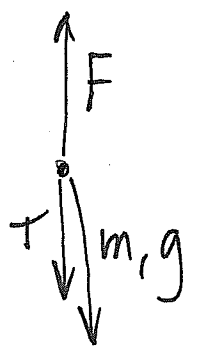
$v_{\text{shadow}} = \frac{Ld}{vt^2}$

Problem B (6 points)

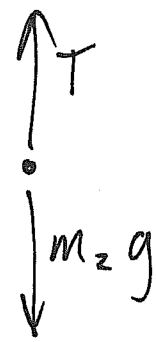
Two masses, m_1 and m_2, are joined by a string. An upward pulling force of magnitude F is applied directly to m_1. As a result, both blocks accelerate upward with acceleration magnitude a. While accelerating, the tension in the string has a magnitude of T.



1. **[1 point]** In the box below, sketch a free-body diagram for m_1, and label the forces. Please label the gravitational force on m_1 as "m_1g", where g is the acceleration due to gravity.



2. **[1 point]** In the box below, sketch a free-body diagram for m_2, and label the forces. Please label the gravitational force on m_2 as "m_2g", where g is the acceleration due to gravity.



3. **[4 points]** If $F = 36$ N, $a = 2.2$ m/s^2, and $T = 24$ N, what are the values of the masses m_1 and m_2, in kg?

from f.b.d. of m_1: Define + = up.

$$F_{net} = F - T - m_1 g = m_1 a$$

$$F - T = m_1(g+a)$$

$$m_1 = \frac{F-T}{g+a} = \frac{36-24}{9.8+2.2} = 1$$

from f.b.d. of m_2:

$$F_{net} = T - m_2 g = m_2 a$$

$$T = m_2(g+a)$$

$$m_2 = \frac{T}{g+a} = \frac{24}{9.8+2.2} = 2$$

$m_1 =$ 1.0 kg

$m_2 =$ 2.0 kg

ROUGH WORK (not marked)