HARLOW Solutions PHY131

Test 2
Fall 2015

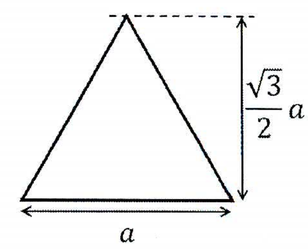
Possibly helpful information for this test:
$\pi = 3.14159$ is the ratio of the circumference to the diameter of a circle
$g = 9.80$ m/s^2 is the acceleration due to gravity near the Earth's surface.

Common Prefixes:
k = "kilo-" = 10^3 c = "centi-" = 10^{-2} m = "milli-" = 10^{-3} μ = "micro-" = 10^{-6}
n = "nano-" = 10^{-9}

60 seconds = 1 minute; 60 minutes = 1 hour; 24 hours = 1 day; 365.25 days = 1 year

The quadratic equation: If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If the base of an equilateral triangle is a, then its height is $\frac{\sqrt{3}}{2}a$.

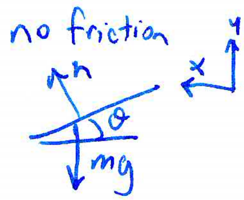


Air resistance may be neglected in all questions, unless otherwise stated.

MULTIPLE CHOICE PART (20 points total)
Choose the best answer for each question. 2 points per question, no penalty for guessing.

1. A banked turn has a radius of 400 m and is designed for cars traveling at 90 km/hr. This means cars traveling at 90 km/hr can stay on the curve without assistance from friction. What is the banking angle of this curve?
   A. 9°
   B. 10°
   C. 13°
   D. 17°
   E. 64°

$v = 90\,\frac{km}{hr}\left(\frac{1000\,m}{1\,km}\right)\left(\frac{1\,hr}{3600\,s}\right) = 25$ m/s



$(F_{Net})_y = 0 = n\cos\theta - mg \Rightarrow n = \frac{mg}{\cos\theta}$

$(F_{Net})_x = \frac{mv^2}{r} = n\sin\theta = mg\frac{\sin\theta}{\cos\theta} = mg\tan\theta$

$\frac{v^2}{r} = g\tan\theta \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{gr}\right) = \tan^{-1}\left(\frac{25^2}{9.8(400)}\right) = 9.06°$

2. You measure the mass of a cart to be 500 ± 5 g, and its speed to be 3.5 ± 0.7 m/s. What is its kinetic energy?
   A. 3 ± 2 J
   B. 3.1 ± 0.7 J
   C. 3.1 ± 1.2 J
   D. 3.06 ± 0.30 J
   E. 3.06 ± 0.61 J

$m = 500\,g\left(\frac{1\,kg}{1000\,g}\right) = 0.5$ kg $\quad u_m = 5\,g\left(\frac{1\,kg}{1000\,g}\right) = 0.005$ kg

$v = 3.5$ m/s $\quad u_v = 0.7$ m/s

$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.5)3.5^2 = 3.0625$ J

Exponent rule $z = v^2$ $\quad \frac{u_z}{z} = n\left(\frac{u_v}{v}\right)^{n-1}$ $\quad u_z = 2\frac{v^2}{v}u_v = 2vu_v = 2(3.5)0.7$

$u_z = 4.9$ m^2/s^2

Product rule $w = mz$ $\quad u_w = w\sqrt{\left(\frac{u_m}{m}\right)^2 + \left(\frac{u_z}{z}\right)^2} = 0.5(12.25)\sqrt{\left(\frac{.005}{0.5}\right)^2 + \left(\frac{4.9}{12.25}\right)^2}$

$u_w = 2.45$

Exact constant $u_K = \frac{1}{2}u_w = 1.23$ J

$K = 3.06 \pm 1.23$

round to tenths place.

3. Here is a table of the results of four repeated measurements of x in m. Which of the choices below shows the best way to report the mean value of x based on these four measurements?

x_1	9.571
x_2	9.396
x_3	10.050
x_4	9.503

A. 9.63 ± 0.14 m
B. 9.62983892 ± 0.28898099 m
C. 9.63 ± 0.29 m
D. 9.63 ± 0.54 m
E. 9.6 ± 1.1 m

$\bar{x} = \frac{x_1+x_2+x_3+x_4}{4} = 9.63$ m

$\sigma = \sqrt{\frac{1}{N-1}\sum(x_i-\bar{x})^2} = \sqrt{\frac{1}{3}(-0.059^2 + -0.234^2 + 0.42^2 + -0.127^2)}$

$\sigma = 0.289$ m

$u_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \frac{0.289}{\sqrt{4}} = 0.144$ m $\Rightarrow \bar{x} = 9.63 \pm 0.144$ round to 100th

4. Initially, Block 1 with mass m_1= 1 kg is moving on a frictionless table with velocity v_{1i} = +1 m/s along the x-axis and block 2 with mass m_2 = 0.5 kg is at rest. Block 1 collides elastically with block 2. The collision is head-on, so that all motion remains along the x-axis. What is the velocity of Block 1 after the collision?

A. −0.7 m/s
B. −0.3 m/s
C. +0.3 m/s
D. +0.7 m/s
E. +1.3 m/s

initial: $\boxed{m_1}$ $\boxed{m_2}$

$v_{1i} = +1$ m/s

$v_{1f} = \left(\frac{m_1 - m_2}{m_1+m_2}\right) v_{1i}$

$= \left(\frac{1-0.5}{1+0.5}\right)(1) = +0.33$ m/s

5. An energy-efficient refrigerator uses 250 W of power when it is running, and, due to good insulation, it only runs 15% of the time. An older refrigerator uses 400 W of power when it is running, and runs 20% of the time. If the cost of electricity is \$0.12/kWh, how much money do you save over 30 days by owning the energy-efficient refrigerator instead of the older one?

$P_1 = 0.250$ kWh/hr $\qquad P_2 = 0.4$ kWh/hr

A. Less than \$0.01
B. \$3.24
C. \$3.67
D. \$6.91
E. \$300

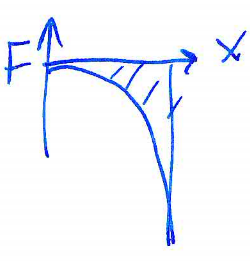
cost to run 1 = $P_1 \cdot (0.15) \cdot t \cdot$ rate $= 0.250(0.15)(24)30(\$0.12) = \$3.24$

cost to run 2 = $P_2(0.2)\,t \cdot$ rate $= 0.4(0.2)(24)(30)(\$0.12) = \$6.91$

difference = 6.91 − 3.24 = \$3.67

6. Consider a mechanical spring that does not obey Hooke's Law. The spring is initially unstretched, with the unconstrained end of the spring at position x = 0. This particular spring provides a force $F = -k_2 x^2$. The nonlinear spring constant is $k_2 = 10^4$ N/m^2. The spring is now compressed so that the unconstrained end moves from x = 0 to x = 0.10 m. What is the work done by the spring as it is compressed?

A. −0.63 J
B. −2.5 J
C. −2.9 J
D. −3.3 J
E. −3.5 J



$W = \int_{x_1}^{x_2} F\,dx = \int -k_2 x^2 dx = \frac{-k_2}{3}\left[x^3\right]_0^{0.1}$

$W = -\frac{10^4}{3}\left[0.1^3 - 0\right] = -3.3$ J

$r_2 = 7600 + 6370 = 13970$ km $r_1 = 630 + 6370 = 7000$ km

7. A satellite in an elliptical orbit has a height above the surface of the Earth which ranges from 630 km at its lowest point, up to 7600 km at its highest. When it is at its lowest point (closest to the surface of the Earth), it's moving at 8.7 km/s. How fast is it moving at its highest point? [Assume the Earth is a sphere of radius 6370 km and mass 5.97×10^{24} kg.]
   A. 2.2 km/s
   B. 4.4 km/s
   C. 6.2 km/s
   D. 8.7 km/s
   E. 17 km/s

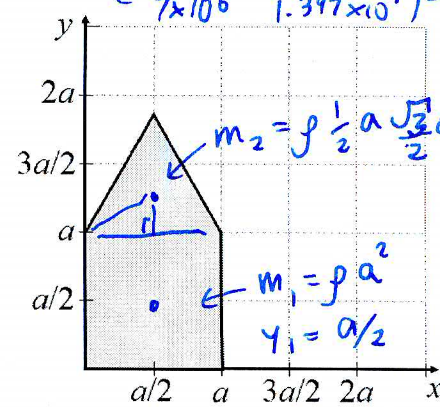
Conservation of energy: $E_1 = E_2$

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$$

$$v_2 = \sqrt{v_1^2 - 2GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$v_2 = \sqrt{(8.7\times10^3)^2 - 2(6.67\times10^{-11})(5.97\times10^{24})\left(\frac{1}{7\times10^6} - \frac{1}{1.397\times10^7}\right)} = 4.35 \text{ km}$$

8. A flat cookie has an even thickness, and is house-shaped, as shown. It is made of a square of side length a, connected underneath an equilateral triangle of side length a (the roof). The house sits so its bottom left corner is at the origin $[x, y] = [0,0]$, as shown. The peak of the roof is at $[x, y] = \left[\frac{a}{2}, \left(1 + \frac{\sqrt{3}}{2}\right)a\right]$. The x-coordinate of the center of mass is $x_{cm} = \frac{a}{2}$. What is the y-coordinate of the center of mass?
   A. 0.5 a
   B. 0.74 a
   C. 0.84 a
   D. a
   E. 1.29 a



$m_2 = \rho \frac{1}{2} a \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{4} \rho a^2$

$m_1 = \rho a^2$

$y_1 = a/2$

$\tan 30° = \frac{y_2 - a}{a/2}$

$y_2 - a$, $30°$, $a/2$

$y_2 = a + \frac{a}{2}\tan 30°$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{\rho a^2 (a/2) + \frac{\sqrt{3}}{4}\rho a^2 (a + a/2 \tan 30°)}{\rho a^2 + \frac{\sqrt{3}}{4}\rho a^2}$$

$$y_{cm} = \left[\frac{1 + \sqrt{3}/2\,(1 + \tan 30°/2)}{2 + \sqrt{3}/2}\right]a = 0.739\, a$$

9. A system consists of a car with momentum 25,000 kg m/s, South, and a truck with momentum 100,000 kg m/s, North. They have a head on collision and stick together. Which statement correctly describes the system immediately after the collision?
   A. The total kinetic energy of the system is converted to thermal energy.
   B. The total mechanical energy of the system is converted to thermal energy.
   C. The total kinetic energy of the system is conserved.
   D. The total mechanical energy of the system is conserved.
   E. Part of the kinetic energy of the system is converted to thermal energy.

10. A wheel with rotational inertia $I = 1.0$ kg m^2 is acted upon by a counterclockwise torque of constant magnitude 1.0 N m. The wheel starts from rest and the torque is applied for 3.0 seconds. What angle, in radians, has it turned through in this time?
   A. 1.5
   B. 3
   C. 4.5
   D. 9
   E. 18

$\alpha = \frac{\tau}{I}$

$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{\tau}{I}\right)t^2$

$\theta = \frac{1}{2}\left(\frac{1}{1}\right)3^2 = 4.5$ rad

FREE-FORM PART (12 points total)

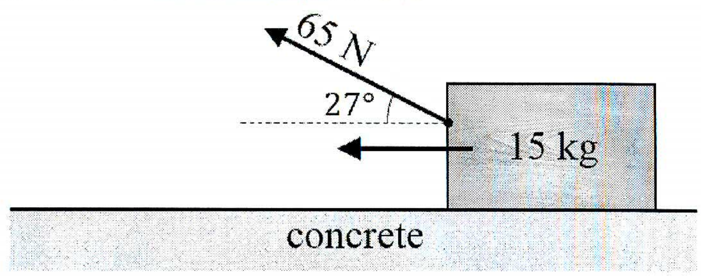
For full marks, you must clearly show all of your work and reasoning in the space provided. State any assumptions you make, and show all the steps of your calculations. Write your final answers in the boxes provided.

For both problems in the Free-form part, assume:

The coefficient of **kinetic** friction between wood and concrete is $\mu_k = 0.15$.
The coefficient of **static** friction between wood and concrete is $\mu_s =$ 0.30

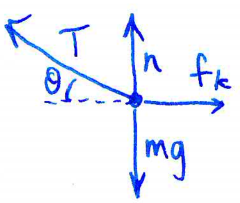
Problem A

A wooden box of mass 15 kg is pulled to the left by a constant tension force of 65 N at an angle of 27° above the horizontal. It starts from rest and slides forward along a smooth, flat, horizontal concrete floor.

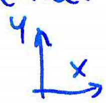


1. (4 points) How much time does it take the box to get up to a speed of 3.0 m/s?

fbd of box:



Define:



$f_k = \mu_k n$

Cannot accelerate along y:

$(F_{net})_y = 0 = T\sin\theta + n - mg \Rightarrow n = mg - T\sin\theta$

x-acceleration:

$(F_{net})_x = ma_x = -T\cos\theta + \mu_k n$

$a_x = \frac{1}{m}\left(-T\cos\theta + \mu_k(mg - T\sin\theta)\right)$

$= \frac{1}{15}\left(-65\cdot\cos 27 + 0.15\left(15(9.8) - 65\sin 27\right)\right)$

$a_x = -2.6861 \ m/s^2$

$\frac{\Delta v}{\Delta t} = a_x \qquad \Delta t = \frac{\Delta v}{a_x} = \frac{-3}{-2.6861} = 1.117 \ s$

$t =$ 1.1 s

2. (2 points) How much thermal energy is generated by friction during this time?

$E_{th} = f_k \cdot d \qquad d = \frac{1}{2}at^2, \ f_k = \mu_k(mg - T\sin\theta)$

$E_{th} = \mu_k(mg - T\sin\theta)\frac{a t^2}{2} = 0.15(15(9.8) - 65\sin 27°)\cdot\frac{2.6861}{2} \times 1.117^2$

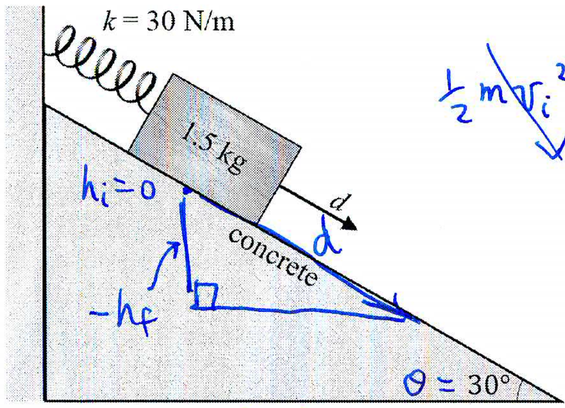
$E_{th} = 29.53 \ J$

$E_{th} =$ 29.5 J

(3.0×10¹ J)

Problem B (6 points)

1. (5 points) An unstretched spring is attached to a 1.5 kg wooden block on a concrete ramp which makes an angle of 30° with respect to the horizontal. The other end of the spring is fixed. The mass is released and it slides down the ramp and stretches the spring. The spring has a constant of 30 N/m. Find the maximum distance that the block slides down the ramp.



$\sin 30^\circ = \frac{-h_f}{d}$

$E_i = E_f$

$$\frac{1}{2}m\cancel{v_i^2}_{0} + mg\cancel{h_i}_{0} + \frac{1}{2}k\cancel{x_i^2}_{0} = \frac{1}{2}m\cancel{v_f^2}_{0} + mgh_f + \frac{1}{2}kx_f^2 + \underbrace{f_k d}_{E_{th}}$$

$x_f = d$, $h_f = -d\sin 30^\circ$

$0 = mg(-d\sin 30^\circ) + \frac{1}{2}kd^2 + \mu_k n d$

$\frac{kd^2}{2} + \mu_k mg\cos\theta\, d = mgd\sin\theta$

$d \neq 0$, so divide both sides by d.

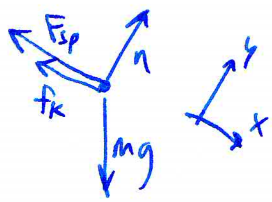
$\frac{kd}{2} = mg[\sin\theta - \mu_k\cos\theta]$

$d = \frac{2mg}{k}[\sin\theta - \mu_k\cos\theta]$

$d = \frac{2(1.5)9.8}{30}[\sin 30 - 0.15\cos 30]$

$d = 0.36269$ m

fbd:

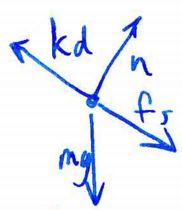


$(F_{net})_y = 0 = n - mg\cos\theta$

$n = mg\cos\theta$

If it stops, f_s holds it there.

fbd:



$(F_{net})_x = 0 = f_s - kd + mg\sin\theta$

$f_s = kd - mg\sin\theta = 3.53$ N ← Needed

f_s max: $\mu_s mg\cos\theta = 3.8\cancel{2}$ N

Needed < max, so it sticks.

$\boxed{d_{max} = 0.36 \text{ m}}$

2.(1 point~~s~~) When the block gets to this position, does it stay still, or will it start back up the ramp? (Circle one)

STAY STILL   START BACK UP