VERSION 1 : HARLOW ANSWERS SP. 2009

Possibly useful constants, equations:
Acceleration due to gravity near the surface of the Earth: $g = 9.80\ m/s^2$
2π radians = 360°
$\pi = 3.14159$
Drag force due to the air: $D = (0.25\ kg/m^3)Av^2$, where A is the cross-section area of the object, v is the speed, and $\vec{D}$ and $\vec{v}$ are in opposite directions.

MULTIPLE CHOICE (16 points total)

1. A woman stands at the edge of a cliff, holding one ball in each hand. At time t_0, she throws one ball straight up with speed v_0 and the other straight down, also with speed v_0. Which ball hits the ground with greater speed? [Neglect air resistance for this question.]
   A. the ball thrown upward
   B. the ball thrown downward
   C. Neither; the balls hit the ground with the same speed.

Version 2 :
2. A

2. Harlow gives a quick, initial push to a heavy textbook with his hands. The textbook then slides across the table to the right for a distance of 2 m, until it stops. Which of these forces are acting on the textbook as it slides?
   1. Gravity, acting downward
   2. The normal force, acting upward
   3. The force of Harlow's hands, acting to the right X
   4. Friction, acting to the left

   A. 1, 2 and 3
   B. 3 only
   C. 1, 2 and 4 but not 3
   D. None of these
   E. All of these

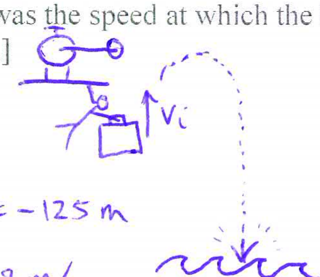
Version 2 :
4. E

3. A sprinter runs a race in which he starts from rest, and crosses the finish line 100.0 m away. Assume that he runs with constant acceleration until reaching his top speed of 10.7 m/s, then maintains that speed through the finish line. If the sprinter reaches his top speed after 2.04 s, what will be his total time between starting and crossing the finish line?
   A. 9.35 s
   B. 18.7 s
   C. 6.28 s
   D. 10.4 s
   E. 11.3 s

See Page 5

Version 2 : 6. A

4. Sawyer is rescued from the ocean by grabbing onto the landing gear of a helicopter. The helicopter tows him upward at a constant velocity. Sawyer is so intent on gripping the landing gear that he lets go of his metal briefcase when he is 125 m above the water. If the briefcase hits the water 5.3 s later, what was the speed at which the helicopter was ascending? [Neglect air resistance for this question.]
   A. 24 m/s
   B. 12 m/s
   C. 1.4 m/s
   D. 140 m/s
   E. 2.4 m/s



$y_f = y_i + v_i t + \frac{1}{2} a_y t^2$, solve for v_i

$v_i = \frac{1}{t}\left[y_f - y_i - \frac{1}{2} a_y t^2\right]$

$y_f - y_i = -125\ m$

$a_y = -9.8\ m/s^2$

$v_i = \frac{1}{5.3}\left[-125\ m - \frac{1}{2}(-9.8)5.3^2\right]$

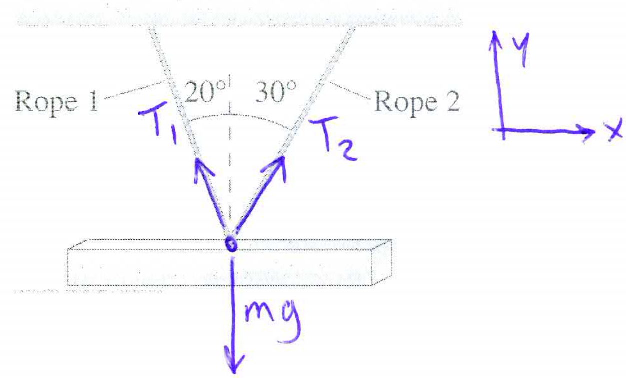
$v_i = +2.4\ m/s$

Version 2 :
8. B

5. A steel beam of mass $M = 440$ kg is supported by two massless ropes, as shown in the figure. The angle between Rope 1 and vertical is 20.0°. The angle between Rope 2 and vertical is 30.0°. The angle between Rope 1 and Rope 2 is 50.0°. What is the tension in Rope 2?
   A. 2.2×10^3 N
   B. 2.1×10^3 N
   C. 1.7×10^3 N
   D. 1.9×10^3 N
   E. 9.7×10^3 N

See Page 6

Version 2: 7.A



6. A 65 kg skydiver can be modeled as a rectangular box with height 160 cm, front-to-back distance 25 cm, and side-to-side distance 35 cm. What is her terminal speed if she falls feet first?
   A. 290 km/hr
   B. 1.0×10^5 km/hr
   C. 35 km/hr
   D. 480 km/hr
   E. 620 km/hr

Version 2: 5.B.

$A = 0.25 \times 0.35 = 0.0875\ m^2$ for feet-first.

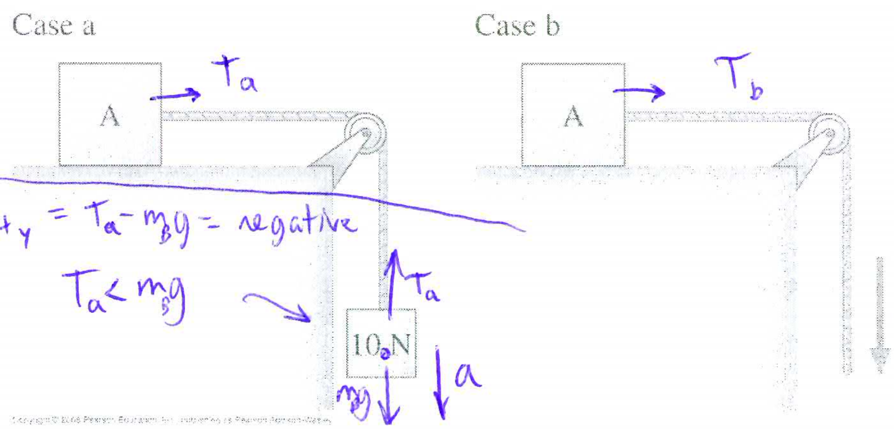
$D = mg$ for equilibrium, terminal speed.

$0.25\ A v^2 = mg$

$v = \sqrt{\dfrac{mg}{0.25A}} = \sqrt{\dfrac{65 \times 9.8}{0.25 \times 0.0875}} = 171\ m/s \left[\dfrac{3600\ s}{1\ hr}\right] \times \left[\dfrac{1\ km}{1000\ m}\right] = 620\ \dfrac{km}{hr}$

7. Consider the figure below. In Case a, block A is accelerated across a frictionless table by a hanging mass whose weight is 10 N (its mass is 1.02 kg). In Case b, block A is accelerated across a frictionless table by a steady 10 N tension in the string. The string is massless, and the pulley is massless and frictionless. The magnitude of the acceleration of Block A is
   A. greater in Case a than in Case b.
   B. less in Case a than in Case b.
   C. the same in Case a as it is in Case b.

Version 2: 1.C.



$F_{Net_y} = T_a - m_B g$ = negative

$T_a < m_B g$

$T_a < T_b$

$m_A a_a < m_A a_b$

$a_a < a_b$

8. A child is sitting on the outer edge of a merry-go-round that is 18 m in diameter and spinning at a constant rate. If the merry-go-round makes 5.4 revolutions per minute, what is the speed of the child in m/s?
   A. 49
   B. 1.0×10^1
   C. 5.1
   D. 97
   E. 17

$\omega = 5.4\ \dfrac{rev}{min}\left[\dfrac{1\ min}{60\ sec}\right]\left[\dfrac{2\pi\ rad}{1\ rev}\right] = 0.565\ \dfrac{rad}{s}$

$r = \dfrac{18m}{2} = 9m$

$V_t = \omega r = 0.565\,(9) = 5.1\ m/s$

Version 2: 3.E.

FREE-FORM IN THREE UNRELATED PARTS (14 points total)

Clearly show your reasoning and work as some part marks may be awarded. Write your final answers in the boxes provided.

PART A (2 points)

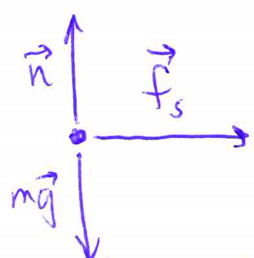
Action: Gravity of Earth on Bob.

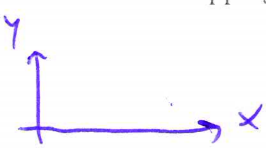
Bob stands on a bathroom scale. There is an ***action*** force on Bob, which is gravity. Gravity pulls Bob down. Identify the ***reaction*** force, which, by Newton's Third Law, must be equal in magnitude but opposite in direction to the force of gravity on Bob.

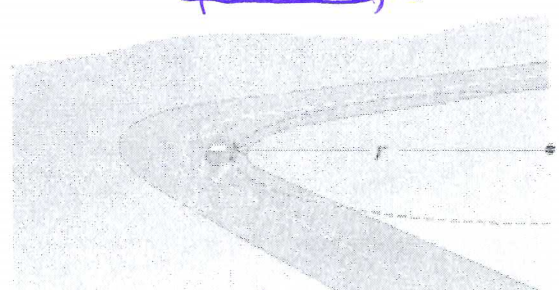
Reaction: Gravity of Bob on Earth.
(pulls Earth up.)

PART B (4 points)

A car of mass $M = 1300$ kg enters a level curve of radius $r = 91.3$ m. The curve is not banked. The coefficient of static friction between the rubber of the car's tires and the concrete road is $\mu_s = 0.90$, and the coefficient of kinetic friction between the rubber of the car's tires and the concrete road is $\mu_k = 0.77$. What is the maximum speed the car can travel around this curve without slipping? Express your final answer in m/s.







	x	y
$\vec{n}$	0	$+n$
$m\vec{g}$	0	$-mg$
$\vec{f}_s$	$+f_s$	0
$\vec{F}_{Net}$	$+f_s$	$n - mg$

y-equilibrium: $(F_{Net})_y = 0 = n - mg$

$\Rightarrow n = mg$

Centripetal acceleration in $+x$ direction

$\Rightarrow F_{Net\,x} = \frac{mv^2}{r} = f_s$, set $f_s = f_{s\,max} = \mu_s n$

$\frac{mv^2}{r} = \mu_s n = \mu_s mg$

$\frac{v^2}{r} = \mu_s g$

$v = \sqrt{\mu_s g r} = \sqrt{0.9(9.8)91.3}$

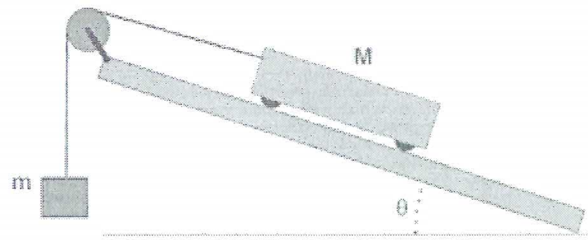
$v_{max} = 28$ m/s

2 sig. figs.

Version 2: 29 m/s

PART C (8 points)

In the figure the frictionless Track is at an angle θ with the horizontal. The Cart has a mass M, and is connected to a hanging mass $m = 0.075 \pm 0.002$ kg by a massless string over a massless, frictionless pulley.



For an angle $\theta = 5.2 \pm 0.1°$ the masses are in equilibrium, i.e. if they are at rest they remain at rest and if they are moving at some speed they continue moving at that speed. Calculate the value of the mass of the cart M. Express your final answer in kg. [You may use the small angle approximation which is that $\sin\theta \approx \tan\theta \approx \theta$ when θ is measured in radians.]

fbd of m:

y-equilibrium: $F_{Net\,y} = 0 = T - mg$

$T = mg$

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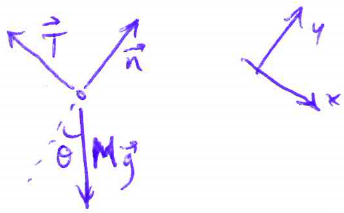
fbd of M:



$F_{Net\,x} = 0 = Mg\sin\theta - T$

$T = Mg\sin\theta$

$T = T \Rightarrow mg = Mg\sin\theta \Rightarrow M = \dfrac{m}{\sin\theta}$, or ,

small angle approximation: $M = \dfrac{m}{\theta}$

where $\theta = 5.2 \pm 0.1° \left[\dfrac{2\pi \text{ rad}}{360°}\right] = 0.0908 \pm 0.0017$ rad.

$$M = \frac{0.075 \pm 0.002}{0.0908 \pm 0.0017} \text{ kg.}$$

$M = 0.826$ kg

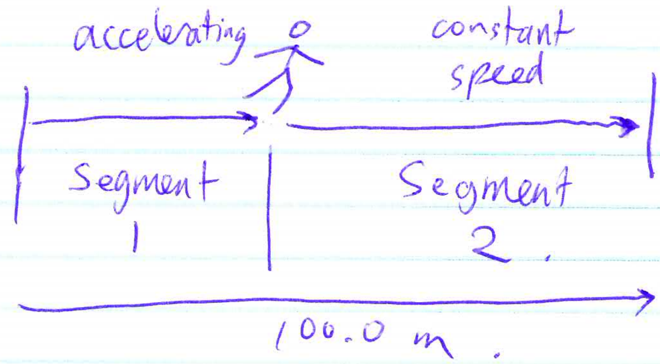
Product Rule: $\Delta M = M\sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta\theta}{\theta}\right)^2} = 0.826\sqrt{\left(\frac{0.002}{0.075}\right)^2 + \left(\frac{0.0017}{0.0908}\right)^2}$

$\Delta M = 0.027$ kg.

$M =$ 0.826 ± 0.027 kg or 0.83 ± 0.03 kg

Version 2: $M = 0.895 \pm 0.030$ kg or 0.90 ± 0.03 kg

3. Sprinter.



<u>Segment 1</u>: $t_1 = 2.04\ s$

$v_{f1} = 10.7\ m/s$

$v_{i1} = 0$

→ can solve for acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_{f1} - v_{i1}}{t_1} = \frac{v_{f1}}{t_1}$$

distance: $$x_{f1} = \overbrace{x_{i1}}^{=0} + \overbrace{v_{i1} t_1}^{=0} + \frac{1}{2} a t_1^2$$

$$x_{f1} = \frac{v_{f1} t_1^2}{2 t_1} = \frac{v_{f1} t_1}{2}$$

<u>Segment 2</u>: $v_{f1} = v_{i2}$, $x_{f1} = x_{i2}$

Constant speed: $x_{f2} = x_{i2} + v_2 t_2$

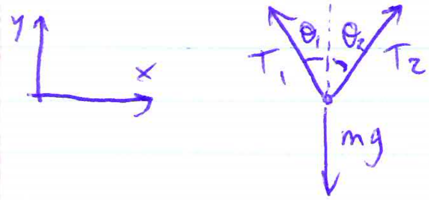
$$t_2 = \frac{x_{f2} - x_{f1}}{v_2} = \frac{1}{v_2}\left[x_{f2} - \frac{v_2 t_1}{2}\right]$$

$$\text{Total Time} = t_1 + t_2 = t_1 + \frac{1}{v_2}\left[x_{f2} - \frac{v_2 t_1}{2}\right]$$

$$= 2.04\ s + \frac{1}{10.7\ m/s}\left[100.0\ m - \frac{(10.7)(2.04)}{2}\right]$$

$$\boxed{t_1 + t_2 = 10.4\ s}$$

5. Steel Beam:



Not accelerating $\Rightarrow$ equilibrium.

	x	y
$\vec{T}_1$	$-T_1 \sin\theta_1$	$+T_1 \cos\theta_1$
$\vec{T}_2$	$+T_2 \sin\theta_2$	$+T_2 \cos\theta_2$
$m\vec{g}$	0	$-mg$
$\vec{F}_{Net}$	$T_2 \sin\theta_2 - T_1 \sin\theta_1$	$T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg$

$F_{Net\,x} = 0 \Rightarrow T_2 \sin\theta_2 - T_1 \sin\theta_1 = 0$, solve for T_1 to eliminate it.

$$\Rightarrow T_1 = T_2 \frac{\sin\theta_2}{\sin\theta_1}$$

$F_{Net\,y} = 0 \Rightarrow T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg = 0$

$$\Rightarrow T_2 = \frac{1}{\cos\theta_2}\left[mg - T_1 \cos\theta_1\right]$$

$$T_2 = \frac{mg}{\cos\theta_2} - T_2\left(\frac{\sin\theta_2}{\cos\theta_2}\right)\left(\frac{\cos\theta_1}{\sin\theta_1}\right)$$

$$T_2\left(1 + \frac{\tan\theta_2}{\tan\theta_1}\right) = \frac{mg}{\cos\theta_2}$$

$$\Rightarrow T_2 = \frac{mg}{\cos\theta_2\left(1 + \left(\tan\theta_2 / \tan\theta_1\right)\right)}$$

$$T_2 = \frac{440\text{ kg} \times 9.8\text{ m/s}^2}{\cos 30\left(1 + \frac{\tan 30}{\tan 20}\right)} = \boxed{1.9 \times 10^3\text{ N}}$$