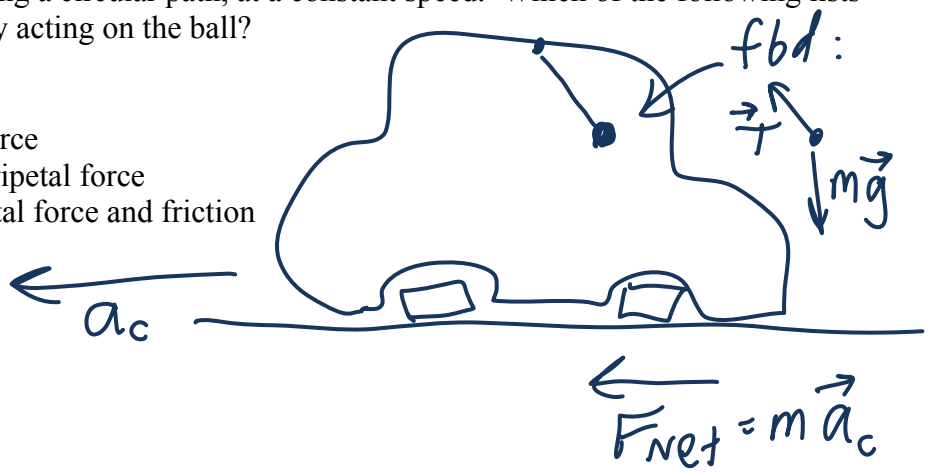
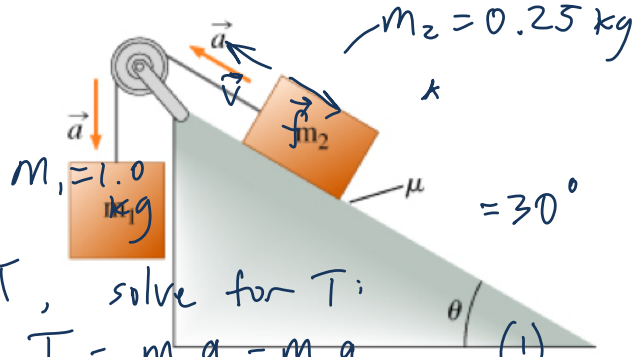
PHY131H1S **Test 2 —version1 Solutions** Tuesday, March 16, 2010

1. A string is attached to the rear-view mirror of a car. A ball is hanging on the other end of the string. The car is driving along a circular path, at a constant speed. Which of the following lists gives all of the forces directly acting on the ball?
   A. tension
   B. tension and gravity
   C. tension and centripetal force
   D. tension, gravity and centripetal force
   E. tension, gravity, centripetal force and friction



2. Block 1, of mass m_1 = 1.0 kg, is connected over an ideal (massless and frictionless) pulley to block 2, of mass m_2 = 0.25 kg, as shown. Assume that the blocks accelerate as shown with an acceleration of magnitude a, and that the coefficient of kinetic friction between block 2 and the plane is μ = 0.20. The velocity of each block at this moment is in the same direction as its acceleration. The angle of the incline is θ = 30.0°. What is the magnitude of the acceleration, a?

A. 4.9 m/s^2
B. 6.5 m/s^2
C. 6.9 m/s^2
D. 8.2m/s^2
E. 9.7 m/s^2



$m_2 = 0.25$ kg

$m_1 = 1.0$ kg

$= 30°$

fbd for m_1:

T

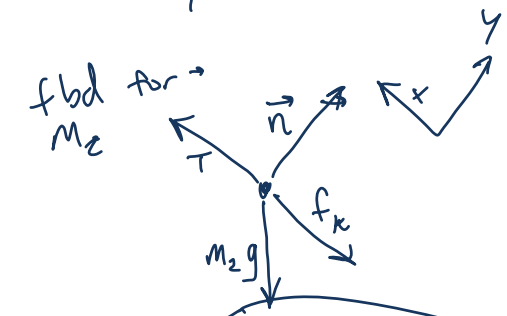
y

$m_1 g$

$(F_{Net})_y = ma = m_1 g - T$, solve for T:

$T = m_1 g - m_1 a$ (1)

fbd for m_2



a = acceleration in +x-dir.

	x	y
n	0	n
T	T	0
$f_k = \mu_k n$	$-\mu_k n$	0
$m_2 g$	$-m_2 g \sin\theta$	$-m_2 g \cos\theta$
$\vec{F}_{Net}$	$(F_{Net})_x = m_2 a$	$(F_{Net})_y = 0$

$\sin\theta = \frac{(m_2 g)_x}{m_2 g}$

$(F_{Net})_y = 0 = n + 0 + 0 - m_2 g \cos\theta$

$n = m_2 g \cos\theta$

$(F_{Net})_x = m_2 a = 0 + T - \mu_k m_2 g \cos\theta - m_2 g \sin\theta$

(1)

$m_2 a = T - m_2 g(\mu_k \cos\theta + \sin\theta)$

$m_2 a = m_1 g - m_1 a - m_2 g(\mu_k \cos\theta + \sin\theta)$

$(m_2 + m_1) a = m_1 g - m_2 g($ " $)$

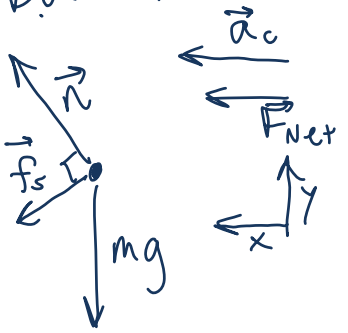
$$a = g\left[\frac{m_1 - m_2 \mu_k \cos\theta - m_2 \sin\theta}{m_1 + m_2}\right]$$

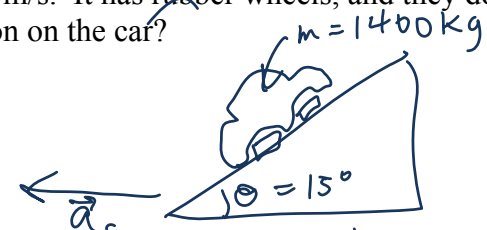
$$a = 9.8\,\text{m/s}^2\left[\frac{1 - (0.25)(0.2)\cos 30° - 0.25 \sin 30}{1.25}\right]$$

$= 6.52$ m/s^2

3. A concrete highway with a curve of radius 70.0 m is banked at a 15° angle. A car of total mass 1400 kg drives around this curve at a constant speed of 21 m/s. It has rubber wheels, and they do not slip. What is the magnitude of the force of static friction on the car?
   A. 5000 N
   B. 8800 N
   C. 3800 N
   D. 3600 N
   E. 13,000 N

f.b.d. of car:



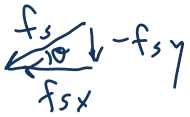


$v = 21\ m/s$

find f_s < $f_{s\,max}$ (does not slip)

assume f_s is down hill for positive f_s.

$\sin\theta = \frac{n_x}{n}$



	x	y
n	$n\sin\theta$	$n\cos\theta$
f_s	$f_s\cos\theta$	$-f_s\sin\theta$
mg	0	$-mg$
F_{Net}	$(F_{Net})_x = \frac{mv^2}{r}$	$(F_{Net})_y = 0$

$$(F_{Net})_y = 0 = n\cos\theta - f_s\sin\theta - mg \quad (1)$$

$$(F_{Net})_x = \frac{mv^2}{r} = f_s\cos\theta + n\sin\theta \quad (2)$$

2 unknowns: f_s, n. → eliminate n, solve for f_s

$$(1) \rightarrow \quad n = \frac{mg + f_s\sin\theta}{\cos\theta}$$

plug into (2):

$$\frac{mv^2}{r} = f_s\cos\theta + \frac{\sin\theta}{\cos\theta}\left[mg + f_s\sin\theta\right]$$

$$\frac{mv^2}{r} - mg\tan\theta = f_s\cos\theta + f_s\frac{\sin^2\theta}{\cos\theta}$$

$$f_s = \frac{1}{\cos\theta + \frac{\sin^2\theta}{\cos\theta}}\left[\frac{mv^2}{r} - mg\tan\theta\right]$$

$$= \frac{1}{\cos 15 + \frac{(\sin 15)^2}{\cos 15}}\left[\frac{1400(21)^2}{70\,m} - 1400(9.8)\tan 15^\circ\right]$$

$$f_s = 4968\ N$$

$f_{s\,Max} = \mu_s n$

rubber on concrete: $\mu_s = 1.0$

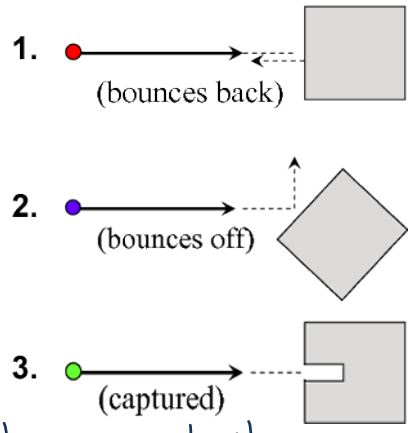
eq(1) gives n:

$f_{s\,max} = 1.0\,(n)$

$f_{s\,max} = 15{,}500\,N$

$f_s < f_{s\,max}$ okay ✓

4. Three identical balls slide on a table and hit a block that is fixed to the table. In the figures we are looking down from above. In each case the ball is going at the same speed before it hits the block. Each collision takes the same amount of time. Rank the magnitudes of the forces exerted on the block by each ball.



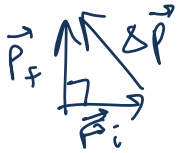
A. $F_1>F_2>F_3$
B. $F_1>F_2=F_3$
C. $F_1<F_2<F_3$
D. $F_1<F_2=F_3$
E. $F_1=F_2=F_3$

Impulse-momentum theorem:

$\Delta p = F\Delta t$

1. $p_f = -p_i$ $|\Delta p| = -p_i - p_i = |-2p_i| = 2p_i = |F_1|\Delta t$

$|F_1| = 2\frac{p_i}{\Delta t}$

2. 

$p_{fy} = p_{ix}$, $p_{fx} = 0$

$|\Delta p| = |\vec{p}_f - \vec{p}_i| = \sqrt{p_i^2 + p_i^2} = \sqrt{2}\,p_i = |F_2|\Delta t$

$|F_2| = \sqrt{2}\frac{p_i}{\Delta t}$

3. $p_f = 0$ $\Delta p = p_i = |F_3|\Delta t$

$|F_3| = \frac{p_i}{\Delta t}$

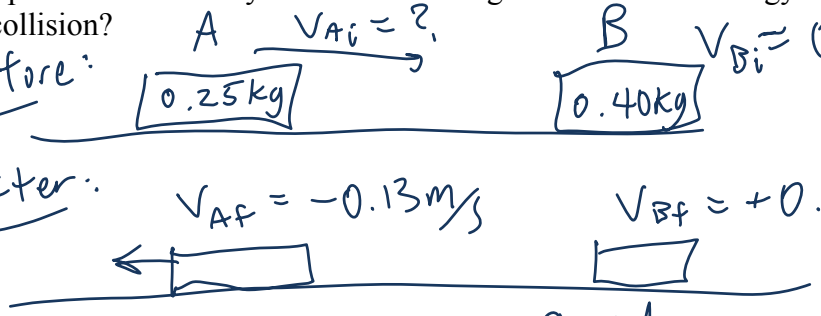
$2 > \sqrt{2} > 1$

$F_1 > F_2 > F_3$

5. On a frictionless horizontal air table, puck A (with mass $m_A = 0.25$ kg) is moving toward puck B (with mass $m_B = 0.40$ kg), which is initially at rest. After the collision, puck A has velocity 0.13 m/s, to the left, and puck B has velocity 0.65 m/s to the right. How much energy is lost from the system during the collision?

A. 0 J
B. 0.017 J
C. 0.087 J
D. 0.082 J
E. 0.91 J

Before:



find ΔK ... need V_{Ai} →

Note No Net external force ⇒ $\vec{P}_f = \vec{P}_i$

$$m_A V_{Af} + m_B V_{Bf} = m_A V_{Ai} + 0$$

solve for V_{Ai}:

$$V_{Ai} = \frac{1}{m_A}\left(m_A V_{Af} + m_B V_{Bf}\right)$$

$$= \frac{1}{0.25}\left(0.25(-0.13) + 0.4(+0.65)\right)$$

$$V_{Ai} = +0.91 \text{ m/s} \quad \text{[to the right]}$$

find kinetic energy:

$$K_i = \frac{1}{2} m_A V_{Ai}^2 + 0 = \frac{1}{2} 0.25 (0.91)^2$$

$$= 0.1035 \text{ J}$$

$$K_f = \frac{1}{2} m_A V_{Af}^2 + \frac{1}{2} m_B V_{Bf}^2$$

$$= \frac{1}{2} 0.25 (-0.13)^2 + \frac{1}{2} 0.4 (0.65)^2 = 0.08661 \text{ J}$$

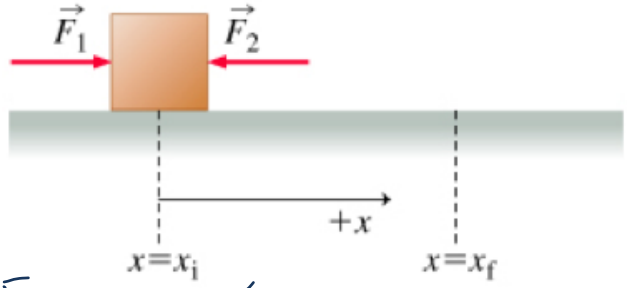
less → energy is lost...

$$\Delta K = K_f - K_i = 0.08661 - 0.1035$$

$$= -0.0169 \text{ J}$$

$$|\Delta K| = 0.017 \text{ J}$$

6. Two forces, of magnitudes F_1 and F_2, act in opposite directions on a block, which sits atop a frictionless surface, as shown in the figure. Initially, the centre of the block is at position x_i. At some later time, the block has moved to the right, and its centre is at a new position, x_f. What is the change in the kinetic energy, ΔK, of the block as it moves from x_i to x_f?



A. zero
B. $(F_1 - F_2)(x_f - x_i)$
C. $(F_2 - F_1)(x_f - x_i)$
D. $(F_1+F_2)(x_f - x_i)$
E. impossible to determine without knowing the mass of the block

Work - Kinetic Energy Theorem:

$\Delta K = W_{Net} = \vec{F}_{Net} \cdot \Delta\vec{r}$

$\Delta\vec{r} = x_f - x_i$ (to the right)

$\vec{F}_{Net} = F_1 - F_2$ (to the right)

$\theta = 0°$

$W_{Net} = (F_1 - F_2)(x_f - x_i)$

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7. Here is a complete python code:

```
t = 0
dt = 1
while t<3:
    print t
    t = t + dt
```

first time through loop: print t=0
set t=1
second time: (1<3) print t=1
set t=2
third time 2<3 print t=2
set t=3
fourth time 3 is not < 3 → loop breaks → end.

0
1
2

What is the output of this code?

A. 0

B. 1 2

C. 0 1 2

D. 1 2 3

E. 0 1 2 3

8. A ping pong ball is a spherical shell, for which the moment of inertia is given by $(2/3)MR^2$. A particular ping pong ball is rolling without slipping along the floor at a speed of 1.0 m/s. Its mass is $M = 0.0027$ kg and its radius is $R = 0.020$ m. What is the total kinetic energy (rotational plus linear) of the ball as it rolls, in milli-Joules [mJ]?
   A. 2.3
   B. 0.90
   C. 1.4
   D. 0
   E. 2.7

$V = 1.0\ m/s$

linear: $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.0027)\,1^2$

$K = 0.00135\ J$

$= 1.35\ mJ$

---

rolling without slipping: $V = \omega R$

rotational $\rightarrow \omega = \frac{V}{R}$

$K_r = \frac{1}{2}I\omega^2 = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}M\cancel{R^2}\right)\left(\frac{v}{\cancel{R}}\right)^2$

$= \frac{1}{3}Mv^2 = 0.90\ mJ$

total: $1.35 + 0.90 = 2.25\ mJ$

~ 2.3

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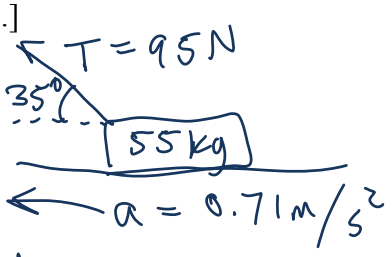
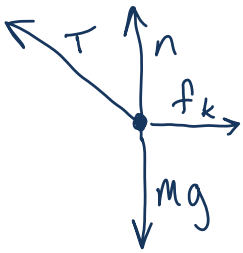
PART A (6 points)

It's a snowy day and you're pulling a friend along a level road on a sled. The sled and your friend together have a total mass of 55 kg. As you walk, you are pulling up and forward on the sled with a rope that is angled at 35° above the horizontal. If you pull with a force of 95 N, you can cause the sled to accelerate along the flat road at 0.71 m/s^2. What is the coefficient of kinetic friction between the sled and the road?

[Please write your final answer in the box provided, and express your answer to 2 significant figures.]



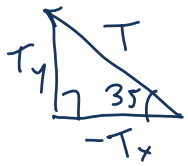
$f_k = \mu_k n$



$a_x = -0.71\ m/s^2$

$a_y = 0$

find μ_k



$\sin\theta = \frac{T_y}{T}$

	x	y
T	$-T\cos\theta$	$T\sin\theta$
n	0	n
f_k	$\mu_k n$	0
mg	0	$-mg$

$(F_{Net})_x = ma_x$

$(F_{Net})_y = 0$

$0 = T\sin\theta + n + 0 - mg$

$n = mg - T\sin\theta$

$n = 55(9.8) - 95(\sin 35)$

$n = 484.5\ N$

$(F_{Net})_x = ma_x = \mu_k n - T\cos\theta$

solve for μ_k:

$\mu_k n = ma_x + T\cos\theta$

$\mu_k = \frac{ma_x + T\cos\theta}{n}$

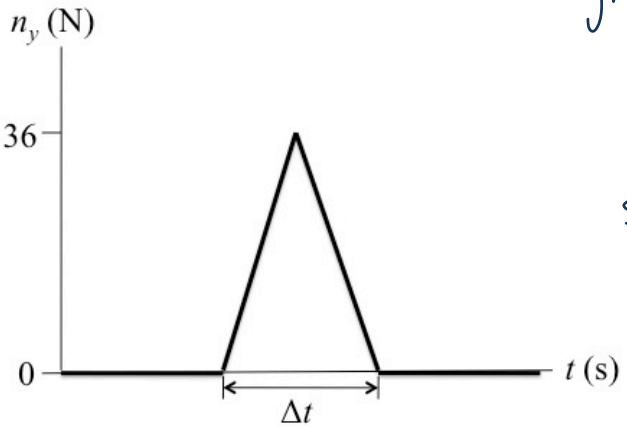
$\mu_k = \frac{55(-0.71) + 95(\cos 35)}{484.5} = 0.080$

$\mu_k =$ 0.080

PART B (6 points)

A 0.057 kg tennis ball is dropped from rest, a height of 1.3 m above the surface of a bathroom scale. It bounces off the scale and returns to a final height of 1.1 m before stopping again. The scale measures the upward normal force on the tennis ball in Newtons, as shown in the plot. What is Δt, the duration of the collision of the tennis ball with the scale?

[Please write your final answer in the box provided, and express your answer to 2 significant figures.]



3 segments: the fall, (1)
the collision (2)
and the rise. (3)

Seg. 1: $v_i = 0$, $-g = a_y$

$v_f^2 = \cancel{v_i^2} + 2a_y y$

$y = -1.3\,m$

$v_f = \sqrt{2(-9.8)(-1.3)} = -5.048\ m/s$

final for seg. 1 becomes initial for seg. 2.

v_i (collision) $= 5.048\ m/s$, down

Seg. 3 $v_i = ?$, positive. $y = 1.1\,m$

$v_f = 0$

$\cancel{v_f^2}^{\,0} = v_i^2 + 2a_y y$

$v_i^2 = -2a_y y$

$v_i = \sqrt{-2(-9.8)(1.1)} = 4.643\ m/s$, up

initial for seg. 3 was final for seg. 2.

v_f (collision) $= 4.643\ m/s$, up.

Collision (seg 2)

$\Delta p = m\,\Delta v = 0.057\,kg\,(4.643 - (-5.048))\ m/s$

$\Delta p = +0.5524\ \frac{kg\,m}{s}$

Area of triangle:

$\frac{1}{2}$ base × height $= \frac{1}{2}(F_{max})\Delta t$

$\Delta p = J = \frac{F_{max}\,\Delta t}{2}$

$\Delta t = \frac{2\,\Delta p}{F_{max}} = \frac{2(0.5524)}{36} = 0.0306\ s$

$\boxed{\Delta t = 0.031\ s}$