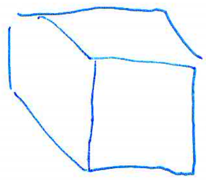
PHY131H1S Term Test —version 1 SOLUTIONS

Tuesday, February 1, 2011 Duration: 80 minutes

1. You are filling boxes with tennis balls for shipment. Each box is a cube, 0.30 m on each side, and you find you can pack a maximum of 125 tennis balls in one box. If you wish to ship 250 tennis balls in a single box, also shaped like a cube, what should be the length of one side of this box?
   A. 0.60 m
   B. 1.2 m
   C. 0.38 m
   D. 0.42 m
   E. 2.4 m



0.3m

125 balls. → double volume

→ double capacity.

$$\text{Volume} \propto l^3$$

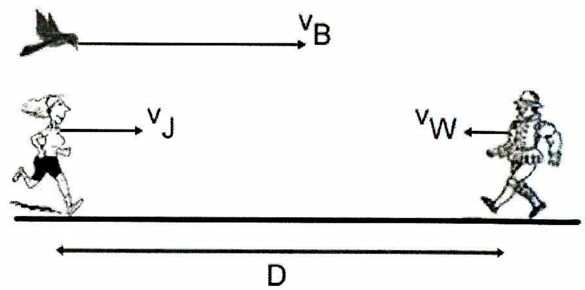
$$l \propto V^{1/3}$$

$$\frac{l_{new}}{l_{old}} = \left(\frac{V_{New}}{V_{old}}\right)^{1/3} = 2^{1/3}$$

$$l_{new} = 0.3\,m(2^{1/3})$$

$$= 0.378\,m$$

2. A jogger runs at a constant speed $v_J = 10$ km/hr to the right. A walker walks at a constant speed $v_W = 5$ km/hr to the left, towards the jogger. When the jogger and the runner are a distance $D = 3$ km away from each other, a bird flying at a constant speed $v_B = 30$ km/hr to the right passes the jogger. When the bird reaches the walker, it turns around and flies back to the jogger at the same speed. When it reaches the jogger it turns around again and flies to the walker. It continues flying back and forth between the jogger and the walker. When the jogger and walker meet each other, how far, in km, has the bird flown?



A. 3
B. 6
C. 1
D. 2
E. 9

$\longrightarrow$ + to the right.

$V_J = +10 \frac{km}{hr}$ $\quad V_W = -5 \frac{km}{hr}$

Relative velocity $V_{JW} = +10 \frac{km}{hr} - \left(-5 \frac{km}{hr}\right)$

$V_{JW} = +15 \frac{km}{hr}$

Time until they meet: $V_{JW} = \frac{D}{t_{meet}}$

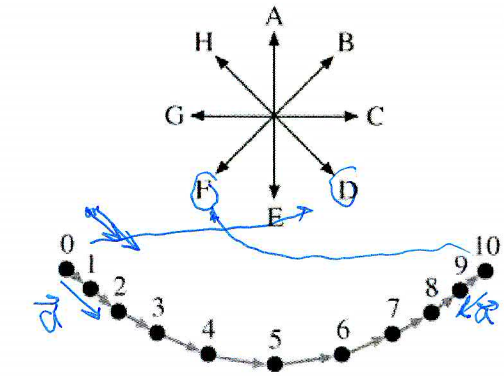
$t_{meet} = \frac{D}{V_{JW}} = \left(\frac{3}{15}\right) hr$

Meanwhile, the bird flies at $30 \frac{km}{hr}$ for $\left(\frac{3}{15}\right)$ hr.

Distance $= V_B(t_{meet}) = 30 \frac{km}{hr} \cdot \frac{3}{15} hr$

$= \frac{3(30)}{15} = \boxed{6\ \cancel{k}\ km}$

3. The motion diagram shown in the figure represents a pendulum released from rest at an angle of 45° from the vertical. The dots in the motion diagram represent the positions of the pendulum bob at eleven moments separated by equal time intervals. Also given is a "compass rose" in which directions are labeled with the letters of the alphabet. What is the direction of the acceleration of the object at moments 0 and 10?



A. D at time moment 0, F at time moment 10
B. E at time moment 0, A at time moment 10
C. B at time moment 0, H at time moment 10
D. D at time moment 0, B at time moment 10
E. zero acceleration at time moment 0, zero acceleration at time moment 10

4. A rocket, initially at rest on the ground, accelerates straight upward from rest with constant acceleration of 26.0 m/s^2. The acceleration period lasts for time 8.50 s until the fuel is exhausted. After that, the rocket is in free fall. What is the maximum height reached by the rocket?

A. 3430 m
B. 4980 m
C. 939 m
D. 2490 m
E. 4040 m

Seg 2 = freefall, $a_y = -g$

Need height at end of Seg. 2. $= y_3$

y_2

Seg.1 $a_y = +26\ m/s^2$

$v_1 = 0$

Seg. 1

$$y_2 = \cancel{y_1}^{0} + \cancel{v_1}^{0} t_1 + \frac{1}{2} a_{y1} t_1^2$$

$$\boxed{y_2 = \frac{1}{2} a_{y1} t_1^2}$$

$t_1 = 8.50\ s$

$a_{y1} = +26.0\ \frac{m}{s^2}$

$$V_2 = \cancel{V_1}^{0} + a_{y1} t_1$$

$$\boxed{V_2 = a_{y1} t_1}$$

Seg. 2 Need height change, don't care about time taken!

$$V_3^2 = V_2^2 + 2 a_{y2} (y_3 - y_2)$$

Max. height when $V_3 = 0$

$$0 = a_{y1}^2 t_1^2 - 2g(y_3 - y_2)$$

$$2g(y_3 - y_2) = a_{y1}^2 t_1^2$$

$$y_3 = y_2 + \frac{a_{y1}^2 t_1^2}{2g}$$

$$y_3 = \frac{1}{2} a_{y1} t_1^2 + \frac{a_{y1}^2 t_1^2}{2g} = \frac{1}{2}(26)8.5^2 + \frac{26^2 (8.5)^2}{2(9.8)}$$

$$= 3431\ m$$

5. A medical technician uses X-ray imaging to make 12 independent measurements of the diameter of a tumor in a patient's left lung. The average of her 12 measurements is 4.81 mm, and the estimated standard deviation is 0.14 mm. The measurements appear to be normally distributed. The technician needs to report a measurement and an error to the supervising physician in order to determine what kind of treatment to recommend for this patient. If her report states that the diameter is 4.81 mm, what should she state as the error in this average?

   A. 0.04 mm
   B. 0.14 mm
   C. 0.005 mm
   D. 0.01 mm
   E. 0.28 mm

$$\Delta \bar{x}_{est} = \frac{\Delta x}{\sqrt{N}}$$

Set $\Delta x = \sigma = 0.14\,mm$

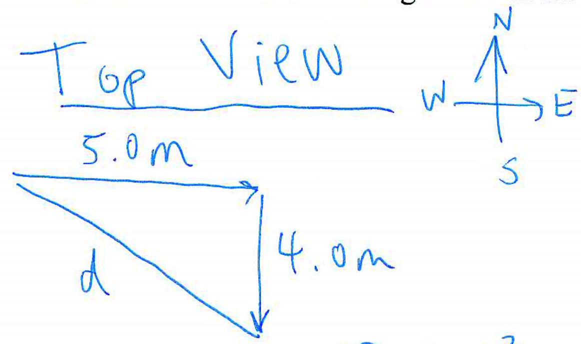
$\Rightarrow$ error in mean is

$$\frac{\sigma}{\sqrt{N}} = \frac{0.14\,mm}{\sqrt{12}}$$

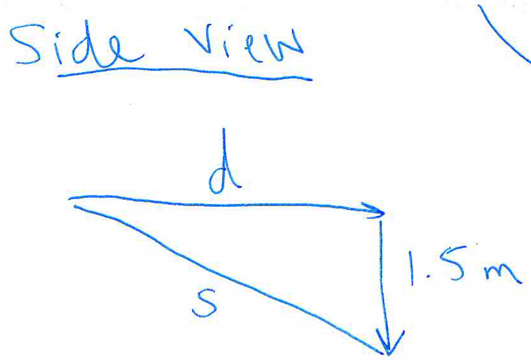
$$= 0.04\ mm$$

6. A field mouse trying to escape a hawk runs East for 5.0 m, then turns right and runs South for 4.0 m, then digs a hole and goes straight down for 1.5 m. What is the magnitude of the net displacement of the mouse?

   A. 6.4 m
   B. 6.6 m
   C. 5.2 m
   D. 5.6 m
   E. 7.4 m



$d^2 = 5^2 + 4^2$



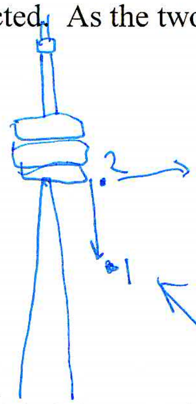
$s^2 = d^2 + 1.5^2$

$s^2 = 5^2 + 4^2 + 1.5^2$

$s = \sqrt{5^2 + 4^2 + 1.5^2}$

$= 6.6$ m

7. You drop a rock from the observation deck of the CN tower. About half a second later, long before the first rock hits the ground, you drop a second rock. Both rocks are initially released at rest, and air resistance may be neglected. As the two rocks fall, the distance between them
   A. increases.
   B. decreases.
   C. stays the same.



released at $t=0$, $v=0$

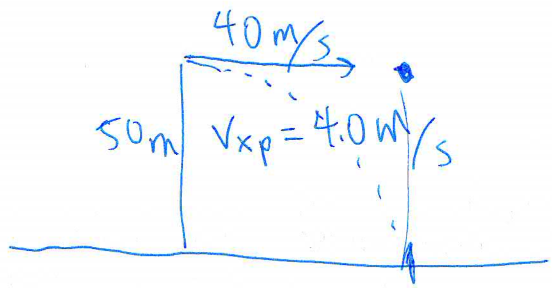
$$d_2 = \frac{1}{2}gt^2$$

$$d_1 = v_{1i}t + \frac{1}{2}gt^2$$

$$|d_2 - d_1| = \cancel{\frac{1}{2}gt^2} - \left(v_{1i}t + \cancel{\frac{1}{2}gt^2}\right)$$

$$= \boxed{v_{1i}t}$$

8. An airplane flies horizontally at 40 m/s at a constant altitude of 50 m. The pilot drops a heavy package, which falls to the flat, horizontal ground below. Where does the package land relative to the airplane's new position at the instant the package lands?
   A. 100 m behind the airplane
   B. 400 m behind the airplane
   C. directly beneath the airplane
   D. 50 m behind the airplane
   E. 500 m behind the airplane



Package is always directly beneath.

FREE-FORM IN TWO UNRELATED PARTS (12 points total)
Clearly show your reasoning and work as some part marks may be awarded. Write your final answers in the boxes provided.

PART A
In Practicals you attach a Fan Accessory to a cart, which causes it to accelerate along a metal track. You release it from rest, and measure that it travels a distance of 1.30 ± 0.04 m in a time of 1.78 ± 0.05 s. From these two measurements, and the assumption that the acceleration of the cart is constant, what do you conclude is the acceleration of the cart? [Please write your final answer in the box provided, with units and error. Both the value and error should have the correct number of significant figures.]

$d = \frac{1}{2}at^2$ Need a:

$2d = at^2$

$a = \frac{2d}{t^2} = \frac{2(1.3)}{1.78^2} = 0.8206 \ m/s^2$

Error in $t^2 =$ $\frac{\Delta z}{z} = 2\frac{\Delta t}{t}$

$\Delta z = 2\Delta t \frac{t^2}{t} = 2\Delta t \, t$

Error in $y = \frac{d}{t^2}$ $\Delta y = y\sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{2\Delta t \, t}{t^2}\right)^2}$

Error in $a = \frac{2d}{t^2}$ $\Delta a = \frac{2d}{t^2}\sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{2\Delta t}{t}\right)^2}$

$= 0.8206\sqrt{\left(\frac{0.04}{1.3}\right)^2 + \left(\frac{2(0.05)}{1.78}\right)^2}$

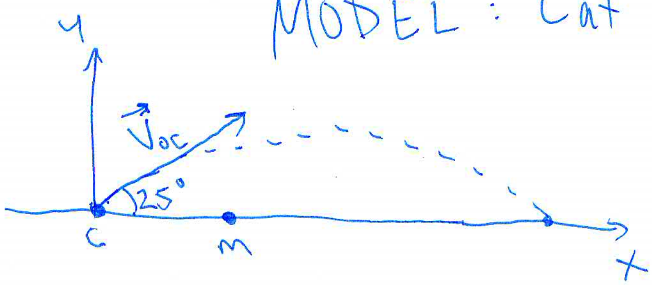
$\Delta a = 0.0526 = 0.053 \ m/s^2$

$a = 0.821 \pm 0.053 \ m/s^2$

Or $0.82 \pm 0.05 \ m/s^2$

PART B

A cat is chasing a mouse. The mouse runs in a straight line along the horizontal floor, directly away from the cat at a speed of 1.2 m/s. At a specific moment, the mouse is 0.80 m in front of the cat, and the cat leaps with an initial velocity at an angle of 25° above the horizontal. At what initial speed must the cat leap in order to land on the poor mouse? [Please write your final answer in the box provided, in units of m/s, to the correct number of significant figures.]



MODEL: Cat & Mouse as particles!

$X_{om} = 0.80\text{ m}$ ← Mouse has head start

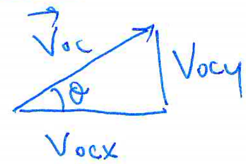
$X_{oc} = 0\text{ m}$

Find: $|\vec{V}_{oc}| = V_{oc}$, such that $X_{fc} = X_{fm}$ ← cat catches mouse

Mouse: runs at constant speed:

$$V_{xm} = \frac{X_{fm} - X_{om}}{t} \rightarrow \boxed{X_{fm} = X_{om} + V_{xm} t}$$

Cat: is a projectile



$\theta = 25°$

Components

x	y
$a_x = 0$	$a_y = -g$
$V_{cx} = V_{ocx} = V_o \cos\theta = \frac{X_{fc} - X_{oc}}{t}$	$Y_{fc} = Y_{oc} + V_{ocy} t + \frac{1}{2} a_y t^2$
$X_{fc} = X_{oc} + V_{oc} \cos\theta\, t = X_{fm}$ to catch mouse.	$Y_{fc} = Y_{oc} = 0 \quad V_{ocy} = V_{oc} \sin\theta$
$V_{oc} t \cos\theta = X_{om} + V_{xm} t$	$V_{oc} t \sin\theta = \frac{1}{2} g t^2$, $t \neq 0$, divide by t
Don't know t! Need V_{oc}	$V_{oc} \sin\theta = \frac{gt}{2}$

$v_{ic} =$

$$\boxed{t = \frac{2 V_{oc} \sin\theta}{g}}$$

$$V_{oc}\left[\frac{2V_{oc}\sin\theta}{g}\right]\cos\theta = X_{om} + V_{xm}\left[\frac{2V_{oc}\sin\theta}{g}\right]$$

solve for V_{oc}.

$$2\sin\theta\cos\theta\, V_{oc}^2 = X_{om}\, g + 2V_{xm}\sin\theta\, V_{oc}$$

$$\underbrace{2\sin\theta\cos\theta}_{a} V_{oc}^2 \underbrace{-2V_{xm}\sin\theta}_{b} V_{oc} \underbrace{-X_{om}g}_{c} = 0$$

Quadratic Equation $x \to V_{oc}$

$a = 2\sin\theta\cos\theta = 2\sin 25^\circ\cos 25^\circ = 0.7660$

$b = -2V_{xm}\sin\theta = 2(1.2)\sin 25^\circ = -1.0143$

$c = -gX_{om} = -9.8(0.8) = -7.84$

$$V_{oc} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Part B continued ...

Quadratic Equation:

$$V_{oc} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1.0143 \pm \sqrt{1.0143^2 - 4(0.7660)(-7.84)}}{2(0.7660)}$$

$$= \frac{1.0143 \pm \sqrt{25.05}}{1.532}$$

$V_{oc} = +3.929$ or -2.605 m/s.

Remember V_{oc} = speed, or magnitude of velocity, which is always positive

→ reject negative solution.

$\Rightarrow \boxed{V_{oc} = 3.9 \text{ m/s.}}$