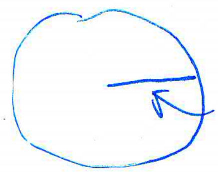
PHY131H1S **Term Test 2 —version 1** **SOLUTIONS**

Tuesday, March 15, 2011 Duration: 80 minutes

1. A 75 kg package of food and medical supplies is dropped from an airplane. The package is shaped like a sphere of radius 18 cm. Estimate terminal speed of this package. Do **not** neglect air resistance for this problem.
   A. 170 m/s
   B. 85 m/s
   C. 120 m/s
   D. 1.7 m/s
   E. 300 m/s

Cross - section Area:

$A = \pi r^2$



$r = 0.18\,m$

$$V_{term} = \sqrt{\frac{4mg}{A}}$$ (Eq. 6.19 from Knight)

$$= \sqrt{\frac{4mg}{\pi r^2}} = \sqrt{\frac{4(75)(9.8)}{3.14159(0.18)^2}}$$

$$= 169.95\ m/s. \simeq 170\ m/s$$

2. A window washer of mass M is sitting on a platform suspended by a system of cables and pulleys as shown . He is pulling on the cable with a force of magnitude F. The cables and pulleys are ideal (massless and frictionless), and the platform has mass, m. What is the magnitude of the minimum force F that allows the window washer to move upward.

A. $\frac{(M+m)g}{3}$

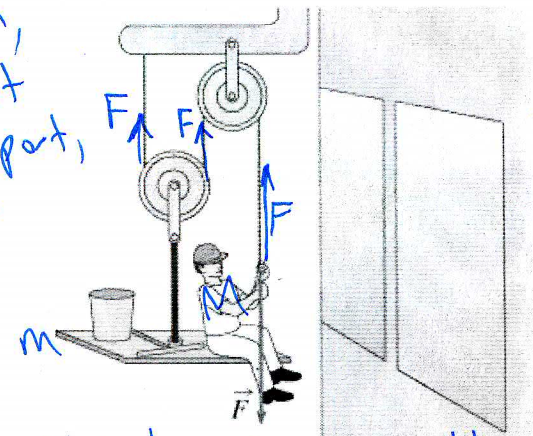
B. $\frac{Mg}{3}$

C. $(2M+m)g$

D. $3(M+m)g$

E. $\frac{Mg}{3}+mg$

In equilibrium, F will be just enough to support, and move the window washer.



2 objects:

Window Washer

$n \uparrow \uparrow F$, $Mg \downarrow$

$F_{net} = 0 = F + n - Mg \rightarrow n = Mg - F$

F = tension in cable.

n = normal force of platform on window washer.

Platform.

$F \uparrow \uparrow F$, $n \downarrow \downarrow mg$

$F_{Net} = 0 = 2F - n - mg = 0$

2 unknowns, 2 eqs.
Eliminate n, solve for F,

$$2F - (Mg - F) - mg = 0$$

$$2F - Mg + F - mg = 0$$

$$3F = mg + Mg = g(M+m)$$

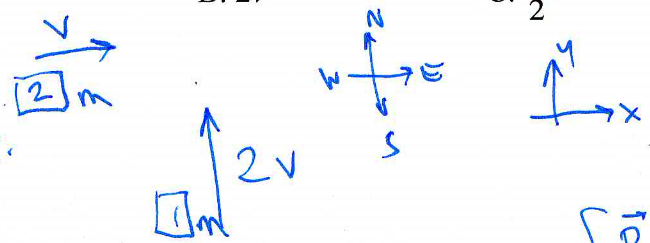
$$F = \frac{(M+m)g}{3}$$

Two cars, both of mass m, collide and stick together. Prior to the collision, one car had been traveling North at speed $2v$, while the second was traveling East at speed v. Neglect horizontal outside forces on the cars, such as friction from the road on the cars.

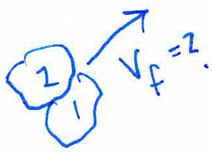
3. Immediately after the collision, what is the speed of the joined cars?

A. v B. $2v$ C. $\frac{3}{2}v$ D. $\sqrt{\frac{3}{2}}v$ E. $\frac{\sqrt{5}}{2}v$

Before:



After:



		x	y
initial	$\vec{p}_1$	0	$2mv$
	$\vec{p}_2$	mv	0
	$\vec{p}_i$	$mv = p_{ix}$	$2mv = p_{iy}$

$$|\vec{p}_f| = |\vec{p}_i| = \sqrt{p_{ix}^2 + p_{iy}^2}$$

$$(2m)v_f = \sqrt{(mv)^2 + (2mv)^2}$$

$$4\cancel{m^2}v_f^2 = \cancel{m^2}v^2 + 4\cancel{m^2}v^2$$

$$4v_f^2 = 5v^2$$

$$v_f^2 = \frac{5}{4}v^2$$

$$v_f = \pm\sqrt{\frac{5}{4}}\,v$$

speed > 0 ⇒ choose positive solution

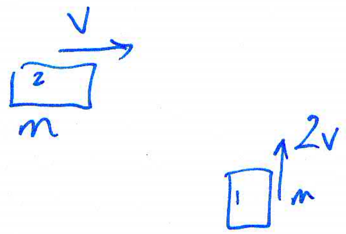
$$\boxed{v_f = \frac{\sqrt{5}}{2}v}$$

Two cars, both of mass m, collide and stick together. Prior to the collision, one car had been traveling North at speed $2v$, while the second was traveling East at speed v. Neglect horizontal outside forces on the cars, such as friction from the road on the cars.

4. What is the ratio, K_f/K_i, of the final to initial kinetic energy of the system just after and before the collision?

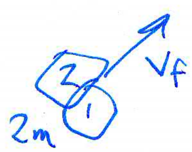
A. 1 B. $\frac{1}{2}$ C. $\frac{1}{4}$ D. $\frac{3}{4}$ E. $\frac{\sqrt{5}}{4}$

initial :



$$K_i = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$
$$= \frac{1}{2}m(2v)^2 + \frac{1}{2}mv^2$$
$$= \frac{4}{2}mv^2 + \frac{1}{2}mv^2$$
$$K_i = \frac{5}{2}mv^2$$

final :



$$v_f = \frac{\sqrt{5}}{2}v$$

$$K_f = \frac{1}{2}(2m)v_f^2 = \frac{2}{2}m\left(\frac{\sqrt{5}}{2}\right)^2 v^2$$
$$= \frac{m\,5}{4}v^2$$
$$K_f = \frac{5}{4}mv^2$$

$$\frac{K_f}{K_i} = \frac{\frac{5}{4}\cancel{mv^2}}{\frac{5}{2}\cancel{mv^2}} = \frac{\cancel{5}\cdot 2}{\cancel{5}\cdot 4} = \boxed{\frac{1}{2}}$$

5. In each of the four elevators in the tower of the Physics building is mounted a spring scale with a 0.750 kg mass hanging from it. As preparation for the 4th Practicals Session, you were advised to take a ride on the elevator and observe the reading on the spring scale. Assume that when the elevator was stationary, you observed the reading on the spring scale to be 7.35 N. You then pressed the "B" button to go down to the basement, and just before reaching the basement, as the elevator was slowing down, you observed the reading on the spring scale to be 8.35 N. What was the acceleration of the elevator at the moment you made this second observation?

   A. 11.1 m/s^2, down
   B. 1.33 m/s^2, down
   C. 1.33 m/s^2, up
   D. 11.1 m/s^2, up
   E. Something is wrong with the data; the reading on the spring scale should have been less than or equal to 7.35 N.

slowing down $\downarrow\vec{v}$ $\uparrow\vec{a}$

fbd for mass

$\uparrow F_{sp}$ $\downarrow mg$ $\uparrow\vec{a}$ = up.

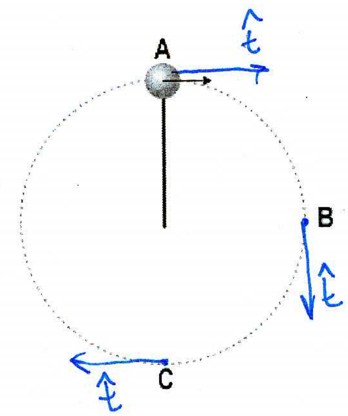
$$\vec{a} = \frac{\vec{F}_{Net}}{m} = \frac{F_{sp} - mg}{m}$$

$$|\vec{a}| = \frac{8.35 - 7.35}{0.75}$$

$$= 1.33\ m/s^2$$

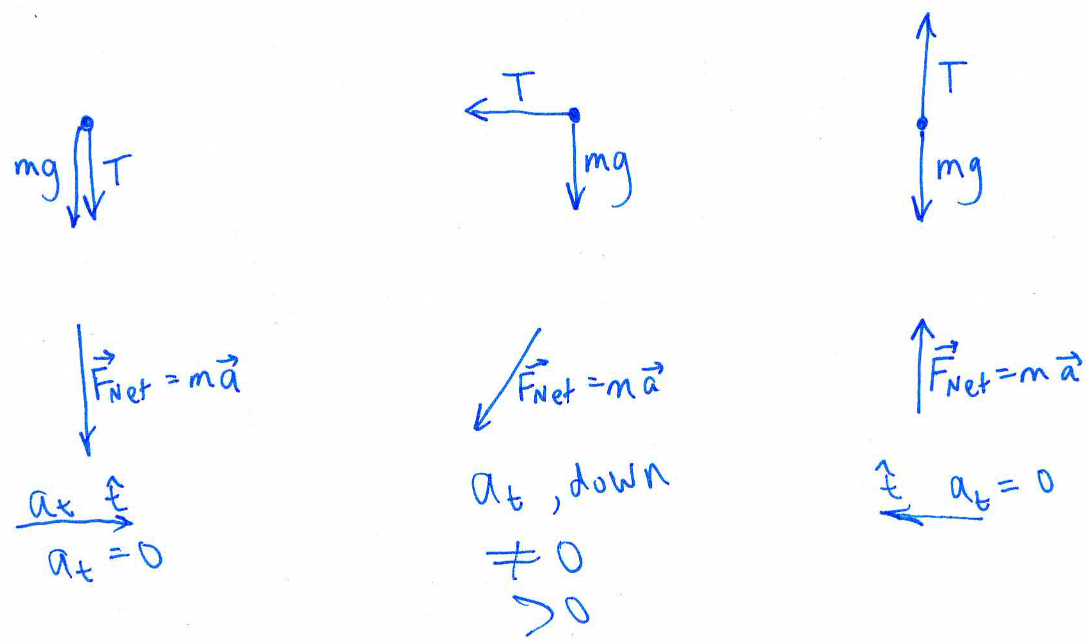
6. A massive ball on a massless string is moving in a vertical circle as shown. Air resistance is negligible. Three points A, B and C on the circle are shown. At which of the three points is the magnitude of the tangential acceleration of the ball, a_t, the largest?

   A. A
   B. B
   C. C
   D. The tangential acceleration has the same non-zero magnitude at all 3 points.
   E. The tangential acceleration is zero at all 3 points.



Free-body diagrams:

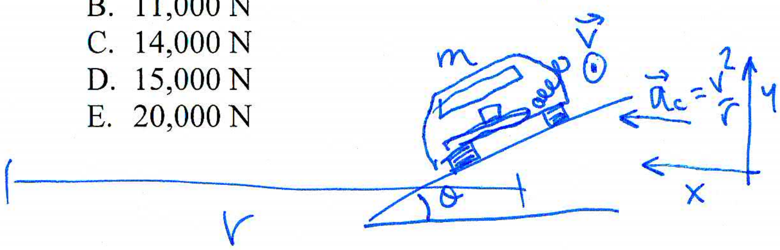
A | B | C



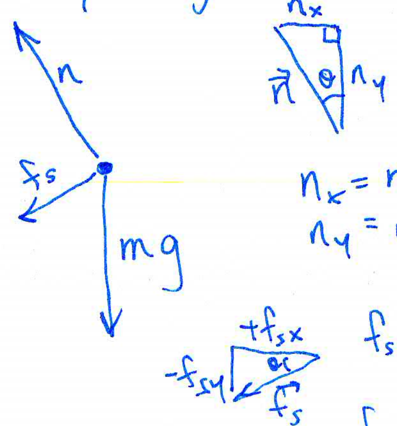
7. A concrete highway curve of radius 70.0 m is banked at a 15° angle. What is the magnitude of the force of static friction on a 1500 kg rubber-tired car traveling at 23 m/s around this curve without slipping?

A. 7100 N
B. 11,000 N
C. 14,000 N
D. 15,000 N
E. 20,000 N

$\mu_s = 1.00 \quad \mu_K = 0.80 \quad \mu_r = 0.02$



free body diagram for car:



$n_x = n \sin\theta$
$n_y = n \cos\theta$

$f_{sx} = f_s \cos\theta$
$f_{sy} = -f_s \sin\theta$

	x	y
n	$n \sin\theta$	$n \cos\theta$
f_s	$f_s \cos\theta$	$-f_s \sin\theta$
mg	0	$-mg$
F_{Net}	$n \sin\theta + f_s \cos\theta = m a_x = \frac{mv^2}{r}$ (✱✱)	$n\cos\theta - f_s \sin\theta - mg = 0$ (✱) since $a_y = 0$

2 eqs, 2 unknowns: n and f_s. Eliminate n, solve for f_s.

$$(✱) \Rightarrow n = \frac{mg + f_s \sin\theta}{\cos\theta}, \text{ plug into } (✱✱):$$

$$\left[\frac{mg + f_s \sin\theta}{\cos\theta}\right]\sin\theta + f_s \cos\theta = \frac{mv^2}{r}$$

$$\frac{mg\sin\theta}{\cos\theta} + \frac{f_s \sin^2\theta}{\cos\theta} + f_s \cos\theta = \frac{mv^2}{r}$$

$$f_s \sin^2\theta + f_s \cos^2\theta = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

$$f_s(\sin^2\theta + \cos^2\theta = 1) = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

$$= 1500\left[\frac{23^2}{70}\cos 15° - 9.8 \sin 15°\right]$$

$$= 7100 \text{ N}$$

NOTE
$n = \frac{mg + f_s \sin\theta}{\cos\theta}$
$n = 17{,}133$ N
$f_{s,max} = \mu_s n$
$= 17{,}133 \text{ N} > f_s$ ✓ okay.

8. You drop a rock from the observation deck of the CN tower. Neglect air resistance. Which one of the following four statements most correct?
   A. The kinetic energy of the rock increases by equal amounts in equal time intervals as it falls.
   B. The kinetic energy of the rock increases by equal amounts in equal distances as it falls.
   C. Both A and B are true.
   D. Neither A nor B are true.

Time interval Δt determines speed:

$$a = g = \frac{V_f - V_i}{\Delta t} \qquad \rightarrow K_f = \frac{1}{2} m V_f^2$$

$$\Rightarrow V_f = V_i + g\Delta t$$

Not linear.

Distance fallen is Δy.

Conservation of Energy:

$$U_f + K_f = U_i + K_i$$

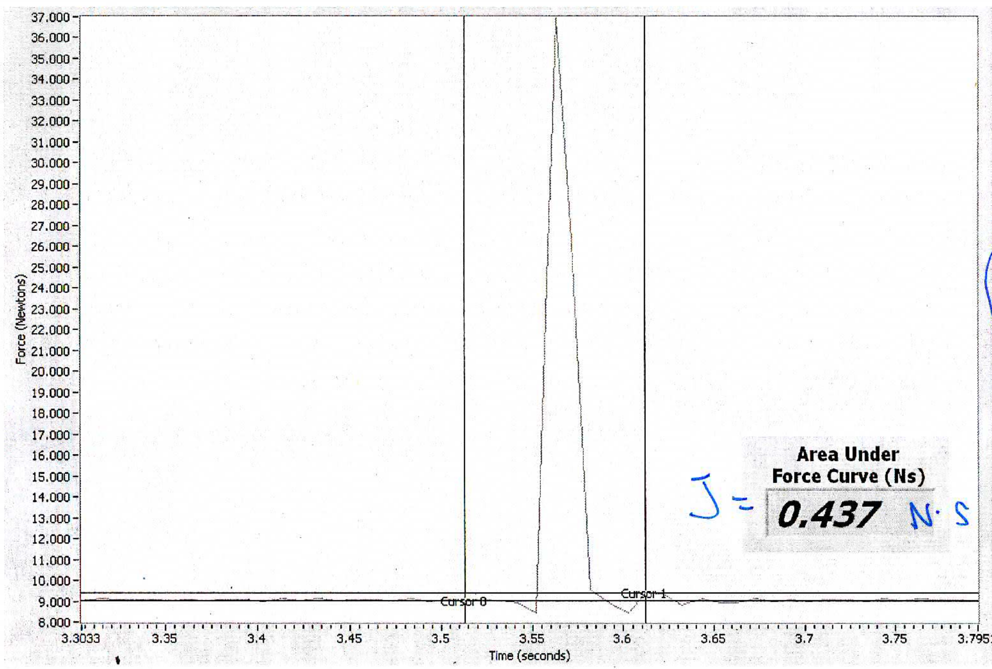
$$K_f = (U_i - U_f) + K_i$$

$$K_f = K_i - \Delta U = K_i - mg(\Delta y)$$

linear.

PART A

In Practicals students rolled a 0.505 kg cart along a low-friction track, and measured its speed with Motion Sensor. According to the computer, just before the cart collided with the bumper of a fixed Force Sensor, its speed was 45.5 cm/s. The cart then collided with the bumper on the Force Sensor, and reversed its direction. Shown is a plot of force versus time as measured by the Force Sensor. The area under the curve, as measured between the cursors which are marked with vertical lines, is 0.437 N s. From this information, predict the speed of the cart immediately after the collision. [Please express your final answer in the box provided in cm/s to 3 significant figures.]



Impulse - Momentum theorem:

Eq. 9.8 from Knight

$\Delta p = J$

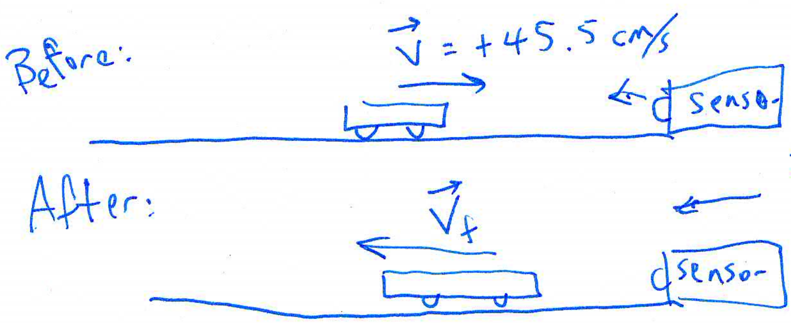
$p = mv \rightarrow \Delta p = m\,\Delta v$

$m\,\Delta v = J$

$\Delta v = \frac{J}{m} = \frac{0.437}{0.505}$

$\Delta v = 0.86535$ m/s

$\vec{\Delta v} = -86.535$ cm/s



$\vec{v}_f = 45.5 - 86.535$

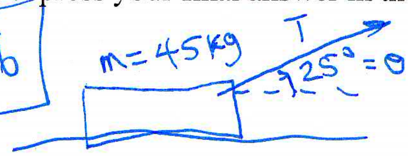
$= -41.035$ cm/s

$v_f =$ 41.0 cm/s

PART B

It's a winter Saturday and you're pulling a friend along a level, snow-covered road on a wooden sled. The rope pulls up on the sled at a 25° angle above the horizontal. The mass of the sled, with your friend on it, is 45 kg. With what force must you pull on the rope in order to walk forward at a steady speed of 1.3 m/s? [Please express your final answer in the box provided in N to 2 significant figures.]

$\boxed{\mu_K = 0.06}$



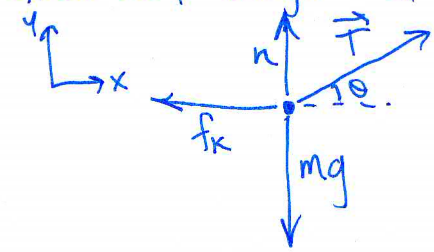
$\vec{v} = 1.3$ m/s

constant

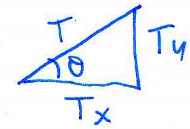
$\Rightarrow \vec{a} = 0$

equilibrium.

free body diagram of sled:



$f_K = \mu_K n$



$T_y = T\sin\theta$

$T_x = T\cos\theta$

	x	y
T	$T\cos\theta$	$T\sin\theta$
mg	0	$-mg$
$f_K = \mu_K n$	$-\mu_K n$	0
n	0	n
F_{Net}	$T\cos\theta - \mu_K n = 0$	$T\sin\theta - mg + n = 0$

$\underline{n = mg - T\sin\theta}$

2 equations, 2 unknowns: n, T

Eliminate n, solve for T.

$$T\cos\theta - \mu_K(mg - T\sin\theta) = 0$$

$$T\cos\theta - \mu_K mg + \mu_K T\sin\theta = 0$$

$$T(\cos\theta + \mu_K\sin\theta) = \mu_K mg$$

$$T = mg\left(\frac{\mu_K}{\mu_K\sin\theta + \cos\theta}\right) = 45(9.8)\left[\frac{0.06}{0.06\sin 25^\circ + \cos 25^\circ}\right]$$

$= 28.40$

$F =$ 28 N