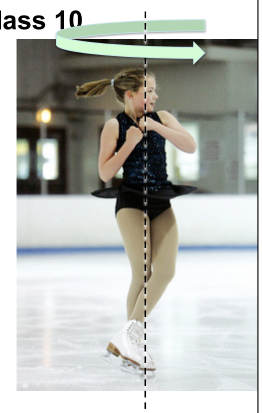
PHY131H1F Summer – Class 10



Today:
- Rotational Motion
- Newton's 2nd Law of Rotation
- Torque
- Moment of Inertia
- Centre of Mass
- Gravitational Torque
- Rotational Kinetic Energy
- Rolling without Slipping
- Equilibrium with Rotation
- Rotation Vectors
- Angular Momentum

Pre-class reading quiz on Chapter 12 (1 of 2)

Moment of inertia **is**

A. the rotational equivalent of mass.
B. the point at which all forces appear to act.
C. the time at which inertia occurs.
D. an alternative term for *moment arm*.

Pre-class reading quiz on Chapter 12 (2 of 2)

A rigid body is in equilibrium if

A. $\vec{F}_{\text{net}} = 0$.
B. $\vec{\tau}_{\text{net}} = 0$.
C. neither A nor B.
D. either A or B.
E. both A and B.

Last day I asked at the end of class:

- Why is a door easier to open when the handle is far from the hinge, and more difficult to open when the handle is in the middle?
- ANSWER:
- Torque is the rotational analog of force:
- Force causes things to accelerate along a line.
- Torque causes things to have angular acceleration.
- Torque = Force × Moment Arm
- Moment arm is the distance between where you apply the force and the hinge or pivot point.
- Putting the handle further from the hinge increases your moment arm, therefore it increases your torque for the same applied force.

Recall from Chapters 1-4:

Linear	**Rotational Analogy**
• s (or x or y) specifies position.	• θ is angular position. The S.I. Unit is radians, where 2π radians = 360°.
• Velocity: $v_x = \frac{d}{dt}(x)$ $v_y = \frac{d}{dt}(y)$	• Angular velocity: $\omega = \frac{d}{dt}(\theta)$
• Acceleration: $a_x = \frac{d}{dt}(v_x)$ $a_y = \frac{d}{dt}(v_y)$	• Angular acceleration: $\alpha = \frac{d}{dt}(\omega)$

Linear / Rotational Analogy

Linear	**Rotational Analogy**
• x	• θ
• v_x	• ω
• a_x	• α
• Force: F_x	• Torque: τ
• Mass: m	• Moment of Inertia: I

Newton's Second Law:

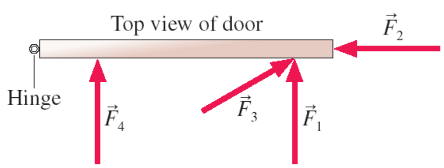
$$a_x = \frac{(F_{net})_x}{m} \qquad \alpha = \frac{\tau_{net}}{I}$$

Example 12.12

- The engine in a small airplane is specified to have a torque of 60.0 N m. This engine drives a propeller whose moment of inertia is 13.3 kg m^2. On start-up, how long does it take the propeller to reach 200 rpm?

Torque

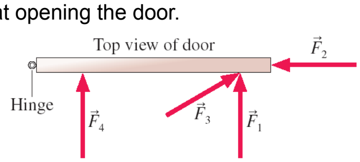
Consider the common experience of pushing open a door. Shown is a top view of a door hinged on the left. Four pushing forces are shown, all of equal strength. Which of these will be most effective at opening the door?



A. F_1
B. F_2
C. F_3
D. F_4

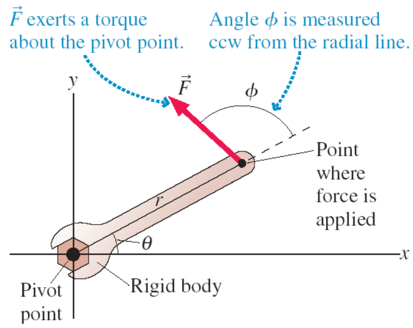
Torque

Consider the common experience of pushing open a door. Shown is a top view of a door hinged on the left. Four pushing forces are shown, all of equal strength. F_1 is most effective at opening the door.



The ability of a force to cause a rotation depends on three factors:
1. the magnitude *F* of the force.
2. the distance *r* from the point of application to the pivot.
3. the angle at which the force is applied.

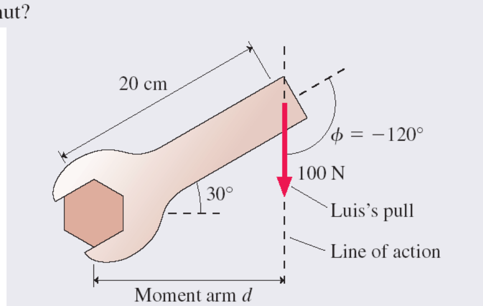
Force $\vec{F}$ exerts a torque about the pivot point.



$$\tau = rF\sin\phi$$

EXAMPLE 12.9 Applying a torque

Luis uses a 20-cm-long wrench to turn a nut. The wrench handle is tilted 30° above the horizontal, and Luis pulls straight down on the end with a force of 100 N. How much torque does Luis exert on the nut?



Consider a body made of N particles, each of mass m_i, where $i = 1$ to N. Each particle is located a distance r_i from the axis of rotation. We define moment of inertia:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum_i m_i r_i^2$$

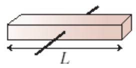
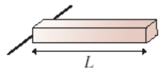
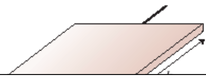
The units of moment of inertia are kg m^2. An object's moment of inertia depends on the axis of rotation.

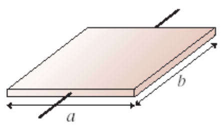
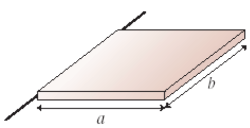
The moment of inertia

$$I = \sum_i m_i r_i^2 = \int r^2\, dm$$

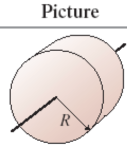
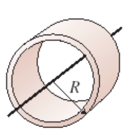
is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If I_{cm} is known, the I about a parallel axis distance d away is given by the **parallel-axis theorem:** $I = I_{cm} + Md^2$.

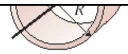
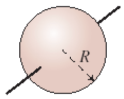
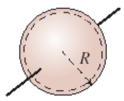
TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	
Thin rod, about center		$\frac{1}{12}ML^2$	C a
Thin rod, about end		$\frac{1}{3}ML^2$	C a
Plane or slab, about center		$\frac{1}{12}Ma^2$	S a

Object and axis	Picture	I	
Plane or slab, about center		$\frac{1}{12}Ma^2$	So ab
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Sp ab

uniform density

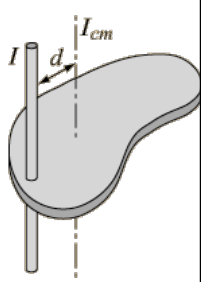
I	Object and axis	Picture	I
$\frac{1}{2}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
			

	about center		
$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

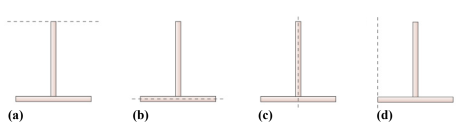
The Parallel-Axis Theorem

- Suppose you know the moment of inertia of an object when it rotates about axis 1: I_1
- You can find the moment of inertia when it is rotating about an axis 2, (I_2) which is a distance d away:

$$I_2 = I_1 + Md^2$$



Four Ts are made from two identical rods of equal mass and length. Rank in order, from largest to smallest, the moments of inertia I_a to I_d for rotation about the dotted line.

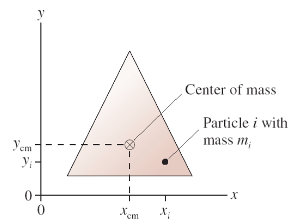


A. $I_a > I_d > I_b > I_c$
B. $I_c = I_d > I_a = I_b$
C. $I_a = I_b > I_c = I_d$
D. $I_a > I_b > I_d > I_c$
E. $I_c > I_b > I_d > I_a$

Center of Mass

The center of mass is the mass-weighted center of the object.

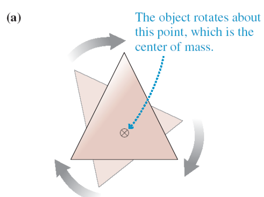
$$x_{cm} = \frac{1}{M}\int x\,dm \quad \text{and} \quad y_{cm} = \frac{1}{M}\int y\,dm$$



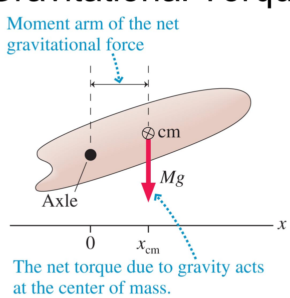
Rotation About the Center of Mass

An unconstrained object (i.e., one not on an axle or a pivot) on which there is no net force rotates about a point called the center of mass. The center of mass remains motionless while every other point in the object undergoes circular motion around it.

FIGURE 12.5 Rotation about the center of mass.



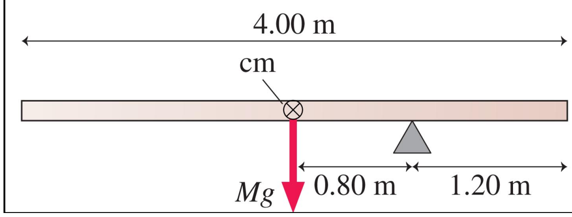
Gravitational Torque



- When calculating the torque due to gravity, you may treat the object as if all its mass were concentrated at the centre of mass.

Example 12.10

- A 4.00 m long, 500 kg steel beam is supported 1.20 m from the right end. What is the gravitational torque about the support?



- A metal hoop has the same mass and radius as a wooden disk. They are both released from rest at the top of an incline, and allowed to roll down, without slipping. Which will roll faster down the incline?

A. Metal hoop
B. Wooden disk
C. Neither; both will roll at the same speed.

"Rolling Without Slipping"

When a round object rolls without slipping, the distance the axis, or centre of mass, travels is equal to the change in angular position times the radius of the object.

$$s = \theta R$$

The speed of the centre of mass is

$$v = \omega R$$

The acceleration of the centre of mass is

$$a = \alpha R$$

Examples:

- What is the acceleration of a slipping object down a ramp inclined at angle θ? [assume no friction]
- What is the acceleration of a solid disk rolling down a ramp inclined at angle θ? [assume rolling without slipping]
- What is the acceleration of a hoop rolling down a ramp inclined at angle θ? [assume rolling without slipping]

Linear / Rotational Analogy

	Linear	Rotational Analogy
	$\vec{s}$, $\vec{v}$, $\vec{a}$	θ, ω, α
	Force: $\vec{F}$	Torque: τ
	Mass: m	Moment of Inertia: I
Newton's 2nd law:	$\vec{a} = \dfrac{\vec{F}_{net}}{m}$	$\alpha = \dfrac{\tau_{net}}{I}$
Kinetic energy:	$K_{cm} = \frac{1}{2}mv^2$	$K_{rot} = \frac{1}{2}I\omega^2$

Rotational Energy

A rotating rigid body has kinetic energy because all atoms in the object are in motion. The kinetic energy due to rotation is called **rotational kinetic energy**.

$$K_{rot} = \frac{1}{2}I\omega^2$$

Example: A 0.50 kg basketball rolls along the ground at 1.0 m/s. What is its *total* kinetic energy? [linear plus rotational]

Summary of some Different Types of Energy:

- Kinetic Energy due to linear motion of centre of mass: $K = ½\, mv^2$
- Gravitational Potential Energy $U_g = mgh$
- Spring Potential Energy: $U_s = ½\, kx^2$
- Rotational Kinetic Energy: $K_{rot} = ½\, I\omega^2$
- Thermal Energy (often created by friction)
  - An object can possess any or all of the above.
  - One way of transferring energy to or out of an object is work:
- Work done by a constant force: $W = Fr\cos\theta$

Updated Conservation of Energy…

Conservation Laws

Energy is conserved for an isolated system.

- Pure rotation $E = K_{rot} + U_g = \frac{1}{2}I\omega^2 + Mgy_{cm}$
- Rolling $E = K_{rot} + K_{cm} + U_g = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{cm}^2 + Mgy_{cm}$

Compare and Contrast Soup Cans

- About same mass
- About same radius and shape

- Thick paste, so when this can is rolling, the contents rotate along with the can as one solid object, like a solid cylinder

- Low viscosity liquid, so the can itself rolls while the liquid may just "slide" along.

Soup Race

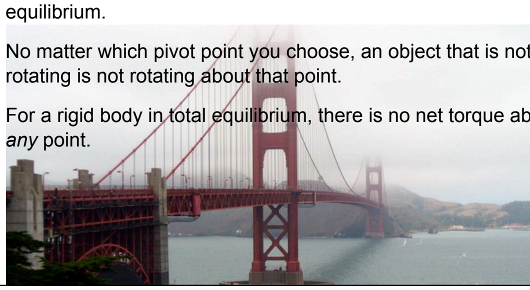
- Two soup cans begin at the top of an incline, are released from rest, and allowed to roll without slipping down to the bottom. Which will win?

  Predict:

A. Cream of Mushroom will win
B. Chicken Broth will win
C. Both will reach the bottom at about the same time.

Equilibrium When Rotation is Possible

- The condition for a rigid body to be in *static equilibrium* is that there is no net force and no net torque.
- An important branch of engineering called *statics* analyzes buildings, dams, bridges, and other structures in total static equilibrium.
- No matter which pivot point you choose, an object that is not rotating is not rotating about that point.
- For a rigid body in total equilibrium, there is no net torque about *any* point.



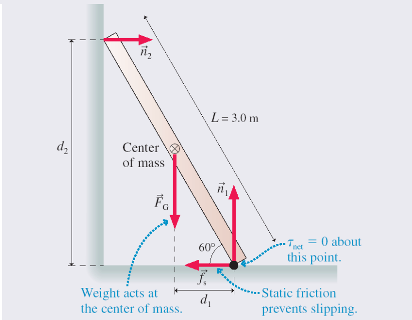
Static Equilibrium Problems

- In equilibrium, an object has no net force and no net torque.
- Draw an extended free-body diagram that shows where each force acts on the object.
- Set up x and y axes, and choose a rotation axis. All of these choices should be done to simplify your calculations.
- Each force has an x and y component and a torque. Sum all of these up.
- Three equations which you can use are:

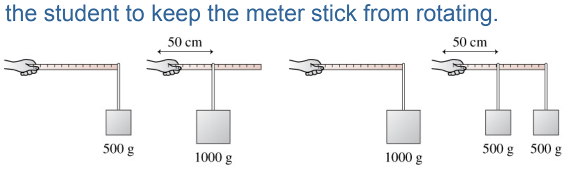
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

EXAMPLE 12.17 Will the ladder slip?

A 3.0-m-long ladder leans against a frictionless wall at an angle of 60°. What is the minimum value of μ_s, the coefficient of static friction with the ground, that prevents the ladder from slipping?



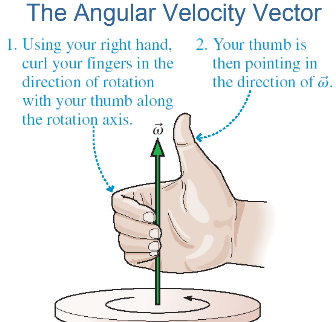
A student holds a meter stick straight out with one or more masses dangling from it. Rank in order, from most difficult to least difficult, how hard it will be for the student to keep the meter stick from rotating.



(a) (b) (c) (d)

A. c > b > d > a
B. b = c = d > a
C. c > d > b > a
D. c > d > a = b
E. b > d > c > a

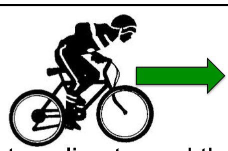
The Angular Velocity Vector



- The magnitude of the angular velocity vector is ω.
- The angular velocity vector points along the axis of rotation in the direction given by the right-hand rule as illustrated above.

Linear / Rotational Analogy

	Linear	Rotational Analogy
	$\vec{s}$, $\vec{v}$, $\vec{a}$	θ, ω, α
	Force: $\vec{F}$	Torque: τ
	Mass: m	Moment of Inertia: I
Newton's 2nd law:	$\vec{a} = \frac{\vec{F}_{net}}{m}$	$\alpha = \frac{\tau_{net}}{I}$
Kinetic energy:	$K_{cm} = \frac{1}{2}mv^2$	$K_{rot} = \frac{1}{2}I\omega^2$
Momentum:	$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$



- A bicycle is traveling toward the right.
- What is the direction of the angular momentum of the wheels?

A. left
B. right
C. into page
D. out of page
E. up

$\vec{L} = I\vec{\omega}$

The Law of Conservation of Momentum

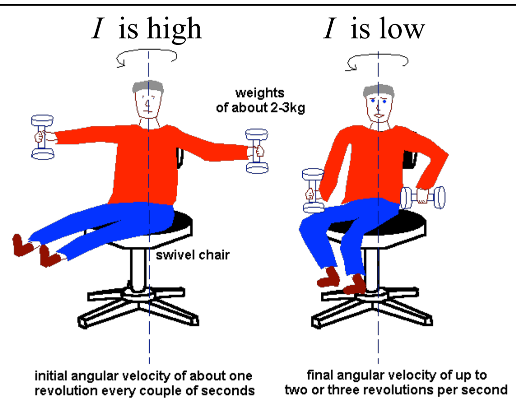
- If there is no net external force on a system, then its momentum is a constant.

The Law of Conservation of Energy

- If there is no work or heat being exchanged with a system and its surroundings, then its energy is constant.

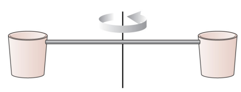
The Law of Conservation of Angular Momentum

- If there is no net external torque on a system, then its angular momentum is a constant.



$\vec{L} = I\vec{\omega}$ is constant when there are no external torques.

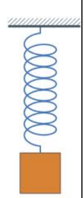
Two buckets spin around in a horizontal circle on frictionless bearings. Suddenly, it starts to rain. As a result,



A. The buckets speed up because the potential energy of the rain is transformed into kinetic energy.
B. The buckets continue to rotate at constant angular velocity because the rain is falling vertically while the buckets move in a horizontal plane.
C. The buckets slow down because the angular momentum of the bucket + rain system is conserved.
D. The buckets continue to rotate at constant angular velocity because the total mechanical energy of the bucket + rain system is conserved.
E. None of the above.

Before Next Class:

- Read Chapter 14 of Knight.
- Note: we are skipping Chapter 13 for this course.
- Complete MasteringPhysics.com Problem Set 8 due by June 20 at 11:59pm



- Something to think about: A block is oscillating on a spring with a period of 2 seconds.
- What is the period if the mass is doubled?
- What is the period if the spring constant is doubled?