PHY131H1F Summer – Class 11

Today:
- Oscillations, Repeating Motion
- Simple Harmonic Motion
- Oscillations follow a sine or cosine function
- Hanging Springs
- The Pendulum
- Damped Oscillations
- Driven Oscillations; Resonance

Italian opera singer Luigi Infantino tries to break a wine glass by singing top 'C' at a rehearsal.

A little pre-class reading quiz on Ch.14… (1 of 2)

The starting conditions of an oscillator are characterized by

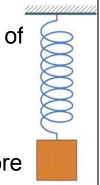
A. the initial acceleration.
B. the phase constant.
C. the phase angle.
D. the frequency.
E. the spring constant.

A little pre-class reading quiz on Ch.14… (2 of 2)

What term is used to describe an oscillator that “runs down” and eventually stops?

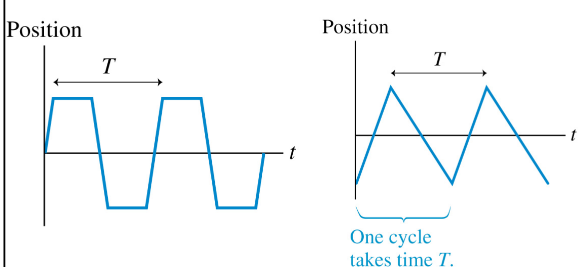
A. Tired oscillator
B. Out of shape oscillator
C. Damped oscillator
D. Resonant oscillator
E. Driven oscillator

Last day I asked at the end of class:



- A block is oscillating on a spring with a period of 2 seconds.
- What is the period if the mass is doubled?
- ANSWER:
- Longer than 2 seconds (2.8 s actually) – a more massive object on the same spring oscillates slower
- What is the period if the spring constant is doubled?
- ANSWER:
- Shorter than 2 seconds (1.4 s actually) – the same object on a stronger spring oscillates faster

Some oscillations are ***not*** sinusoidal:



Period, frequency, angular frequency

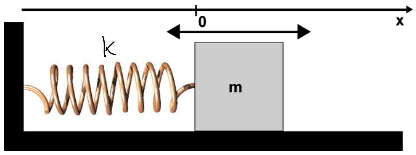
- The time to complete one full cycle, or one oscillation, is called the period, T.
- The frequency, f, is the number of cycles per second. Frequency and period are related by

$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

- The oscillation frequency f is measured in cycles per second, or Hertz.
- We may also define an angular frequency ω in radians per second, to describe the oscillation.

$$\omega \text{ (in rad/s)} = \frac{2\pi}{T} = 2\pi f \text{ (in Hz)}$$

The Spring-Mass System



The force exerted on the mass by the spring:

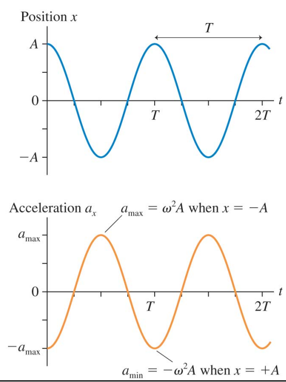
$F = -k\,x$ (Hooke's Law)

$F = m\,a$ (Newton's Second Law)

Combine to form a differential equation:

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

FIGURE 14.13 Position and acceleration graphs for an oscillating spring. We've chosen $\phi_0 = 0$.



$$F_s = -kx$$

$$\Downarrow$$

$$a = \frac{F_{Net}}{m}$$

$$\Downarrow$$

$$a_x = -\frac{k}{m}x$$

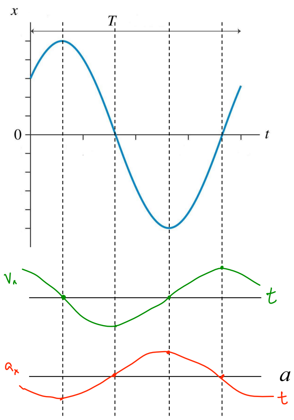
Simple Harmonic Motion

If the initial position of an object in SHM is not A, then we may still use the cosine function, with a phase constant measured in radians.

In this case, the two primary kinematic equations of SHM are:

$$x(t) = A\cos(\omega t + \phi_0)$$

$$v_x(t) = -\omega A\sin(\omega t + \phi_0) = -v_{max}\sin(\omega t + \phi_0)$$

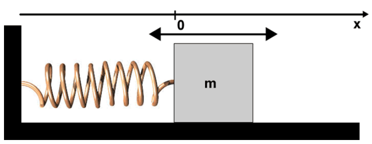


Simple Harmonic Motion (SHM)

$$x = A\cos(\omega t + \phi_0)$$

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi_0)$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi_0)$$



A mass is oscillating on a spring in S.H.M. When it passes through its equilibrium point, an external "kick" suddenly decreases its speed, but then it continues to oscillate. As a result of this slowing, the frequency of the oscillation

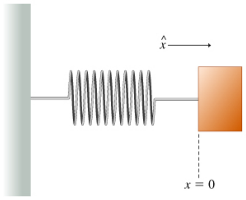
A. goes up
B. goes down
C. stays the same

S.H.M. notes.

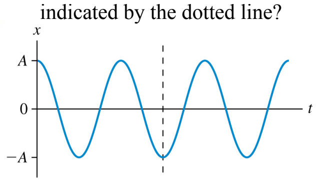
- The frequency, f, is set by the properties of the system. In the case of a mass m attached to a spring of spring-constant k, the frequency is always

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

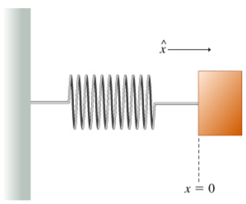
- A and ϕ_0 are set by the initial conditions: x_0 (initial position) and v_0 (initial velocity).
- A turns out to be related to the total energy of the spring oscillator system: $E = \frac{1}{2} k A^2$.



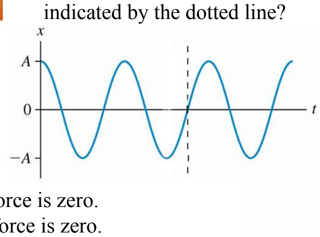
This is the position graph of a mass on a spring. What can you say about the velocity and the force at the instant indicated by the dotted line?



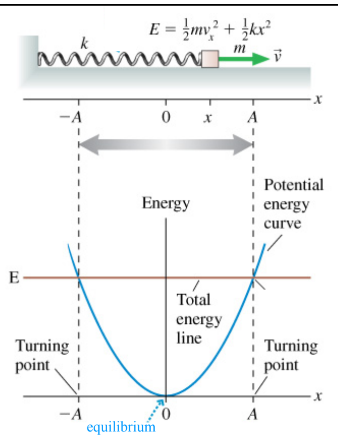
A. Velocity is positive; force is zero.
B. Velocity is negative; force is zero.
C. Velocity is negative; force is to the right.
D. Velocity is zero; force is to the right.
E. Velocity is zero; force is to the left.



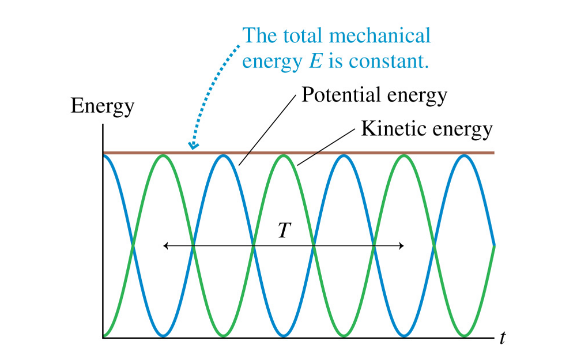
This is the position graph of a mass on a spring. What can you say about the velocity and the force at the instant indicated by the dotted line?

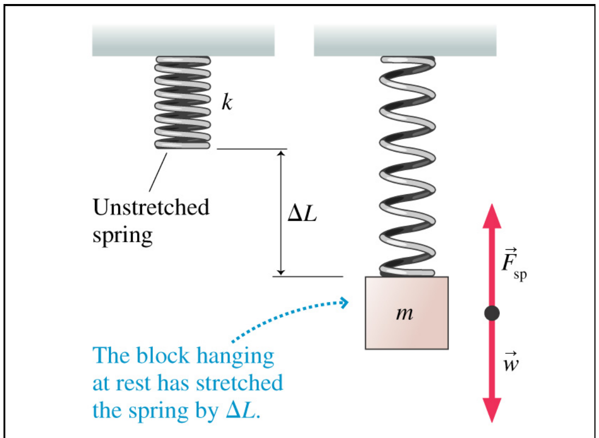


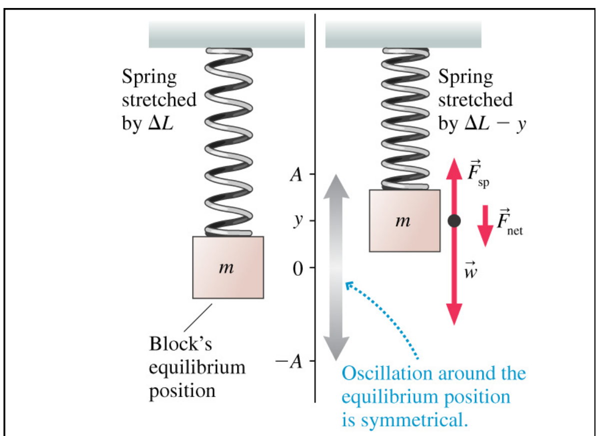
A. Velocity is positive; force is zero.
B. Velocity is negative; force is zero.
C. Velocity is negative; force is to the right.
D. Velocity is zero; force is to the right.
E. Velocity is zero; force is to the left.



$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(v_{max})^2 \quad \text{(conservation of energy)}$$

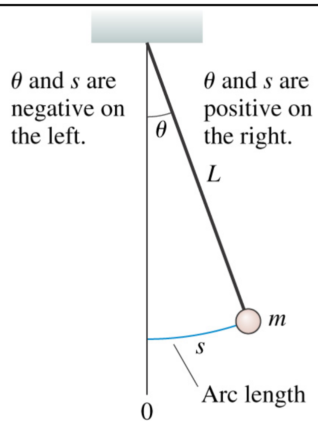






EXAMPLE 14.7 Bungee oscillations

An 83 kg student hangs from a bungee cord with spring constant 270 N/m. The student is pulled down to a point where the cord is 5.0 m longer than its unstretched length, then released. Where is the student, and what is his velocity 2.0 s later?



The Pendulum

Suppose we restrict the pendulum's oscillations to small angles (< 10°). Then we may use the **small angle approximation** $\sin\theta \approx \theta$, where θ is measured in radians. Since $\theta = s/L$, the net force on the mass is

$$(F_{net})_t = -\frac{mg}{L}s$$

and the angular frequency of the motion is found to be

$$\omega = 2\pi f = \sqrt{\frac{g}{L}}$$

Mass on Spring versus Pendulum

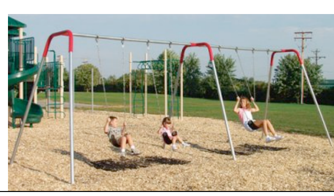
	Mass on a Spring	Pendulum
Condition for S.H.M.	Small oscillations	Small angles
Angular frequency	$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{L}}$
Period	$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$

Two pendula have the same length, but different mass. The force of gravity, F=mg, is larger for the larger mass. Which will have the longer period?

A. the larger mass
B. the smaller mass
C. neither

A person swings on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead, two people sit side-by-side on the same swing, the natural frequency of the swing is

A. greater
B. the same
C. smaller

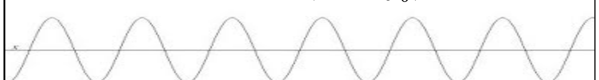


A person swings on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead, the person stands on the swing, the natural frequency of the swing is

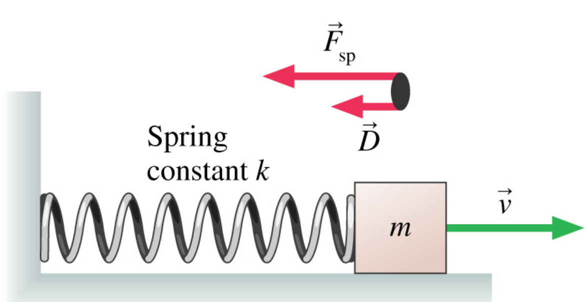
A. greater
B. the same
C. smaller

Simple Harmonic Motion (SHM)

- If the net force on an object is a linear restoring force (ie a mass on a spring, or a pendulum with small oscillations), then the position as a function of time is related to cosine: $x = A\cos(\omega t + \phi_0)$



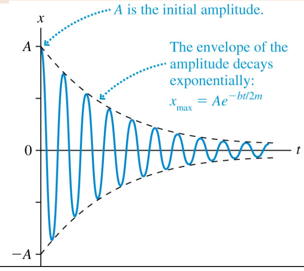
- Cosine is a function that goes forever, but in real life, due to friction or drag, all oscillations eventually slow down.



Damped Oscillations

When a mass on a spring experiences the force of the spring as given by Hooke's Law, as well as a drag force of magnitude $|D|=bv$, the solution is

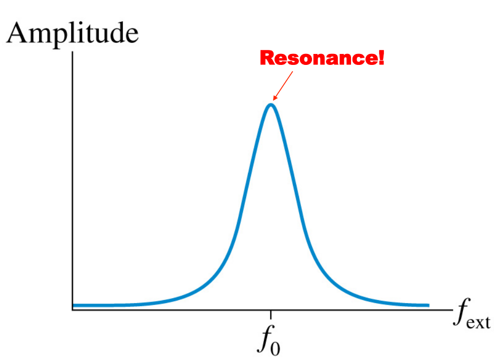
$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0) \quad \text{(damped oscillator)}$$



Driven Oscillations and Resonance

- Consider an oscillating system that, when left to itself, oscillates at a frequency f_0. We call this the **natural frequency** of the oscillator.
- Suppose that this system is subjected to a *periodic* external force of frequency f_{ext}. This frequency is called the **driving frequency**.
- The amplitude of oscillations is generally not very high if f_{ext} differs much from f_0.
- As f_{ext} gets closer and closer to f_0, the amplitude of the oscillation rises dramatically.

14.8 Externally Driven Oscillations



Before Next Class (the last class!):

- Read Chapter 15, sections 15.1 to 15.4 only.
- Note we are skipping sections 15.5 and 15.6 for this course.
- Complete MasteringPhysics.com Problem Set 9 due tomorrow by 11:59pm
- Something to think about: If you stand on a waterproof bathroom scale in a wading pool, so that part of your legs are immersed in the water, will your measured weight be different than normal? If so, why?