Relativity Module
Student Guide

Concepts of this Module

- Events that can cause other events
- Synchronising clocks
- Simultaneity
- Time dilation and inertial reference frames
- Rigid bodies and relativity
- Tachyons
- Geometric approaches to relativity
- Photons and relativity
- Mass-energy equivalence
- The Equivalence Principle
- Geometry

The Activities

Course Concepts *Activity 1*

Event 1 occurs at $x = 0$ and $t = 0$.

A. Event 2 occurs at x = 1200 *m* and is caused by Event 1. When is the earliest that Event 2 can occur. Explain.
B. Event 0 occurred at x = -4,000 *m* at time t = -1.0 μs. Could Event 0 have caused Event 1? Explain.

Activities 2 and 3 refer to the following situation.

Assume Toronto and Montreal are exactly 500 km apart in the Earth frame of reference. Assume that Kingston is exactly half way between Toronto and Montreal. They are all, of course, stationary relative to each other and the Earth. Ignore any effects due to the Earth's rotation on its axis. Ignore any effects due to the Earth's gravitational field. Assume the surface of the Earth is flat. Assume that the speed of light in the air is exactly equal to c.

Note that if any object is moving relative to you, its length along the direction of motion is $L = L_0\sqrt{1 - v^2/c^2}$, where L_0 is the "rest-length" of the object. For everyday speeds, L

is almost exactly equal to L_0, which is why we don't normally notice length-contraction. However, for speeds approaching the speed of light, $L < L_0$, and as the speed of an object approaches c, its length approaches zero!

A powerful searchlight is in Toronto, pointed towards Montreal. A second searchlight is in Montreal, pointed towards Toronto. Initially the two searchlights are turned off. Assume that both searchlights are visible from each other and from any point between Toronto and Montreal.

Course Concepts *Activity 2*

A. You are stationary in Kingston. With two powerful telescopes you are looking at a clock in Toronto and another clock in Montreal. You see that they read the same time. Are the clocks synchronized? If yes, explain. If no, which clock is ahead and by how much?
B. You are on top of the CN Tower in Toronto right beside a clock. With a powerful telescope you look at a clock in Montréal, and see that it reads exactly the same time you see on the clock on the CN Tower. Are the clocks synchronized? If yes, explain. If no, which clock is ahead and by how much?
C. You are in Montreal with a clock and see that it reads a time of 12.000000 s. With a powerful telescope you simultaneously look at a clock in Toronto. If the two clocks are synchronized, what time should you see the Toronto clock read?

Course Concepts *Activity 3*

A. You are in a very fast bullet-train traveling from Toronto to Montreal at 0.8 c relative to the Earth. [This is a thought-experiment; if your train was actually moving this fast in real-life you could get from Toronto to Montreal in 2 milliseconds!] For you, what is the distance between Toronto and Montreal? For you what is the distance between Toronto and Kingston? [Note that in your own personal reference frame, you and the train are stationary, and it is the earth and all the cities on it that are moving at 0.8 c.]
B. The two searchlights in Toronto and Montreal are quickly turned on and off, both emitting quick flashes of light. **For you, the two flashes were emitted simultaneously.** Imagine you were just leaving Toronto when the searchlight in Toronto is turned on. You see the flash from the searchlight in Toronto instantaneously. Sketch the positions of the two flashes of light a very brief moment after they are emitted, indicating their speeds and the distance between them for you. How long should it take before you see the flash from Montreal? Be sure to clearly indicate how you arrived at your answer.

C. Another member of your Team took a slightly earlier train, and is currently traveling from Toronto to Montreal at 0.8 c. What is your Teammate's speed relative to you? Imagine your Teammate is traveling through Kingston when the flashes are emitted. Are the two flashes of light emitted simultaneously for your Teammate? Will he/she see the flashes simultaneously? If yes, how long after the flashes will he/she see them? If no, which flash will he/she see first and by how much? Add your Teammate to your sketch from Part B.
D. Your Instructor lives in a house in Kingston, and is stationary relative to Kingston and the Earth. What is your Instructor's speed relative to you? Will your Instructor see the flashes from the two searchlights of Part B simultaneously? If no, which flash will he/she see first and by how much as measured by you? Explain. You may find it useful to add your Instructor to the sketch from Parts B and C.
E. Imagine your Instructor has an ipod that will begin playing music when it receives a flash of light, and quits when it receives a second flash of light. In the instructor's reference frame, stationary relative to the Earth, does the ipod turn on, and if so, for how long? Is there any music? In a train's reference frame, moving at 0.8 c, does the ipod turn on, and if so, for how long? Is there any music?

Activity 4

George and Helen are twins, born at the same time (a biological impossibility). George stays at home on Earth, which we assume is a good inertial reference frame. Helen is an astronaut who blasts off for a distant star, travels to the star at high speed, and then turns around and returns to Earth. After she lands on Earth she will be younger than George.

A. Draw a spacetime diagram for a reference frame stationary relative to George. Include George's worldline and Helen's worldline. What can you conclude about the relation between the length of the two worldlines and which twin ends up younger?
B. Draw a spacetime diagram for an inertial reference frame in which Helen is stationary during her trip from Earth to the distant star. Include George's worldline and Helen's worldline. What can you conclude about the relation between the length of the two worldlines and which twin ends up younger? Is this consistent with your conclusion from Part A?
C. Do you think this is a general result: that when analysed from *any* inertial reference frame Helen's worldline is longer than George's, and she will end up younger than George?

Activity 5

In 1971 Hafele and Keating tested the predictions of the Theories of Relativity by flying cesium beam atomic clocks around the world on regularly scheduled commercial airline flights. One clock remained in their laboratory outside Washington DC, one was flown to the East, and the other to the West. They got the data on speeds and altitudes of the planes flying the Eastbound and Westbound clocks from the flight recorders.

When the got all three clocks back in the laboratory they compared the measured times. Here are the predicted and experimental results for the elapsed times compared to the clock that stayed in the laboratory.

	Eastbound clock (ns)	**Westbound clock (ns)**
Special Relativity prediction	lose 184 ± 18	gain 96 ± 10
General Relativity prediction	gain 144 ± 14	gain 179 ± 18
Total predicted effect	lose 40 ± 23	gain 275 ± 21
Measured	lost 59 ± 10	gained 273 ± 7

The Special Relativity prediction is because that theory predicts that moving clocks run slowly. The General Relativity prediction is because that theory predicts that clocks in gravitational fields run slowly.

A. Why does Special Relativity predict that the Eastbound clock gains time and the Westbound clock loses time? Shouldn't they both either gain time or lose time?
B. Do the predicted effects agree with the experimental data? Explain.
C. Are the calculated errors in the total predicted effect correct? How were these errors calculated?

Course Concepts *Activity 6*

Here is another thought-experiment. Imagine we have a 120 m long car and a 100 m long garage, both as measured at rest relative to the car and the garage. We will assume the garage has both a front door and a back door, making it possible for the car to drive straight through the garage and out the other side.

A. Make a scale-diagram in your notebook of two rectangles: one representing the car, and one representing the garage, both at rest. Use a scale such that 1 cm in your notebook represents 50 m in real life. When parked, does this car fit in the garage?

Instead of buying a smaller car, you might be able to squeeze this huge car into the garage by driving it extremely fast! If the car is driving at 70% of the speed of light, its length will be contracted.

B. Assume the car is moving at 0.7 c in a direction parallel to its length. How long will the car be in the frame of reference of the garage? Make a scale-diagram in your notebook of two rectangles representing the car and garage, from the **garage** reference frame. Does the car fit in the garage?

But if we are riding along with the car, the car is at rest relative to us, so its length is not contracted. In fact, in the car's reference frame, the garage is actually moving in the opposite direction at 70% the speed of light. Therefore, the garage should be length contracted.

C. According to an observer sitting in the car, the garage is moving towards them at 0.7 c. How long will the garage be in the frame of reference of the car? Make a scale-diagram in your notebook of two rectangles representing the car and garage, from the **car** reference frame. Does the car fit in the garage?

There is a nice applet available at http://www.physics.uq.edu.au/people/mcintyre/applets/relativity/relativity.html .

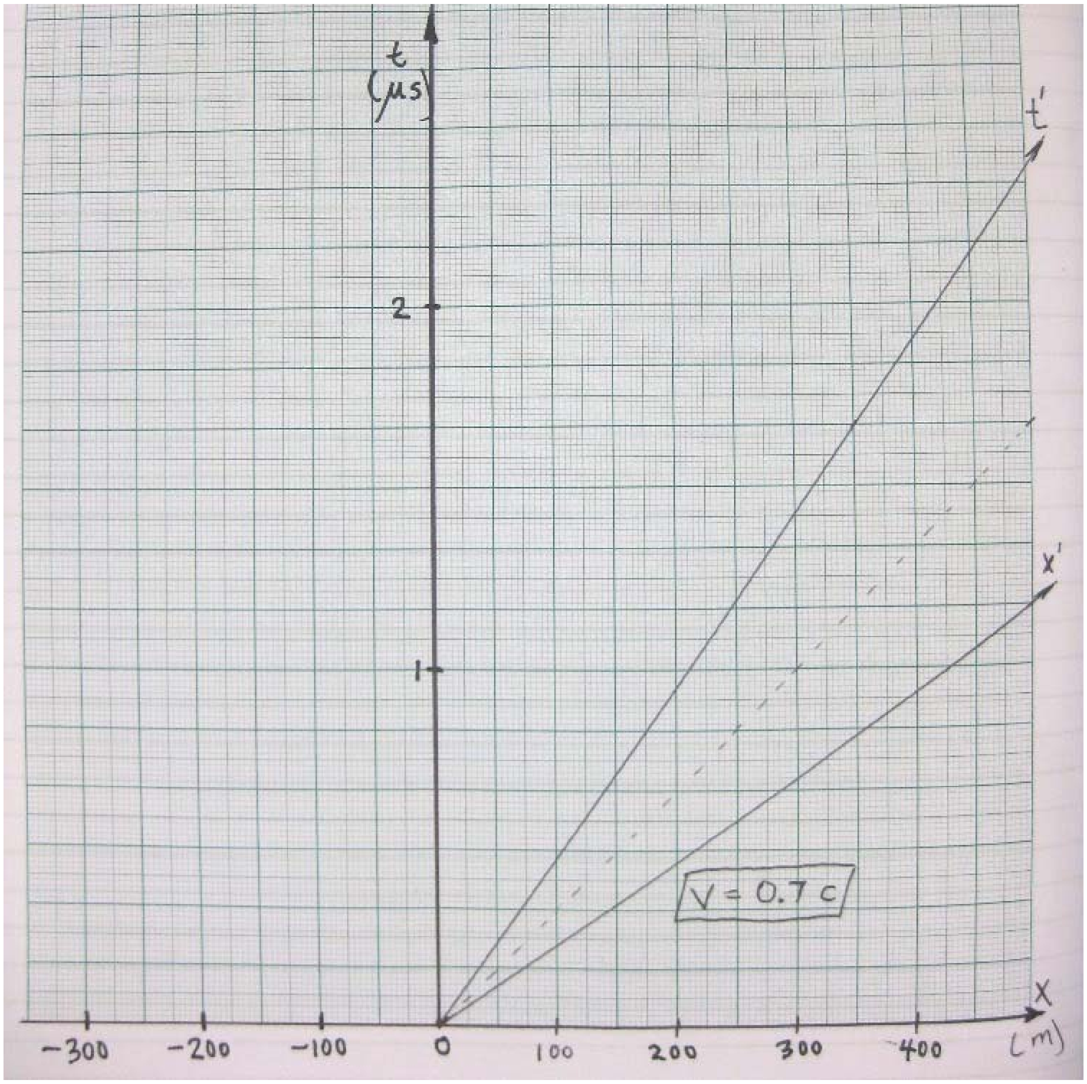
When you first open the page, or reload it, the blue car is pointed toward the red garage, with its center a distance of 200 m to the left of the center of the garage. The origin is defined to be the center of the garage, and the car is 120 m long when at rest, so the car's front begins at a position of $x = -140$ m, and the car's back begins at a position of $x = -260$ m.

D. Use the slider to adjust the speed of the car to be 0.7 c. The simulation shrinks the blue car, showing the instantaneous situation as observed by an observer at rest relative to the garage. Start the simulation, then pause it just after the car passes through the garage. A space-time diagram is plotted in the frame of reference of the garage. The blue lines show the front and back of the car, the red lines show the front and back of the garage. The ct axis defines where x=0, and it is parallel to the worldline for any object that is stationary in the garage frame. The x axis defines where t=0, and is parallel to any line connecting simultaneous events in the garage frame. Note where the back of the car enters the garage: call this event A. Note where the front of the car exits the garage: call this event B. Which event has a greater value of t as measured in the garage frame, A or B? So which event happened first? Does that mean the car was entirely in the garage at some time?

E. Make a space-time diagram to scale in your notebook in the frame of reference of the garage, similar to the one in the simulation. x is plotted on the horizontal axis, and t is plotted on the vertical axis, as shown in the photo on the next page.

   Use a horizontal scale of 1 cm on the page = 50 m in real life. Use a vertical scale of 6 cm on the page = 1 μs of time. Place the origin at the bottom of the graph, and leave 10 cm of space to the right of the origin (corresponding to 500 m). A dashed line extending from the origin up and to the right at a 45° angle represents the worldline of beam of light traveling along the x-axis.

The t' axis represents the position of the origin in the frame of reference of the car which is traveling at 0.7 c. It has a slope of 10/7 on this scale. The t' axis is parallel to the worldline for any object that is stationary in the car frame. The x' axis defines where $t'=0$, and is parallel to any line connecting simultaneous events in the car frame. The x' axis has a slope of 7/10 on this scale.



F. As was done in the applet, add and label lines that show the front and back of the car, and lines that show the front and back of the garage. Note where the back of the car enters the garage: label this event A. Note where the front of the car exits the garage: label this event B.

Add lines that pass through A and B and are parallel to the x' axis (with slopes of 7/10). Note where these lines pass the t' axis. In the car frame, events that lie along these lines are simultaneous. Which event has a greater value of t' as measured in the car frame, A or B? So which event happened first? Does that mean the car was entirely in the garage at some time?

Course Concepts *Activity 7*

For a long time people interpreted the Special Theory of Relativity to mean that *nothing* can travel faster than the speed of light. In 1967 Feinberg showed that this is not correct. There is room in the theory for objects whose speed is always greater than *c*. Feinberg called these hypothetical objects *tachyons*; the word has the same root as, say, tachometer. Attempts have been made to observe tachyons: so far all such experiments have failed.

A. The relation between energy E and mass m for an object traveling at speed u relative to us is given by:
$$E = \frac{mc^2}{\sqrt{1 - u^2/c^2}}$$
If the energy of a tachyon is a real number, what kind of number must its mass m be?

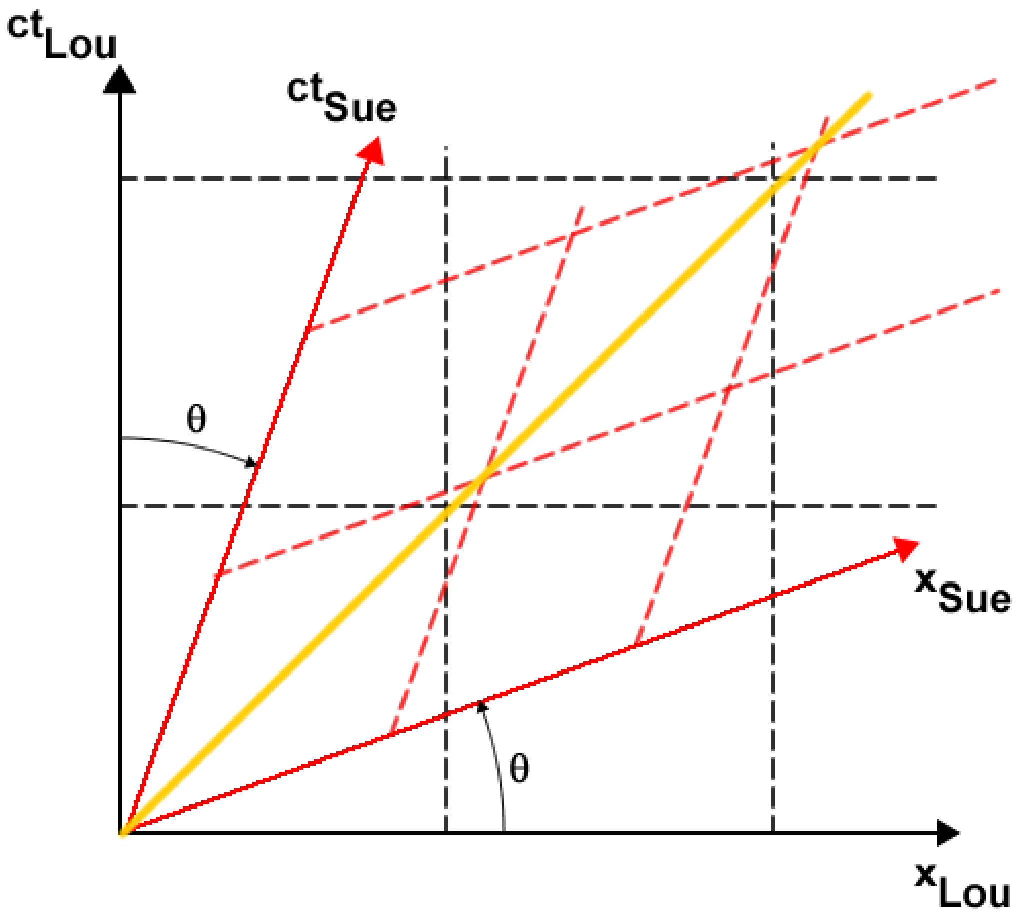
B. Draw energy versus speed axes with the speed going from 0 to 5 c. Sketch the relation between the energy E and speed u for an ordinary object. On the same plot sketch the relation between E and u for a tachyon. For ordinary objects we say: “It takes infinite energy to speed the object up to a speed equal to the speed of light.” What is the equivalent statement for a tachyon? For ordinary objects we can also say: “The minimum value of the energy is when it is at rest, and has a value equal to mc^2.” What is the equivalent statement for a tachyon?

C. A tachyon is produced by some apparatus in the laboratory at $x = 0$ at $t = 0$ and travels at $u_x = 20\ c$ in the +x direction to a detector at $x = 1{,}000\ m$. You are traveling at $v = 0.1\ c$ in the +x direction relative to the laboratory. The formula for addition of velocities is: $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$. What is the speed of the tachyon relative to you?

D. For Part C describe in words what you see the tachyon doing. What does this say about the tachyon being produced by some apparatus causing its later detection by the detector.

Activity 8

Sue and Lou are moving relative to each other at speed *v*. We can relate their spacetime diagrams as shown.



In the above diagram:

$$\tan(\theta) = \frac{v}{c}$$

The axes are rotated relative to each other, but they are rotating in opposite directions. Also shown in yellow is the worldline for light traveling at *c* relative to both Sue and Lou.

You will be using this diagram below, and may find it useful to print a few copies of it to staple into your lab book.

A. Explain how the diagram shows that the speed of light is the same value for both Sue and Lou.

B. Place two dots on the diagram representing the positions and times of two events that are simultaneous for Lou. Are the two events simultaneous for Sue? If no, which event occurs first? Place two more dots on the diagram representing the positions and times of two events that are simultaneous for Sue. Are the two events simultaneous for Lou? If no, which event occurs first?
C. Sketch the worldline of an object that is stationary relative to Lou. What is the direction of motion of the object relative to Sue? Explain.
D. Sketch the worldline of an object moving at some speed $u < c$ relative to Lou. Show geometrically that the object is moving at speed $u' < c$ relative to Sue.

Course Concepts *Activity 9*

As you may know, in some circumstances we can treat light as a particle called a *photon*.

A. Relativistic time dilation means that an unstable particle which decays in time $\Delta\tau$ when it is rest relative to some observer will live a longer time Δt for an observer for whom it is moving with speed v where:

$$\Delta t = \frac{\Delta\tau}{\sqrt{1 - v^2/c^2}} > \Delta\tau$$

If you apply this formalism to a photon, what does it predict about its lifetime relative to any observer?
B. The relation between energy E and mass m for an object traveling at speed u relative to us is given by:

$$E = \frac{mc^2}{\sqrt{1 - u^2/c^2}}$$

A photon has a real non-zero energy. What does this equation say about the value of its mass m? What do you think your high school math teacher would say about your answer?

Course Concepts **Activity 10**

Four elementary particles, an electron, a muon, a proton, and a neutron have the rest energies and relativistic total energies as shown.

Particle	**Rest energy**	**Total energy**
Electron	0.511 MeV	0.511 Mev
Muon	106MeV	212 MeV
Proton	938 MeV	4,690 MeV
Neutron	940 MeV	2,820 MeV

Rank in order from the largest to the smallest the particle's speeds.

Activity 11

In Jules Verne's **From the Earth to the Moon** (1865) a huge cannon fires a projectile at the moon. Inside the projectile was furniture, three people and two dogs. The figure is from the original edition.

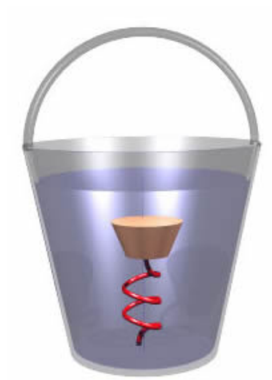
Verne reasoned that at least until the projectile got close to the Moon it would be in the Earth's gravitational field during its journey. Thus the people and dogs would experience normal gravity, and be able to, for example, sit on the chairs just as if the projectile were sitting on the Earth's surface.

One of the dogs died during the trip. They put the dog's body out the hatch and into space. The next day the people looked out the porthole and saw that the dog's body was still floating just beside the projectile.

A. Is there a contradiction between the inhabitants inside the projectile experiencing normal gravity and the dog's body outside the projectile not falling back to the Earth?
B. If your answer to Part A is yes, where did Verne make his mistake? If your answer is no, explain.

Activity 12

A bucket of water has a spring soldered to the bottom, as shown. A cork is attached the spring, and is therefore suspended under the surface of the water.



You are on top of the CN tower, holding the bucket, and step off. While falling towards the ground, do you see the cork move towards the top of the water, towards the bottom of the bucket, or stay where it is relative to the bucket and the water?

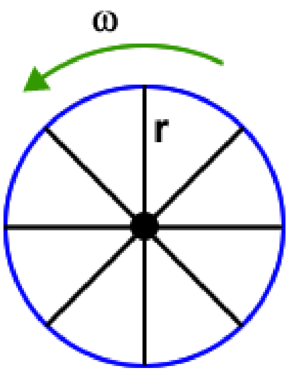
Explain your answer using Einstein's Equivalence Principle.

Course Concepts Activity 13

As you know, according to plane geometry the circumference C_0 of a circle is related to its radius r according to:

$$C_0 = 2\pi r$$

Imagine we have a wheel of radius r whose circumference when it is stationary is C_0. But the wheel is rotating with constant angular velocity ω. According to Special Relativity, the rim of the wheel will be contracted. Assume the rim has negligible thickness.



A. Will the length of the spokes of the wheel be contracted too? Explain.
B. What is the circumference C of the wheel according to Special Relativity? Express your answer in terms of the centripetal (radial) acceleration a of each point on the rim.
C. Following Einstein, we say that the geometry of the rotating wheel is not the geometrical equation given above. Express the difference in the geometries of the rotating and non-rotating wheel, i.e. $C - C_0$.

Course Concepts Activity 14

The word *geometry* literally means the measure of the Earth. As you know, according to plane geometry the sum of the angles of a triangle is 180º.

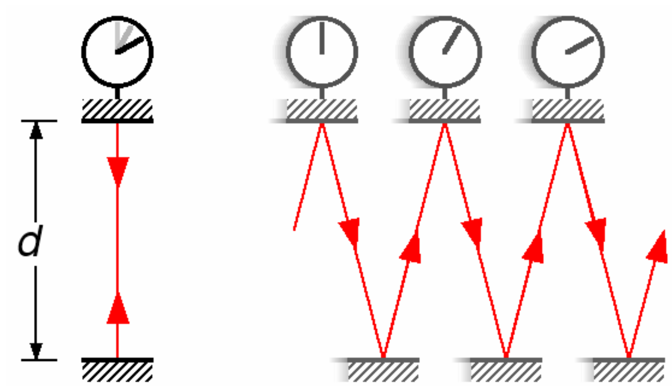
A. Assume the Earth is a perfect smooth sphere. Imagine you construct a triangle on the *surface* of the Earth. What is the sum of the angles of this triangle? You may wish to consider an isosceles triangle with the apex at the North Pole and the base along the equator.
B. From Part A, you may have discovered that geometry on a spherical surface is different than the geometry on a plane. To describe the geometry around a massive object, which geometry is the best bet to be correct? Explain.

Activity 15

Course Concepts

A thought-experiment, sometimes called a Gedanken experiment, is an experiment that you can imagine in order to test or explore theories in physics or other fields. Typically, thought-experiments might be very inconvenient or practically impossible to set up in real life, and you might have no intention of actually setting up the experiment. Nevertheless, thought-experiments can be very helpful in testing and discussing theories.

One famous thought experiment is the "Light-Clock". A light clock is made up of two parallel mirrors, separated by a vacuum and held at a fixed distance of d, as shown in the figure. A short pulse of light bounces between the mirrors. Each time the light pulse reflects off the top mirror, the clock "ticks". The time between ticks for a stationary light clock then is the time for a round-trip of the light pulse: $t = 2d/c$, where c is the speed of light.



One of the most fundamental and surprising principles of Einstein's Theory of Relativity is "**light travels at speed *c* in all inertial reference frames.**" Here an inertial reference frame is just one that is not accelerating.

If the light clock is moving toward the right at speed v, the time between ticks is longer, because the light pulse must travel along the diagonal. This time-dilation, or slowing of time, can be computed using the Pythagorean theorem. This is done on page 1157 of Knight Physics for Scientists and Engineers 2nd Edition, and there is a nice applet showing this derivation at http://physics.ucsc.edu/~snof/Tutorial/ .

A. Please open this tutorial with your browser. Click on the [1] to view the #1 Tutorial. It should take about 2 minutes to go through this tutorial and see the derivation of equation 37.22 from Knight: $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$. At the end of the tutorial, it says "Type a number 1 through 7 to set the speed of the clock, then click 'Play' to watch it." When you click 1, what is the value of γ? What is the corresponding value of v? Using a stopwatch, measure the round-trip time of reflections of the pulse from the top mirror.

B. When you click 5, what is the value of γ? What is the corresponding value of v? Using a stopwatch, measure the round-trip time of reflections of the pulse from the top mirror.

C. Click on the [2] to view the #2 Tutorial. It should take about 1 minute to go through this tutorial and see why objects must be shorter along the direction of motion in order for light to obey the rule: "**light travels at speed *c* in all inertial reference**

frames." This is called Length Contraction. Note that while the authors of this applet state that the stick "appears" shorter, this is not an optical illusion or an effect caused by the delay-time of light as it travels to reach our eyes. The moving object truly is shorter as carefully measured by an observer that is at rest. Please describe, in your own words, using one or two sentences, why the horizontal mirrors must be closer than the vertical mirrors when the system is moving toward the right.

The Relativity Module Student Guide Activities 1-14 were written by David M. Harrison, Dept. of Physics, Univ. of Toronto in January 2009. Activity 15 was written by Jason Harlow in July 2009.

Activity 3 is based on Rachel E. Scherr, Peter S. Shaffer, and Stamatis Vokos, "The challenge of changing deeply held student beliefs about the relativity of simultaneity," American Journal of Physics **70** (12), December 2002, 1238 – 1248.

David learned about the geometric approach to Special Relativity of Activity 8 from Edwin F. Taylor and John Archibald Wheeler, **Spacetime Physics** (W.H. Freeman, 1963). This classic is highly recommended.

Activities 1, 7 and 9 also appear in David Harrison and William Ellis, **Student Activity Workbook** that accompanies Hans C. Ohanian and John T. Markert, **Physics for Engineers and Scientists**, 3rd ed. (W.W. Norton, 2007).

The applet used in Activity 6 was written by Patrick Leung and Tim McIntyre at The University of Queensland, 2007.

Activities 11 and 12 also appear in Mechanics Module 3.

The image in Relativity Module Activity 15 was downloaded on July 10, 2009 from http://commons.wikimedia.org/wiki/File:Light-clock.png and was created by Michael Schmid on October 18, 2005.

The applet used in Activity 15 was developed at the University of California at Santa Cruz (UCSC). Devin Kelly-Sneed programmed the applets and built this site as an undergraduate Computer Science student at UCSC. Joel Primack is a Professor of Physics at UCSC. He originally designed the programs that make up Einstein's Rocket in the early 1980s with the help of Eric Eckert, who programmed them in 6502 assembly language.

Last revision: October 9, 2009 by Jason Harlow