

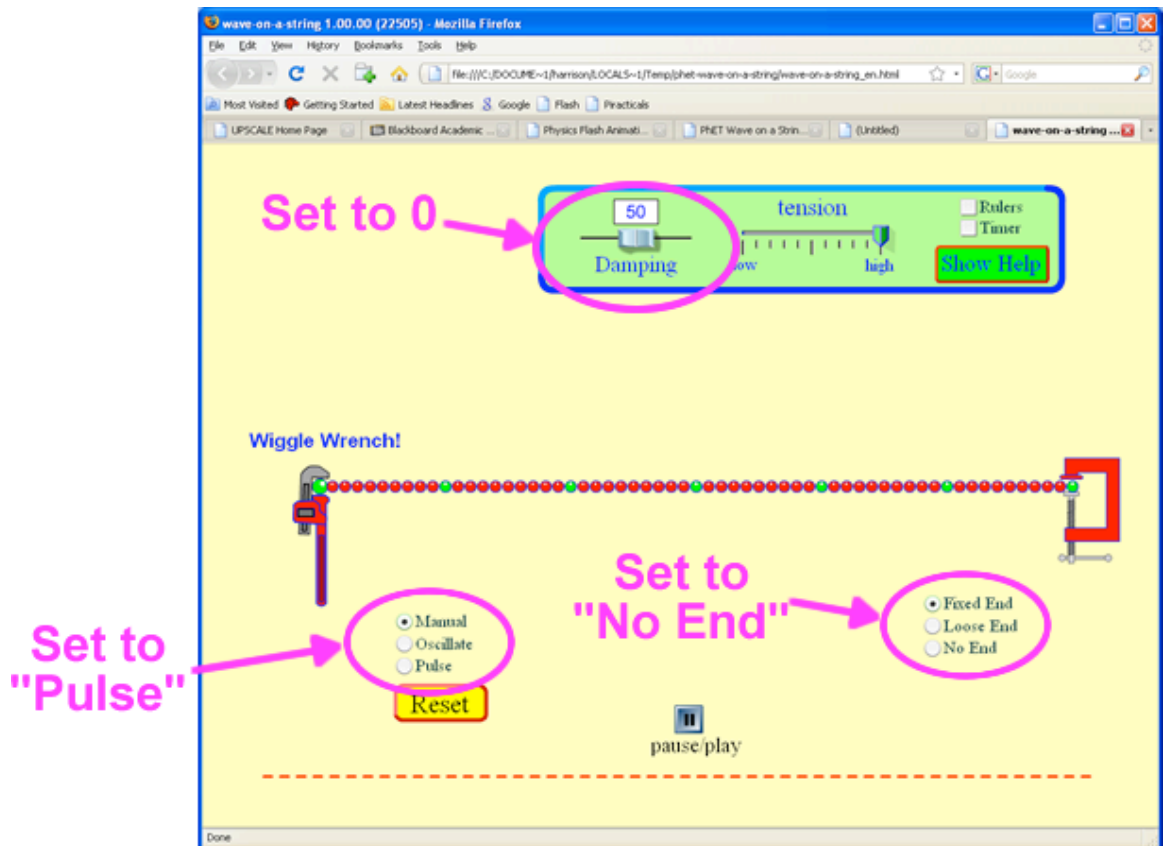
Concepts of this Module

- Traveling waves
- Intensity
- Reflection
- Superposition
- Standing Waves



Activity 1

- A. Open the Java applet `wave-on-a-string.jar` which is at:
Feynman:Public/Modules/Waves.



The screenshot shows the 'wave-on-a-string' Java applet interface. Key features and annotations include:

- Control Panel (Top Right):** Contains a 'Damping' slider set to 0 (annotated with 'Set to 0'), a 'tension' slider, and checkboxes for 'Rulers' and 'Timer'. A 'Show Help' button is also present.
- Wiggle Wrench! (Middle Left):** A wrench icon is shown. Below it, three radio buttons are selected: 'Manual' (annotated with 'Set to "Pulse"'), 'Oscillate', and 'Pulse'. A 'Reset' button is located below these options.
- End Conditions (Middle Right):** Three radio buttons are shown: 'Fixed End', 'Loose End', and 'No End'. The 'No End' option is selected (annotated with 'Set to "No End"').
- String and Wrench:** A horizontal string is shown with a wrench on the left end and a fixed support on the right end.
- Pause/Play:** A 'pause play' button is located at the bottom center.

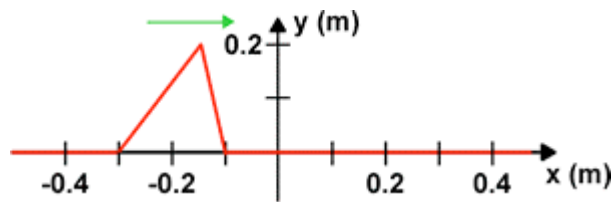
- Set the Damping to 0
- Set the wave type to “Pulse”
- Set the end to “No End” which will replace the vise on the right side with an open window for the waves to go through.

Click on the *Pulse* button that will appear. Imagine you are standing right beside the window as the pulse goes out of it, measuring the amplitude as a function of time as it goes by you. Sketch the amplitude as a function of time.

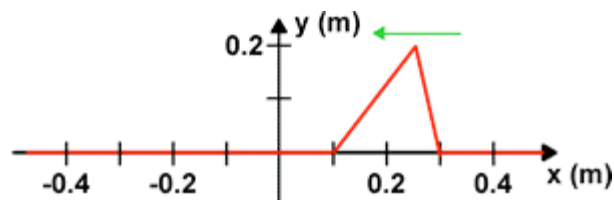
- B. Click on the *Rulers* control in the upper-right corner of the simulation. The rulers that appear can be moved with the mouse. Estimate the speed, width and amplitude of the wave pulse. Add labeled tick marks on the axes of the sketch of Part A.

This is a good time to experiment with different values of the Damping and tension. What happens as the Damping is increased? What happens as the tension in the string is decreased? You may wish to explore some of the other settings of the simulation too.

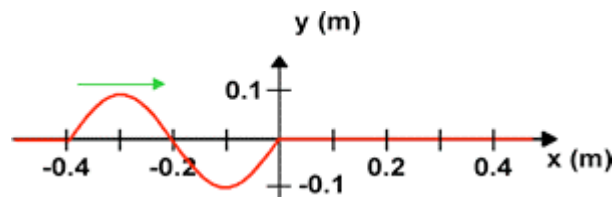
- C. The triangular pulse of Parts A and B was symmetric. Here is a plot of an asymmetric triangular pulse traveling from left to right. At the moment shown the time $t = 0$. The wave is traveling with a speed of 0.5 m/s. Sketch the amplitude of the pulse at $x = 0$ as a function of time t as the pulse goes by. Include labeled tick marks on both the y and t axes.



- D. Here is the same triangular pulse as Part B, but it is traveling from right to left at 0.5 m/s. At the moment shown the time $t = 0$. Sketch the amplitude of the pulse at $x = 0$ as a function of time t as the pulse goes by. Include labeled tick marks on both the y and t axes. Compare to the sketch from Part C.

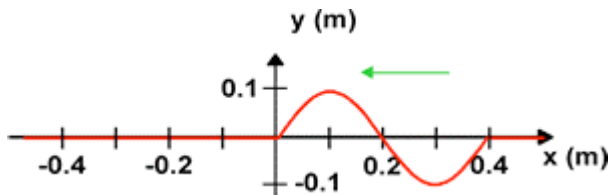


- E. Here is a sinusoidal wave pulse traveling from left to right at $v = 0.5$ m/s. At the moment shown $t = 0$. Sketch the amplitude of the pulse at $x = 0$ as a function of time t



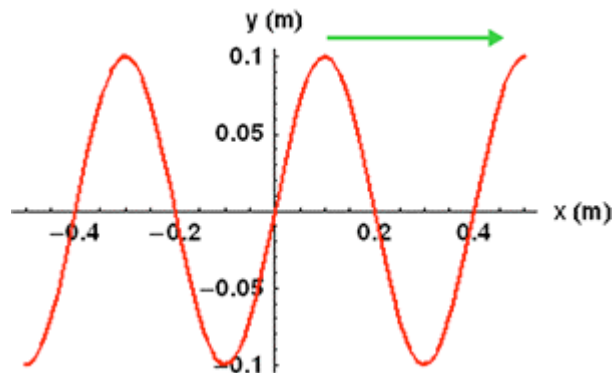
as the pulse goes by. Include labeled tick marks on both the y and t axes. What is the wavelength λ of the pulse? From your sketch what is the period T , frequency f , and angular frequency ω of the sinusoidal pulse? What is the relation between λ , f and v ?

- F. Here is a sinusoidal wave pulse traveling from right to left at $v = 0.5$ m/s. At the moment shown $t = 0$. Sketch the amplitude of the pulse at $x = 0$ as a function of time t



as the pulse goes by. Include labeled tick marks on both the y and t axes. What is the wavelength λ of the pulse? From your sketch what is the period T , frequency f , and angular frequency ω of the sinusoidal pulse? What is the relation between λ , f and v ?

- G. Here is a sine wave traveling from left to right with $v = 0.5$ m/s. The wave extends to infinity in both directions along the x axis. At the moment shown the time $t = 0$. At the moment shown the amplitude as a function of position is:



$$\Psi(x, t = 0) = 0.1 \sin\left(2\pi \frac{x}{\lambda}\right)$$

In your own words, explain the factor 2π in the above equation. We can describe the amplitude as the wave passes $x = 0$ either as $\Psi(x = 0, t) = 0.1 \sin\left(2\pi \frac{t}{T}\right)$ or as

$\Psi(x = 0, t) = 0.1 \sin\left(-2\pi \frac{t}{T}\right)$. Which form is correct? Explain your own words.

Write down a form of $\Psi(x, t)$ which is valid for all values of x and t . You may find the following Flash animation useful in visualizing this situation:

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/TravelWaves/TravelWaves.html>

The above link is to a fixed size animation which works nicely if only one person is viewing it. If more than one person is viewing the animation, a version which can be resized is better. Here is a link to a resizable version of the same animation:

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/TravelWaves/TravelWaves.swf>


Course Concepts **Activity 2**

Here is a link to a simple little Flash animation of a plane wave traveling through two different mediums:

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Waves/TwoMediums/TwoMediums.html>

Here is a link to a resizable version of the same animation, which is nicer if more than one person is trying to view it:

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Waves/TwoMediums/TwoMediums.swf>

Open one of the versions of the animation.

- A. At what rate do the wave fronts from the left strike the medium in the centre? What is the period, frequency, and angular frequency of the wave to the left of the medium in the centre?
- B. For the medium in the centre, at what rate do the wave fronts leave the left-hand side? Is this the same as your answer to Part A? Explain. Do the wave fronts strike the right side of the medium in the centre at this same rate? What is the period, frequency, and angular frequency of the wave while it is traveling through the medium in the centre?
- C. How does the wavelength of the wave traveling from the left to the medium in the center compare to the wavelength of the wave while it is traveling through the medium in the centre? Show how you arrived at your answer.
- D. The wave leaves the medium in the centre and travels off to the right. How do the period, frequency, angular frequency, and wavelength of the wave traveling to the right of the medium in the centre compares to the same quantities for the waves in the other regions?


Course Concepts **Activity 3**

In Activity 2 the waves strike the interface between the two mediums straight on, with zero angle of incidence. Here is a link to a Flash animation where the angle of incidence is not zero.

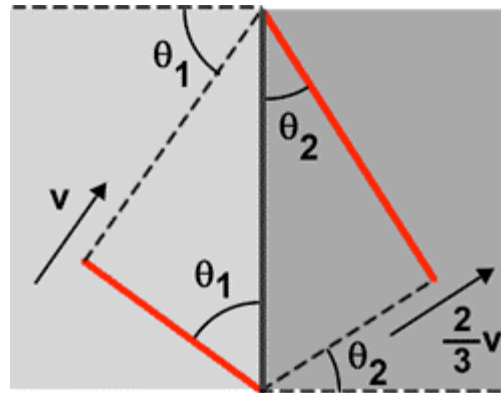
<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Waves/Refraction/Refraction.html>

Here is a link to a resizable version:

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Waves/Refraction/Refraction.swf>

Open one of the versions of the animation.

- A. At what rate do the wave fronts from the left strike the medium in the centre? Is the rate the same regardless of what vertical position you are considering? What is the period, frequency, and angular frequency of the wave to the left of the medium in the centre?
- B. For the medium in the centre, at what rate do the wave fronts leave the left-hand side? Is this the same as your answer to Part A? Explain. Do the wave fronts strike the right side of the medium in the centre at this same rate? What is the period, frequency, and angular frequency of the wave while it is traveling through the medium in the centre?
- C. How does the wavelength of the wave traveling from the left to the medium in the center compare to the wavelength of the wave while it is traveling through the medium in the centre? Show how you arrived at your answer.
- D. The wave leaves the medium in the centre and travels off to the right. How do the period, frequency, angular frequency, and wavelength of the wave traveling to the right of the medium in the centre compares to the same quantities for the waves in the other regions?
- E. The figure to the right shows a portion of two wave fronts of the animation. What is the relation between θ_1 and θ_2 ? Notice that there are two right triangles in the figure with a common hypotenuse.



Course Concepts Activity 4

Here is a Flash animation of a molecular view of a sound wave traveling through the air:

http://faraday.physics.utoronto.ca/IYearLab/Intros/StandingWaves/Flash/long_wave.html

The above link is to a fixed size animation which works nicely if only one person is viewing it. If more than one person is viewing the animation, a version which can be resized is better. Here is a link to a resizable version of the same animation:

http://faraday.physics.utoronto.ca/IYearLab/Intros/StandingWaves/Flash/long_wave.swf

Open one of the versions of the animation.

- A. The bottom shows the motion of the air molecules. You may wish to imagine that the molecules are connected to their nearest neighbors by springs, which are not shown. There is a wave of increasing and decreasing density of the molecules. Is the wave moving to the right or to the left? Explain. Is the wave longitudinal or transverse? Explain.

- B. Often instead of describing the wave as one of density we talk about a *pressure* wave. Does the higher density of molecules correspond to higher or lower pressure? Can you explain?
- C. The top shows the displacement of the molecules from their equilibrium positions. It too is a wave, often called a *displacement* wave. Is the wave moving to the right or to the left? Explain. Is the wave longitudinal or transverse? Explain.
- D. Use the *step* controls, pause the animation and position molecules 3 and 9 at their equilibrium position with molecule 6 at maximum displacement. The amplitude of the displacement wave is zero for molecules 3 and 9. Is the amplitude of the pressure wave at the position of molecule 3 also zero, or is it a maximum or a minimum? What about the pressure wave at the position of molecule 9?
- E. Use the *step* controls to position molecules 3 and 9 at their equilibrium position with molecule 6 at minimum displacement. Is the amplitude of the pressure wave at the position of molecule 3 zero, or is it a maximum or a minimum? What about the pressure wave at the position of molecule 9?
- F. From your results for Parts D and E, what is the phase angle between the pressure wave and the displacement wave?



Course Concepts **Activity 5**

- A. Open the Java applet `wave-on-a-string.jar` which is at:
`Feynman:Public/Modules/Waves`. Part A of Activity 1 shows a screen shot of the applet.
- Set the Damping to 0
 - Set the wave type to “Oscillate”
 - Set the end to “No End” which will replace the vise on the right side with an open window for the waves to go through.

How does the amplitude of the wave change as it propagates down the string? Is this a one dimensional, two dimensional, or three dimensional wave? You may wish to look over Parts B and C before answering this question.

- B. A two dimensional wave, such as a water wave, is propagating away from its source equally in all directions. Assume damping is negligible. How does the amplitude of the wave change with distance from the source?
- C. A three dimensional wave, such as a sound wave, is propagating away from its source equally in all directions. Assume damping is negligible. How does the amplitude of the wave change with distance from the source?
- D. What physical principle or conservation law gives the answers to Parts A – C? Explain


Course Concepts Activity 6

- A. Open the Java applet `wave-on-a-string.jar` which is at:
`Feynman:Public/Modules/Waves`. Part A of Activity 1 shows a screen shot of the applet.
- Set the Damping to 0
 - Set the wave type to “Pulse”
 - Leave the end in its default state of “Fixed End” which clamps the right side of the string with a C-clamp.

Click on the *Pulse* button. What is the behavior of the wave pulse when it is reflected by a fixed end?

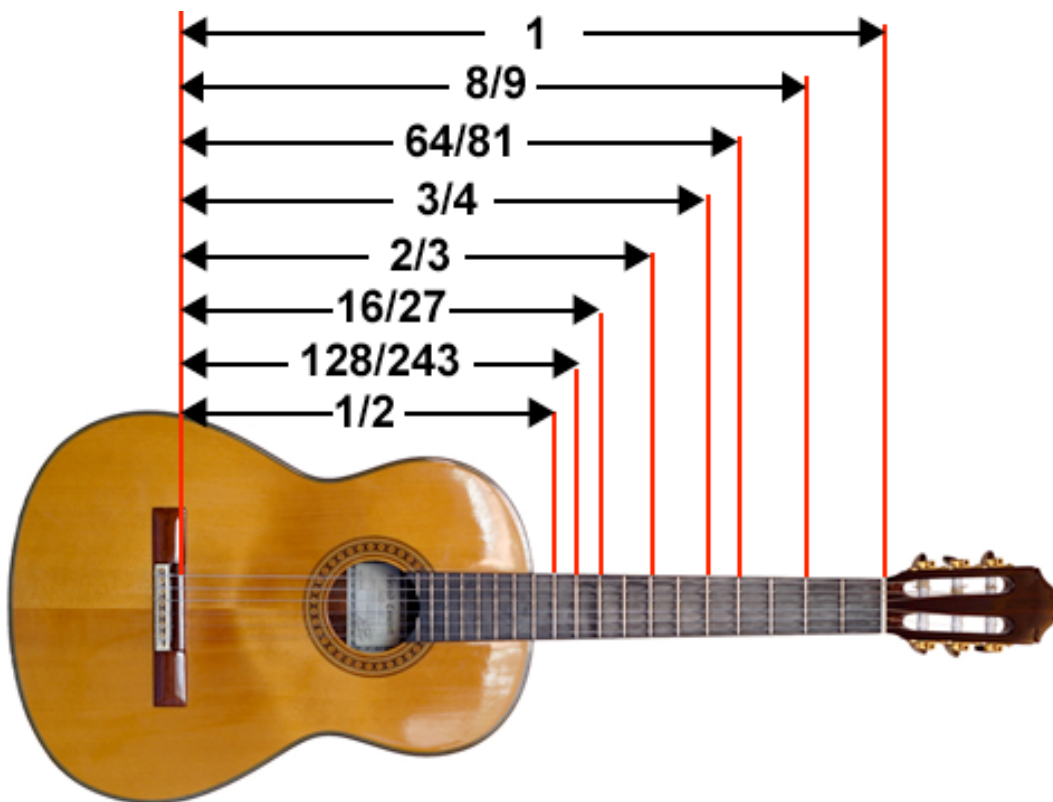
- B. Change the end of the string to “Loose End” which terminates the right hand side of the string with a frictionless loop around a vertical rod. Click on *Reset* and then on *Pulse*. What is the behavior of the wave pulse when it is reflected by a free end?
- C. Set the end of the string back to “Fixed End,” click on *Reset* and then on *Pulse*. Use the *pause/play* button and then the *step* one to step the wave pulse through a complete reflection at one end of the string. There is a point where the wave pulse nearly disappears. Where did the wave go? Where did the wave’s energy go? Explain what is happening.
- D. Set the end of the string back to “Loose End,” click on *Reset* and then on *Pulse*. Click on the *Rulers* control in the upper-right corner of the simulation. The rulers that appear can be moved with the mouse. Measure the maximum amplitude of the wave pulse; you may already have done this measurement in Activity 1 Part B. Use the *pause/play* button and then the *step* one to step the wave pulse through a complete reflection at one end of the string. There was a point where the amplitude of the wave at the position of the free end was large. Use the ruler to estimate its amplitude. Explain your result.
- E. Set the end of the string back to “Fixed End.” Set the *Damping* to 10. Set the wave type to “Oscillate” and click on *Reset*. You will see a “standing wave” on the right hand side of the string. Use the *pause/play* button and then the *step* one to step the wave pulse through a complete reflection at the right end of the string. There is a point where the wave pulse near the right hand side nearly disappears. Where did the wave go? Explain what is happening.
- F. Set the end of the string back to “Loose End,” click on *Reset*. You will once again see a “standing wave” on the right hand side of the string. Is there a difference between this standing wave and the one you saw in Part E? Explain. Is there a point where the wave amplitude near the right hand side nearly disappears, as in Part E? Explain.
- G. Set the end of the string back to “Fixed End.” Leave the wave type as “Oscillate.” Set the *Damping* to 0. Click on *Reset*. What happens? Explain.

Although we have used the *wave on a string* applet in Activities 1, 5, and now here, there is still lots more Physics that you can learn from it. You are invited to explore further.

Course Concepts **Activity 7**

“Music is a hidden practice of the soul, that does not know it is doing mathematics.” --Leibniz

If Pythagoras had a guitar, it might have looked like this:



We assume that Pythagoras was a large man, so the length of the strings from the bridge to the nut is 1 m, as shown.

The second string of six from the top is conventionally tuned to A two octaves below concert A. This is often written as A_2 , and has a frequency of 110 Hz.

The notes of an A scale starting at A_2 are: $A_2 - B_2 - C_1\# - D_1 - E_1 - F\#_1 - G\#_1 - A_1$. Here are the frequencies of these notes in a Pythagorean tuning; also shown are the frequencies of the notes in an equally tempered tuning which is more common today.

Note	Pythagorean Tuning (Hz)	Equally Tempered (Hz)
A ₂	110.00	110.00
B ₂	123.75	123.47
C ₁ #	139.22	138.59
D ₁	146.67	146.83
E ₁	165.00	164.81
F ₁ #	185.63	185.00
G ₁ #	208.83	207.65
A ₁	220.00	220.00

You will need to know that, as discussed in the textbook, the speed of a traveling wave on string with tension T_s is

$$v_{string} = \sqrt{\frac{T_s}{\mu}}$$

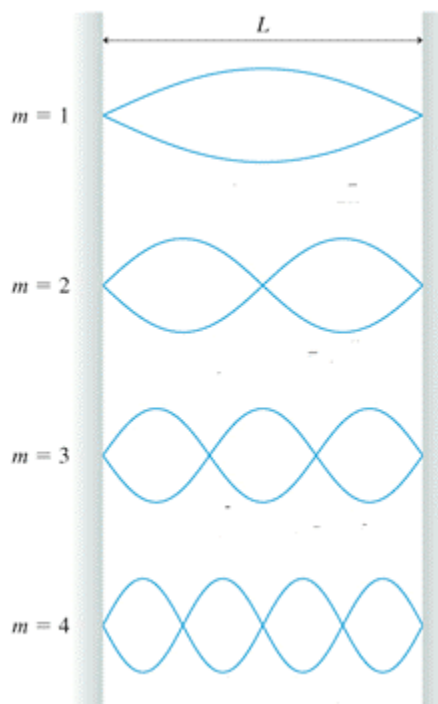
where μ is the string's mass-to-length ratio

$$\mu = \frac{m}{L}$$

Note that the speed is independent of the frequency.

To the right are shown the first four normal modes of a vibrating string.

Here is a link to a simple Flash animation that shows the actual motion of the string for the first three normal modes:



<http://faraday.physics.utoronto.ca/IYearLab/Intros/StandingWaves/Flash/sta2fix.html>

- When the second string from the top is playing the note A₂ = 110 Hz the frequency f of the first normal mode is also 110 Hz. What is the wavelength of the first normal mode? What is the speed of a traveling wave on the string? Explain why increasing the tension in the string increases the frequency of the note the string plays.
- What is the wavelength of the *second* normal mode of the string? What is the frequency of the standing wave?
- If you place your finger just to the right of the 12th fret the effective length of the string becomes 0.5 m, as shown in the figure. What is the wavelength of the first normal mode, and the frequency? What musical note is the string playing? How do your values compare to your result for Part B? Explain.

- D. If you place your finger just to the right of the 7th fret the effective length of the string becomes $2/3$ m, as shown in the figure. What is the wavelength of the first normal mode and the frequency? What musical note is the string playing? Explain.
- E. Is there a pattern between the positions of the labeled frets in the figure of the guitar and the notes of the A scale in a Pythagorean tuning? What is the pattern? Are the lengths shown in the figure rational or irrational numbers? You may wish to know that if the guitar frets were set up to be equally tempered, except for the 12th fret the lengths would not be rational numbers.
- F. As indicated in the table, the frequencies of the notes in a scale are slightly different in the Pythagorean tuning and the equally tempered tuning commonly used today. You may see if you can hear the difference by listening to a scale played with the Pythagorean tuning in the file `Pythagorean.mid` and an equally tempered tuning in `EqualTempered.mid`; both files are located at `Feynman:public/Modules/Waves`. You may also wish to explore further at: <http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Temperament/Temperament.html>

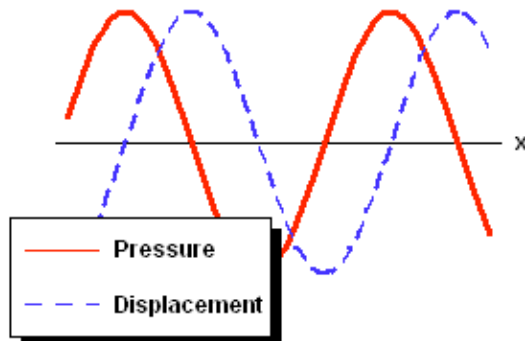


Expt Activity 8

As investigated in Activity 4, we can think of a sound wave two different ways:

1. A pressure wave. The pressure oscillates around atmospheric pressure.
2. A displacement wave. The displacements of the air molecules oscillate around their equilibrium positions.

These two waves are 90 degrees out of phase: when one has a maximum or minimum the other is at zero amplitude.

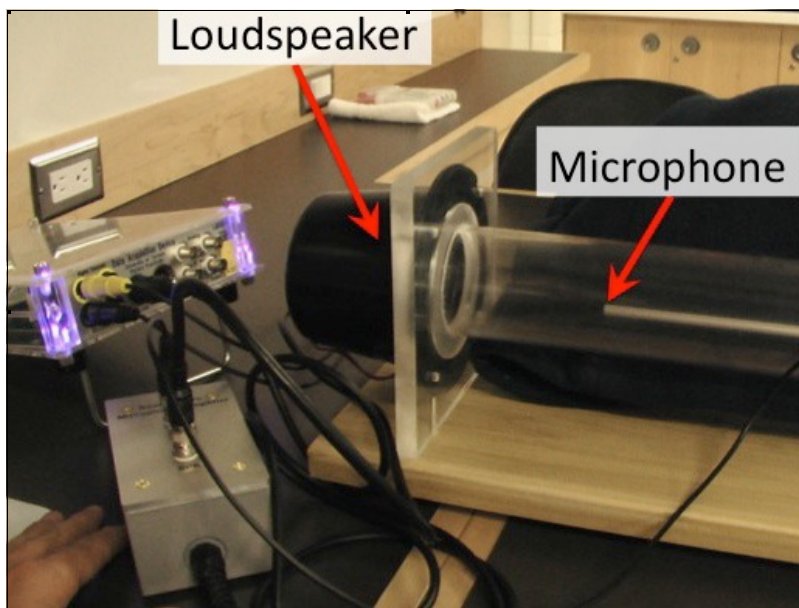


You will want to know that microphones measure the *pressure* wave. You will also want to know that the speed of sound is:

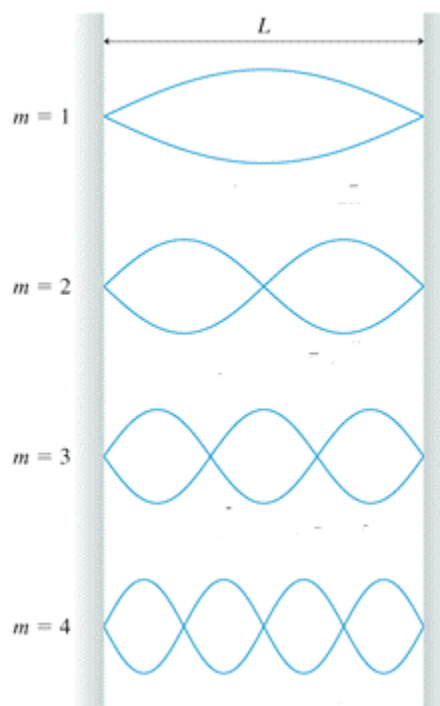
$$v_{\text{accepted}} = 331 + 0.61T \text{ (m/s)}$$

where T is the temperature of the air in Celsius.

In this Activity you will set up standing sound waves in a tube filled with air. A loudspeaker generates the sound wave. A rod inside the tube has a small microphone on the end, so the sound wave inside the tube can be measured at different positions. The part of the tube with the loudspeaker is shown in the figure below.



When the tube is closed at both ends, the possible *displacement* standing waves are the same as those for a standing waves on a string that is fixed at both ends: there is a node at each end of the tube. The figure to the right shows the first four possible standing waves. These are the same standing waves that for a string we called *normal modes* in Activity 7, and in fact this is the same figure that appears there!



- A. What are the wavelengths of the shown standing waves? What is the wavelength of the $m = 5$ standing wave which is not shown? Generalise to a formula for the wavelengths for any value of m .
- B. For the first two or three displacement standing waves, sketch the corresponding *pressure* standing wave.

Here is a link to a simple Flash animation that shows the displacement wave for the first three standing waves:

<http://faraday.physics.utoronto.ca/IYearLab/Intros/StandingWaves/Flash/sta2fix.html>

Be careful not to push the Sound Sensor all the way into the speaker, as the speaker is made of paper!

Set up the tube on the table with the speaker closer to the computer.

The Data Acquisition Device (DAQ) is designed to interface between the computer and physical apparatus. It has three purposes for this experiment:

1. The DAQ generates a variable voltage which drives the magnet in the speaker and creates a sound. This is the ***input*** signal for the sound-tube.
2. The DAQ amplifies and measures the small variable voltage generated by the Sound Sensor, or microphone. This is the ***output*** signal from the sound-tube, which you will be measuring.
3. The DAQ measures the rotation of a gear on a rotary motion sensor and converts it to a displacement, or change in position of the microphone. You will be plotting sound strength versus microphone position along the tube.

Plug the wire from the Sound Tube Microphone Amplifier box into the **A (ai2) Analog Sensor** plug in the DAQ. The MIC and SPK plugs in the box should be wired to the microphone and speaker, respectively.

Plug the yellow and black wires from the Rotary Motion Sensor into the digital channel 0 plugs in the DAQ.

On the computer, open the Labview program which generates the output signal to the speaker and also measures and displays the input from the microphone and motion sensor. A shortcut to this program is at: My Computer → public on 'feynman' (P:) → LabView, and is called “Speed of Sound Tube”.

Notes:

- You have to ‘Run’ the program to get anything to happen (the button with the small white arrow)
- To adjust the frequency, either type a new frequency in the green box, or adjust the circular knob.
- To start the speaker and begin collecting distance versus sound pressure, click the ‘START’ button.

Measuring a Standing Wave in the Tube

Have the tube closed at both ends. We have found from experience with these tubes that the fundamental frequency for closed-closed modes is about 270 Hz.

- C. Set the frequency to 270 Hz. Slide the rotary motion sensor to the closest it will go to the sound tube. Start collecting data. Slowly slide the rotary motion sensor away from the sound tube. As you slide, the computer will plot sound

- intensity versus distance. In order to get a good graph, you should slide the sensor slowly and steadily, and it should take you about 30 seconds to 1 minute to get from start to finish. When the rotary motion sensor gets as far as it can from the sound tube, stop collecting data. Now click on “Fit Data”, and print the graph, if it is good, and staple it into your notebook. Label the nodes and antinodes, and describe what the m value is of this mode.
- D. Recall equation 21.17 from pge 644 of Knight, Chapter 21: $f_m = mv/2L$ for a closed-closed tube, where L is the length of the tube. Solve this equation for v , the speed of sound, and use your measurements to determine the speed of sound in the tube. Does this match with the equation for $v_{accepted}$, above?
- E. Repeat step C for a different frequency, for example: 540 Hz, 810 Hz or 1080 Hz.



Expt Activity 9

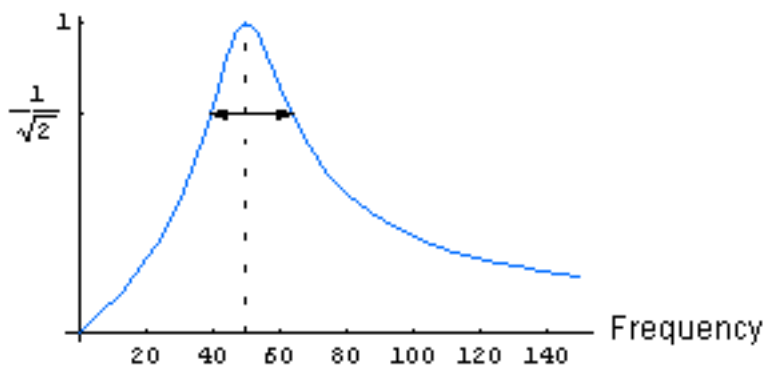
If the apparatus of Activity 8 were perfect, then when the tube is closed on both ends we would not hear any sound outside the tube. Similarly, if the air inside the tube were perfect, all molecule-molecule collisions would be perfectly elastic; this means that as a sound wave travels through the air none of its energy would be converted to heat energy of the air. However, neither the apparatus nor the air is perfect, The *Quality Factor Q* measures the degree of “perfection” of the system.

Say we have a standing wave when the frequency is f_0 . For frequencies close to the "resonant frequency" f_0 the amplitude A of the sound wave at the position where there was an maximum in the pressure wave is given by:

$$A(f) \neq A_0 \frac{1}{\sqrt{1 + Q^2 \left(\frac{f}{f_0} - \frac{f_0}{f} \right)^2}}$$

Note in the above that the amplitude $A(f)$ is equal to A_0 when the frequency f is equal to the resonant frequency.

The figure to the right shows $A(f)$ for A_0 equal to 1, Q equal 2, and for a resonant frequency of 50 Hz. Note that we have indicated the width of the curve where the maximum amplitude is



$1/\sqrt{2}$ times the maximum amplitude A_0 .

A nearly trivial amount of algebra shows that the amplitude A is $1/\sqrt{2}$ times the maximum amplitude A_0 for positive frequencies when the frequency is:

$$f = \frac{f_0}{2Q} (\sqrt{1 + 4Q^2} \pm 1)$$

Thus, if the width of the curve is Δf , then Q is:

$$Q = \frac{f_0}{\Delta f}$$

- A. For a given resonant frequency f_0 how does the width of the curve of amplitude versus frequency depend on the Quality Factor Q ?
- B. When the Quality Factor Q is zero, the maximum amplitude A_0 is zero. When Q is infinite so is the maximum amplitude. Explain.
- C. In the same directory where you found “Speed of Sound Tube”, you should find a program called “Sound Sweep.” Open this program. Place the microphone somewhere in the middle of the tube. Run a sweep of frequencies from 50 Hz - 3 kHz. What are the resonances? Choose a resonance and calculate the quality factor of this resonance in this tube.

This Student Guide was written by David M. Harrison, Dept. of Physics, Univ. of Toronto in the Fall of 2008. Last revision: September 23, 2010 by Jason J.B. Harlow.

The Java applet used in Activities 1, 5 and 6 was written by the Physics Education Technology (PhET) group at the University of Colorado, <http://phet.colorado.edu/index.php>. Retrieved November 9, 2008.

The figure of normal modes of a vibrating string in Activity 7 is slightly modified from Figure 21.22 of Randall D. Knight, **Physics for Scientists and Engineers**, 2nd edition (Pearson Addison-Wesley, 2008), pg. 640. The same figure is used in Activity 8.

The Pythagorean and equally tempered scales used in Activity 6 are from Wikipedia, http://en.wikipedia.org/wiki/Pythagorean_tuning. Retrieved November 15, 2008.

Activities 8 and 9 are based on a Student Guide written by David M. Harrison in October 1999 and revised in June 2001, then again in September 2010.