UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2009 EXAMINATION — version 2
PHY132H1F
Duration – 2 hours

Aids allowed: A non-programmable calculator without text storage. A single, hand-written aid-sheet prepared by the student, no larger than 8.5"x11", written on both sides.

Solutions posted by Jason Harlow, October 2010

Possibly useful constants and information:
Acceleration due to gravity near the surface of the Earth: $g = 9.80$ m/s^2
Speed of light in a vacuum: $c = 3.00 \times 10^8$ m/s.
The speed of sound in air (unless otherwise specified): $v = 343$ m/s
Mass of an electron: $m_e = 9.11 \times 10^{-31}$ kg
Fundamental unit of charge: $e = 1.60 \times 10^{-19}$ C
2π radians = 360°
$\pi = 3.14159$
Common Prefixes: nano- (n) = 10^{-9} , micro- (μ) = 10^{-6} , milli- (m) = 10^{-3}

MULTIPLE CHOICE (24 points total)

1. Two harmonic waves, with the same frequency and amplitude, travel in opposite directions and interfere to produce a standing wave described by $y = 3 \sin(2x) \cos(5t)$, where x and y are in m and t is in s. What is the approximate wavelength in m of the interfering waves?
   A. 6
   B. 12
   C. 1
   D. 2
   E. 3

This is what was described in Eqs 21.6 & 21.7 of Knight 2e.

$A(x) = 2a \sin kx = 3\sin(2x)$ in our case.

$y = A(x)\cos\omega t$.

so $\omega = 5$, $k = 2$ and $a = 1.5$.

The wavelength comes from k:

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$$

$$\pi \simeq 3$$

2. Electrons are sent through a double-slit apparatus, and an interference pattern is observed. If protons were sent through the same apparatus with the same speed as that of the electrons, the distance between the fringes would be
   A. the same.
   B. smaller.
   C. larger.

The distance between fringes in a double-slit interference pattern is proportional to wavelength.

de Broglie wavelength is: $\lambda = \frac{h}{p}$ [Eq. 25.8]

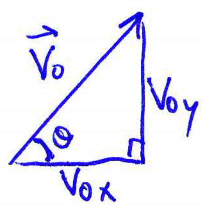
as m increases, $p = mv$ increases.

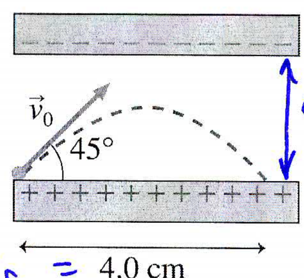
as p increases, λ decreases.

$\Rightarrow$ dist. between fringes decreases.

3. An electron is launched at an angle of $\theta = 45°$ and a speed of $v_0 = 5.0 \times 10^6$ m/s from the positive plate of a parallel-plate capacitor, as shown. The electron lands 4.0 cm away. The spacing between the plates is 2.5 cm. What is the voltage of the capacitor? [You may neglect the force of gravity on the electron in this problem.]

   A. 180 V
   B. 3.6×10^3 V
   C. 1.8×10^{-5} V
   D. 72 V
   E. 89 V





$d = 2.5 \times 10^{-2}$ m

$x_f = 4.0$ cm

[Eq. 29.25]

$\boxed{|\vec{E}| = V/d \text{ for a capacitor}}$

$V_{0x} = V_0 \cos\theta$

$V_{0y} = V_0 \sin\theta$

Acceleration is in negative-y direction: $a_y = -\frac{F}{m} = -\frac{Eq}{m} = -\frac{Vq}{md}$

Consider the y-component of the motion:

By symmetry, $V_{fy} = -V_{0y}$. $V_{fy} = V_{0y} + a_y t$

$\Rightarrow a_y t = -2V_{0y}$

Consider the x-component. $a_x = 0$

$V_x = V_{0x} = \frac{x_f - x_0}{t} \Rightarrow \boxed{t = \frac{x_f}{V_0 \cos\theta}}$

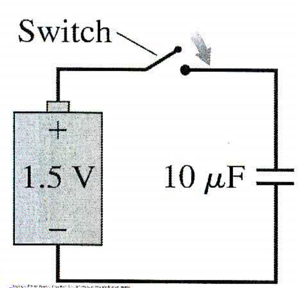
$\Rightarrow \left(\frac{-Vq}{md}\right)\frac{x_f}{v_0 \cos\theta} = -2v_0 \sin\theta \Rightarrow V = \frac{2v_0^2 m d \sin\theta\cos\theta}{x_f\, q}$

$V = 2(5\times10^6)^2(9.11\times10^{-31})(2.5\times10^{-2})(\sin 45)\cos 45$

$V = 88.96$ V

4. Initially, the switch shown is open and the capacitor is uncharged. How much charge flows through the switch after the switch is closed?

A. 6.7×10^{-5} C
B. 1.5 C
C. 1.5×10^{-6} C
D. 6.7×10^{-6} C
E. 1.5×10^{-5} C



Initially, capacitor has zero voltage. Current will flow until ΔV across capacitor is -1.5 V, so circuit obeys loop-rule,

$$Q = \Delta V C \quad [\text{Eq. } 30.18]$$
$$= (1.5)(10 \times 10^{-6} \text{ F})$$
$$Q = 1.5 \times 10^{-5} \text{ C}$$

5. A charged particle moving within a static, uniform magnetic field

A. may or may not experience a magnetic force, but this force, if it exists, can never cause the particle's speed to change.
B. will never experience a magnetic force.
C. will always experience a magnetic force, and this force can cause the particle's speed to change.
D. will always experience a magnetic force, but this force cannot cause the particle's speed to change.
E. may or may not experience a magnetic force, and this force, if it exists, can cause the particle's speed to change.

Magnetic Force: $\vec{F} = q\vec{v} \times \vec{B}$ [Eq. 33.17]

Cross-Product: $\vec{F} = 0$ if $\vec{v} \parallel \vec{B}$

F is maximum if $\vec{v} \perp \vec{B}$, and $\vec{F}$ is always perpendicular to both $\vec{B}$ and $\vec{v}$.

Therefore, $\vec{F}$ provides the centripetal force.

→ causes the direction to change, but never the speed.

6. How much work W must be done on a particle of mass m to accelerate it from a speed of 0.980 c to a speed of 0.990 c?

A. 5.03 mc^2
B. 7.09 mc^2
C. 1.03 mc^2
D. 1.44 mc^2
E. 2.06 mc^2

Work- Kinetic Energy theorem is:

$W = K_f - K_i$ [Eq. 11.9]

Relativistic Kinetic Energy is

$K = (\gamma - 1)mc^2$ [Eq. 37.44]

$$\Rightarrow W = (\gamma_f - 1)mc^2 - (\gamma_i - 1)mc^2$$
$$= \gamma_f mc^2 - \cancel{mc^2} - \gamma_i mc^2 + \cancel{mc^2}$$
$$= (\gamma_f - \gamma_i)mc^2$$
$$= \left[\frac{1}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{(0.98c)^2}{c^2}}}\right]mc^2$$

$$W = 2.064\ mc^2$$

7. A mother hawk screeches as she dives at you, while you sit, stationary relative to the air. You recall from biology that female hawks screech at 750 Hz, but you hear the screech at 820 Hz. How fast is the hawk approaching?

A. 56 m/s
B. 660 m/s
C. 29 m/s
D. 32 m/s
E. 38 m/s

Doppler Effect, approaching source.

Eq. 20.39: $f_+ = \dfrac{f_0}{1 - v_s/v}$, solve for v_s:

$$v_s = v\left(1 - \frac{f_0}{f_+}\right) = 343\,\text{m/s}\left(1 - \frac{750}{820}\right)$$

$$v = 29.3\ \text{m/s}$$

8. Martina has myopia. The far point of her left eye is 230 cm. What power lens will restore normal vision? [Neglect the small space between the lens and her eye.]
   A. +0.91 D
   B. +3.3 D
   C. –3.3 D
   D. –3.1 D
   E. –0.43 D

Following example 24.5:

Martina wishes to focus on a very far object : $S_o \to \infty$.

To do this, a virtual image must be formed, a distance of $-230\text{ cm} = S_i$.

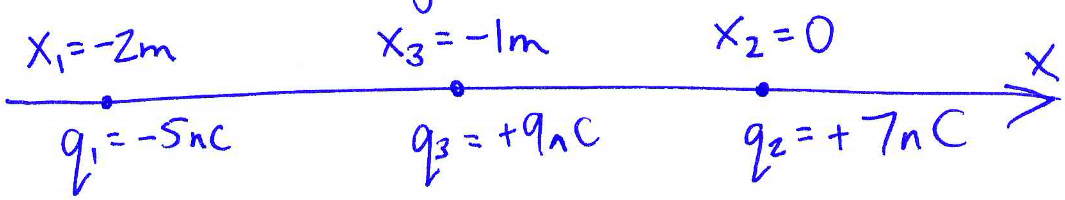
$$\text{Power} = \frac{1}{f} = \frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{\infty} + \frac{1}{-230\text{cm}} = -4.3\times10^{-3}\text{ cm}^{-1}$$

$$\text{Power} = -0.43\text{ m}^{-1}$$

$$= \boxed{-0.43\text{ diopters}}$$

9. Two point charges are located on the x-axis: one charge, $q_1 = -5$ nC, is located at $x_1 = -2$ m; the second charge, $q_2 = 7$ nC, is at the origin ($x_2 = 0$). What is the net force exerted by these two charges on a third charge $q_3 = 9$ nC placed at $x_3 = -1$ m?
   A. -1×10^{-6} N
   B. $+3 \times 10^{-5}$ N
   C. –500 N
   D. -3×10^{-5} N
   E. -5×10^{-7} N

Draw a diagram:



Force of q_1 on q_3: $F_1 = \dfrac{-k|q_1 q_3|}{(1m)^2} = \dfrac{-9\times10^9(5\times10^{-9})(9\times10^{-9})}{1}$

$F_1 = -4.05\times10^{-7}$ N

Force of q_2 on q_3: $F_2 = \dfrac{-k|q_2||q_3|}{(1m)^2} = -9\times10^9(7\times10^{-9})(9\times10^{-9})$

$F_2 = -5.67\times10^{-7}$

$F_{tot} = -9.7\times10^{-7}\text{ N} \simeq -10^{-6}\text{ N}$

10. What is the magnitude of the change in electric potential along the length of a 12 m long wire in the walls of your house? [The wire is made of solid aluminum, has a diameter of 2.0 mm, and carries 8.0 A of current. The resistivity of aluminum is 2.8×10^{-8} Ω m.]

A. 6.8 V
B. 19 V
C. 0.21 V
D. 0.86 V
E. 1.6 V

Eq. 31.23: $R = \frac{\rho L}{A}$

where $A = \pi r^2$

Ohm's Law, Eq. 31.24: $I = \frac{\Delta V}{R}$

$\Rightarrow \Delta V = IR = \frac{I \rho L}{\pi r^2} = \frac{8.0(2.8 \times 10^{-8})(12)}{\pi (0.001)^2}$

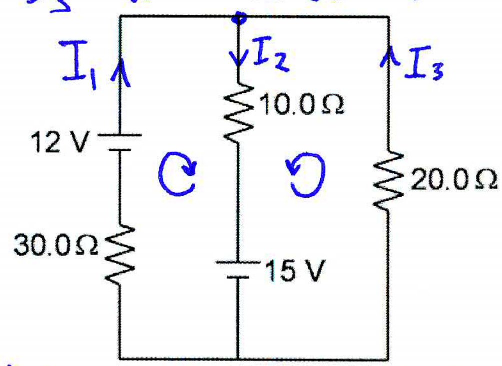
$\Delta V = 0.856$ V

Ch. 32 Question: Kirchoff's loop & junction laws.

11. What is the magnitude of the current through the 10.0 Ω resistor in the circuit shown?

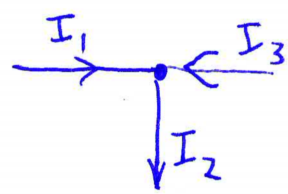
Define I_1, I_2, I_3 with directions as follows:

A. 0.55 A
B. 5.0 A
C. 0.020 A
D. 0.20 A
E. 0.46 A



Set up two loops: left loop goes clockwise, right loop goes counterclockwise. That way current in centre wire, I_2, is down for both loops.

Junction Rule for top dot:



$$\Rightarrow \boxed{I_1 + I_3 = I_2} \quad (1)$$

Loop Rule for left loop:

$$\Rightarrow \boxed{12 - 10 I_2 - 15 - 30 I_1 = 0} \quad (2)$$

Loop Rule for right loop:

$$\Rightarrow \boxed{-15 - 20 I_3 - 10 I_2 = 0} \quad (3)$$

→ 3 equations, 3 unknowns.

Solve for I_2 by eliminating I_1 & I_3.

$$(2) \Rightarrow 30 I_1 = -10 I_2 - 3$$

$$\boxed{I_1 = -\frac{I_2}{3} - 0.1}$$

$$(3) \Rightarrow 20 I_3 = -10 I_2 - 15$$

$$\boxed{I_3 = \frac{-I_2}{2} - \frac{3}{4}}$$

plug into (1): $I_2 = -\frac{I_2}{3} - 0.1 - \frac{I_2}{2} - 0.75$

$$6 I_2 = -2 I_2 - 3 I_2 - 5.1$$

$$11 I_2 = -5.1$$

$$\boxed{I_2 = -0.464 \text{ A}}$$

← means our direction is wrong → going up.

12. Initially unpolarized light of intensity 350 W/m^2 travels through three polarizing filters. The axis of the first is vertical, the axis of the second is 30.0° from vertical, and the axis of the third is horizontal. What is the intensity of the light which emerges from the third filter?

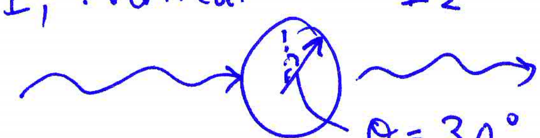
A. 180 W/m^2
B. zero
C. 33 W/m^2
D. 44 W/m^2
E. 66 W/m^2

First filter: when unpolarized light falls on a polarizing filter, the intensity is reduced by ½:

$$I_1 = \frac{I_0}{2}. \qquad (\text{Eq. } 35.43)$$

Second Filter

I_1 : vertical $\qquad\qquad I_2$



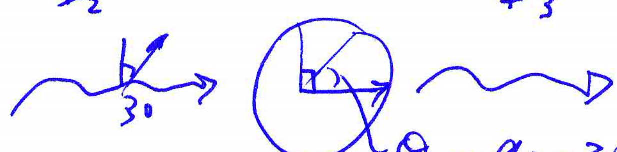
$\theta_2 = 30°.$

use Malus's Law Eq. 35.42 ∴

$$I_2 = I_1 \cos^2\theta_2 = \frac{I_0}{2}\cos^2(30°)$$

Third Filter

I_2 : 30° $\qquad\qquad I_3$



$\theta_3 = 90 - 30 = 60°$

Malus's Law:

$$I_3 = I_2 \cos^2\theta_3 = \frac{I_0}{2}\cos^2(30°)\cos^2(60°)$$

$$I_3 = I_0(0.09375)$$

$$= \boxed{32.8 \text{ W/m}^2}$$

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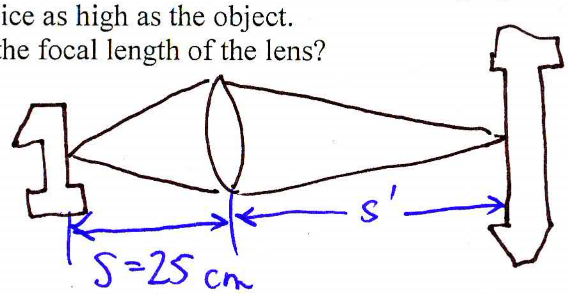
FREE-FORM IN FOUR UNRELATED PARTS (16 points total)

Clearly show your reasoning and work as some part marks may be awarded. Please draw a box around your final answer for each question.

PART A (4 points)

An object is located 25.0 cm from a certain converging lens. The lens forms a real image that is exactly twice as high as the object.

1. What is the focal length of the lens?



$s' = 2s = 50\ \text{cm}$

$f = \left[\frac{1}{25\,\text{cm}} + \frac{1}{50\,\text{cm}}\right]^{-1}$

$\boxed{f = +16.7\ \text{cm}}$

3 sig. figs

lateral magnification, Eq. 23.14:

$|m| = \frac{s'}{s} = 2$

$\Rightarrow s' = 2s$

2. Now replace the lens used in part A.1 with another lens. The new lens is a diverging lens whose focal points are at the same distance from the lens as the focal points of the first lens. If the object is 5.00 cm high, what is the magnitude of the height of the image formed by the new lens? The object is still located 25.0 cm from the lens.

Same $|f|$, but negative: $f = -16.667\ \text{cm}$

$s = 25\ \text{cm}$

$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1} = \left(\frac{1}{-16.667} - \frac{1}{25}\right)^{-1}$

$s' = -10\ \text{cm}$

$m \equiv \frac{h'}{h} = \frac{s'}{s} \Rightarrow$ $|h'| = \left|\frac{h s'}{s}\right| = \frac{(5\ \text{cm})(10\ \text{cm})}{25\ \text{cm}} = \boxed{2.00\ \text{cm}}$

3 sig figs.

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PART B (3 points)

Two identical particles of mass *m* and charge *q* are very far apart, and traveling directly toward one another. When the distance between the particles is very large, they both have speed *v*. Assume *v* is much less than *c*. How close will the particles be to one another at the moment they stop, just before they turn around?

initial: $\xrightarrow{v}$ $\xleftarrow{v}$

$d \to \infty$, $P.E._0 = 0$

$K.E._0 = 2\left(\frac{1}{2}mv^2\right) = mv^2$

final • $\overset{d}{\longleftrightarrow}$ •

$K.E._f = 0$

Eq. 29.14: electric potential energy of two point charges $\rightarrow$ $P.E._f = \frac{Kqq}{d}$

Conservation of energy: $E_f = E_0$

$K.E._f + P.E._f = K.E._0 + P.E._0$

$0 + \frac{Kqq}{d} = mv^2 + 0$

$\boxed{d = \frac{Kq^2}{mv^2}}$ or $\boxed{d = \frac{q^2}{4\pi\epsilon_0 mv^2}}$

PART C (3 points)

~~CNFY~~ CFNY "The Edge" broadcasts from the top of the CN Tower at 102.1 MHz with a power of 35 kW. Assume that the radiation is emitted uniformly in all directions. Here in this room at U of T, we are 2.1 km away from the top of the CN Tower.

1. What is the intensity of the CFNY signal in this room?

Eq. 35.38: $I = \frac{P}{4\pi r^2} = \frac{35\times10^3\ W}{4\pi(2.1\times10^3)^2} = \boxed{6.3\times10^{-4}\ W/m^2}$

2 sig. figs.

2. What is the electric field amplitude of the CFNY signal in this room?

Eq. 35.37: $I = \frac{c\epsilon_0 E_0^2}{2}$, solve for E_0:

$E_0^2 = \frac{2I}{c\epsilon_0}$

$E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2\times6.316\times10^{-4}}{3\times10^8\times8.85\times10^{-12}}}$

$\boxed{E_0 = 0.69\ V/m}$ or $\boxed{E_0 = 0.69\ N/C}$

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PART D (6 points)

Earth and Mars are 7.8×10^{10} m apart at the closest approach in their orbits. Imagine a space shuttle which travels along this shortest straight line from Earth to Mars at a constant speed of 2.90×10^{8} m/s. [You may neglect the relatively small orbital velocities of Earth and Mars in this problem.]

1. How long does it take the shuttle to travel from Earth to Mars, as measured in the reference frame of the Sun?

$$v = \frac{d}{t} \Rightarrow t = \frac{d}{v} = \frac{7.8 \times 10^{10}\ \text{m}}{2.9 \times 10^{8}\ \text{m/s}} \approx \boxed{270\ \text{s}}$$

or $\boxed{t = 4.5\ \text{minutes}}$

2. How long does it take the shuttle to travel from Earth to Mars, as measured in the reference frame of an observer on the moving shuttle?

Do Part D.3 first

$$t' = \frac{L'}{v} = \frac{2.00 \times 10^{10}\ \text{m}}{2.9 \times 10^{8}\ \text{m/s}} = \boxed{69\ \text{s}}$$

or $\boxed{t' = 1.1\ \text{minutes}}$

3. What is the distance from Earth to Mars, as measured in the reference frame of an observer on the moving shuttle?

$$L' = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$= 7.8 \times 10^{10}\ \text{m} \sqrt{1 - \left(\frac{2.9 \times 10^{8}}{3 \times 10^{8}}\right)^2}$$

$$= 1.997 \times 10^{10}\ \text{m}$$

$\boxed{L' = 2.0 \times 10^{10}\ \text{m}}$