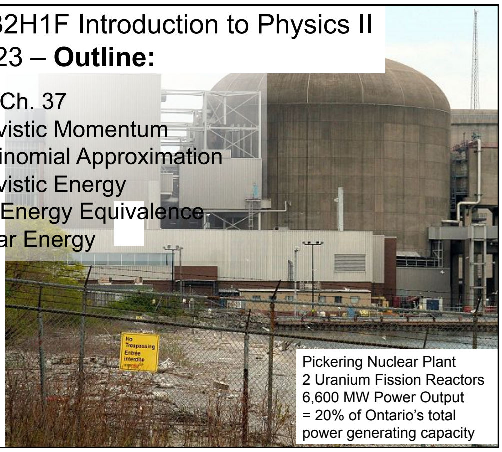
PHY132H1F Introduction to Physics II
Class 23 – **Outline:**

Today, Ch. 37
- Relativistic Momentum
- The Binomial Approximation
- Relativistic Energy
- Mass-Energy Equivalence
- Nuclear Energy

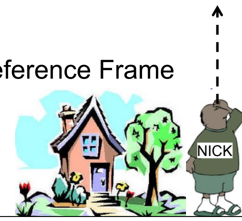


Pickering Nuclear Plant
2 Uranium Fission Reactors
6,600 MW Power Output
= 20% of Ontario's total power generating capacity

Review of time dilation: Molly flies her rocket past Nick at constant velocity *v*. Molly and Nick both measure the time it takes the rocket, from nose to tail, to pass Nick. Which of the following is true?

A. Nick measures a shorter time interval than Molly.
B. Molly measures a shorter time interval than Nick.
C. Both Molly and Nick measure the same amount of time.

Nick's Reference Frame



Last time (ICDQ3): Molly flies her rocket past Nick at constant velocity *v*. Molly and Nick both measure the time it takes the rocket, from nose to tail, to pass Nick. Which of the following is true?

A. Nick measures a shorter time interval than Molly.

Molly's Reference Frame

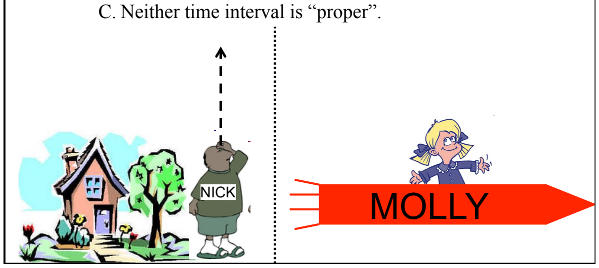
Both observers agree on the *relative speed*.

But it's a further distance in Molly's frame. (she isn't length contracted in her own frame).

So, $\Delta t_{\text{Molly}} > \Delta t_{\text{Nick}}$

But who is right??! Molly flies her rocket past Nick at constant velocity *v*. Molly and Nick both measure the time it takes the rocket, from nose to tail, to pass Nick. Which of the following is true?

A. Nick measures the "proper" time interval.
B. Molly measures the "proper" time interval.
C. Neither time interval is "proper".



Time Dilation

The time interval between two events that occur at the *same position* is called the **proper time** $\Delta\tau$. In an inertial reference frame moving with velocity $v = \beta c$ relative to the proper time frame, the time interval between the two events is

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}} \geq \Delta\tau \qquad \text{(time dilation)}$$

The "stretching out" of the time interval is called time dilation.

Length Contraction

The distance L between two objects, or two points on one object, measured in the reference frame S in which the objects are at rest is called the proper length ℓ. The distance L' in a reference frame S′ is

$$L' = \sqrt{1-\beta^2}\,\ell \leq \ell$$

NOTE: Length contraction does not tell us how an object would *look*. The visual appearance of an object is determined by light waves that arrive simultaneously at the eye. Length and length contraction are concerned only with the *actual* length of the object at one instant of time.

Recall the Galilean Transformations

Consider two reference frames S and S′. The coordinate axes in S are x, y, z and those in S′ are x', y', z'. Reference frame S′ moves with velocity v relative to S along the x-axis. Equivalently, S moves with velocity −v relative to S′.

The *Galilean transformations of position* are:

$$x = x' + vt \qquad x' = x - vt$$
$$y = y' \quad \text{or} \quad y' = y$$
$$z = z' \qquad z' = z$$

The *Galilean transformations of velocity* are:

$$u_x = u'_x + v \qquad u'_x = u_x - v$$
$$u_y = u'_y \quad \text{or} \quad u'_y = u_y$$
$$u_z = u'_z \qquad u'_z = u_z$$

The Lorentz Transformations

Consider two reference frames S and S′. An event occurs at coordinates x, y, z, t as measured in S, and the same event occurs at x', y', z', t' as measured in S′. Reference frame S′ moves with velocity $+v$ relative to S, along the x-axis.

The Lorentz transformations for the coordinates of one event are:

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

The Lorentz Velocity Transformations

Consider two reference frames S and S′. An object moves at velocity u along the x-axis as measured in S, and at velocity u' as measured in S′. Reference frame S′ moves with velocity v relative to S, also along the x-axis.

The Lorentz velocity transformations are:

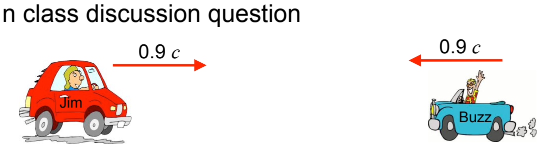
$$u' = \frac{u - v}{1 - uv/c^2} \quad \text{and} \quad u = \frac{u' + v}{1 + u'v/c^2}$$

NOTE: It is important to distinguish carefully between v, which is the relative velocity between two reference frames, and u and u' which are the velocities of an *object* as measured in the two different reference frames.

Example

- Jim and Buzz drive toward each other. Each is driving at 0.6 c.
- In the reference frame of Jim, how fast is Buzz approaching him?

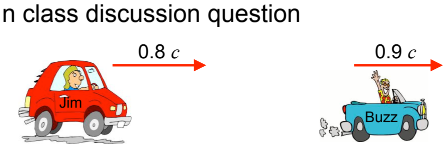
In class discussion question



- Jim and Buzz drive toward each other at 0.9 c.
- In the reference frame of Jim, Buzz is traveling at speed:

A. 1.8 c
B. 0.994 c
C. 0.9 c
D. zero
E. c

In class discussion question



- Jim is driving along the road at 0.8 c. Suddenly, Buzz passes him! Buzz is driving in the same direction as Jim, but at speed 0.9 c.
- In the reference frame of Jim, Buzz is traveling at speed:

A. 0.1 c
B. 0.06 c
C. 0.36 c
D. zero
E. c

Relativistic Momentum

The momentum of a particle moving at speed u is

$$p = \gamma_p m u$$

$$\gamma_p = \frac{1}{\sqrt{1 - u^2/c^2}}$$

where the subscript p indicates that this is γ for a particle, not for a reference frame.

If $u << c$, the momentum approaches the Newtonian value of $p = mu$. As u approaches c, however, p approaches infinity.

- For this reason, a force cannot accelerate a particle to a speed higher than c, because the particle's momentum becomes infinitely large as the speed approaches c.

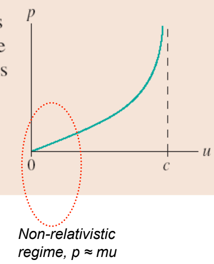
Relativistic Momentum

Momentum

The law of conservation of momentum is valid in all inertial reference frames if the momentum of a particle with velocity u is $p = \gamma_p m u$, where

$$\gamma_p = 1/\sqrt{1 - u^2/c^2}$$

The momentum approaches ∞ as $u \rightarrow c$.



Relativistic Energy

The total energy E of a particle is

$$E = \gamma_p mc^2 = E_0 + K = \text{rest energy} + \text{kinetic energy}$$

This total energy consists of a rest energy

$$E_0 = mc^2$$

and a relativistic expression for the *kinetic energy*

$$K = (\gamma_p - 1)mc^2 = (\gamma_p - 1)E_0$$

This expression for the kinetic energy is very nearly $\frac{1}{2}mu^2$ when $u << c$.

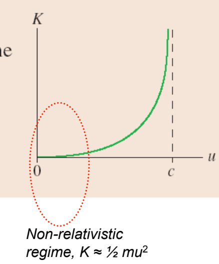
Relativistic Energy

Energy

The law of conservation of energy is valid in all inertial reference frames if the energy of a particle with velocity u is $E = \gamma_p mc^2 = E_0 + K$

Rest energy $E_0 = mc^2$

Kinetic energy $K = (\gamma_p - 1)mc^2$.



Conservation of Energy

The creation and annihilation of particles with mass, processes strictly forbidden in Newtonian mechanics, are vivid proof that neither mass nor the Newtonian definition of energy is conserved.

The *total* energy—the kinetic energy *and* the energy equivalent of mass—remains a conserved quantity.

Law of conservation of total energy The energy $E = \sum E_i$ of an isolated system is conserved, where $E_i = (\gamma_p)_i m_i c^2$ is the total energy of particle i.

Mass and energy are not the same thing, but they are *equivalent* in the sense that mass can be transformed into energy and energy can be transformed into mass as long as the total energy is conserved.

Conservation of Mass/Energy

Mass-energy equivalence

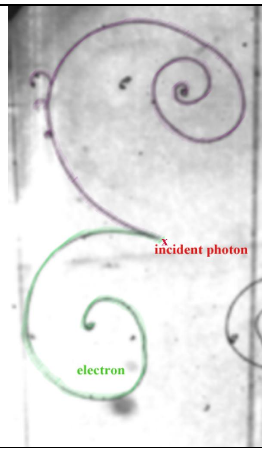
Mass m can be transformed into energy $E = mc^2$.

Energy can be transformed into mass $m = \Delta E/c^2$.

If you could convert the mass in 1 kg of water into energy, about how much energy could you get, in principle?

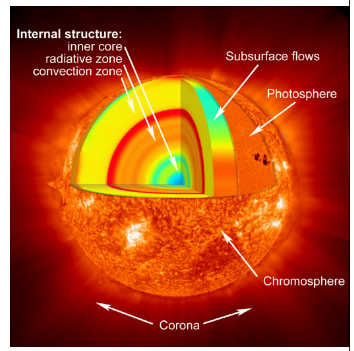
A. 10^8 J
B. 10^{11} J
C. 10^{14} J
D. 10^{17} J
E. 10^{20} J

Energy converting to Mass



High Energy Photons can transform into a particle / anti-particle pair

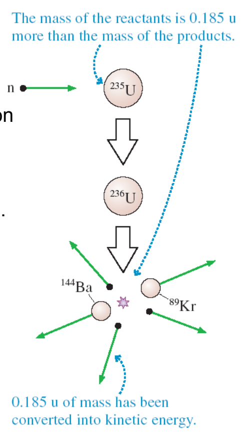
Mass converting to Energy



High pressure, high temperature Hydrogen gas at the core of the sun can be converted into Helium. (Nuclear Fusion)

The mass decreases in the process, releasing high energy photons.

Nuclear Fission is easier to control in reactors than nuclear fusion.



The Barium and Krypton isotopes produced in the process are very radioactive, and must be stored carefully, so as not to be a health-hazard to humans.

Next Class

- We meet here on Wednesday 11am-12pm.
- I will discuss any questions about Problem Set 9, due Wednesday evening.
- I will finish up any outstanding issues with Chapter 37.
- I will review the course. – Please email me if there is anything specific you'd like me to cover!