PHY132H1F
Term Test 2 —version 1
Tuesday, November 16, 2010   Duration: 80 minutes
SOLUTIONS

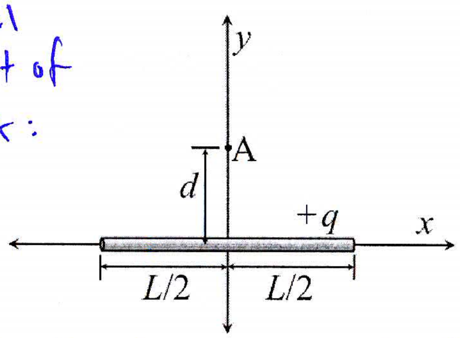
Possibly helpful information for this test:

Coulomb's constant is $K = 8.99 \times 10^9$ N m^2/C^2 Fundamental unit of charge $e = 1.60 \times 10^{-19}$ C

$\int \sin(x)dx = -\cos(x)$ $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$

Prefixes: "c" = 10^{-2} "m" = 10^{-3} "μ" = 10^{-6} "n" = 10^{-9}

1. A finite rod of length L has total charge q, distributed uniformly along its length. The rod lies on the x-axis and is centered at the origin. Thus one endpoint is located at $(-L/2, 0)$, and the other is located at $(L/2, 0)$. Define the electric potential to be zero at an infinite distance away from the rod. What is V_A, the electric potential at point A (see the figure), located a distance d above the midpoint of the rod on the y-axis? [Note that in the expressions below, K is the Coulomb constant, and ln(x) is the natural logarithm of x.]



A. $\frac{Kq}{L}\ln\left(L+\sqrt{L^2+d^2}\right)$

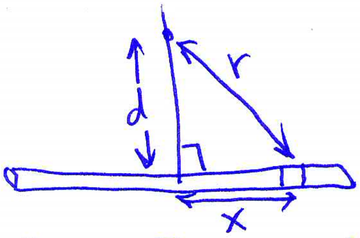
[✓ B.] $\frac{2Kq}{L}\ln\left(\frac{L}{2d}+\sqrt{\frac{L^2}{4d^2}+1}\right)$

C. $\frac{2Kq}{L}\ln\left(\frac{L}{2}+\sqrt{\frac{L^2}{4}+d^2}\right)$

D. $\frac{2Kq}{Ld}\ln\left(\frac{L}{2}+\sqrt{\frac{L^2}{4}+d^2}\right)$

E. $\frac{2Kq}{L}\ln\left(\frac{\sqrt{(L^2+4d^2)}+L}{2d}\right)$

Consider the potential due to a small part of the rod at position, x:



$r = \sqrt{d^2+x^2}$

length of small part $= dx$

charge of small part $= dq$

$dq = \left(\frac{q}{L}\right)dx$.

potential due to small part is $dV = \frac{K\,dq}{r}$

Note: By symmetry, Potential at A will be potential of right half of rod + potential of left half = 2 × Potential of right half.

$$V = 2\int_{x=0}^{x=L/2} dV = 2\int_0^{L/2} K\left(\frac{q}{L}\right)dx\,\frac{1}{\sqrt{x^2+d^2}} = \frac{2Kq}{L}\int_0^{L/2}\frac{dx}{\sqrt{x^2+d^2}}$$

$$V = \frac{2Kq}{L}\left[\ln\left(x+\sqrt{x^2+d^2}\right)\right]_0^{L/2} = \frac{2Kq}{L}\left[\ln\left(\frac{L}{2}+\sqrt{\frac{L^2}{4}+d^2}\right)-\ln(d)\right]$$

Note: $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$:

$$V = \frac{2Kq}{L}\ln\left(\frac{L}{2d}+\sqrt{\frac{L^2}{4d^2}+1}\right)$$

2. According to Wikipedia, *"In the field of cell biology, potassium channels are the most widely distributed type of ion channel and are found in virtually all living organisms."* [en.wikipedia.org/wiki/Potassium_channel] A Potassium ion is near a cell membrane and experiences the electric field of a dipole within the cell. This draws the ion directly toward the dipole. As the ion approaches the cell, its distance from the dipole decreases by a factor of ½. As this happens, the electric field strength experienced by the ion due to the dipole
   A. does not change.
   B. increases by a factor of 2.
   C. increases by a factor of 4.
   [✓ D.] increases by a factor of 8.
   E. increases by a factor of 16.

Note $\vec{E}_{dipole} \propto \frac{1}{r^3}$

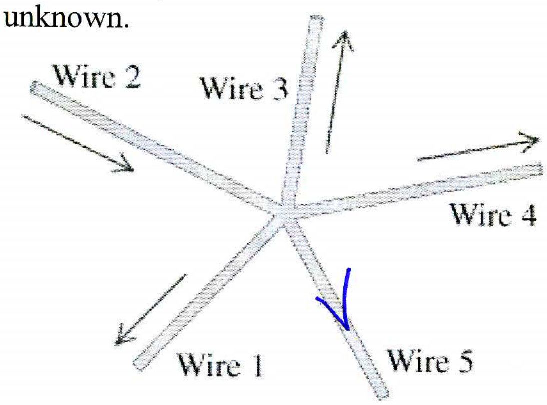
[Eq. 27.11 and Eq. 27.12].

So $r \to \frac{r}{2}$

$$\Rightarrow \vec{E} \to 2^3 E = \boxed{8E}$$

3. Consider a junction of five wires, as shown in the figure. The arrows indicate the direction of current flow. The information about the magnitudes of the current density and the diameters for wires 1, 2, 3, and 4 is given in the table. Some of the values are unknown.

Wire	Current density [A/mm^2]	Diameter [mm]	Total Current [A]
1	2.1	1.3	???
2	???	2.0	5.9
3	3.9	0.73	???
4	1.0	???	12



What is the current I_5 in wire 5? [Assume that the current out of the junction is positive and that the current into the junction is negative.]

A. −9.1 A
B. +9.1 A
[✓ C.] −11 A
D. +11 A
E. +15 A

By Junction Rule

$I_{in} = I_{out}$

$I_2 = I_1 + I_3 + I_4 + I_5$

$\Rightarrow I_5 = I_2 - (I_1 + I_3 + I_4)$

Also Eq. 31.21: $I = JA = J\pi r^2 = \frac{J\pi D^2}{4}$.

$\Rightarrow I_1 = \frac{(2.1)\pi(1.3^2)}{4} = 2.787\ A$

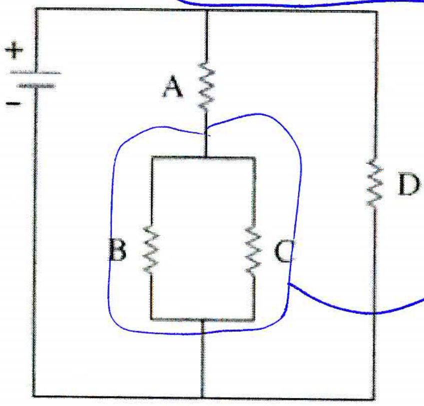
$I_3 = \frac{(3.9)\pi(0.73^2)}{4} = 1.632\ A$

$\Rightarrow I_5 = 5.9 - (2.787 + 1.632 + 12)$

$= -10.519$ or $I_5 = -11\ A$ to 2 sig figs.

4. Rank the power consumption of the 4 identical resistors shown.

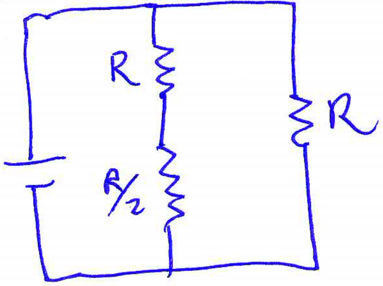
- A. $P_B = P_A = P_C = P_D$
- B. $P_D > P_B = P_C > P_A$
- C. $P_A > P_B = P_C > P_D$
- D. $P_D > P_A = P_B = P_C$
- [✓ E.] $P_D > P_A > P_B = P_C$



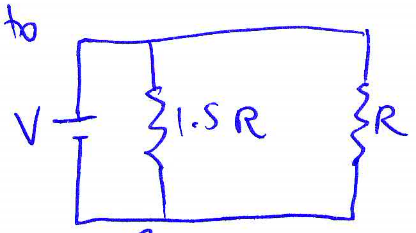
All have resistance R.

$$R_{eq_{BC}} = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{R}{2}$$

↓ equivalent to



equivalent to ↓



The potential drop across both ends is V.

⇒ Power dissipated by D is $P_D = \frac{V^2}{R}$

Summary

$P_D = 1\ V^2/R$

$> P_A = 0.7\ V^2/R$

$> P_B = P_C = 0.1\ V^2/R$

current through middle path is given by $I_A = \frac{V}{1.5R} = 0.667\left(\frac{V}{R}\right)$

⇒ Power dissipated by A is $P_A = I_A^2 R = \left(0.667\frac{V}{R}\right)^2 R$

$P_A = 0.444\ \frac{V^2}{R}$

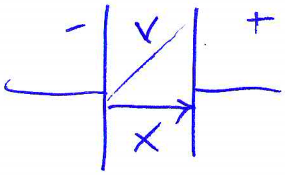
voltage drop across A is $V_A = I_A R = 0.667\left(\frac{V}{R}\right)R = 0.667\,V$.

⇒ voltage drop across B & C is $V - 0.667\,V = 0.333\,V$.

⇒ Power dissipated by B & C is $P_B = P_C = \frac{(0.333\,V)^2}{R} = 0.111\ \frac{V^2}{R}$

5. The electric potential inside a charged capacitor

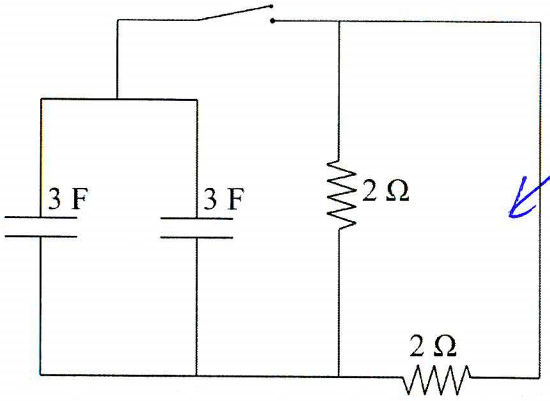
   A. is constant.
   [✓ B.] increases linearly from the negative to the positive plate.
   C. decreases linearly from the negative to the positive plate.
   D. decreases inversely with distance from the negative plate.
   E. decreases inversely with the square of the distance from the negative plate.



$V = Es$ [Eq. 29.25]

6. What is the time constant for the discharge of the capacitors in the figure below?

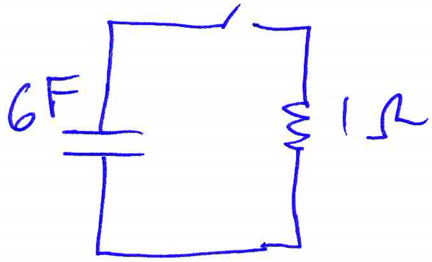
A. 2×10^{-3} s
B. 0.17 s
C. 1.3 s
[✓ D. 6 s]
E. 24 s



$R_{eq} = \left(\frac{1}{2} + \frac{1}{2}\right)^{-1}$

$R_{eq} = 1\,\Omega$

equivalent to:



$\tau = RC = (1\Omega)(6F) = 6\,s$

7. The electron in a hydrogen atom maintains an average distance from the proton of 0.053 nm. What is the electric field at the position of the electron?

   A. 8.2×10^{-8} V/m, toward the proton
   B. 27 V/m, toward the proton
   C. 27 V/m, away from the proton
   ~~D.~~ 5.1×10^{11} V/m, toward the proton
   [✓ E.] 5.1×10^{11} V/m, away from the proton

$$\vec{E} = \frac{Kq}{r^2} = \frac{(8.99\times10^{9})(1.6\times10^{-19})}{(0.053\times10^{-9})^2}$$

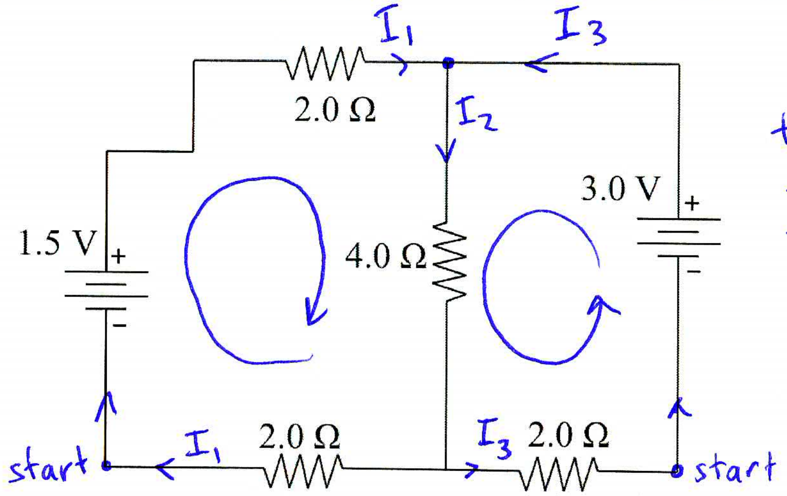
$$E = 5.12\times10^{11}\ \frac{N}{C}$$

same as V/m.

away from proton

8. What is the magnitude of the current through the 4.0 Ω resistor in the circuit below?

A. 0.15 A
B. 0.19 A
[✓ C. 0.47 A]
D. 0.50 A
E. 1.1 A



choose loop direction to match natural flow through batteries → for both loops assumed current is down in common wire & 4 Ω resistor.

Kirchhoff's Loop Law for left loop:

$$+1.5\text{V} - 2.0\,I_1 - 4.0\,I_2 - 2.0\,I_1 = 0 \qquad \text{(Eq.1)}$$

Kirchhoff's Loop Law for right loop:

$$+3.0\,\text{V} - 4.0\,I_2 - 2.0\,I_3 = 0 \qquad \text{(Eq.2)}$$

Kirchhoff's Junction Law for top junction

$$\Sigma I_{in} = \Sigma I_{out}$$

$$I_1 + I_3 = I_2 \qquad \text{(Eq.3)}$$

$$\text{(Eq.1)} \Rightarrow 4.0\,I_1 = 1.5 - 4.0\,I_2$$

$$\boxed{I_1 = \frac{1.5}{4} - I_2}$$

$$\text{(Eq.2)} \Rightarrow 2.0\,I_3 = 3.0 - 4.0\,I_2$$

$$\boxed{I_3 = \frac{3}{2} - 2I_2}$$

$$I_2 = \left(\frac{1.5}{4} - I_2\right) + \left(\frac{3}{2} - 2I_2\right)$$

$$I_2 = \frac{3}{8} + \frac{12}{8} - 3I_2$$

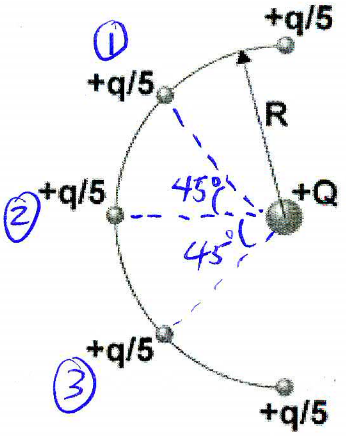
$$4\,I_2 = \frac{15}{8} \Rightarrow I_2 = \frac{15}{32} = 0.46875$$

FREE-FORM IN TWO UNRELATED PARTS (12 points total)
Clearly show your reasoning and work as some part marks may be awarded. Write your final answers in the boxes provided.

Mag. of all forces: $F = \frac{K(q/5)Q}{R^2}$

PART A

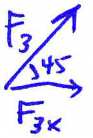
1. A point charge +*Q* is located a distance *R* away from five identical point charges, each of charge +*q*/5, equally distributed along a semicircular arc of radius *R* as shown. What is the total force, magnitude and direction, exerted on +*Q*? [Please express your answer in terms of *Q*, *q*, *R*, the Coulomb constant, *K*, and numerical constants.]



- By symmetry, force from top and bottom charges cancel.
- Also by symmetry, vertical components will all cancel, so net force will be horizontal.

$F_{1x} = F_1 \cos 45$

$F_{2x} = F_2$



$F_{3x} = F_3 \cos 45$

$$F_{Netx} = F_{1x} + F_{2x} + F_{3x} = (\cos 45 + 1 + \cos 45)\left(\frac{K(q/5)Q}{R^2}\right)$$

$$= \left(\frac{2}{\sqrt{2}} + 1\right)\frac{1}{5}\,\frac{KQq}{R^2}$$

$$\vec{F}_{Net} = \left(\frac{1+\sqrt{2}}{5}\right)\frac{KQq}{R^2}, \text{ to the right.}$$

$$= 0.483\,\frac{KQq}{R^2}, \text{ to the right.}$$

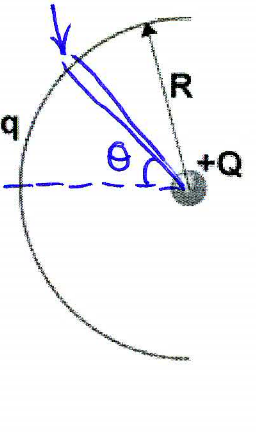
$$\vec{F} = \left(\frac{1+\sqrt{2}}{5}\right)\frac{KQq}{R^2}, \text{ to the right}$$

PART A

2. A point charge $+Q$ is located a distance R away from a semicircular arc that is uniformly charged with a total harge of $+q$ as shown. The charge per length along the semicircle is:

$$\lambda = \frac{+q}{\pi R}$$

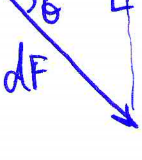
What is the total force, magnitude and direction, exerted on $+Q$? [Please express your answer in terms of Q, q, R, the Coulomb constant, K, and numerical constants.]



- By symmetry, force from top & bottom half will be the same, so $F = 2F_{top}$
- horizontal force due to segment $d\theta$ is dF_x

$$F_{top} = \int_{\theta=0}^{\theta=\pi/2} dF_x$$

- Arc length along $d\theta$ is $ds = R\,d\theta$
- Charge of this segment is $dq = \lambda\, ds = \frac{q\,R\,d\theta}{\pi R} = \frac{q}{\pi}\,d\theta$



$$dF_x = dF\cos\theta$$

Magnitude is : $dF = \frac{K(dq)\,Q}{R^2}$

$$\Rightarrow dF_x = \frac{KQ}{R^2}\left(\frac{q}{\pi}\,d\theta\right)\cos\theta = \frac{KQq}{\pi R^2}\cos\theta\,d\theta$$

$$F = 2F_{top} = 2\left(\frac{KQq}{\pi R^2}\right)\int_0^{\pi/2}\cos\theta\,d\theta = \frac{2KQq}{\pi R^2}\Big[\sin\theta\Big]_0^{\pi/2}$$

$$\vec{F} = \frac{2KQq}{\pi R^2}\text{, to the right}$$

$$\boxed{\vec{F} = \left(\frac{2}{\pi}\right)\frac{KQq}{R^2}\text{, to the right}}$$

$$\vec{F} = 0.637\,\frac{KQq}{R^2}\text{, to the right}$$

PART B

The flash unit in a camera uses a 1.5 V battery to charge a 33 μF capacitor. The capacitor is then discharged through a flashlamp. The discharge takes 8.0 μs. During this time interval, what is the average power dissipated by the flashlamp?

initial voltage $(\Delta V)_0 = 1.5\,V$

$\Rightarrow$ initial potential energy, from

Eq-30.26: $U_{c0} = \dfrac{Q_0^2}{2C}$, where $Q_0 = (\Delta V)_0 C$

$$U_{c0} = \frac{(\Delta V)_0^2 C^2}{2C} = \frac{(\Delta V)_0^2 C}{2}$$

$$U_{c0} = \frac{1.5^2 (33 \times 10^{-6})}{2}$$

$U_{c0} = 3.713 \times 10^{-5}$ Joules.

Final voltage & charge = zero $\Rightarrow$ $U_{cf} = 0$.

$\Rightarrow$ Power dissipated = $\dfrac{|\text{change in energy}|}{\text{time interval}}$

$$= \frac{3.713 \times 10^{-5} \text{ Joules}}{8 \times 10^{-6} \text{ s}}$$

$P_{avg} = 4.64$ J/s

2 sig figs:

$$\boxed{P_{\text{avg}} = 4.6 \text{ W}}$$