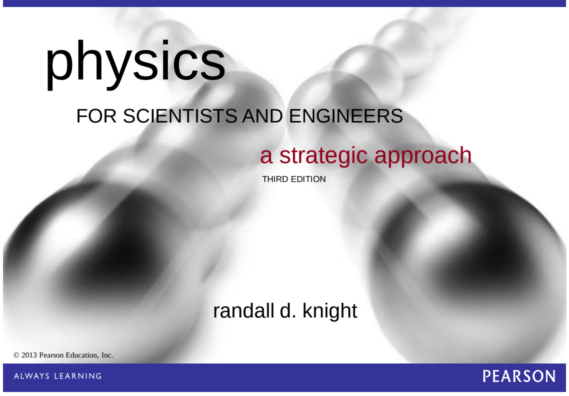
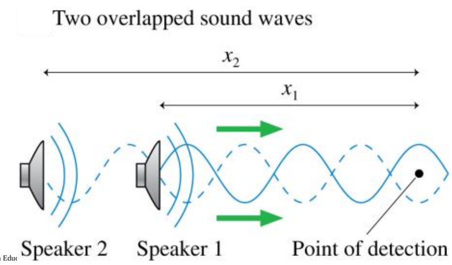
Class 3, Sections 21.5-21.8 Preclass Notes



Interference in One Dimension

The pattern resulting from the superposition of two waves is often called **interference**. In this section we will look at the interference of two waves traveling in the *same* direction.

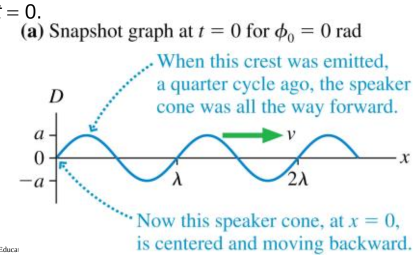


Interference in One Dimension

- A sinusoidal wave traveling to the right along the x-axis has a displacement:

$$D = a\sin(kx - \omega t + \phi_0)$$

- The phase constant ϕ_0 tells us what the source is doing at $t = 0$.

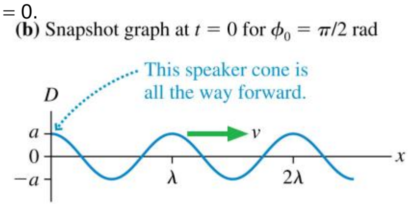


Interference in One Dimension

- A sinusoidal wave traveling to the right along the x-axis has a displacement:

$$D = a\sin(kx - \omega t + \phi_0)$$

- The phase constant ϕ_0 tells us what the source is doing at $t = 0$.



Constructive Interference

$$D_1 = a\sin(kx_1 - \omega t + \phi_{10})$$

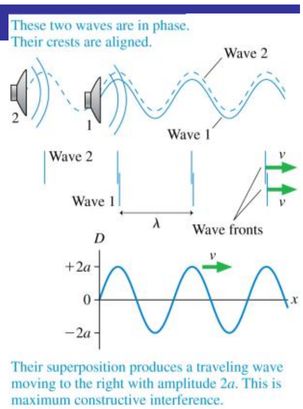
$$D_2 = a\sin(kx_2 - \omega t + \phi_{20})$$

$$D = D_1 + D_2$$

- The two waves are **in phase**, meaning that

$$D_1(x) = D_2(x)$$

- The resulting amplitude is $A = 2a$ for *maximum constructive interference*.



Destructive Interference

$$D_1 = a\sin(kx_1 - \omega t + \phi_{10})$$

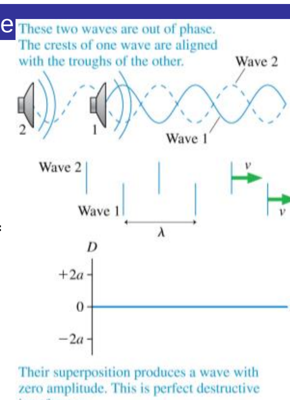
$$D_2 = a\sin(kx_2 - \omega t + \phi_{20})$$

$$D = D_1 + D_2$$

- The two waves are **out of phase**, meaning that

$$D_1(x) = -D_2(x).$$

- The resulting amplitude is $A = 0$ for *perfect destructive interference.*



The Mathematics of Interference

- As two waves of equal amplitude and frequency travel together along the *x*-axis, the net displacement of the medium is:

$$D = D_1 + D_2 = a\sin(kx_1 - \omega t + \phi_{10}) + a\sin(kx_2 - \omega t + \phi_{20})$$
$$= a\sin\phi_1 + a\sin\phi_2$$

- We can use a trigonometric identity to write the net displacement as:

$$D = \left[2a\cos\left(\frac{\Delta\phi}{2}\right)\right]\sin(kx_{avg} - \omega t + (\phi_0)_{avg})$$

Where $\Delta\phi = \phi_1 - \phi_2$ is the phase difference between the two waves.

The Mathematics of Interference

$$D = \left[2a\cos\left(\frac{\Delta\phi}{2}\right)\right]\sin(kx_{avg} - \omega t + (\phi_0)_{avg})$$

- The amplitude has a maximum value $A = 2a$ if $\cos(\Delta\phi/2) = \pm 1$.
- This is **maximum constructive interference**, when:

$$\Delta\phi = m \cdot 2\pi \quad \text{(maximum amplitude } A = 2a\text{)}$$

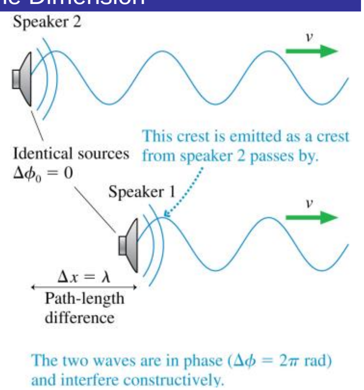
where m is an integer.

- Similarly, the amplitude is zero if $\cos(\Delta\phi/2) = 0$.
- This is **perfect destructive interference**, when:

$$\Delta\phi = \left(m + \tfrac{1}{2}\right) \cdot 2\pi \qquad \text{(minimum amplitude } A = 0\text{)}$$

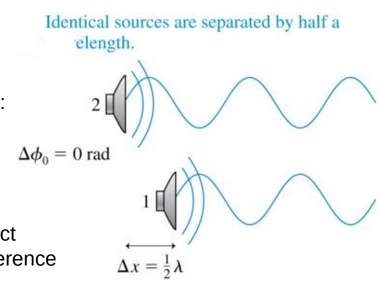
Interference in One Dimension

- Shown are two identical sources located one wavelength apart: $\Delta x = \lambda$
- The two waves are "in step" with $\Delta\phi = 2\pi$, so we have maximum constructive interference with $A = 2a$.

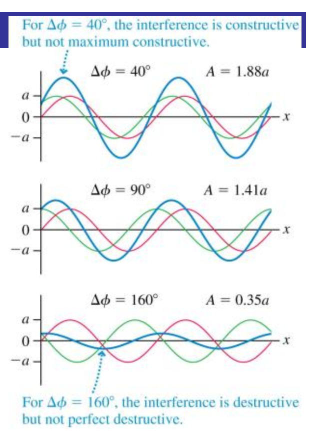


Interference in One Dimension

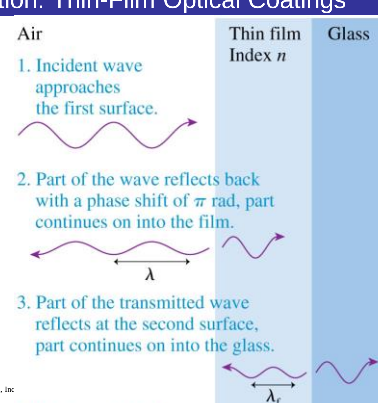
- Shown are two identical sources located half a wavelength apart: $\Delta x = \lambda/2$
- The two waves have phase difference $\Delta\phi = \pi$, so we have perfect destructive interference with $A = 0$.



- It is entirely possible, of course, that the two waves are neither exactly in phase nor exactly out of phase.
- Shown are the calculated interference of two waves that differ in phase by 40°, 90° and 160°.



Application: Thin-Film Optical Coatings



Application: Thin-Film Optical Coatings

- The phase difference between the two reflected waves is:

$$\Delta\phi = 2\pi\frac{2d}{\lambda/n} = 2\pi\frac{2nd}{\lambda}$$

where n is the index of refraction of the coating, d is the thickness, and λ is the wavelength of the light in vacuum or air.

- For a particular thin-film, constructive or destructive interference depends on the wavelength of the light:

$$\lambda_C = \frac{2nd}{m} \qquad m = 1, 2, 3, \ldots \qquad \text{(constructive interference)}$$

$$\lambda_D = \frac{2nd}{m - \frac{1}{2}} \qquad m = 1, 2, 3, \ldots \qquad \text{(destructive interference)}$$

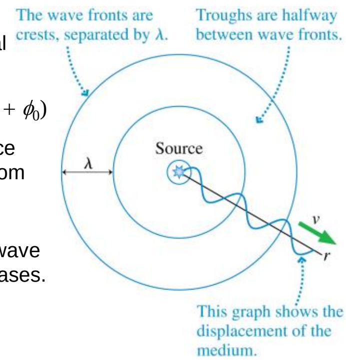
A Circular or Spherical Wave

- A circular or spherical wave can be written:

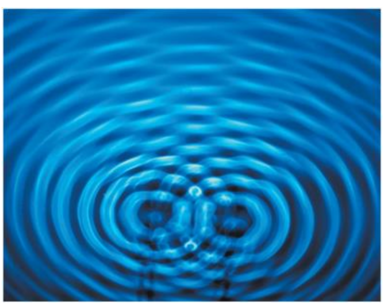
$$D(r, t) = a\sin(kr - \omega t + \phi_0)$$

where r is the distance measured outward from the source.

- The amplitude a of a circular or spherical wave diminishes as r increases.



Interference in Two and Three Dimensions



Two overlapping water waves create an interference pattern.

Interference in Two and Three Dimensions

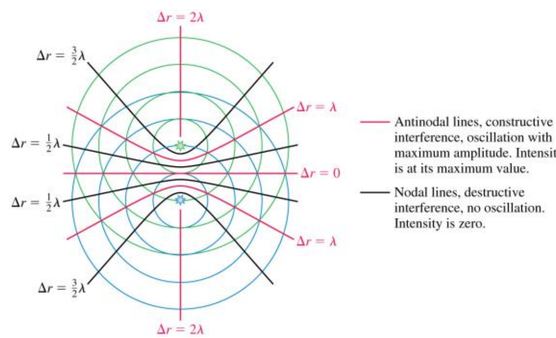


Figure 21.30, page 612

Interference in Two and Three Dimensions

- The mathematical description of interference in two or three dimensions is very similar to that of one-dimensional interference.
- The conditions for constructive and destructive interference are:

Maximum constructive interference:

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = m \cdot 2\pi$$

Perfect destructive interference:

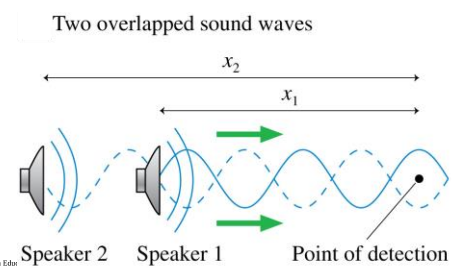
$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi$$

$$m = 0, 1, 2, \ldots$$

where Δr is the *path-length difference*.

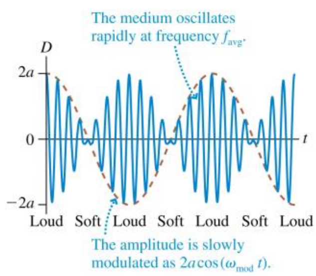
Interference in One Dimension: Beats

The pattern resulting from the superposition of two waves is often called **interference**. In this section we will look at the interference of two waves traveling in the *same* direction.



Beats

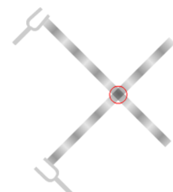
The figure shows the history graph for the superposition of the sound from two sources of equal amplitude a, but slightly different frequency.



Beats

http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/Beats/Beats.html

Beats



"Beats" is a phenomenon that arises when 2 waves, such as sound waves, have almost identical frequencies.

To the left we show 2 tuning forks, each generating a sound pressure wave. The frequencies of the the 2 forks are slightly different.

We will be interested in the region where the 2 sound waves overlap each other; this region has a red circle around it in the figure.

This animation uses sound. You may wish to click on the button to the left and adjust the sound level of your speakers.

Next Scene — This is Scene 1 of 6

Beats

Two Drums

We begin by thinking about periodic motion. Our example here is striking two drums.

The two drums are being hit at slightly different rates: the upper snare drum is being struck at a higher rate than the bass drum.

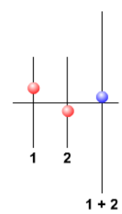
When this scene was first entered, they were being struck simultaneously. Every 20 seconds they are again being struck simultaneously, i.e. they are "in phase" with each other.

10 seconds after being in phase, they are "out of phase:" when one drum is being struck the other one's striker is at a maximum distance away from the drum itself.

Previous Scene — Next Scene

Beats

Two Oscillators



Now we have 2 oscillators, 1 and 2, whose periods are double the times between the striking of the two drums in the previous scene. The sum of the amplitudes of the 2 oscillators is shown as the blue ball.

Just as with the drums, when this scene was first entered the 2 oscillators were in phase. At that time the amplitude of the blue ball was twice the amplitude of each oscillator.

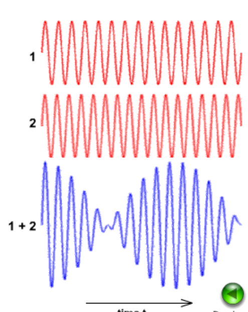
After about 40 seconds, the 2 oscillators are again in phase with each other.

About 20 seconds after the oscillators are in phase, they are out of phase with each other. At this time the amplitude of the sum is zero.

Previous Scene — Next Scene

Beats

Visualising the Sum



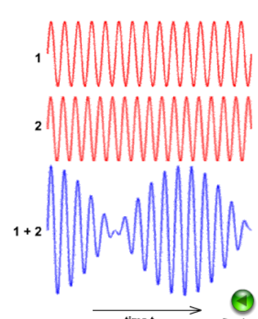
Now we will start thinking of the "oscillators" as being the amplitude of 2 sound waves at some position in space. To the left we show the amplitudes as a function of time of the 2 sound waves, and their sum.

The principle of superposition says that the sum of the 2 waves is the total sound wave.

Previous Scene — Next Scene

Beats

The Mathematics



$y_1 = A \sin(\omega_1 t)$

$y_2 = A \sin(\omega_2 t)$

$$y_1 + y_2 = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right) * \sin\left(\frac{\omega_1 + \omega_2}{2}t\right)$$

The sum has a frequency equal to the average of the 2 simple waves, but its maximum amplitude is "modulated" by a frequency = 1/2 the difference in frequencies.

Previous Scene — Next Scene

Beats

Hearing the Sum

In the animations, we had very low frequencies of oscillation, so we could see what was happening.

Here we have generated two sound waves of frequencies 440 Hz and 442 Hz. Clicking on the round buttons to the left plays one or the other or both. Playing both clearly demonstrates the phenomenon of *beats*. All clips play for 5 sec.

1
440 Hz

2
442 Hz

1 + 2

Stop

Clicking on the "Stop" button stops the sound.

The previous scene indicated that the beats should occur with a frequency of 1 Hz, but you can hear that it is 2 Hz. Why?

People with a good sense of pitch may be able to hear the difference between the 440 Hz tone and the 442 Hz one.

First Scene

Previous Scene