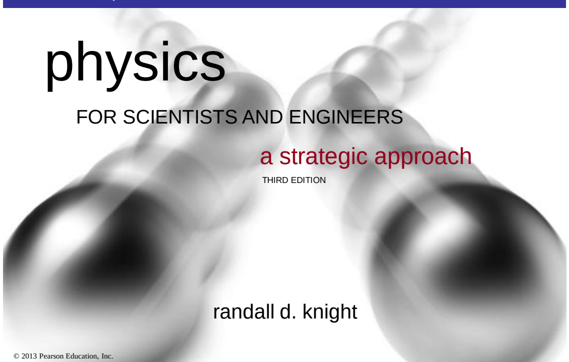
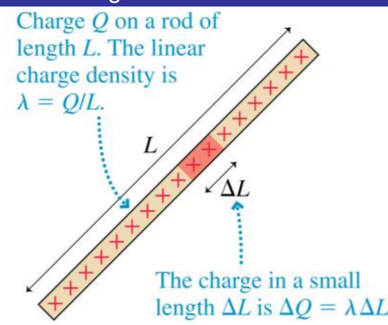
Class 10, Sections 26.3 - 26.7 Preclass Notes



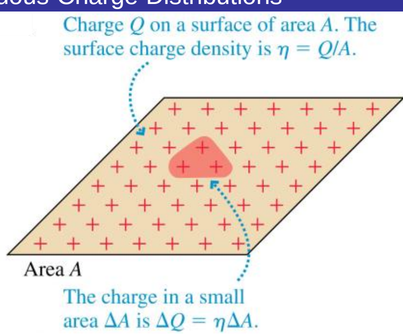
ALWAYS LEARNING PEARSON

Continuous Charge Distributions



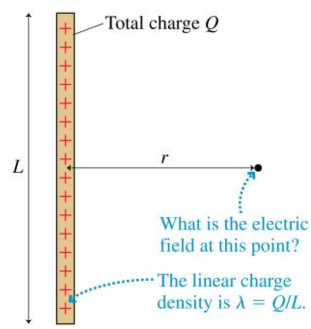
Linear charge density λ, which has units of C/m, is the amount of charge *per meter* of length.

Continuous Charge Distributions



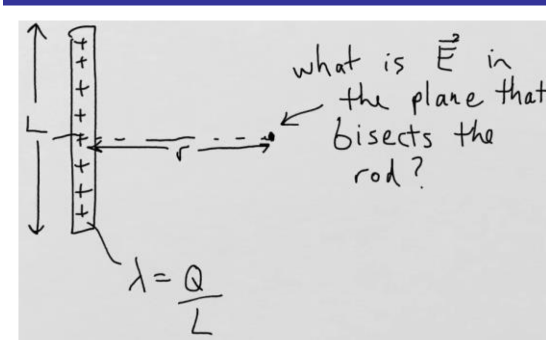
Surface charge density η, with units C/m^2, is the amount of charge *per square meter*.

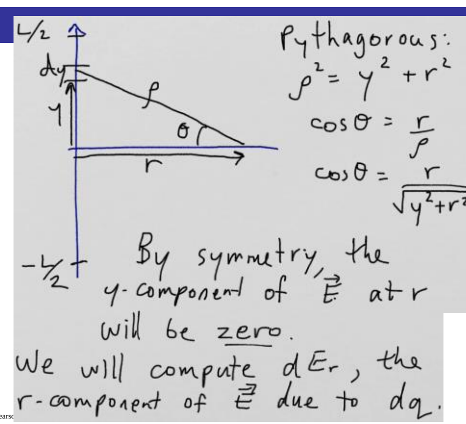
The Electric Field of a Finite Line of Charge

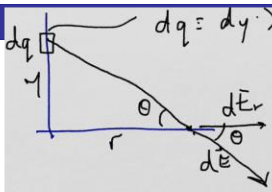


Example 26.3 in the text uses integration to find the electric field strength at a radial distance r in the plane that bisects a rod of length L with total charge Q:

$$E_{rod} = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$







$$dq = dy \cdot \lambda = \frac{dy\,Q}{L}$$

$$\cos\theta = \frac{dE_r}{|d\vec{E}|}$$

$$dE_r = \cos\theta\,|d\vec{E}|$$

$$|d\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{dq}{\rho^2}$$

$$dE_r = \frac{\cos\theta}{4\pi\epsilon_0}\frac{dq}{\rho^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{r}{\sqrt{y^2+r^2}}\left(\frac{dy\,Q}{L}\right)\left(\frac{1}{y^2+r^2}\right)$$

$$dE_r = \frac{Qr}{4\pi\epsilon_0 L}\frac{1}{(y^2+r^2)^{3/2}}\,dy$$

$$E_r = \frac{Q}{4\pi\epsilon_0 L}\int_{-L/2}^{L/2}\frac{r\,dy}{(y^2+r^2)^{3/2}}$$

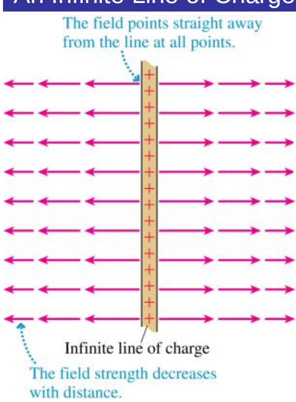
Integration table. (r is constant)

$$E_r = \frac{Q}{4\pi\epsilon_0 L}\left[\frac{y}{r\sqrt{y^2+r^2}}\right]_{-L/2}^{L/2}$$

$$E_r = \frac{Q}{4\pi\epsilon_0 L}\left[\frac{L/2}{r\sqrt{(L/2)^2+r^2}} - \frac{-L/2}{r\sqrt{(-L/2)^2+r^2}}\right]$$

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{(L/2)^2+r^2}}$$

An Infinite Line of Charge



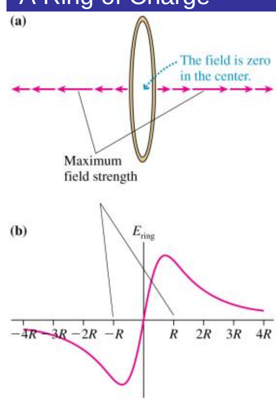
The electric field of a thin, uniformly charged rod may be written:

$$E_{rod} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}\frac{1}{\sqrt{1+4r^2/L^2}}$$

If we now let $L \to \infty$, the last term becomes simply 1 and we're left with:

$$E_{line} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}$$

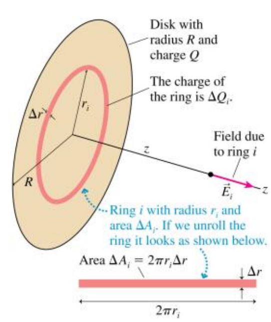
A Ring of Charge



- Consider the on-axis electric field of a positively charged ring of radius R.
- Define the z-axis to be the axis of the ring.
- The electric field on the z-axis points away from the center of the ring, increasing in strength until reaching a maximum when $|z| \approx R$, then decreasing:

$$(E_{ring})_z = \frac{1}{4\pi\epsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$

A Disk of Charge



- Consider the on-axis electric field of a positively charged disk of radius R.
- Define the z-axis to be the axis of the disk.
- The electric field on the z-axis points away from the center of the disk, with magnitude:

$$(E_{disk})_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{1+R^2/z^2}}\right]$$

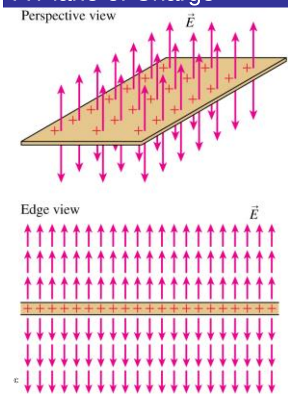
A Plane of Charge

- The electric field of a plane of charge is found from the on-axis field of a charged disk by letting the radius $R \to \infty$.
- The electric field of an infinite plane of charge with surface charge density η is:

$$E_{plane} = \frac{\eta}{2\epsilon_0} = \text{constant}$$

- For a positively charged plane, with $\eta > 0$, the electric field points *away from* the plane on both sides of the plane.
- For a negatively charged plane, with $\eta < 0$, the electric field points *towards* the plane on both sides of the plane.

A Plane of Charge



$$(E_{\text{plane}})_z = \begin{cases} +\dfrac{\eta}{2\epsilon_0} & z > 0 \\ -\dfrac{\eta}{2\epsilon_0} & z < 0 \end{cases}$$

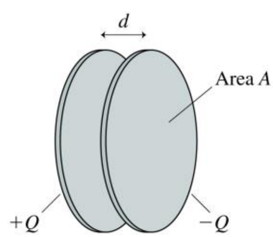
A Sphere of Charge

A sphere of charge Q and radius R, be it a uniformly charged sphere or just a spherical shell, has an electric field *outside* the sphere that is exactly the same as that of a point charge Q located at the center of the sphere:

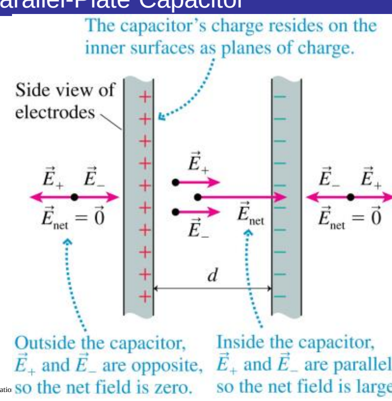
$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \qquad \text{for } r \geq R$$

The Parallel-Plate Capacitor

- The figure shows two electrodes, one with charge $+Q$ and the other with $-Q$ placed face-to-face a distance d apart.
- This arrangement of two electrodes, charged equally but oppositely, is called a **parallel-plate capacitor.**
- Capacitors play important roles in many electric circuits.



The Parallel-Plate Capacitor



The Parallel-Plate Capacitor

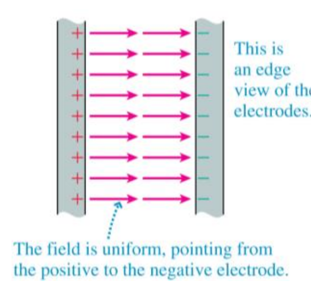
The electric field inside a capacitor is

$$\vec{E}_{\text{capacitor}} = \vec{E}_+ + \vec{E}_- = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right) = \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right)$$

where A is the surface area of each electrode.

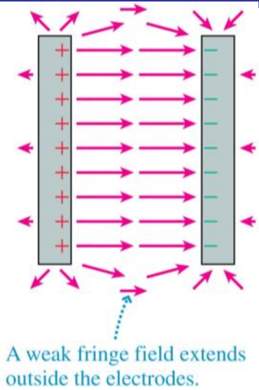
The Ideal Capacitor

- The figure shows the electric field of an ideal parallel-plate capacitor constructed from two infinite charged planes
- The ideal capacitor is a good approximation as long as the electrode separation d is much smaller than the electrodes' size.

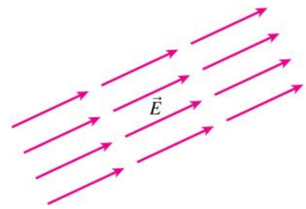


A Real Capacitor

- Outside a real capacitor and near its edges, the electric field is affected by a complicated but weak **fringe field.**
- We will keep things simple by always assuming the plates are very close together and using $E = \eta/\epsilon_0$ for the magnitude of the field inside a parallel-plate capacitor.



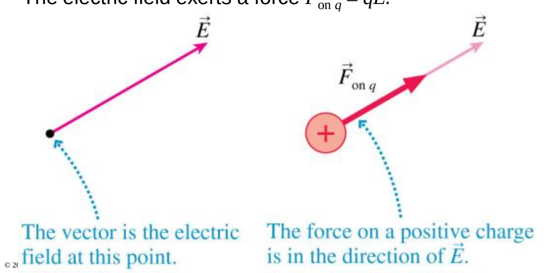
Uniform Electric Fields



- The figure shows an electric field that is the *same*—in strength and direction—at every point in a region of space.
- This is called a **uniform electric field.**
- The easiest way to produce a uniform electric field is with a parallel-plate capacitor.

Motion of a Charged Particle in an Electric Field

- Consider a particle of charge q and mass m at a point where an electric field $\vec{E}$ has been produced by *other* charges, the source charges.
- The electric field exerts a force $\vec{F}_{\text{on } q} = q\vec{E}$.



Motion of a Charged Particle in an Electric Field

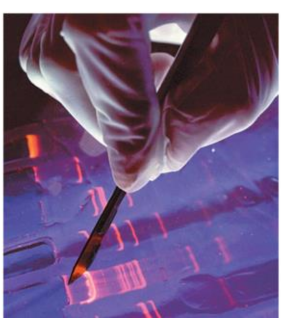
- The electric field exerts a force $\vec{F}_{\text{on } q} = q\vec{E}$ on a charged particle.
- If this is the only force acting on q, it causes the charged particle to accelerate with

$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E}$$

- In a uniform field, the acceleration is constant:

$$a = \frac{qE}{m} = \text{constant}$$

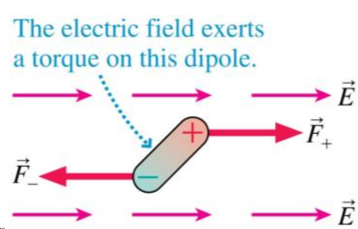
Motion of a Charged Particle in an Electric Field



- "DNA fingerprints" are measured with the technique of *gel electrophoresis.*
- A solution of negatively charged DNA fragments migrate through the gel when placed in a uniform electric field.
- Because the gel exerts a drag force, the fragments move at a terminal speed inversely proportional to their size.

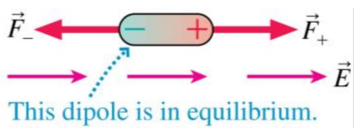
Dipoles in a Uniform Electric Field

- The figure shows an electric dipole placed in a *uniform* external electric field.
- The net force on the dipole is zero.
- The electric field exerts a *torque* on the dipole which causes it to *rotate.*



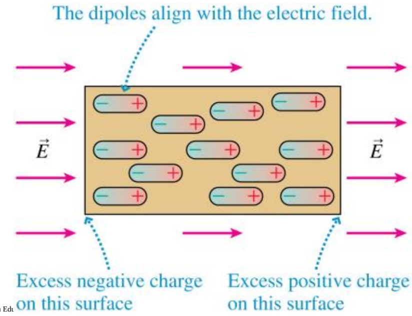
Dipoles in a Uniform Electric Field

- The figure shows an electric dipole placed in a *uniform* external electric field.
- The torque causes the dipole to rotate until it is aligned with the electric field, as shown.
- Notice that the positive end of the dipole is in the direction in which $\vec{E}$ points.



Dipoles in a Uniform Electric Field

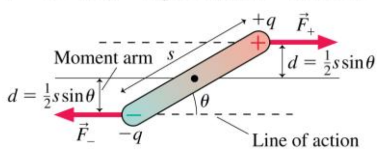
- The figure shows a sample of permanent dipoles, such as water molecules, in an external electric field.



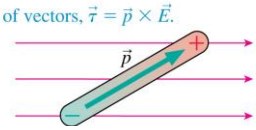
The Torque on a Dipole

The torque on a dipole placed in a uniform external electric field is:

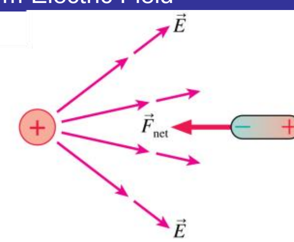
$$\tau = 2 \times dF_+ = 2(\tfrac{1}{2}s\sin\theta)(qE) = pE\sin\theta$$



In terms of vectors, $\vec{\tau} = \vec{p} \times \vec{E}$.



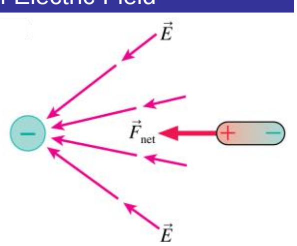
Dipoles in a Nonuniform Electric Field



- Suppose that a dipole is placed in a nonuniform electric field, such as the field of a positive point charge.
- The first response of the dipole is to rotate until it is aligned with the field.
- Once the dipole is aligned, the leftward attractive force on its negative end is slightly stronger than the rightward repulsive force on its positive end.
- This causes a net force to the *left*, toward the point charge.

Dipoles in a Nonuniform Electric Field

- A dipole near a negative point charge is also attracted toward the point charge.
- The net force on a dipole is toward the direction of the strongest field.



- Because field strength increases as you get closer to any finite-sized charged object, we can conclude that a **dipole will experience a net force toward any charged object.**