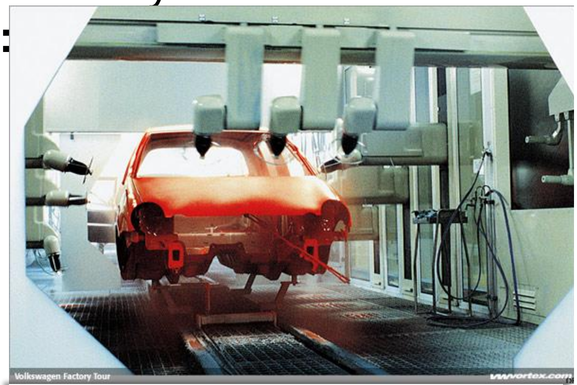
PHY132 Introduction to Physics II
Class 10 – **Outline:**

- Finishing off chapter 26
- Electric Field of:
  - Continuous Charge Distribution
  - Rings, Planes and Spheres
  - Parallel Plate Capacitor
- Motion of a Charged Particle in an Electric Field
- Motion of a Dipole in an Electric Field

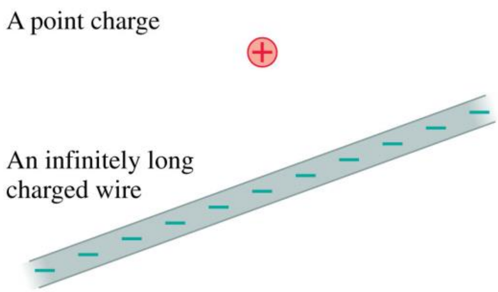


Volkswagon Factory Tour: Ionized paint droplets are transferred in an electrostatic field to the body, and adheres to the metal in an even coat.

Image from http://www.vwvortex.com/artman/publish/vortex_news/article_329.shtml?page=4

Electric Field Models

- Most of this chapter will be concerned with the *sources* of the electric field.
- We can understand the essential physics on the basis of simplified *models* of the sources of electric field.
- The drawings show models of a positive point charge and an infinitely long negative wire.
- We also will consider an infinitely wide charged plane and a charged sphere.

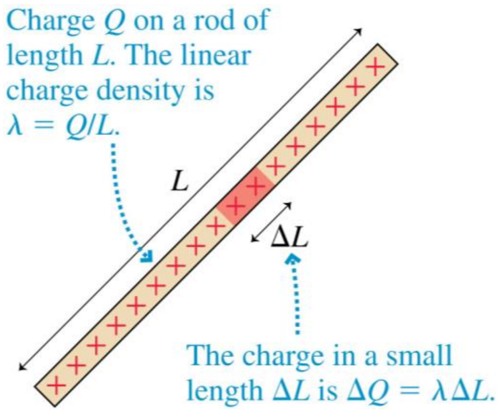


Continuous Charge Distributions

The linear charge density of an object of length L and charge Q is defined as

$$\lambda = \frac{Q}{L}$$

Linear charge density, which has units of C/m, is the amount of charge *per meter* of length.



If 8 nC of charge are placed on the square loop of wire, the linear charge density will be

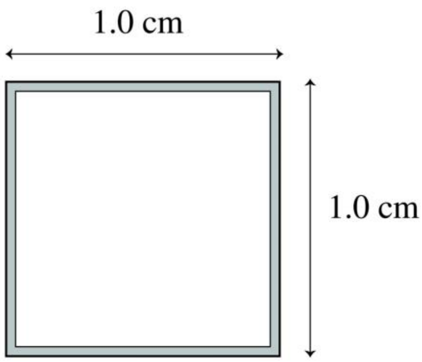
A. 800 nC/m.

B. 400 nC/m.

C. 200 nC/m.

D. 8 nC/m.

E. 2 nC/m.

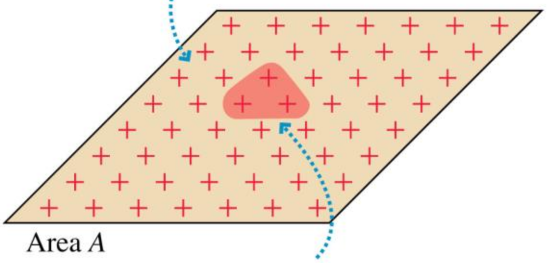


Continuous Charge Distributions

η = "eta"

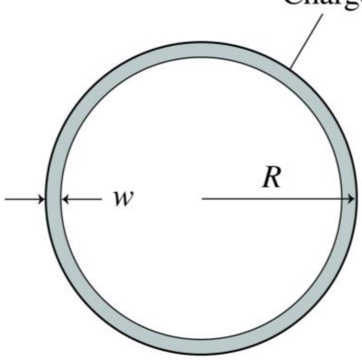
The surface charge density of a two-dimensional distribution of charge across a surface of area A is defined as:

$$\eta = \frac{Q}{A}$$



Surface charge density, with units C/m^2, is the amount of charge *per square meter*.

A flat circular ring is made from a very thin sheet of metal. Charge Q is uniformly distributed over the ring. Assuming $w << R$, the surface charge density η on the top side, facing out of the page, is



A. $Q/2\pi Rw$.

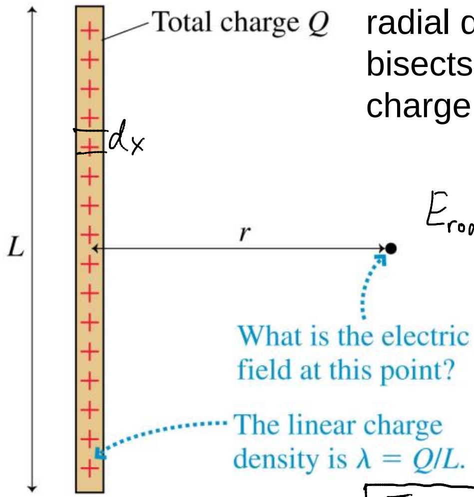
B. $Q/4\pi Rw$.

C. $Q/\pi R^2$.

D. $Q/2\pi R^2$.

E. $Q/\pi Rw$.

The Electric Field of a Finite Line of Charge



The electric field strength at a radial distance r in the plane that bisects a rod of length L with total charge Q:

$$K = \frac{1}{4\pi\epsilon_0}$$

$$E_{rod} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$

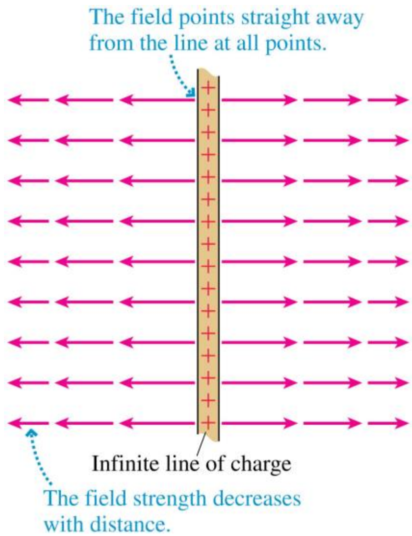
$$E_{rod} = \frac{K}{r}\frac{Q}{\frac{L}{2}\sqrt{\frac{4r^2}{L^2}+1}}$$

$$E_{rod} = \frac{2K}{r}\frac{Q}{L}\frac{1}{\sqrt{\frac{4r^2}{L^2}+1}}$$

$\frac{Q}{L} = \lambda$, $\frac{4r^2}{L^2} \to 0$ as $L \to \infty$

$$\boxed{E_{line} = \frac{2K\lambda}{r}}$$

An Infinite Line of Charge



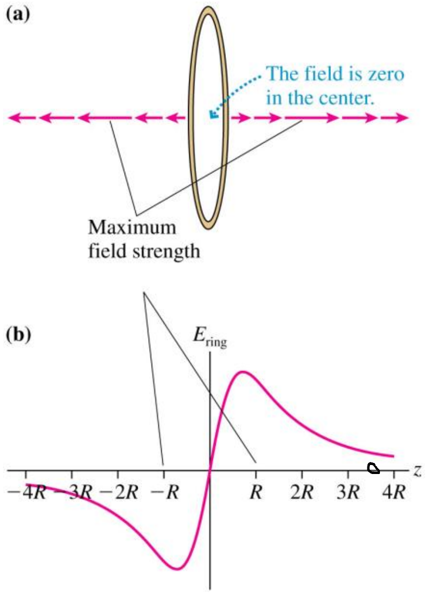
The electric field of a thin, uniformly charged rod may be written:

$$E_{rod} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}\frac{1}{\sqrt{1 + 4r^2/L^2}}$$

If we now let $L \to \infty$, the last term becomes simply 1 and we're left with:

$$E_{line} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}$$

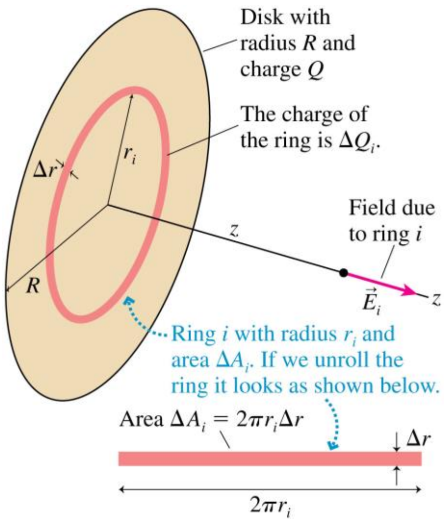
A Ring of Charge



- Consider the on-axis electric field of a positively charged ring of radius R.
- Define the z-axis to be the axis of the ring.
- The electric field on the z-axis points away from the center of the ring, increasing in strength until reaching a maximum when $|z| \approx R$, then decreasing:

$$(E_{\text{ring}})_z = \frac{1}{4\pi\epsilon_0} \frac{zQ}{(z^2 + R^2)^{3/2}}$$

A Disk of Charge



- Consider the on-axis electric field of a positively charged disk of radius R.
- Define the z-axis to be the axis of the disk.
- The electric field on the z-axis points away from the center of the disk, with magnitude:

$$(E_{\text{disk}})_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{1 + R^2/z^2}}\right]$$

Infinite plane: let $R \to \infty$

$$\frac{1}{\sqrt{1+\frac{R^2}{z^2}}} \to 0$$

$$E_{plane} = \frac{\eta}{2\epsilon_0}$$ ← does not depend on z!

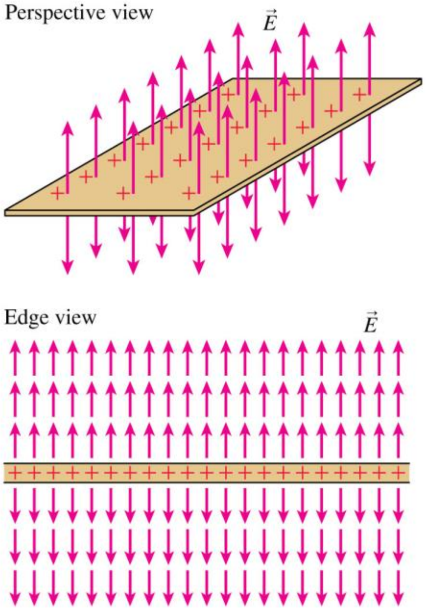
A Plane of Charge

- The electric field of a plane of charge is found from the on-axis field of a charged disk by letting the radius $R \to \infty$.
- The electric field of an infinite plane of charge with surface charge density η is:

$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant}$$

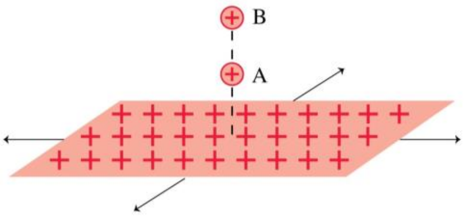
- For a positively charged plane, with $\eta > 0$, the electric field points *away from* the plane on both sides of the plane.
- For a negatively charged plane, with $\eta < 0$, the electric field points *towards* the plane on both sides of the plane.

A Plane of Charge



$$(E_{\text{plane}})_z = \begin{cases} +\dfrac{\eta}{2\epsilon_0} & z > 0 \\ -\dfrac{\eta}{2\epsilon_0} & z < 0 \end{cases}$$

Two protons, A and B, are next to an infinite plane of positive charge. Proton B is twice as far from the plane as proton A. Which proton has the larger acceleration?



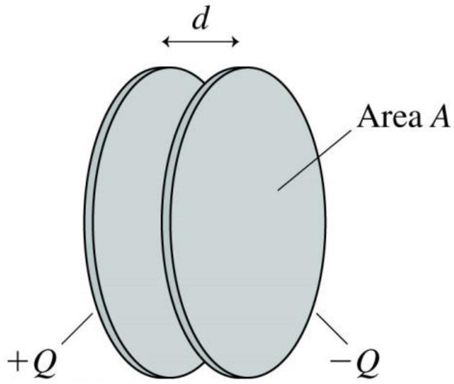
A. Proton A.

B. Proton B.

C. Both have the same acceleration.

The Parallel-Plate Capacitor

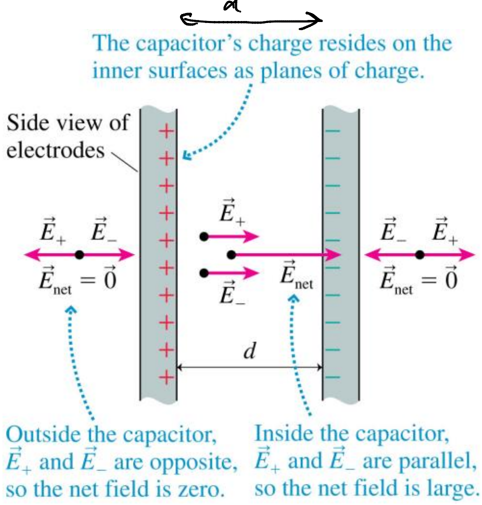
- The figure shows two electrodes, one with charge $+Q$ and the other with $-Q$ placed face-to-face a distance d apart.
- This arrangement of two electrodes, charged equally but oppositely, is called a **parallel-plate capacitor.**
- Capacitors play important roles in many electric circuits.



The Parallel-Plate Capacitor

If d << size of plates

- The figure shows two capacitor plates, seen from the side.
- Because opposite charges attract, all of the charge is on the *inner* surfaces of the two plates.
- Inside the capacitor, the net field points toward the negative plate.
- Outside the capacitor, the net field is zero.



The Parallel-Plate Capacitor

The electric field inside a capacitor is

$$\vec{E}_{capacitor} = \vec{E}_+ + \vec{E}_- = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$

$$\frac{\eta}{2\epsilon_0} + \frac{\eta}{2\epsilon_0}$$

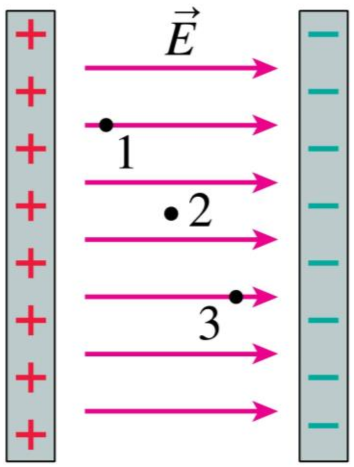
$$= \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right)$$

where A is the surface area of each electrode. Outside the capacitor plates, where E_+ and E_- have equal magnitudes but *opposite* directions, the electric field is zero.

Assume infinite plates.

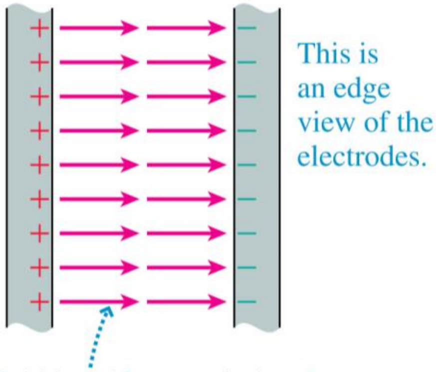
Three points inside a parallel-plate capacitor are marked. Which is true?

A. $E_1 > E_2 > E_3$
B. $E_1 < E_2 < E_3$
C. $E_1 = E_2 = E_3$
D. $E_1 = E_3 > E_2$



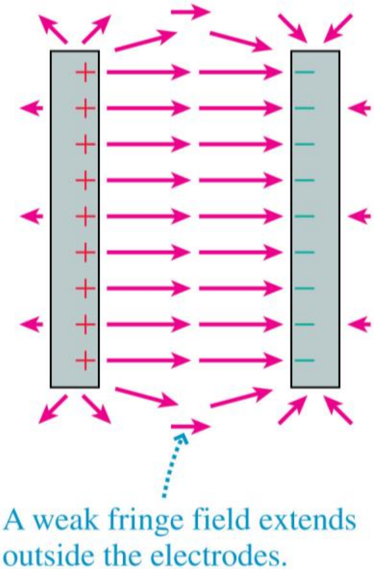
The Ideal Capacitor

- The figure shows the electric field of an ideal parallel-plate capacitor constructed from two infinite charged planes
- The ideal capacitor is a good approximation as long as the electrode separation d is much smaller than the electrodes' size.

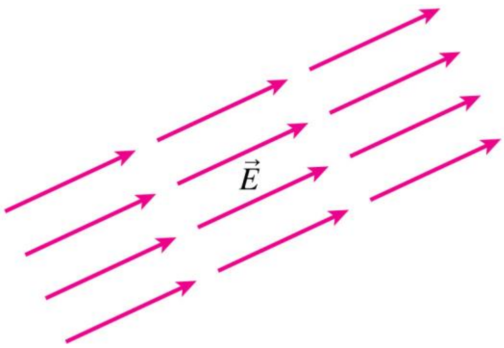


A Real Capacitor

- Outside a real capacitor and near its edges, the electric field is affected by a complicated but weak **fringe field.**
- We will keep things simple by always assuming the plates are very close together and using $E = \eta/\epsilon_0$ for the magnitude of the field inside a parallel-plate capacitor.



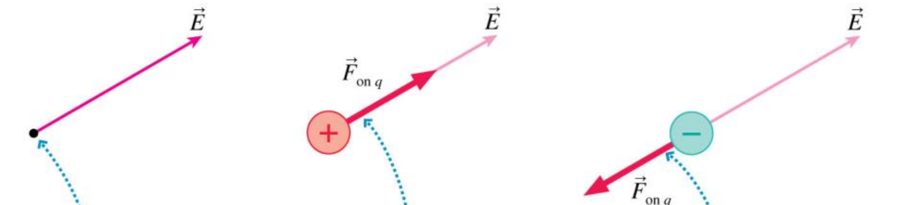
Uniform Electric Fields



- The figure shows an electric field that is the *same*—in strength and direction—at every point in a region of space.
- This is called a **uniform electric field.**
- The easiest way to produce a uniform electric field is with a parallel-plate capacitor.

Motion of a Charged Particle in an Electric Field

- Consider a particle of charge q and mass m at a point where an electric field $\vec{E}$ has been produced by *other* charges, the source charges.
- The electric field exerts a force $\vec{F}_{\text{on } q} = q\vec{E}$.



Motion of a Charged Particle in an Electric Field

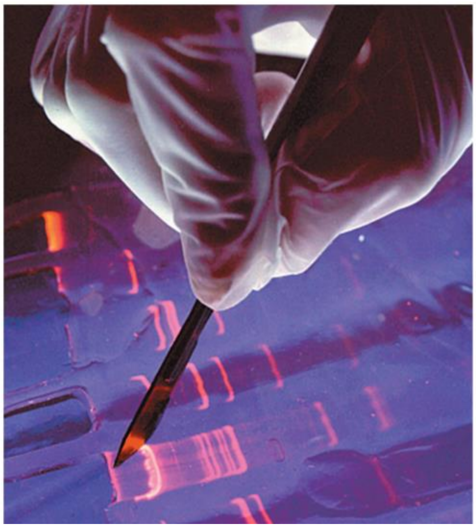
- The electric field exerts a force $\vec{F}_{\text{on } q} = q\vec{E}$ on a charged particle.
- If this is the only force acting on q, it causes the charged particle to accelerate with

$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E}$$

- In a uniform field, the acceleration is constant:

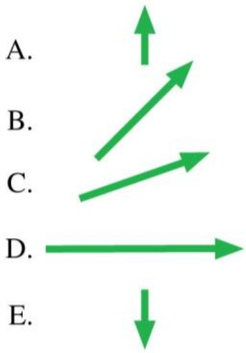
$$a = \frac{qE}{m} = \text{constant}$$

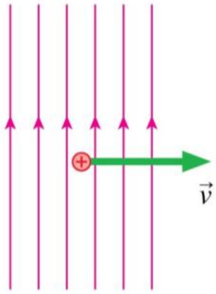
Motion of a Charged Particle in an Electric Field



- "DNA fingerprints" are measured with the technique of *gel electrophoresis.*
- A solution of negatively charged DNA fragments migrate through the gel when placed in a uniform electric field.
- Because the gel exerts a drag force, the fragments move at a terminal speed inversely proportional to their size.

A proton is moving to the right in a vertical electric field. A very short time later, the proton's velocity is

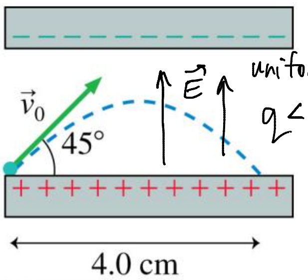




Problem 26.50

An electron is launched at a 45° angle at a speed of 5×10^6 m/s from the positive plate of the parallel plate capacitor shown. The electron lands 4 cm away. What is the electric field strength inside the capacitor?

Let's neglect gravity (for now)..



uniform $\vec{E}$

$q<0$, $\vec{F}\downarrow$

$\vec{F}=q\vec{E}$

$F_{Net} = ma = qE$

$\vec{a} = \frac{qE}{m}$, down ← find a

$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

x-direction $a_x = 0$

$x = v_{0x} t$

$t = \frac{x}{v_0 \cos\theta} = \frac{0.04\text{m}}{5\times10^6 \text{m/s} \cos 45°}$

$t = 1.1314 \times 10^{-8}$ s

y-direction $a_y =$ constant

Use symmetry:

$v_{fy} = -v_{0y}$

$v_{fy} = v_{0y} - at$

$F_{Net} = ma = F_e = qE$

$\vec{a} = \frac{qE}{m}$, down

$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

x-direction $a_x = 0$

$x = v_{0x} t$

$t = \frac{x}{v_0 \cos\theta} = \frac{0.04\text{m}}{5\times10^6 \frac{m}{s} \cos 45}$

$t = 1.1314 \times 10^{-8}$ s

y-direction $a_y = ?$

$v_f = -v_0$

$v_f = v_0 - at$

$a = \frac{v_{0y} - (-v_{0y})}{t} = \frac{2v_{0y}}{t}$

$a = \frac{2(5\times10^6 \text{m/s})}{1.1314\times10^{-8}\text{s}}$

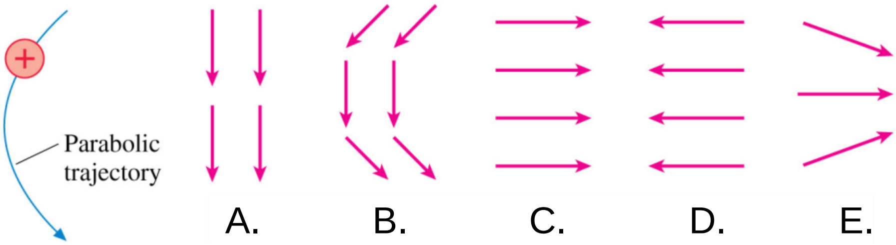
$a = 6.25\times10^{14}\ \text{m/s}^2$

$a \gg 9.8\ \text{m/s}^2$, so neglecting gravity is justified.

$E = \frac{ma}{q} = \frac{(9.1\times10^{-31}\text{kg})\times 6.25\times10^{14}}{1.6\times10^{-19}\text{C}}$

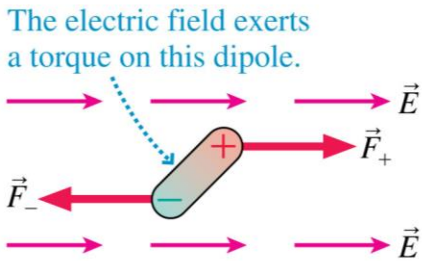
$E = 3550\ \frac{N}{C}$

Which electric field is responsible for the proton's trajectory?



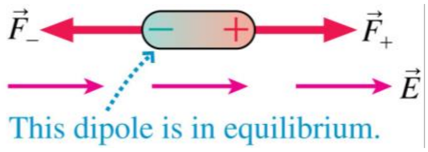
Dipoles in a Uniform Electric Field

- The figure shows an electric dipole placed in a *uniform* external electric field.
- The net force on the dipole is zero.
- The electric field exerts a *torque* on the dipole which causes it to *rotate.*



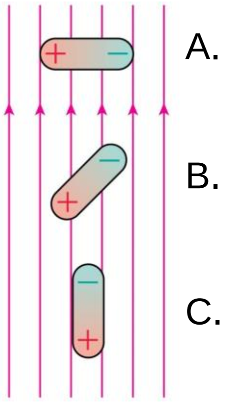
Dipoles in a Uniform Electric Field

- The figure shows an electric dipole placed in a *uniform* external electric field.
- The torque causes the dipole to rotate until it is aligned with the electric field, as shown.
- Notice that the positive end of the dipole is in the direction in which $\vec{E}$ points.



Which dipole experiences no net force in the electric field?

A. Dipole A.
B. Dipole B.
C. Dipole C.
D. Both dipoles A and C.
E. All three dipoles.



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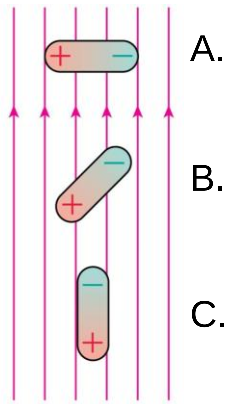
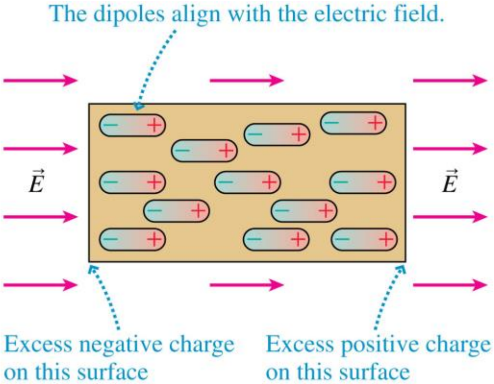


Dipoles in a Uniform Electric Field

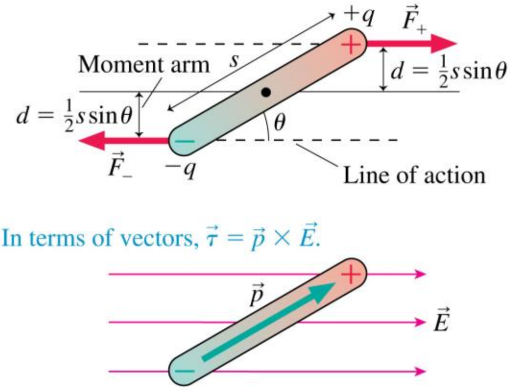


- The figure shows a sample of permanent dipoles, such as water molecules, in an external electric field.
- All the dipoles rotate until they are aligned with the electric field.
- This is the mechanism by which the sample becomes *polarized.*

The Torque on a Dipole

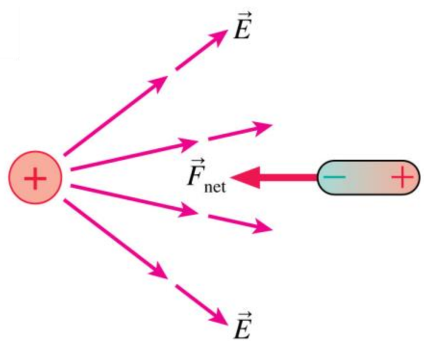
The torque on a dipole placed in a uniform external electric field is

$$\tau = 2 \times dF_+ = 2(\tfrac{1}{2}s\sin\theta)(qE) = pE\sin\theta$$



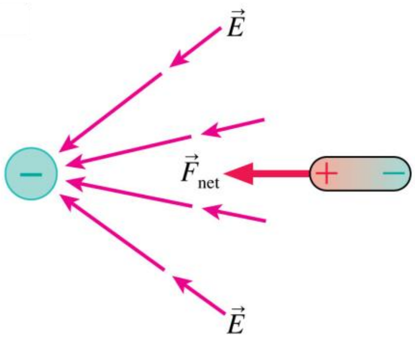
Dipoles in a Nonuniform Electric Field

- Suppose that a dipole is placed in a nonuniform electric field, such as the field of a positive point charge.
- The first response of the dipole is to rotate until it is aligned with the field.



- Once the dipole is aligned, the leftward attractive force on its negative end is slightly stronger than the rightward repulsive force on its positive end.
- This causes a net force to the *left*, toward the point charge.

Dipoles in a Nonuniform Electric Field



- A dipole near a negative point charge is also attracted toward the point charge.
- The net force on a dipole is toward the direction of the strongest field.
- Because field strength increases as you get closer to any finite-sized charged object, we can conclude that a **dipole will experience a net force toward any charged object.**

Before Class 11 on Monday

- Complete Problem Set 4 on MasteringPhysics due Sunday at 11:59pm on Ch. 26.
- Please read Knight Pgs. 810-818: Ch. 28, sections 28.1-28.3
- Please do the short pre-class quiz on MasteringPhysics by Sunday night.

- Something to think about: If a fixed charge repels a moving charge, does it do **work** on the charge? Does this increase the **energy** of the system?