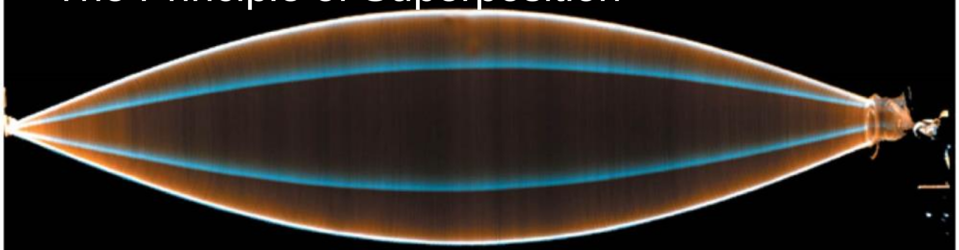
PHY132 Introduction to Physics II

Class 3 – **Outline:**

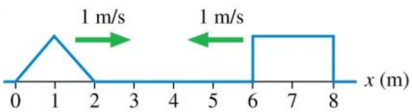
- Ch. 21, sections 21.1-21.4
- The Principle of Superposition



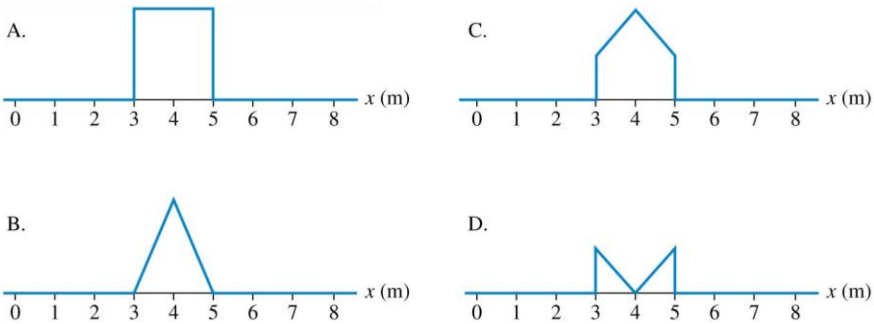
- Standing Waves
- Nodes and Antinodes
- Musical Instruments

i-Clicker Discussion Question

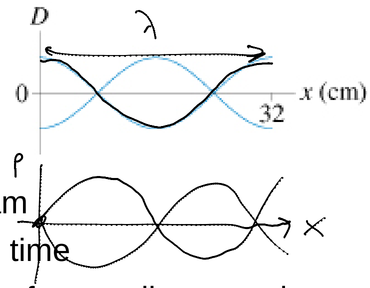
Two wave pulses on a string approach each other at speeds of $1\ \mathrm{m/s}$. How does the string look at $t = 3\ \mathrm{s}$?



Approaching pulses at $t = 0$ s



Class 3 Preclass Quiz on MasteringPhysics



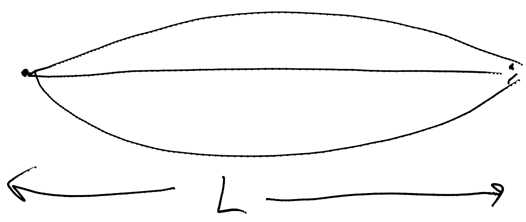
- This was due this morning at 8:00am
- 679 students submitted the quiz on time
- The figure shows the displacement of a standing sound wave in a 32-cm-long horizontal tube of air open at both ends.
- 67% of students answered correctly: The mode is **m = 2** (count pressure antinodes)
- The air molecules are moving **horizontally.** That's because sound is a longitudinal wave.
- The displacement antinodes are at **0, 16, 32 cm**
- The pressure antinodes are at **12, 24 cm**

Class 3 Preclass Quiz on MasteringPhysics

- 81% of students answered correctly: A guitar string is fixed at both ends. If you tighten it to increase its tension **the frequencies of its vibrational modes will increase but its wavelengths will not be affected.**

$m =$

$v = \lambda f$



$v \uparrow$, $f \uparrow$ $L = \lambda/2$ is fixed.

Class 3 Preclass Quiz – Student Comments…

- *"Do you play the guitar??"*
- **Harlow answer:** Yes, I do, but very poorly. I have a stand in my living room and sometimes I pick it up and play little "3-chord busker" songs for my kids..
- *"whats the difference between regular waves and standing waves?"*
- **Harlow answer:** A regular wave is travels along in a particular direction, carrying energy from one place to another. A standing wave is a very specific kind of wave: a sinusoidal wave trapped in a cavity with reflecting walls and a certain 'resonance' frequency.
- *"The concept of superposition is super confusing!"*

Class 3 Preclass Quiz – Student Comments…

- *"Does the length of string have anything to do with the wave's properties(wavelength, frequency...), like piano?"*
- **Harlow answer:** YES! The length of the string is $\lambda/2$!
- *"I was particularly confused about the changing the tension on a guitar string question. I understand the frequency changes (as that's what changes the sound to a higher pitch), so do the wavelengths get shorter as well? If that's the case, then can $v = f\lambda$ work?"*
- **Harlow answer:** No: for a standing wave, the wavelength depends on the length of the string (or tube, in the case of sound waves), and the mode number. If these are fixed, then the wavelength is fixed, and changing v by tightening the string changes f.

Class 3 Preclass Quiz – Student Comments…

- *"the video moved too fast when talking about modes of waves and how to calculate frequency."*
- **Harlow suggestion:** If the video goes too quickly or you miss something, press pause, rewind a bit, and go over it again. Try that 2 or 3 times. If it doesn't work, try reading the section in the book. There's also your friends, Tas, my office hours, etc. Don't give up!

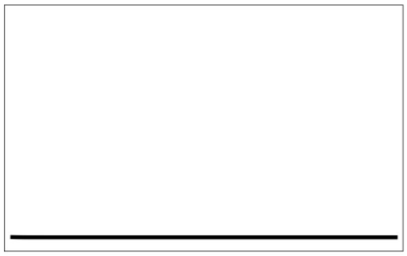
Class 3 Preclass Quiz – Student Comments…

- *"I cant differentiate between the velocity of the particles and the velocity of waves.I mean of course the formulas are given for it but I dont undertsand the concewpt.Isnt the particle moving same speed as the wave when a disturbance is caused>?"*
- **Harlow thought:** This is an important point to figure out. The particles are always oscillating so v is not constant. The wave is a pattern that moves with a fixed speed. These are **completely** different.
- *"Shouldn't air be able to move vertically and horizontally?"*
- **Harlow answer:** Well it can, but it doesn't support a wave that way. There has to be a restoring force. For vibrations in the direction of the wave, the pressure in an ideal gas tries to push particles in a compression back into a rarefaction.

Particles and Waves

- Particles cannot occupy the same space. They **collide**.

- Waves pass right through each other. They **interfere**.



[Animations from http://www.physicsclassroom.com/mmedia/newtlaws/mb.cfm and http://www.acs.psu.edu/drussell/demos/superposition/superposition.html]

The Principle of Superposition

If two or more waves combine at a given point, the resulting disturbance is the *sum* of the disturbances of the individual waves.

$$D = D_1 + D_2$$

Reflection of Transverse Wave Pulse

- A pulse traveling to the right on a heavy string attached to a lighter string
- Speed suddenly increases

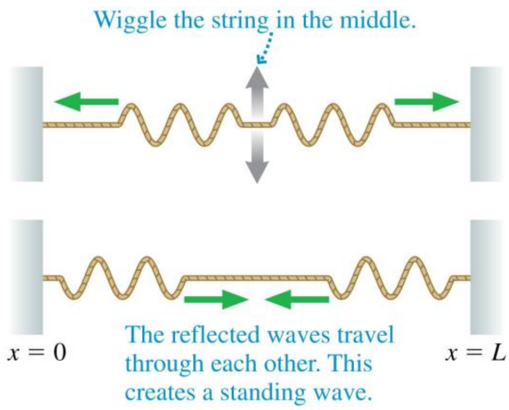
[Animation courtesy of Dan Russell, Penn State]

Reflection of Transverse Wave Pulse

- A pulse traveling to the right on a light string attached to a heavier string
- Speed suddenly decreases

[Animation courtesy of Dan Russell, Penn State]

Standing Waves on a String



Reflections at the ends of the string cause waves of *equal amplitude and wavelength* to travel in opposite directions along the string, which results in a standing wave.

The Mathematics of Standing Waves

According to the principle of superposition, the net displacement of a medium when waves with displacements D_R and D_L are present is

$$D(x,t) = D_R + D_L = a\sin(kx - \omega t) + a\sin(kx + \omega t)$$

We can simplify this by using a trigonometric identity, and arrive at:

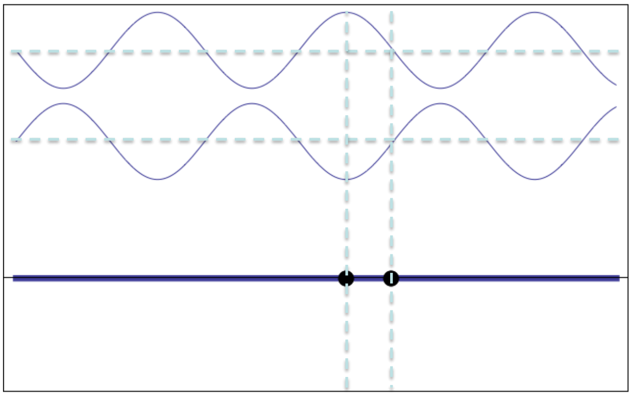
$$D(x,t) = A(x)\cos(\omega t)$$

where $$A(x) = 2a\sin(kx)$$

For a standing wave, the pattern is not propagating!

Standing Wave:

The superposition of two 1-D sinusoidal waves traveling in opposite directions.



[Animation courtesy of Dan Russell, Penn State]

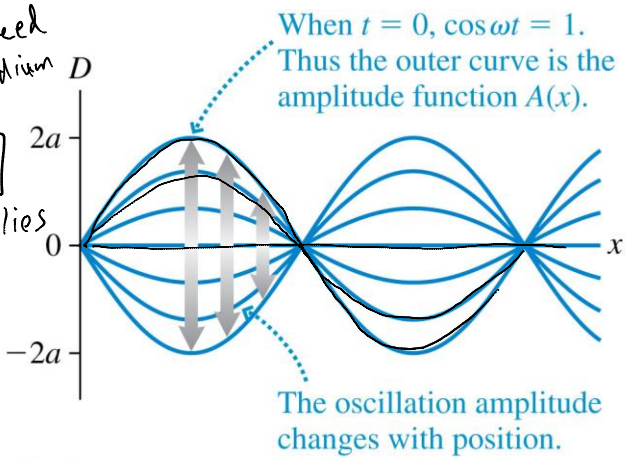
The Mathematics of Standing Waves

The amplitude reaches a maximum value of $A_{max} = 2a$ at points where sin $(kx) = 1$.

wave speed for medium

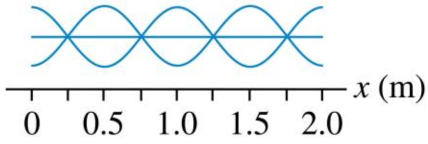
$\boxed{v = f\lambda}$

still applies



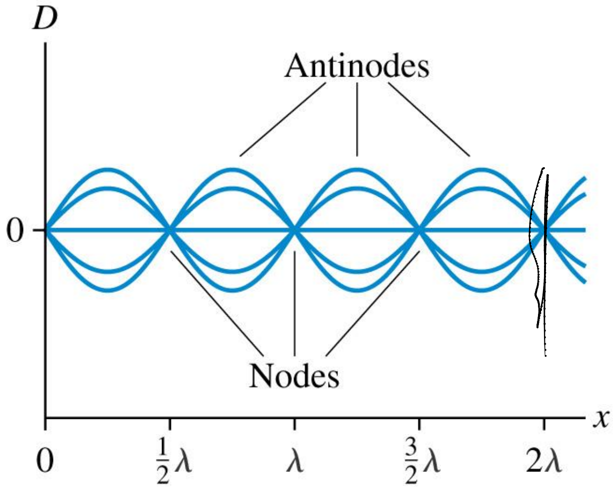
i-Clicker Discussion Question

What is the wavelength of this standing wave?



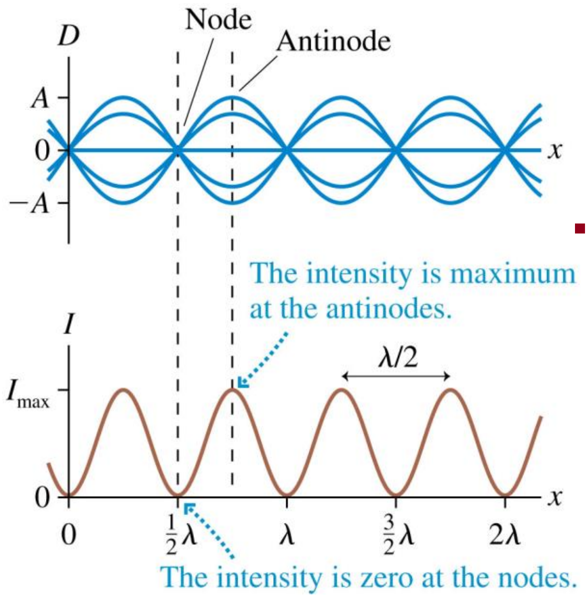
A. 0.25 m.
B. 0.5 m.
C. 1.0 m.
D. 2.0 m.
E. Standing waves don't have a wavelength.

Node Spacing on a String

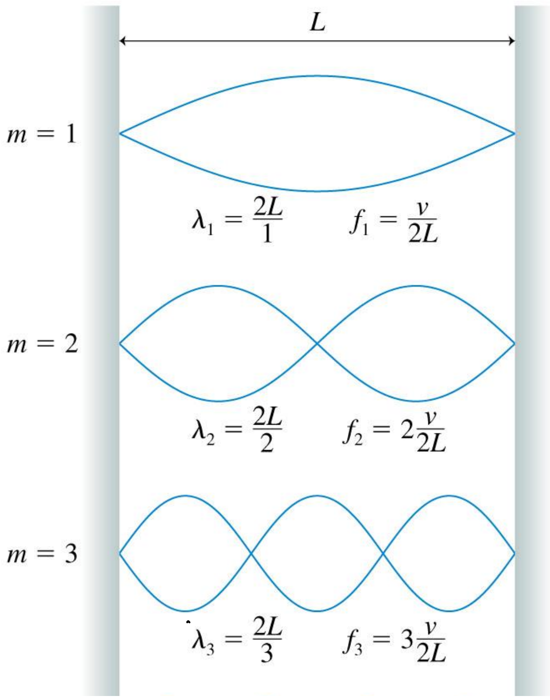


The nodes and antinodes are spaced $\lambda/2$ apart.

Standing Waves



- In Chapter 20 you learned that the *intensity* of a wave is proportional to the square of the amplitude: $I \propto A^2$.
- Intensity is maximum at points of constructive interference and zero at points of destructive interference.



On a string of length L with fixed end points,

$$D(0,t) = 0 \text{ and } D(L,t) = 0$$

Only oscillations with specific wavelengths are allowed.

i-Clicker Discussion Question

What is the mode number of this standing wave?

A. 4.
B. 5.
C. 6.
D. Can’t say without knowing what kind of wave it is.

Standing Waves on a String

There are three things to note about the normal modes of a string.

1. m is the number of *antinodes* on the standing wave.
2. The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
3. The frequencies of the normal modes form a series: f_1, $2f_1$, $3f_1$, …These are also called **harmonics**. $2f_1$ is the “second harmonic”, $3f_1$ is the “third harmonic”, etc.

Musical Instruments

- Instruments such as the harp, the piano, and the violin have strings fixed at the ends and tightened to create tension.
- A disturbance generated on the string by plucking, striking, or bowing it creates a **standing wave** on the string.
- The fundamental frequency is the musical note you hear when the string is sounded:

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T_s}{\mu}}$$

where T_s is the tension in the string and μ is its linear density.

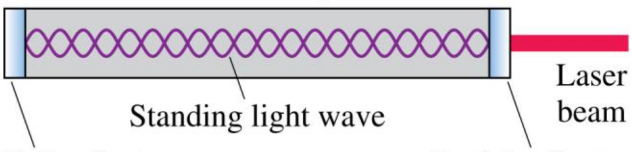
i-Clicker Discussion Question

The frequency of the third harmonic of a string is

A. One-third the frequency of the fundamental.
B. Equal to the frequency of the fundamental.
C. Three times the frequency of the fundamental.
D. Nine times the frequency of the fundamental.

Standing Electromagnetic Waves

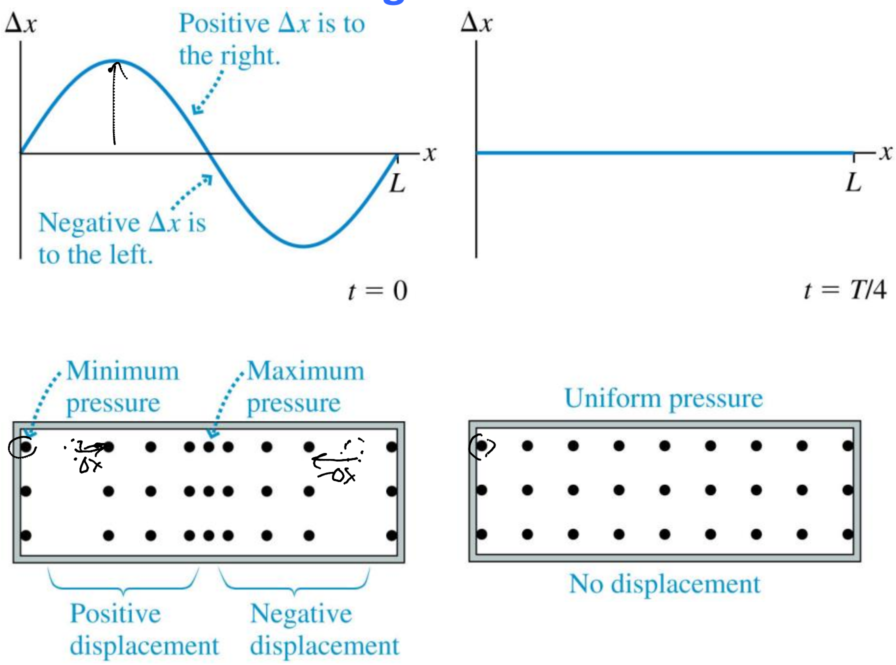
- Standing electromagnetic waves can be established between two parallel mirrors that reflect light back and forth.
- A typical laser cavity has a length $L \approx 30$ cm, and visible light has a wavelength $\lambda \approx 600$ nm.
- The standing light wave in a typical laser cavity has a mode number m that is $2L/\lambda \approx 1{,}000{,}000$!



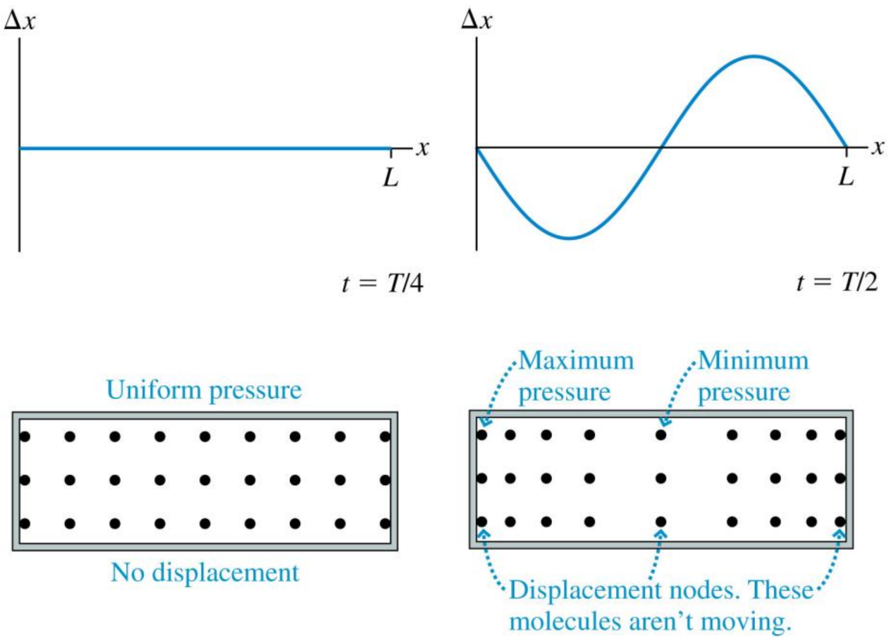
Standing Sound Waves

- A long, narrow column of air, such as the air in a tube or pipe, can support a longitudinal standing sound wave.
- A closed end of a column of air must be a displacement node. Thus the boundary conditions — nodes at the ends — are the same as for a standing wave on a string.
- It is often useful to think of sound as a pressure wave rather than a displacement wave. The pressure oscillates around its equilibrium value.
- The nodes and antinodes of the pressure wave are interchanged with those of the displacement wave.

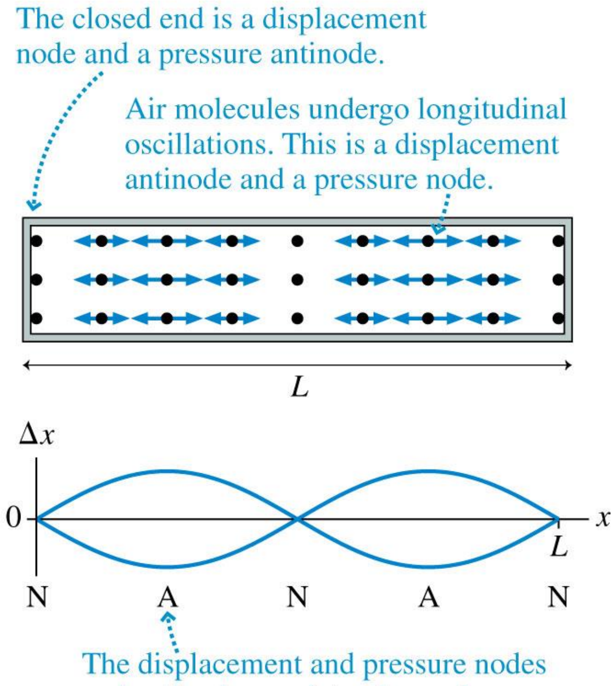
Standing Sound Waves



Standing Sound Waves

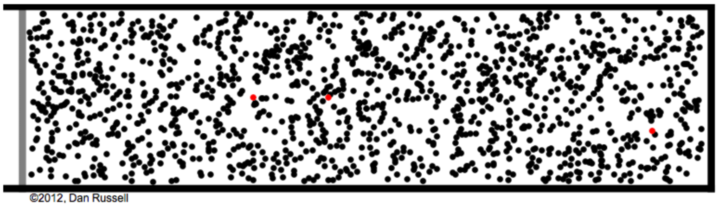


- Displacement Δx and pressure graphs for the $m = 2$ mode of standing sound waves in a closed-closed tube.
- The nodes and antinodes of the pressure wave are interchanged with those of the displacement wave.

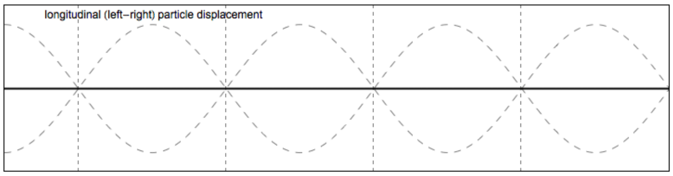


Class 3 Preclass Quiz – Student Comments…

- *"In vibration of air molecules in a tube (regardless of open ends or closed ends), why does maximum pressure corresponds to minimum displacement (still do not quite get that idea)."* [LOTS of questions like this]
- **Harlow answer:** This is tricky. Let's look carefully at the animation on the next page…



Δx



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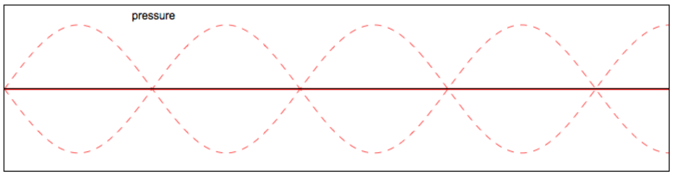
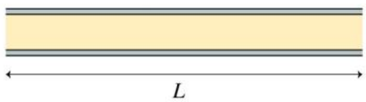
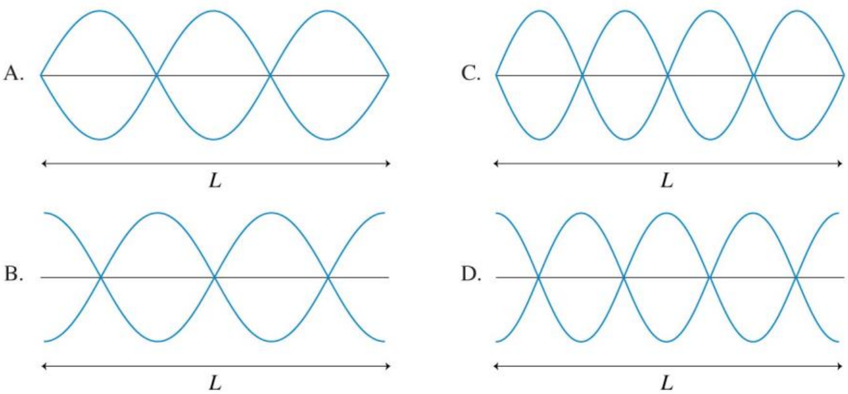


i-Clicker Discussion Question

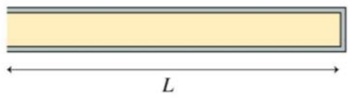
An open-open tube of air has length L. Which is the displacement graph of the $m = 3$ standing wave in this tube?

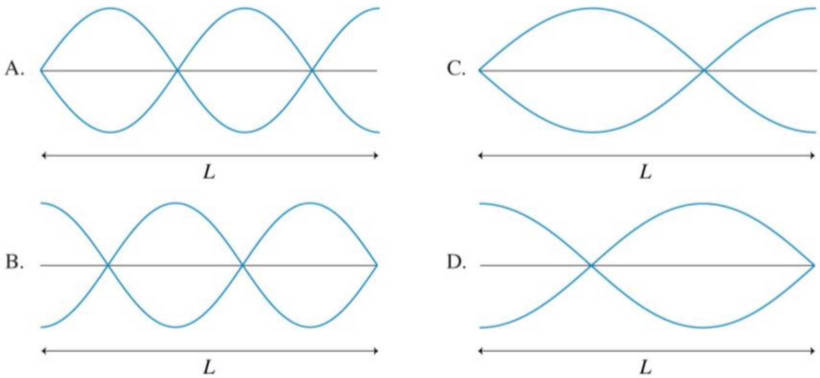


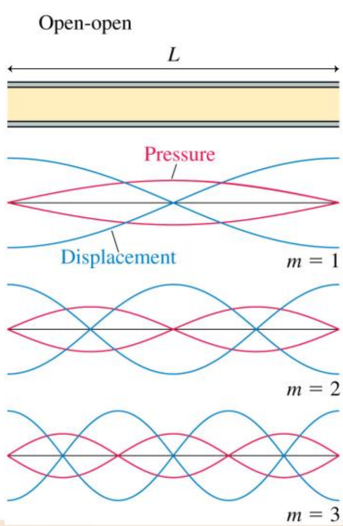


i-Clicker Discussion Question

An open-closed tube of air of length L has the closed end on the right. Which is the displacement graph of the $m = 3$ standing wave in this tube?





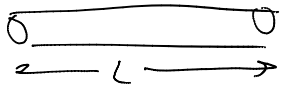


$$\begin{cases} \lambda_m = \dfrac{2L}{m} \\ f_m = m\dfrac{v}{2L} = mf_1 \end{cases}$$

$m = 1, 2, 3, 4, \ldots$
(open-open or closed-closed tube)

Example from a past test

A metal pipe, open at both ends, can create a standing wave in the second harmonic with a frequency of 483 Hz. What is the length of the pipe?



$m = 2$, $f_2 = 483\ Hz$

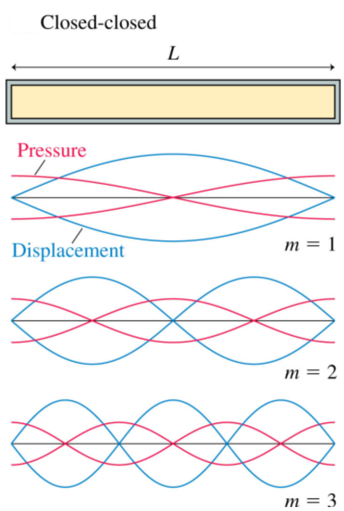
Use: $f_m = \frac{mv}{2L}$

$v = 343\ m/s$, speed of sound.

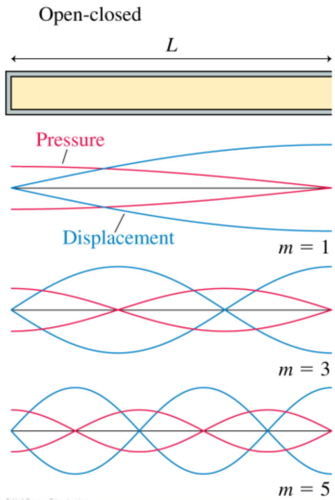
Solve for L:

$$L = \frac{mv}{2f_m} = \frac{2(343\ m/s)}{2(483\ Hz)}$$

$$\boxed{L = 0.710\ m}$$



$$\begin{cases} \lambda_m = \dfrac{2L}{m} \\ f_m = m\dfrac{v}{2L} = mf_1 \end{cases} \qquad m = 1, 2, 3, 4, \ldots \quad \text{(open-open or closed-closed tube)}$$



$$\begin{cases} \lambda_m = \dfrac{4L}{m} \\ f_m = m\dfrac{v}{4L} = mf_1 \end{cases} \qquad \begin{array}{l} m = 1, 3, 5, 7, \ldots \\ \text{(open-closed tube)} \end{array}$$

Musical Instruments

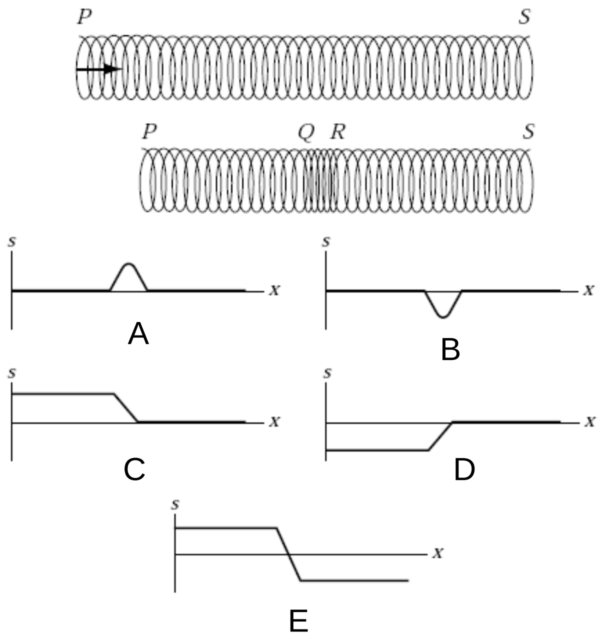
- With a wind instrument, blowing into the mouthpiece creates a standing sound wave inside a tube of air.
- The player changes the notes by using her fingers to cover holes or open valves, changing the length of the tube and thus its fundamental frequency:

$$f_1 = \frac{v}{2L}$$ for an open-open tube instrument, such as a flute

$$f_1 = \frac{v}{4L}$$ for an open-closed tube instrument, such as a clarinet

- In both of these equations, v is the speed of sound in the air *inside* the tube.
- Overblowing wind instruments can sometimes produce higher harmonics such as $f_2 = 2f_1$ and $f_3 = 3f_1$.

A longitudinal wave is sent along a long spring by moving the left end rapidly to the right and keeping it there. The figure shows the wave pulse at *QR*—part *RS* of the long spring is as yet undisturbed. Which of the graphs A–E correctly shows the relation between displacement *s* and position *x*? (Displacements to the right are positive.)



Before Class 4 on Wednesday

- Please read Knight Ch. 21, sections 21.5-21.8 (finish the chapter)
- Please do the short pre-class quiz on MasteringPhysics by Tuesday evening.
- Something to think about: What is “constructive interference”? How can you interfere with something and **increase** its strength?