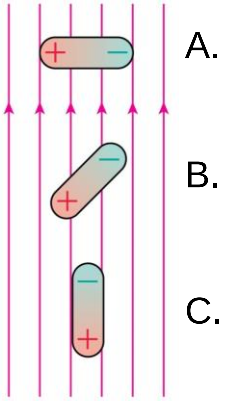
PHY132 Introduction to Physics II
Class 11 – **Outline:**

- Finishing Chapter 26 on dipoles..
- Electric Potential Energy of:
  - Point Charges
  - Dipoles
- Electric Potential: V
- Voltage: ΔV

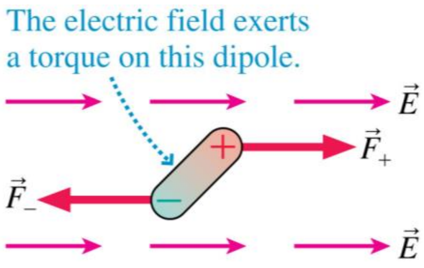
Which dipole experiences no net force in the electric field?

A. Dipole A.
B. Dipole B.
C. Dipole C.
D. Both dipoles A and C.
E. All three dipoles.



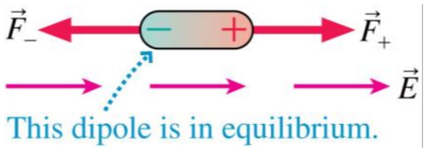
Dipoles in a Uniform Electric Field

- The figure shows an electric dipole placed in a *uniform* external electric field.
- The net force on the dipole is zero.
- The electric field exerts a *torque* on the dipole which causes it to *rotate.*


The electric field exerts a torque on this dipole.

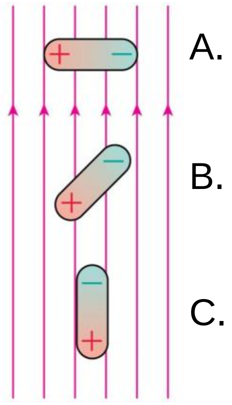
Dipoles in a Uniform Electric Field

- The figure shows an electric dipole placed in a *uniform* external electric field.
- The torque causes the dipole to rotate until it is aligned with the electric field, as shown.
- Notice that the positive end of the dipole is in the direction in which $\vec{E}$ points.

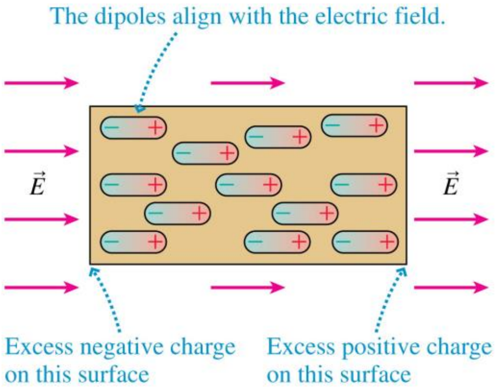

This dipole is in equilibrium.

Which dipole experiences no net **torque** in the electric field?

A. Dipole A.
B. Dipole B.
C. Dipole C.
D. Both dipoles A and C.
E. All three dipoles.



Dipoles in a Uniform Electric Field

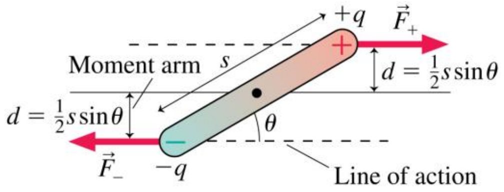


- The figure shows a sample of permanent dipoles, such as water molecules, in an external electric field.
- All the dipoles rotate until they are aligned with the electric field.
- This is the mechanism by which the sample becomes *polarized.*

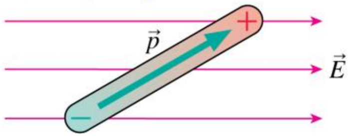
The Torque on a Dipole

The torque on a dipole placed in a uniform external electric field is

$$\tau = 2 \times dF_+ = 2(\tfrac{1}{2}s\sin\theta)(qE) = pE\sin\theta$$

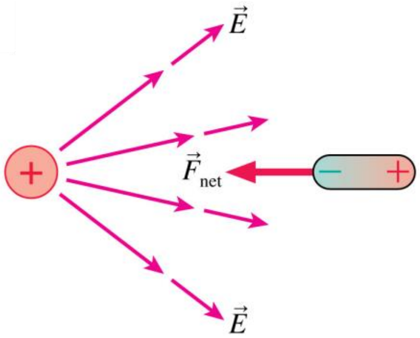


In terms of vectors, $\vec{\tau} = \vec{p} \times \vec{E}$.



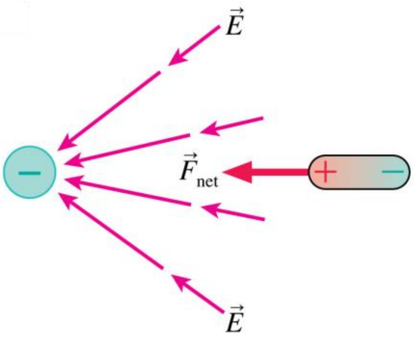
Dipoles in a Nonuniform Electric Field

- Suppose that a dipole is placed in a nonuniform electric field, such as the field of a positive point charge.
- The first response of the dipole is to rotate until it is aligned with the field.



- Once the dipole is aligned, the leftward attractive force on its negative end is slightly stronger than the rightward repulsive force on its positive end.
- This causes a net force to the *left*, toward the point charge.

Dipoles in a Nonuniform Electric Field



- A dipole near a negative point charge is also attracted toward the point charge.
- The net force on a dipole is toward the direction of the strongest field.
- Because field strength increases as you get closer to any finite-sized charged object, we can conclude that a **dipole will experience a net force toward any charged object.**

What is Potential Energy?

A. mgh

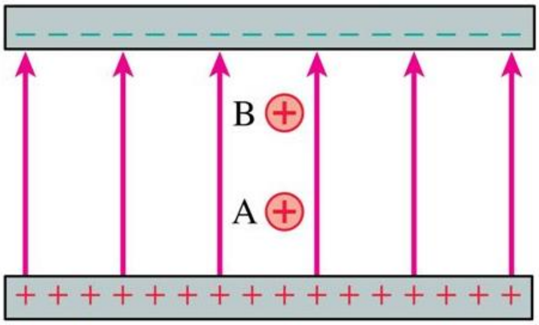
B. When an object has the ***potential*** to speed up.

C. Voltage

D. $\frac{1}{2} k(\Delta x)^2$

Class 11 Preclass Quiz on MasteringPhysics

- 74% got: Two positive charges are equal. **Charge A** has *more* electric potential energy.



Both of these charges have the **potential** to accelerate toward the negative plate, speeding up.

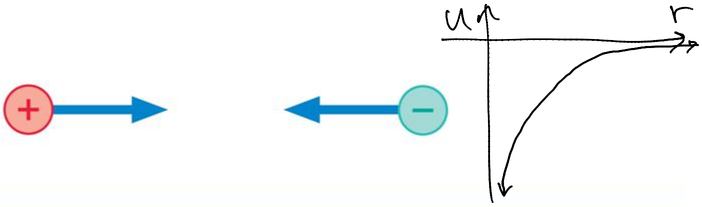
Class 11 Preclass Quiz on MasteringPhysics

- 70% of students got: The electric potential energy of a system of two point charges is proportional to the **inverse of the distance between the two charges.**

$$U_{\text{elec}} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

Class 11 Preclass Quiz on MasteringPhysics

- 49% of students got: A positive and a negative charge are released from rest in vacuum. They move toward each other. As they do a **negative potential energy** becomes **more negative**.

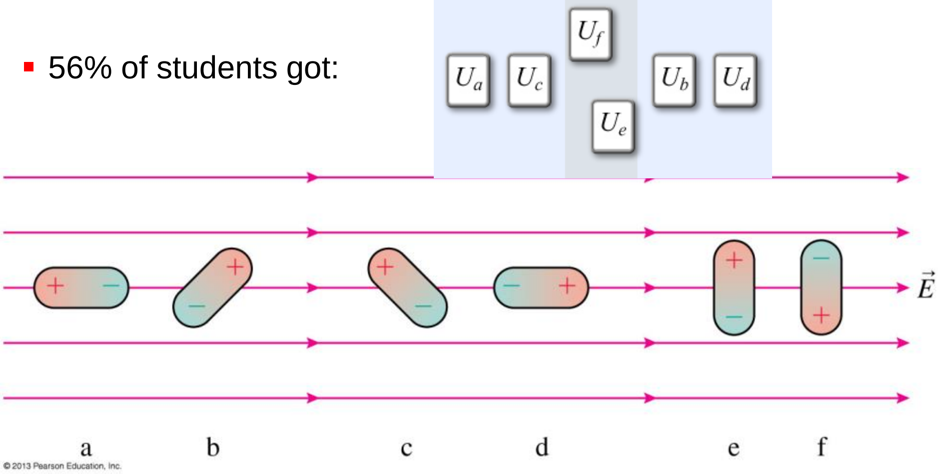


$$U_{\text{elec}} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

$U_e \to 0$ as $r \to \infty$

Class 11 Preclass Quiz on MasteringPhysics

- 56% of students got:



"Dipole d has the smallest potential energy because it is aligned with the electric field. The greater the angle between the positive to the direction of the electric field, the greater the electric potential."

Class 11 Preclass Quiz – Student Comments…

- *“If charged particle moves perpendicular to the electric field direction, does its potential not change then?”*
- **Harlow answer:** Correct. If the charge moves perpendicular to the electric field, then the electric force does zero work and the electric potential energy is unchanged.
- *“for the equation of electric potential energy in a uniform field, is the Uo always zero?”*

$$U_{\text{elec}} = U_0 + qEs$$

- **Harlow answer:** No! U_0 is arbitrary. You choose a convenient value of s where $U_0 = 0$. (similar to the zero-point in gravitational potential energy.)

Class 11 Preclass Quiz – Student Comments…

- *“What is the name of Sherlock Ohms' assistant...Dr. WATTSon!”*
- *“Q: What would you call a power failure?*
- *A: A current event.”*

Energy

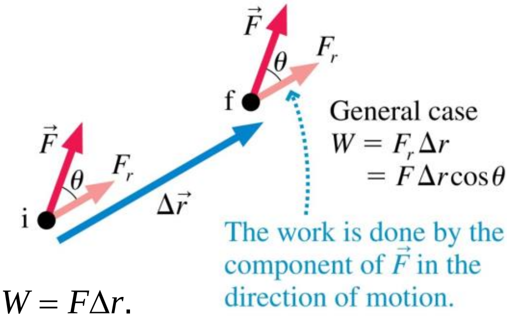
- The kinetic energy of a system, K, is the sum of the kinetic energies $K_i = 1/2 m_i v_i^2$ of all the particles in the system.
- The potential energy of a system, U, is the *interaction energy* of the system.
- The change in potential energy, ΔU, is -1 times the work done by the interaction forces:

$$\Delta U = U_\mathrm{f} - U_\mathrm{i} = -W_{\text{interaction forces}}$$

- If all of the forces involved are *conservative forces* (such as gravity or the electric force) then the total energy $K + U$ is *conserved*; it cannot be created or destroyed.

Work Done by a Constant Force

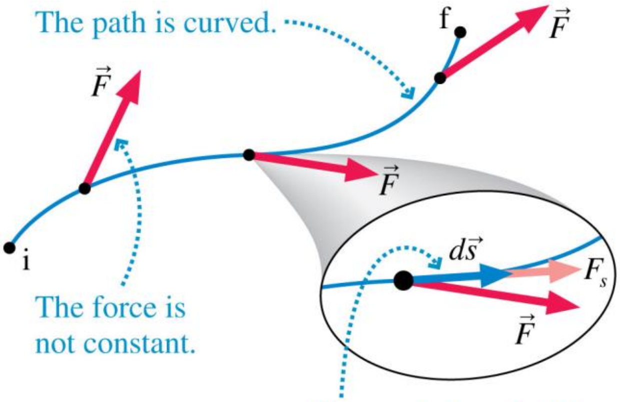
- Recall that the work done by a constant force depends on the angle θ between the force F and the displacement Δr.



- If $\theta = 0°$, then $W = F\Delta r$.
- If $\theta = 90°$, then $W = 0$.
- If $\theta = 180°$, then $W = -F\Delta r$.

Work

If the force is *not* constant or the displacement is *not* along a linear path, we can calculate the work by dividing the path into many small segments.



The path is curved.

The force is not constant.

The work done in this small segment of the motion is $F_s\,ds = \vec{F}\cdot d\vec{s}$.

$$W = \sum_j (F_s)_j\,\Delta s_j \rightarrow \int_{s_i}^{s_f} F_s\,ds = \int_i^f \vec{F}\cdot d\vec{s}$$

Electric Potential Energy in a Uniform Field

- A positive charge q inside a capacitor speeds up as it “falls” toward the negative plate.
- There is a constant force $F = qE$ in the direction of the displacement.
- The work done is:

$$W_{\text{elec}} = qEs_{\text{i}} - qEs_{\text{f}}$$

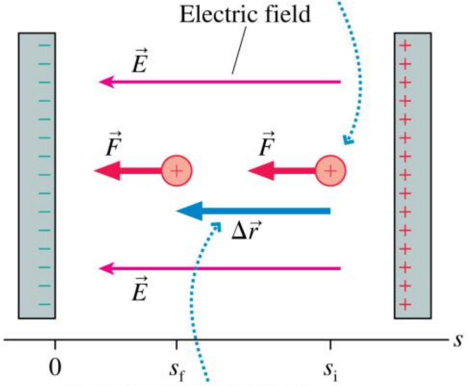
- The change in **electric potential energy** is:

$$\Delta U_{\text{elec}} = -W_{\text{elec}}$$

where

$$U_{\text{elec}} = U_0 + qEs$$

The electric field does work on the particle. We can express the work as a change in electric potential energy.

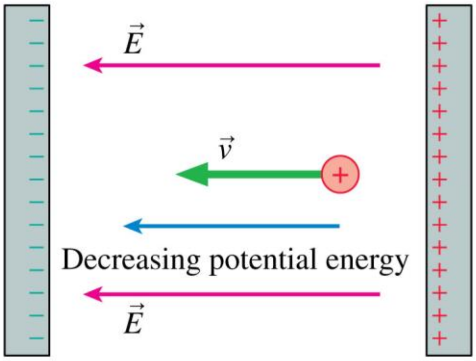


The particle is “falling” in the direction of $\vec{E}$.

Electric Potential Energy in a Uniform Field

$$U_{\text{elec}} = U_0 + qEs$$

A positively charged particle gains kinetic energy as it moves in the direction of decreasing potential energy.



The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.

Electric Potential Energy in a Uniform Field

q<0

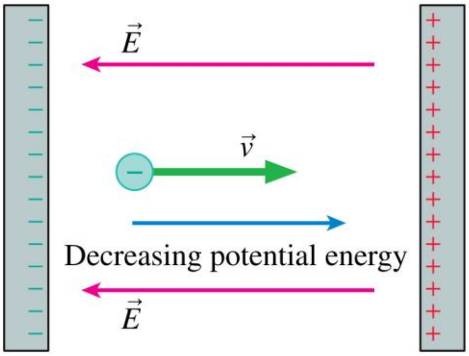
$$U_{\text{elec}} = U_0 + qEs$$

A negatively charged particle gains kinetic energy as it moves in the direction of decreasing potential energy.

As s increases,

U_{elec} decreases

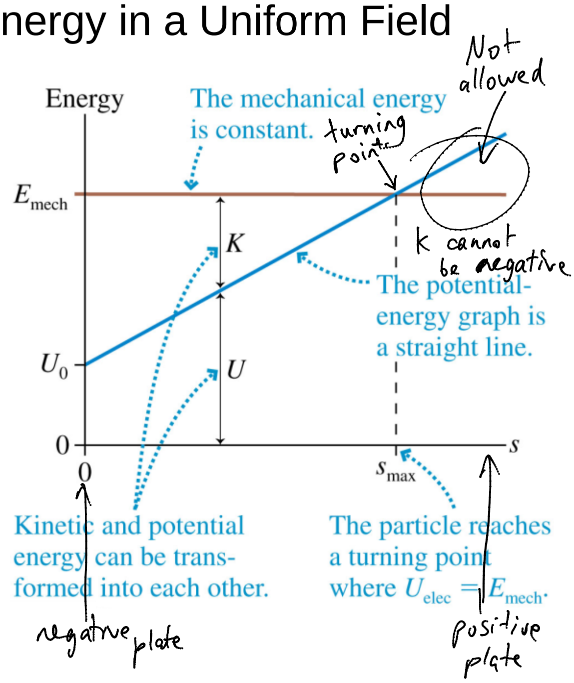
∴ K increases.



The potential energy of a negative charge decreases in the direction opposite to $\vec{E}$. The charge gains kinetic energy as it moves away from the negative plate.

Electric Potential Energy in a Uniform Field

- The figure shows the **energy diagram** for a positively charged particle in a uniform electric field.
- The potential energy increases linearly with distance, but the total mechanical energy E_{mech} is fixed.



The mechanical energy is constant.

The potential-energy graph is a straight line.

Kinetic and potential energy can be transformed into each other.

The particle reaches a turning point where $U_{\text{elec}} = E_{\text{mech}}$.

Not allowed

turning point

K cannot be negative

negative plate

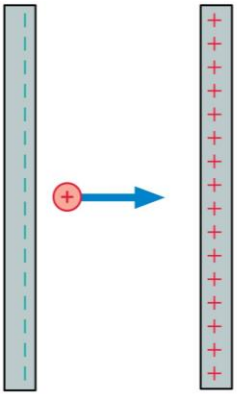
positive plate

A positive charge moves as shown. Its kinetic energy

A. Increases.

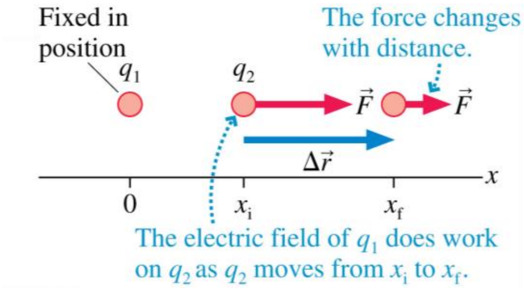
B. Remains constant.

C. Decreases.



The Potential Energy of Two Point Charges

- Consider two like charges q_1 and q_2.
- The electric field of q_1 *pushes* q_2 as it moves from x_i to x_f.
- The work done is:



$$W_{\text{elec}} = \int_{x_i}^{x_f} F_{1\text{ on }2}\,dx = \int_{x_i}^{x_f} \frac{Kq_1q_2}{x^2}\,dx = Kq_1q_2\frac{-1}{x}\bigg|_{x_i}^{x_f} = -\frac{Kq_1q_2}{x_f} + \frac{Kq_1q_2}{x_i}$$

- The change in electric potential energy of the system is $\Delta U_{\text{elec}} = -W_{\text{elec}}$ if:

$$U_{\text{elec}} = \frac{Kq_1q_2}{x} + \text{"a constant"}$$

The Potential Energy of Two Point Charges

Consider two point charges, q_1 and q_2, separated by a distance r. The electric potential energy is

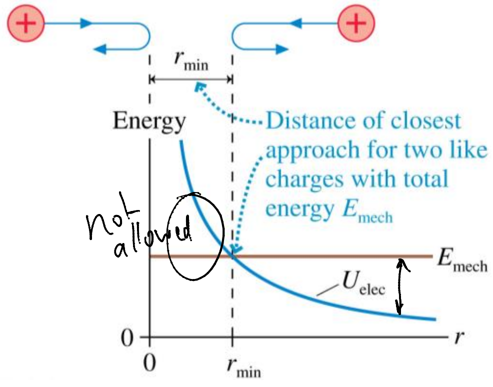
$$U_{\text{elec}} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

- This is explicitly the energy of *the system*, not the energy of just q_1 or q_2.
- Note that the potential energy of two charged particles approaches zero as $r \to \infty$.

Historical convention: we set integration constant such that $U \to 0$ as $r \to \infty$

The Potential Energy of Two Point Charges

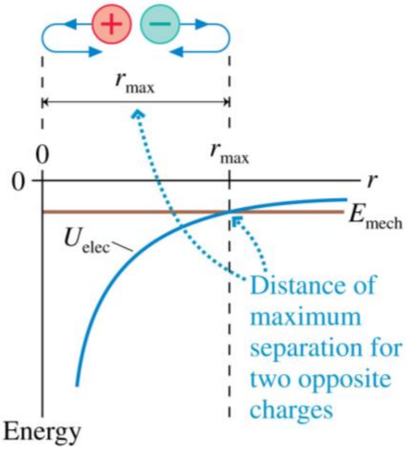
- Two like charges approach each other.
- Their total energy is $E_{\text{mech}} > 0$.
- They gradually slow down until the distance separating them is r_{min}.
- This is the *distance of closest approach.*



$$U_{\text{elec}} = \frac{Kq_1q_2}{r}$$

The Potential Energy of Two Point Charges

- Two opposite charges are shot apart from one another with equal and opposite momenta.
- Their total energy is $E_{\text{mech}} < 0$.
- They gradually slow down until the distance separating them is r_{max}.
- This is their *maximum separation.*



$$U_{\text{elec}} = \frac{Kq_1q_2}{r}$$

The Potential Energy of Multiple Point Charges

Consider more than two point charges, the potential energy is the sum of the potential energies due to all pairs of charges:

$$U_{\text{elec}} = \sum_{i<j} \frac{Kq_iq_j}{r_{ij}}$$

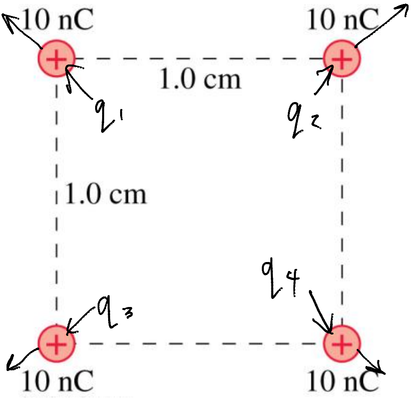
$i = 1, 2, 3, \ldots, N$

$j = 1, 2, 3, \ldots, N$

where r_{ij} is the distance between q_i and q_j.
The summation contains the $i < j$ restriction to ensure that each pair of charges is counted only once.

Problem 28.37

The four 1.0 g spheres shown in the figure are released simultaneously and allowed to move away from each other. What is the speed of each sphere when they are very far apart?



Conservation of Energy:

$$E_f = E_i$$

$$K_f + \cancel{U_f}^{\to 0} = \cancel{K_i}^{\to 0} + U_i$$

$U_f \to 0$ as the spheres get very far apart.

$K_i = 0$ ← released from rest.

$$\Rightarrow K_f = U_i$$

By symmetry, Energy is split equally among 4 spheres:

(ie) $K_1 = \frac{K_{total}}{4}$

Use $U_i = \sum_{i<j} \frac{Kq_iq_j}{r_{ij}}$

Conservation of energy: $E_f = E_i$

$K_f + U_f = K_i + U_i$

$U_f \to \infty$ as the spheres get very far apart.

$K_i = 0 \Leftarrow$ released from rest.

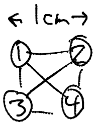
$\Rightarrow K_f = U_i$

split equally among 4 spheres

$K_1 = \frac{K_{total}}{4}$.

Use: $U_i = \sum_{i<j} \frac{Kq_iq_j}{r_{ij}}$

$$U_i = \frac{Kq_1q_2}{r_{12}} + \frac{Kq_1q_3}{r_{13}} + \frac{Kq_1q_4}{r_{14}} + \frac{Kq_2q_3}{r_{23}} + \frac{Kq_2q_4}{r_{24}} + \frac{Kq_3q_4}{r_{34}}$$



$q_1 = q_2 = q_3 = q_4 = q = 10nC$

$r_{12} = r_{13} = r_{24} = r_{34} = 0.01m$

$r_{14} = r_{23} = \sqrt{2}(0.01m)$

$$U_i = Kq^2\left(4\frac{1}{0.01} + 2\frac{1}{\sqrt{2}(0.01)}\right)$$

$$= \frac{9\times10^9(10\times10^{-9})^2}{0.01}\left(4 + \frac{2}{\sqrt{2}}\right)$$

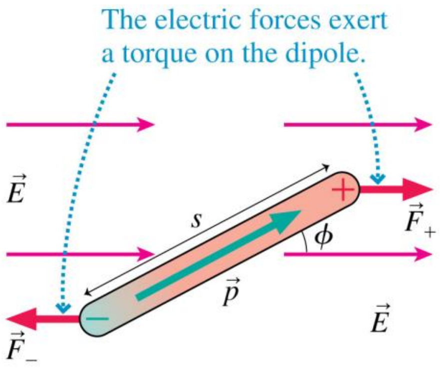
$U_i = 4.87\times10^{-4}$ Joules.

$K_f = \frac{U}{4} = \frac{1}{2}mv_f^2 = 1.2\times10^{-4}J$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(1.2\times10^{-4})}{10^{-3}kg}} = \boxed{0.49\frac{m}{s}}$$

The Potential Energy of a Dipole

- Consider a dipole in a uniform electric field.
- The forces F_+ and F_- exert a torque on the dipole.
- The work done is:


The electric forces exert a torque on the dipole.

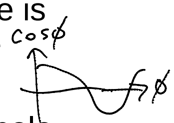
$$W_{\text{elec}} = -pE\int_{\phi_i}^{\phi_f} \sin\phi\, d\phi = pE\cos\phi_f - pE\cos\phi_i$$

- The change in electric potential energy of the system is $\Delta U_{\text{elec}} = -W_{\text{elec}}$ if:

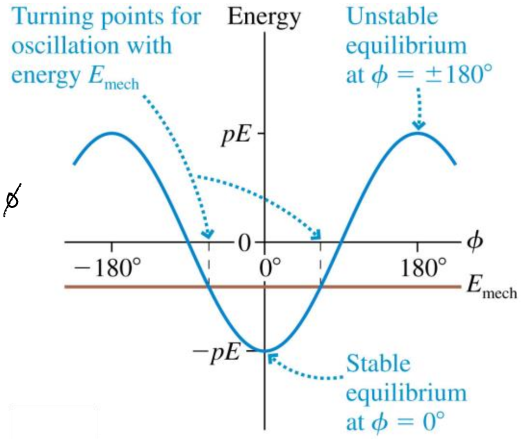
$$U_{\text{dipole}} = -pE\cos\phi = -\vec{p}\cdot\vec{E}$$

The Potential Energy of a Dipole

- The potential energy of a dipole is $\phi = 0°$ minimum at where the dipole is aligned with the electric field.



- A frictionless dipole with mechanical energy E_{mech} will oscillate back and forth between turning points on either side of $\phi = 0°$.



$$U_{\text{dipole}} = -pE\cos\phi = -\vec{p}\cdot\vec{E}$$

The Electric Potential

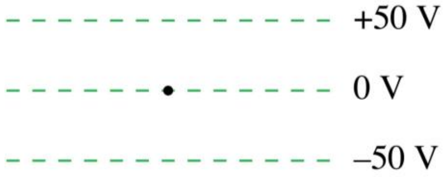
- We define the electric potential V (or, for brevity, just the potential) as

$$V \equiv \frac{U_{q+\text{sources}}}{q}$$

- This is NOT the same as electric potential energy. (different units, for one thing).
- The unit of electric potential is the joule per coulomb, which is called the volt V:

$$1\ \text{volt} = 1\ \text{V} \equiv 1\ \text{J/C}$$

A proton is released from rest at the dot. Afterward, the proton



A. Remains at the dot.
B. Moves upward with steady speed.
C. Moves upward with an increasing speed.
D. Moves downward with a steady speed.
E. Moves downward with an increasing speed.

Before Class 12 on Wednesday (my last class…)

- Please finish reading Knight Ch. 28
- Please do the short pre-class quiz on MasteringPhysics by tomorrow night.
- Something to think about. A battery is designed to supply a steady amount of which of the following quantities?
  - Energy
  - Power
  - Electric potential difference
  - Electric current

(**Hint:** only *one* of these is correct!)