SOLUTIONS – December 10, 2004

Question 1

A fish on a hook is hanging from a long fishing line, and is swinging back and forth with simple harmonic motion. The fishing line does not stretch. When the fish is at maximum displacement from equilibrium, which of the following is also at a maximum?

(A) its speed
(B) its period
(C) its frequency
→(D) its height **RIGHT ANSWER**
(E) its kinetic energy

Question 2

A spring with negligible mass hangs from a hook in the ceiling. It has a spring constant of k=150 N/m, and an equilibrium length when hanging of 0.25 m. A 0.15 kg mass is attached to the end of the spring when the spring is at its equilibrium length, and then released from rest. As the mass falls, it pulls the spring down, then begins oscillating up and down with simple harmonic motion. What is the maximum kinetic energy of the mass?

→(A) 0.0072 J **RIGHT ANSWER**
(B) 0.37 J
(C) 4.7 J
(D) 32 J
(E) 1.0×10^3 J

Question 3

Ocean waves with a wavelength of 120 m are moving East at a speed of 16 m/s. A boat is traveling West with a speed of 4 m/s. What is the frequency of waves as observed by a person on the boat?

(A) 0.1 waves per minute.
(B) 2 waves per minute.
(C) 6 waves per minute.
(D) 11 waves per minute.
→(E) 10 waves per minute. **RIGHT ANSWER**

Question 4

Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves that are exactly out of phase. In other words, their phase difference is π. The frequency of the waves emitted by each speaker is 86 Hz. You are 8.0 m from speaker A. What is the closest you can be to speaker B and be at a point of constructive interference?

(A) 1.0 m
→ (B) 2.0 m **RIGHT ANSWER**
(C) 4.0 m
(D) 6.0 m
(E) 7.0 m

Question 5

The wavelength of the lowest frequency harmonic at which resonance occurs in the air in a tube of length L closed at one end, open at the other is

(A) ½ L

(B) ¼ L

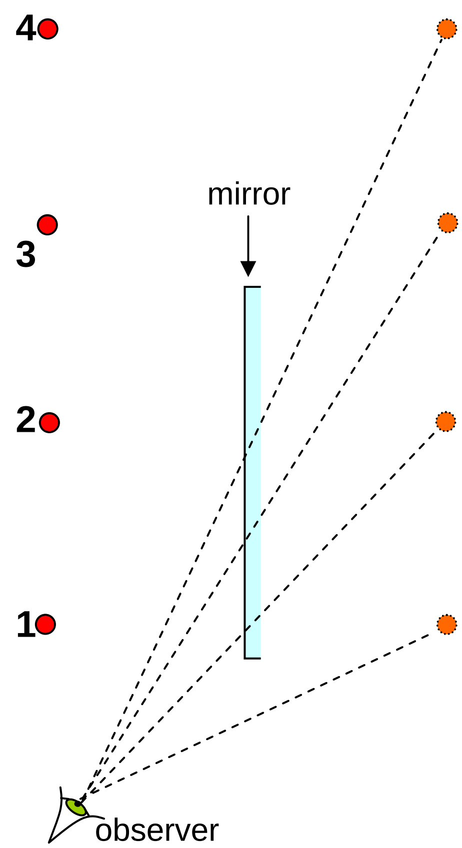
(C) 2 L

→(D) 4 L **RIGHT ANSWER**

(E) 3 L

Question 6

An observer, shown as an eye in cross-section in the drawing, attempts to look at four objects in a mirror. Which (if any) of the objects have a reflection that can be seen by this observer?



(A) 1 and 2

(B) 2 and 3

→(C) 2, 3 and 4 **RIGHT ANSWER**

(D) all of the objects have a reflection that can be seen

(E) none of the objects have a reflection that can be seen

Question 7

A beam of light from a monochromatic laser shines perpendicularly down into a piece of glass. The glass has a thickness L and an index of refraction n. The frequency of the laser light in a vacuum is f. How many wavelengths of light fit into the thickness of the glass?

(A) Lc/f

→(B) Lfn/c **RIGHT ANSWER**

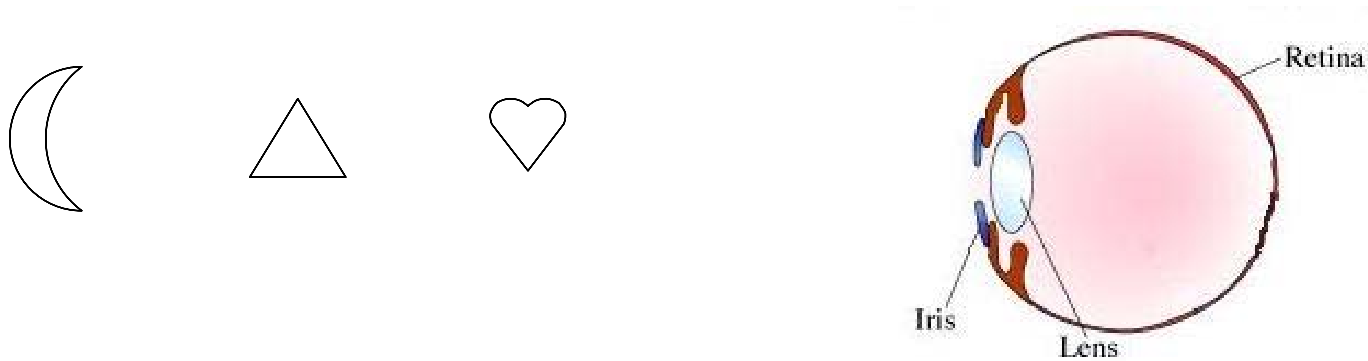
(C) cn/fL

(D) c/fL

(E) Lf/cn

Question 8

Shown in the diagram is a simplified model of the human eyeball gazing at three shapes at different distances from the eye: a moon, a triangle and a heart. The light enters the eye through the lens, and then strikes the retina on the back of the eye. The ciliary muscles surrounding the lens can be expanded and contracted to change the curvature of the lens, which in turn changes the effective focal length of the lens. This in turn changes the location of the image of any object in one's field of view. Images formed on the retina appear in focus. This particular eye is focused on the triangle. Which of the following statements is true regarding the locations of the images of the moon and heart?



→(A) The image of the moon is formed between the lens and retina and the image of the heart is formed to the right of the retina. **RIGHT ANSWER**

(B) The image of the moon is formed to the right of the retina and the image of the heart is formed to the right of the retina.

(C) The image of the moon is formed between the lens and retina and the image of the heart is formed between the lens and retina.

(D) The image of the moon is formed to the right of the retina and the image of the heart is formed between the lens and retina.

(E) The image of the moon is formed on the retina and the image of the heart is formed on the retina.

LONG ANSWER (36 marks total)

The high "E-note" string on a guitar has a linear mass density of μ. It is tightened between two fixed points a distance L apart to a tension of T.

PART A

When the guitarist gently touches a finger to the centre of the string, it forces a node at that point. What is the lowest frequency of a standing wave that can be produced in this way? Please express your answer in terms of numerical constants and any of the following: μ, L, T and the speed of sound in air, v.

The guitarist is forcing the first harmonic, as shown in Figure 14.8(c) on page 479. The frequency is $f_2 = 2v_{string}/(2L)$, where v_{string} is the speed of waves on the string, which is v_{string}=sqrt(T/μ). So the frequency is f=(1/L)*sqrt(T/μ):

$$\boxed{f = \frac{1}{L}\sqrt{\frac{T}{\mu}}}$$

PART B

The high note from Part A is played on a guitar by a performer standing on a stage. A spectator rushes toward the stage at high speed. What is the minimum speed the spectator must go so that the observed frequency is above the human range of hearing, f_{max}? Please express your answer in terms of numerical constants and any of the following: μ, L, T, f_{max} and the speed of sound in air, v.

We were asked to use the note from Part A, so $f=(1/L)*\text{sqrt}(T/\mu)$ is the emitted frequency. The source is stationary, so vs=0, but the observer is moving with some minimum speed vs=vmin so that the observed frequency is the maximum of the human range, f'=fmax.

$$f' = f_{\max} = f\left(\frac{v+v_o}{v-v_s}\right) = f\left(\frac{v+v_o}{v}\right)$$

Solving for v_o, we get:

$$v_o = v_{\min} = v\left(\frac{f_{\max}}{f} - 1\right)$$

Substituting in our previous value for f found in Part A:

$$\boxed{v_{\min} = v\left(f_{\max} L\sqrt{\frac{\mu}{T}} - 1\right)}$$

PART C

Express your answer to Part B numerically using the following values:

$\mu = 7.3\times10^{-4}$ kg/m
L = 0.66 m
T = 530 N
f_{max} = 10,000 Hz
v = 344 m/s

$$\boxed{v_{min} = 2000 \text{ m/s}}$$