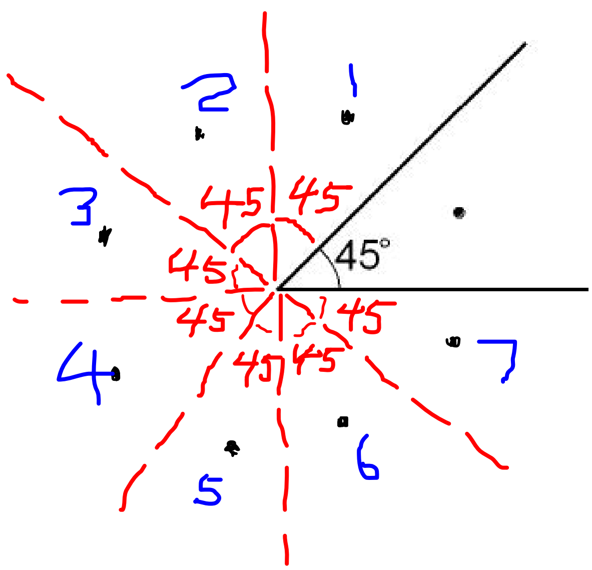
PHY138Y Second-Quarter Test – December 5, 2006-12-06

Multiple Choice: Version 1:
1. C
2. B
3. A
4. E
5. C
6. A
7. D
8. C
9. B

Multiple Choice: Version 2:
1. E
2. E
3. A
4. B
5. A
6. E
7. B
8. D
9. C

Long Answer Solution

Part A.
Question: *Two plane mirrors intersect with an angle of 45°, as seen from above in the diagram below. A small circular object sits between them. How many images of the object will form? Please sketch the approximate locations of these images in the diagram.*
Solution:



Number of images = 7

Part B.
Question: *You wish to make a lens that is perfectly flat (no curvature) on one side, and has a focal length of −20 cm, using glass with index of refraction n = 1.50. What should be the radius of curvature of the other side, and should it be concave or convex (circle one in the box)?*
Solution:

Use the lensmaker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Assuming the flat side faces the object, set $R_1 = \infty$ (flat side) so that $1/R_1 = 0$, and solve for R_2:

$$\frac{1}{f} = (n-1)\left(0 - \frac{1}{R_2}\right)$$

$R_2 = -(n-1)\,f = -(1.5-1)(\,-20\text{ cm})$
$R_2 = 10\text{ cm}$

To make a diverging lens with one flat side, the other side must be concave. That way it is thinner in the middle than at the edges. In fact, the way it is solved above, by Knight's sign convention (Table 23.4, pg.743) surface 2 is "convex toward the object", but surface 2 faces *away* from the object, so, as viewed from outside the lens, surface 2 is concave.

Another way to solve the problem is to assume that the curved side faces the object, so we set $R_2 = \infty$ (flat side) so that $1/R_2 = 0$, and solve for R_1:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - 0\right)$$

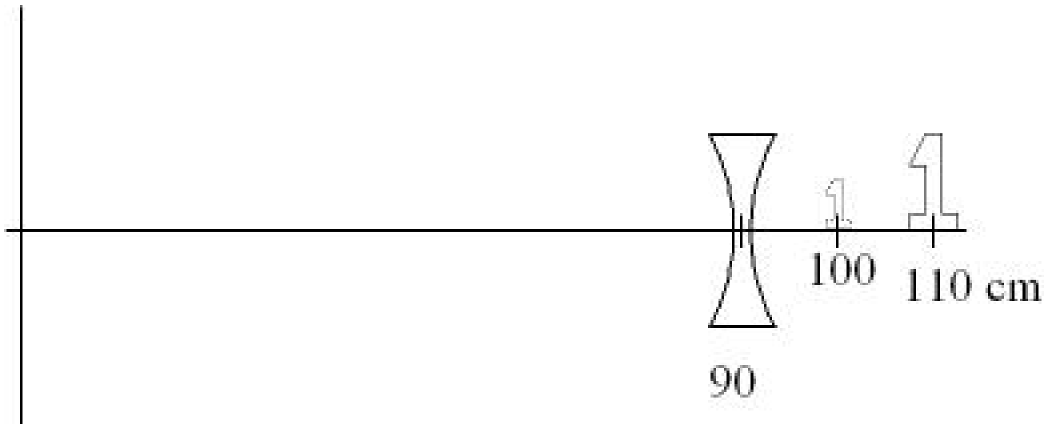
$R_1 = (n-1)\,f = (1.5-1)(\,-20\text{ cm})$
$R_1 = -10\text{ cm}$
By Knight's sign convention (Table 23.4, pg.743) surface 1 is "concave toward the object", and since this surface faces the object, it is in fact concave.

$R = +10$ cm (or −10 cm is fine)
CONCAVE / CONVEX

Part C.

Question: *A viewing screen sits parallel to the y-z plane at x = 0. A 1.0 cm tall object sits on the x-axis at x = 110 cm. A diverging lens with focal length −20 cm is centred on the x-axis, with its optical axis along the x-axis, at x = 90 cm. At what value of x will the image form? Is it a real or virtual image (circle one in the box)?*

Solution:



s = 20 cm, the distance of the object to the lens, (110 cm − 90 cm). f = −20 cm. Solve for s' using the thin lens equations:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$s' = \left[\frac{1}{f} - \frac{1}{s}\right]^{-1} = \left[\frac{1}{-20} - \frac{1}{20}\right]^{-1}$$

s' = − 10 cm. This means the image is on the same side of the lens (to the right of the lens) and it is virtual. The x position is 90 + 10 = 100 cm.

Note also that the magnification is m = −s'/s = +0.5, so the image will be upright, and half the height of the object, or 0.5 cm tall.

x_{object} = 100 cm
REAL / VIRTUAL

Part D.
Question: *You wish to add a second lens to the system from Part C which will form an image on the screen that is well focused and 2.0 cm tall. What should be the position and focal length of the second lens? Will the final image be upright or inverted (circle one)?*
Solution:

Use the image from Part C to become the object for this part. The object for this part is upright at x = 100 cm, and has a height of 0.5 cm.

The magnitude of the magnification is given by:

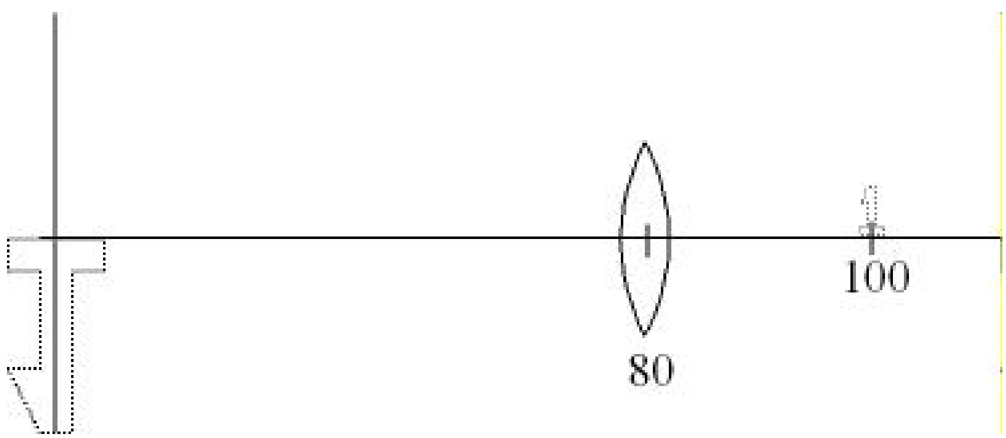
$$|m| = \frac{s'}{s} = \frac{h'}{h}$$

To make a 2 cm image from a 0.5 cm object, $h'/h = 2 / 0.5 = 4$. So $s'/s = 4$.

Also we must make the image form on the screen. Putting the lens between the object and image, we have $s' + s = 100$ cm (distance from object to screen.) Now we have two equations and two unknowns, we can solve for s' and s:

$$s' = 4s$$
$$4s + s = 100 \text{ cm}$$
$$s = 100 / 5 = 20 \text{ cm}$$
$$s' = 80 \text{ cm}$$

Both s' and s are positive, so the lens is in between the object and image. The lens is therefore 20 cm to the left of the object, at $x_{lens} = 100 - 20 = 80$ cm. And the magnification is actually $m = -s'/s = -80/20 = -4$, so the image is inverted (see diagram).



The focal length can be found from the thin-lens equation:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{20} + \frac{1}{80}$$
$$f = 16 \text{ cm}$$

(f is positive, so it is a converging lens.)

x_{lens} = 80 cm

f_{lens} = 16 cm

UPRIGHT / INVERTED