Multiphoton Localization and Propagating Quantum Gap Solitons in a Frequency Gap Medium

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The many-particle spectrum of an isotropic frequency gap medium doped with impurity resonance atoms is studied using the Bethe ansatz technique. The spectrum is shown to contain pairs of quantum correlated "gap excitations" and their heavy bound complexes ("gap solitons"), enabling the propagation of quantum information within the classically forbidden gap. In addition, multiparticle localization of the radiation and the medium polarization occurs when such a gap soliton is pinned to the impurity atom. [S0031-9007(97)03751-4]

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Light localization is a classical effect predicted [1] to occur in strongly scattering dielectric microstructures. In the context of photonic band gap (PBG) materials [2,3], nonclassical forms of localization such as photon-atom bound states have been predicted [4] when the resonant transition frequency of an impurity atom lies within a gap. This bound state is an eigenstate of the quantum electrodynamic Hamiltonian for a realistic PBG crystal exhibiting a general anisotropic photon dispersion relation. In this state, a virtually emitted photon may tunnel many wavelengths away from the atom before being reabsorbed, leading to non-Markov memory effects [5] in collective light emission from many atoms. It was recently shown that an effective model [6] describing both isotropic PBG systems and frequency dispersive media (DM) [7] doped with resonance atoms exhibits hidden integrability [8] and is diagonalized exactly [6,8] by means of the Bethe ansatz technique [9]. This suggests the possibility of a rich multiparticle spectrum in real physical systems exhibiting a frequency gap, when such systems are doped with impurity atoms.

In this paper, we demonstrate the existence of nonclassical states of light which may be generated, for instance, through the interaction of an external laser field with an impurity atom placed within a polariton gap [10] of a DM. In addition to ordinary polaritons and their bound complexes (ordinary solitons) occurring outside of the gap, the subgap spectrum of the system is shown to contain propagating pairs of correlated "gap excitations" and their heavy bound complexes (gap solitons). The individual gap excitations comprising the pair are correlated such that the probability amplitude of finding them far apart decreases exponentially with the ratio of their separation distance to the classical penetration length of the radiation into the medium. In addition to heavy gap solitons propagating within the gap, the spectrum contains multiphoton localized states pinned to the atom. Under external perturbations a pinned gap soliton may dissociate into propagating gap excitations. We evaluate the dispersion relations, the effective masses, and the dissociation energies of quantum gap solitons and we show that they are stable with respect to weak perturbations. Our results demonstrate a clear distinction, at the quantum level, between fermionic gap systems (such as electronic semiconductors) and bosonic gap systems. In a semiconductor, propagation within the energy gap is strictly forbidden. In the bosonic gap, however, certain nonclassical many-body gap states are allowed to propagate. This may have important consequences for the transmission of quantum information within a bosonic gap medium. Although the Bethe ansatz method made use of the *isotropic* one-particle dispersion relation, qualitative similar results may hold in a realistic, *anisotropic* PBG material which also exhibits a small nonresonant Kerr nonlinearity [11].

Consider the model Hamiltonian $\hat{H} = H_0 + V$, where

$$H_0 = \omega_{12}(\sigma^z + 1/2) + \int_C \frac{d\omega}{2\pi}\, \omega\, p^\dagger(\omega) p(\omega), \qquad (1a)$$

$$V = -\sqrt{\gamma} \int_C \frac{d\omega}{2\pi}\, z(\omega)[p(\omega)\sigma^+ + p^\dagger(\omega)\sigma^-]. \qquad (1b)$$

Here ω_{12} is the transition frequency of the two-level impurity atom placed within a bosonic frequency gap medium. The spin operators $\vec{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$, $\sigma^\pm = \sigma^x \pm i\sigma^y$ satisfy the standard commutation algebra $[\sigma^i, \sigma^j] = \epsilon_{ijk}\sigma^k$ and act on the atomic variables of the system. The operators $p^\dagger(\omega)$ $[p(\omega)]$ create (annihilate) bosons of frequency ω in a specific (electric dipole) spherical harmonic state and satisfy the algebra $[p(\omega), p^\dagger(\omega')] = 2\pi\delta(\omega - \omega')$. The integration contour consists of two parts, $C = C_- \oplus C_+$, where $C_- = (0, \Omega_\perp)$ and $C_+ = (\Omega_\parallel, \infty)$ correspond to the lower and upper branches of the medium excitations, respectively. The interaction term (1b) describes emission and absorption of bosons by the atom. Here $\gamma = 4\omega_{12}^3 d^2/3$ is the inverse lifetime of the excited atom in free space with the dipole transition moment d, while the atomic form factor $z(\omega)$ contains the information about the polariton spectrum. Since the polariton gap persists for small wave vectors, $k \to 0$, we can neglect anisotropies of the underlying ionic crystal and take the dispersion relation to be isotropic. Also, we choose units in which $\hbar = c = 1$.

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