Height-dependent behaviour of meandering syrup

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In this experiment, the behaviour of a viscous thread in a 'fluid-mechanical sewing machine' setup was examined, as a function of nozzle height, over a range of 10.0000 ± 0.0001 cm to 6.5995 ± 0.0001 cm. Several periodic states were observed. Furthermore, the system exhibited disordered behaviour at heights in the ranges of 7.4996 - 8.2997 ± 0.0001 cm and 8.5998 - 8.9999 ± 0.0001 cm. A window of periodic behaviour which would require more exploration in order for it to be classified was observed in the range of $h = 8.3998 \pm 0.0001$ cm to $h = 8.4998 \pm 0.0001$ cm.

I. INTRODUCTION

When viscous fluid is dribbled onto a flat surface, it exhibits a coiling behaviour. Breaking rotational symmetry by dribbling the fluid onto a moving belt produces many interesting patterns. Applications of such a setup have been seen in certain 3-D printing and manufacturing methods.[1]

Different behaviour can be observed in this system depending on several parameters which can be varied. It has been previously shown that nozzle height, belt speed, and flow rate all affect the state of the system.[2][3][4]

In this experiment, the flow rate and belt speed were held constant, and the height was varied.

II. APPARATUS AND SETUP

The apparatus used in this experiment is a setup similar to the one used by Welch et al.[5] A notable difference between this experiment and that of Welch et al, however, is that the images in this experiment were captured at a rate of 400 frames per second, a much higher framerate than those used in previous experiments.[5][4] Hence, the data collected has a much higher time resolution than previous experiments.

The fluid used in the experiment is Dow Corning 200 Fluid, with a viscocity of 30 000 CST. (0.03 m^2/s.) This fluid has been used in previous experiments, namely those by Welch et al and Morris et al.[5][4]

The nozzle height was adjusted using a programmable translation stage, as pictured in Figure 1. The height range examined was 10.0000 ± 0.0001 cm to 6.5995 ± 0.0001 cm, in increments of 0.1000 ± 0.0001 cm. This height range was chosen to overlap with some of the height range in previous experiments, so that some comparison coucld be made between the results of this experiment and past results.[4]

The belt speed, which was controlled by a stepper motor, was fixed at 1.75 ± 0.01 m/s, a speed which was determined to have some interesting behaviour in the 6.5995 cm to 8.0000 cm height range, which was previously examined by Morris et al.

The flow rate was controlled by a syringe pump, as pictured in Figure 1. The calibration of the syringe pump was verified by weighing the amount of fluid dispensed by the nozzle over two minutes for a variety of syringe pump flow rate settings, and then comparing the flow rates indicated by the pump to the observed flow rates. For this experiment, the flow rate was set to 1.61 ± 0.01 mL/min; the same flow rate as that used by Morris et al.

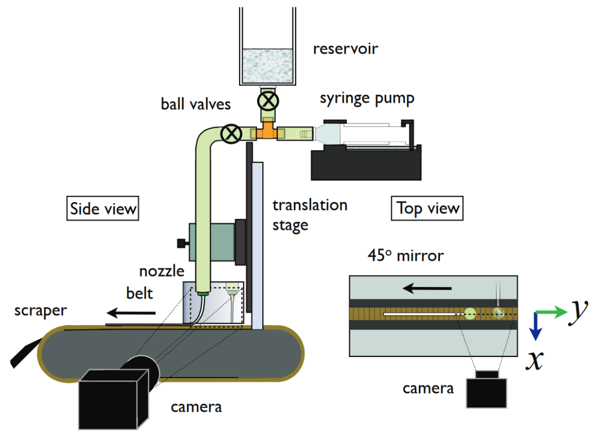


FIG. 1: A diagram of the apparatus used in the experiment.[5] The nozzle height, belt speed, and flow rate were all adjustable, but for the purpose of this experiment, the belt speed and nozzle height were fixed.

III. IMAGE ANALYSIS

The images taken by the camera were 16-bit TIFF images which were converted to 8-bit BMP images. An edge-detection algorithm was implemented in Python to determine the position of the threads in the images. For each image, the algorithm proceeded as follows. The program extracted a horizontal line from the image as an array of floating-point values, and interpolated between the points in the extracted line using the scipy.interpolate.interp1d method. The line extracted from the images was 45 px from the top of the image, as this was a height low enough to reflect the motion of the strand while being high enough to not be affected by the strand folding over itself so that it a horizontal line would cross through it twice. This 'folding' behaviour was observed in many of the nonlinear states; an example of it is shown in Figure 3.

Figure 2 is a plot comparing the interpolated values with the actual values of the plot. The points were interpolated because there were too few points in the original horizontal line array to take a numerical derivative which could accurately pinpoint the local minima.

The program calculated the numerical derivative of the array using central differences. The dark bands corresponding to the edges of the strands take on extreme values; hence, their positions would be at zeros of the derivative of the array of floating-point image values.

Figure 4 shows a plot showcasing the accuracy of the edge detection algorithm, and Figure 5 shows an example of centres of strands found by the algorithm.

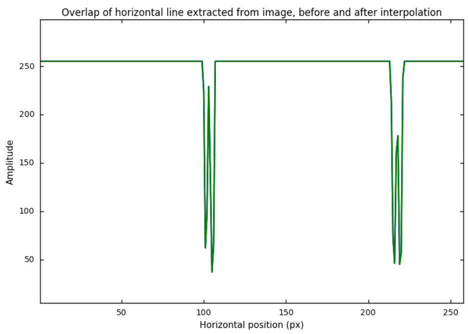


FIG. 2: Interpolation vs. actual values

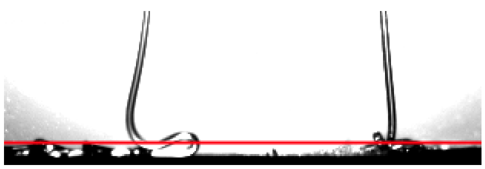


FIG. 3: Folding behaviour; the red line is an example of a horizontal line that would pass through the strand more than once.

IV. DATA PROCESSING

The edge-detection algorithm was run in a loop for all the images from a given run, and the resulting x and y positions were gathered in arrays, which would then be displayed in plots. The resulting data had some aliasing due to the low image resolution, so the data plots were smoothed by interpolating them with a Gaussian for viewing purposes. The error produced by this process was in the range of $0.12 - 0.18$ cm; small enough that the error can be disregarded when considering the shapes of the plots.

A comparison of the Lissajous figures for the raw data and the antialiased data can be found in Figure 6 Note that the visual difference in the overall shapes of the plots is small.

In order to determine the periodicity of the data, Fourier transforms were taken in the x and y coordinates. These Fourier transforms were taken from data which was

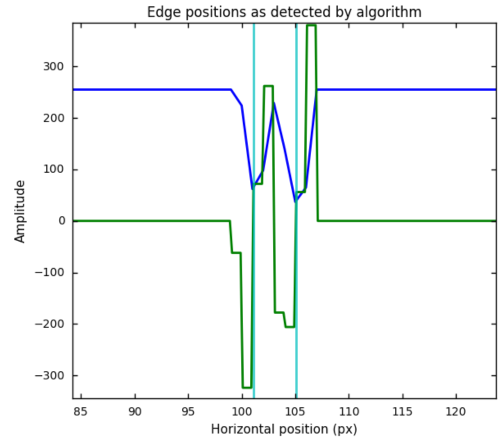


FIG. 4: Edge positions as detected by the algorithm, zoomed in on one strand. The dark blue line corresponds to the brightness of the pixels, the green line is the numerically-calculated derivative, and the light blue lines indicate the calculated horizontal positions of the minima.

FIG. 5: Centres of strands, as detected by the algorithm

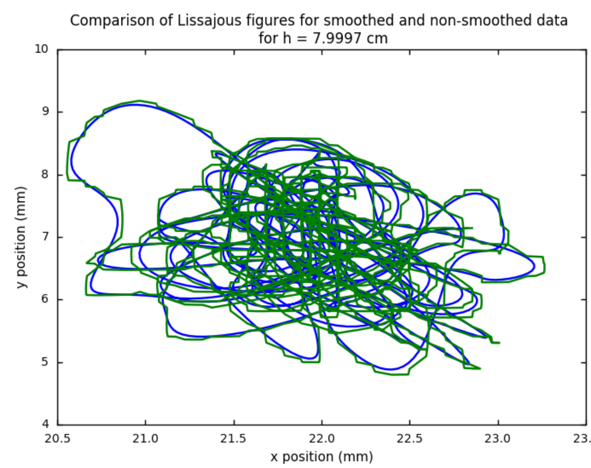


FIG. 6: Comparison of Lissajous figures, with (blue) and without (green) antialiasing. The maximum difference between the two lines for this plot was 0.18 mm.

not antialiased, so the antialiasing did not affect the analysis of the data.

The reconstructions of the patterns were constructed by plotting the x position against the shifted (longitudinal) y position, which was calculated using the formula

$y = y_0 + vt$, where y_0 is the unshifted y position, v is the belt speed, and t is the time elapsed from the start of the data capture. This simulated an image of the path traced out by the thread in the reference frame of the belt, as opposed to the motion in the lab frame, which is pictured in the Lissajous figures.

V. RESULTS

A few periodic states were observed, although some aperiodic, nonlinear behaviour was observed as well. Most of the periodic states observed in this experiment were also observed by other experiments, including, notably, those conducted by Chiu-Webster et al.[3] The states observed in this experiment that correspond to states observed in previous research are presented below in Figure 7.

Based on the Fourier spectra of the states, it was possible to discern if they were periodic or disordered. The Fourier spectra of periodic states show a clear spike with a low amplitude in the surrounding frequencies, whereas the Fourier spectra of chaotic states show a less pronounced spike with a stronger signal in the surrounding frequencies, considerably higher than any baseline noise. Figure 8 shows Fourier spectra for the states shown in Figure 7. Note that although Figure 8c has a spike with a considerably smaller amplitude than Figure 8a and Figure 8b, the sharpness of the spike and lack of strong signal in the surrounding frequencies suggests that this is nevertheless a periodic state. On the other hand, Figure 8d has a spike with a relatively low amplitude, as well as a strong signal in the surrounding frequencies. This suggests that the state is indeed chaotic.

In the region of state space explored, there is a window of periodic behaviour around $h = 8.3998 \pm 0.0001$ cm to $h = 8.4998 \pm 0.0001$ cm. This state appears to be some sort of coiling, although from the present results, it is not clear if it would be better described as translated coiling (as in Figure 7a) or as slanted coiling (as in Figure 7c), or, perhaps, as a different sort of coiling altogether. A plot of this behaviour, along with the corresponding Fourier spectrum, is shown in Figure 9.

Figure 10 shows a diagram of the state space explored in this experiment. The state in Figure 9 is labelled as 'Intermediate coiling'.

VI. DISCUSSION

The states observed show a fairly good agreement with the results obtained by Morris et al for the height ranges of 6.5999 ± 0.0001 cm to 7.9997 ± 0.0001 cm.[4] This suggests that the experimental results are a good reflection of the behaviour of the system.

The behaviour of the system around $h = 8.3998 \pm 0.0001$ cm to $h = 8.4998 \pm 0.0001$ cm appears to be an intermediate state between translated coiling and slanted coiling. Further investigation of this height range would provide more insight into the behaviour of the system in this range.

The current experimental setup is capable of examining a higher 'height resolution'; i.e. using a smaller increment when varying the height. Hence, in future experiments, it would be of interest to use a smaller increment when changing the nozzle height. This would be beneficial in exploring the regions where the state changes. In particular, this would prove useful for observing the behaviour in the region around $h = 8.3998 \pm 0.0001$ cm to $h = 8.4998 \pm 0.0001$ cm.

It would also be interesting to vary other parameters in future experiments, namely the belt speed and the flow rate. This would enable further exploration of the state space.

VII. CONCLUSION

The state of the system shows a clear dependence on height, as is expected from previous experiments. There is a fairly good agreement with the results from this experiment with previous results for certain heights.[4] Disordered behaviour was observed in the height ranges of 7.4996 - 8.2997 ± 0.0001 cm, and 8.5998 - 8.9999 ± 0.0001 cm. Most of the stable states observed in this experiment were also observed in previous experiments. A window of periodic behaviour with a coiling state was also observed, although more observation is necessary to properly categorize this state.

[1] Klein, Johhn, Michael Stern, et al, **Additive Manufacturing of Optically Transparent Glass**, *3D Printing and Additive Manufacturing*, **2(3)**, (2015).

[2] Brun, P.-T., Basile Audoly, et al, **Liquid Ropes: A Geometrical Model for Thin Viscous Jet Instabilities**, *Physics Review Letters*, **114**, 174501 (2015).

[3] Chiu-Webster, S. and J. R. Lister, **The fall of a viscous thread onto a moving surface: a 'fluid-mechanical sewing machine,'** *J. Fluid Mech.*, 569, **89**, (2006).

[4] Morris, Stephen W., Jonathan H. P. Dawes, Neil M. Ribe, and John R. Lister, **The meandering instability of a viscous thread**,*Physical Review E*, **77**, 066218 (2008).

[5] Welch, Robert L., Billy Szeto, and Stephen W. Morris, **Frequency structure of the nonlinear instability of a dragged viscous thread**, *Physical Review E*, **85**, 066209 (2012).

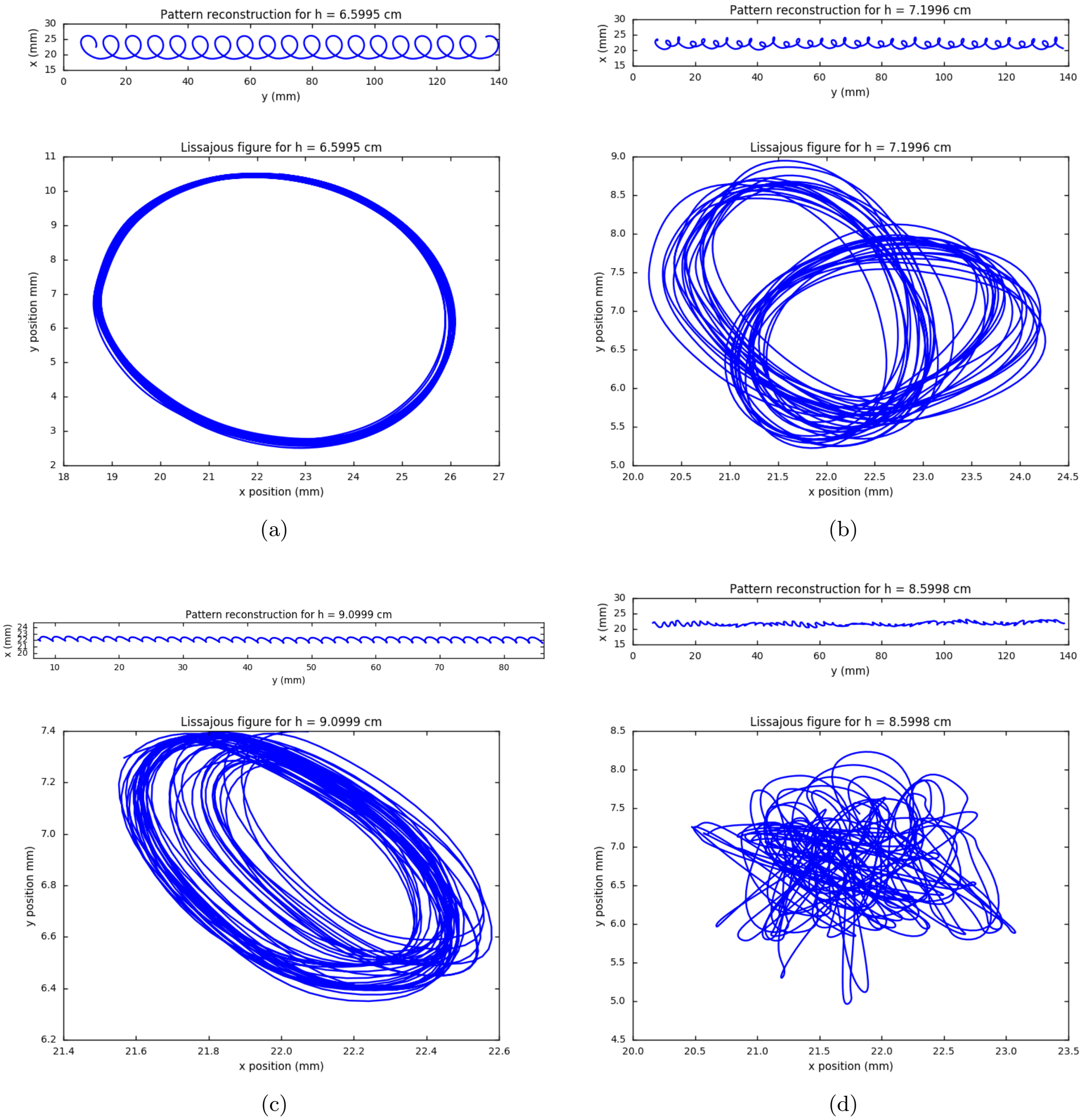
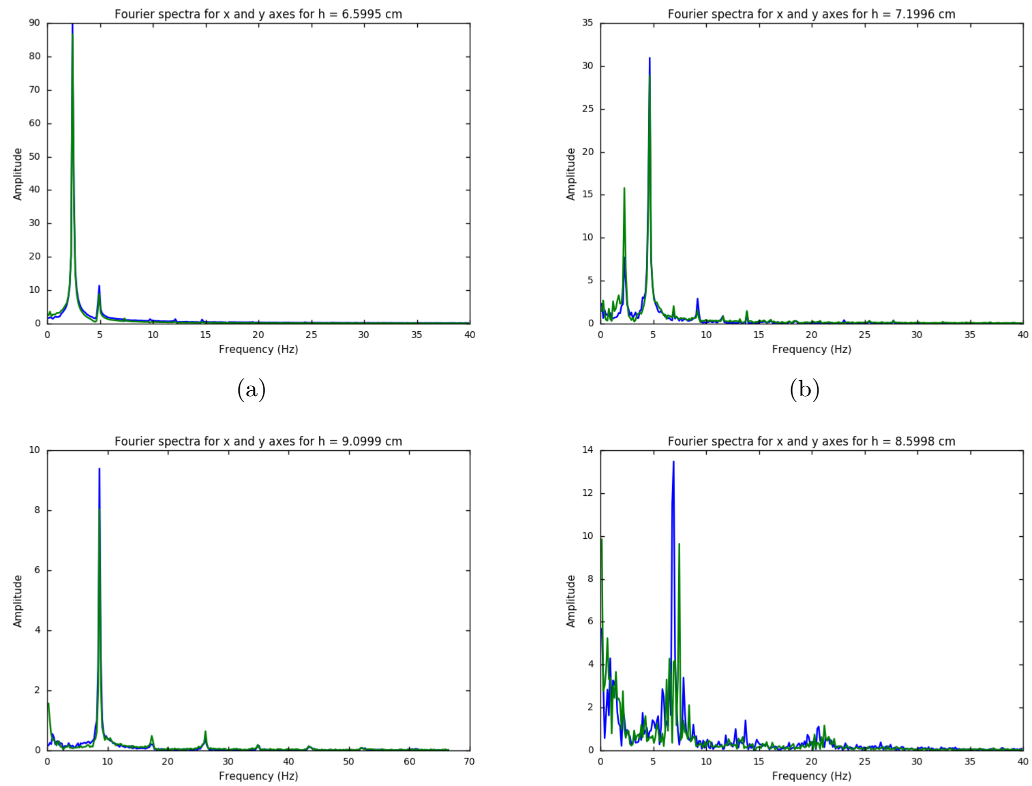


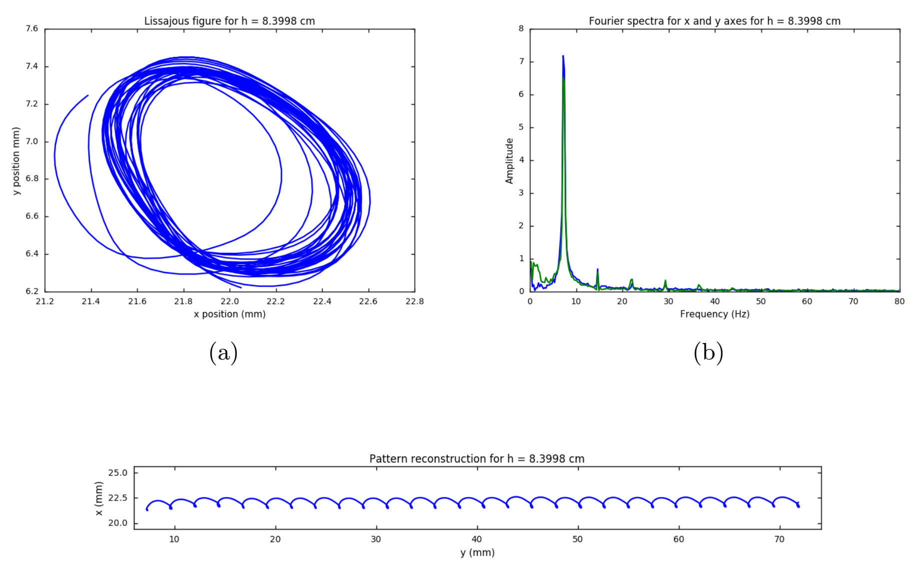
FIG. 7: Reconstructions of the patterns observed along with their corresponding Lissajous figures. (a) Translated coiling (b) Double coiling (c) Slanted coiling (d) Disordered behaviour. Note that plot (c) has a higher zoom level in order to better highlight the pattern.



(a) (b)

(c) (d)

FIG. 8: Fourier spectra observed for (a) the translated coiling state (b) the double coiling state (c) the diagonal coiling state (d) the disordered state. The spectra of the x-positions are in blue, and the spectra of the y-positions are in green.



(a) (b)

(c)

FIG. 9: Lissajous figure (a), Fourier spectra (b), and pattern reconstruction (c) of the ambiguous coiling state at $h = 8.3998 \pm 0.0001$ cm. In (b), the spectrum of the x-direction is in blue, and the spectrum of the y-direction is in green.

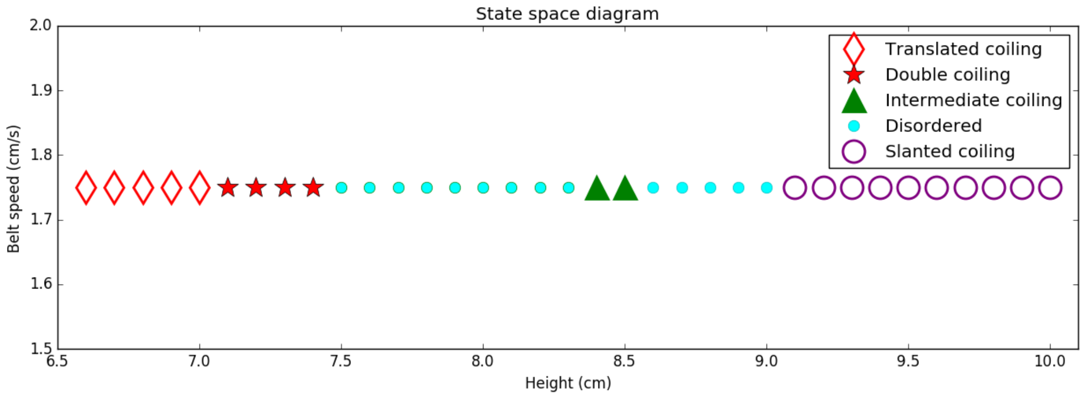


FIG. 10: State space diagram obtained from experimental results.