Wave Phenomena

1. **INTRODUCTION**

The ripple tank provides an ideal laboratory context for a first study of wave phenomena. Wave phenomena are ubiquitous in the natural world and have been as a result been studied extensively. Two-dimensional waves in water display a more complicated behavior than acoustic or electromagnetic waves, but they are an accessible system with which various wave properties can be observed. In the following experiment you will experimentally investigate reflection, refraction, interference, and diffraction. In addition to this, the wave phase velocity can be studied at different water depths.

Theoretical Background

Water surface waves travel along the boundary between air and water. The restoring forces of the wave motion are surface tension and gravity. At different water depths these two forces play different roles. A two-dimensional traveling wave is a disturbance of a medium (in this case the water air boundary), which can be expressed as a periodic function:

$$\psi(x, y, t) \tag{1}$$

Here x and y are position coordinates, while t is time. The wave speed is in simple cases defined as: $v = \frac{\omega}{k} = f\lambda$ where ω is the angular frequency, f is the frequency, λ is the wavelength, and k is the wave number.

The wave velocity v is technically called *phase velocity*. A non-dispersive wave has a constant phase velocity. Dispersion is characterized by a dependence of the phase velocity on λ. While in general surface water waves can be dispersive, we shall primarily consider shallow water waves, where gravity provides the restoring force and the phase velocity is independent of wavelength: $v = \sqrt{gd}$, where d is the water depth.

The Wave Equation

The wave equation provides a concise mathematical description of a wave. In cartesianr coordinates (x, y),

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \tag{2}$$

where ψ is the amplitude of the wave as a function of position and time, and v is the velocity of the wave.

If we assume a unidimensional wave solution: $\psi = A_o cos(kx - \omega t)$ and substitute it into the wave equation, we find that $v^2 = \frac{\omega^2}{k^2}$. This is called a *dispersion relation*.

The plane waves have this type of simple dispersion relation. These waves are generated by oscillations of a straight (flat) object whose dimension is much larger than the wavelength of the generated wave. More complex dispersion relations arise in situations where the structure of the wave propagation medium is more complex.

For dispersive waves, the speed of propagation is typically a non-trivial function of the frequency and wavelength. In certain cases waves may be more conveniently represented in spherical or cylindrical coordinates depending on the geometry of the wave setup. In the ripple tank, two dimensional spherical wave patterns will be quite common and are mathematically represented by a wave equation in spherical coordinates (r, θ, ϕ) reduced to planar geometry.

Complete wave equation in spherical coordinates is:

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi}{\partial r}) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \tag{3}$$

Huygens' Principle

1) Each point on a wave front is the source of a new spherical wave that spreads out at the wave speed,

2) At a later time the shape of the original wave front is the tangent to all the secondary spherical waves.

Huygens' principle is not a mathematical formulation of wave phenomena, but rather an intuitive tool useful in understanding the physics of wave propagation. Waves display a number of phenomena which can be understood using this simple picture.

Wave Reflection

As seen in Figure 1, a finite plane wave of width H approaches a reflective barrier at angle of incidence θ_i. The portion of the plane wave which is closest to the barrier will hit first and we can imagine a new spherical wave spreading out from the barrier at the speed of the incident wave.

As the wave cannot exist inside the barrier, the wave spreads out as if it was generated by the barrier. The portion of the wave which was farthest would radiate out with the same speed and a delay between their respective wave fronts equal to the additional time it took that portion of the original plane wave to encounter the barrier. If one looks closely at these expanding plane waves they turn out to add up only in one single direction, a direction defined as: $\theta_i = \theta_r$:

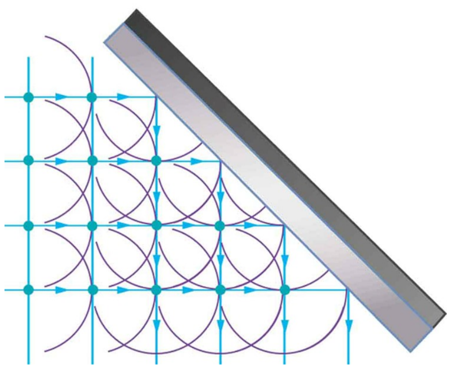

Figure 1: Huygens' principle applied to wave reflection

Wave Refraction

Similarly, refraction can easily be understood using Huygens' Principle. Refraction is the effect where a wave appears to bend as it crosses a boundary between two mediums. An example of this effect is when laser light appears to be bent by going through a piece of glass.

Refraction occurs when a wave crosses a boundary where the speed of propagation is different. If we consider a plane wave approaching a medium, then as each portion of some slice of the wave encounters the boundary it will become a new point source for a spherical wave. These spherical waves expand out at the speed of propagation in the medium which may be considerably slower (or faster) than the speed of propagation in the original medium. This results in a bending effect as the parts of the wave which haven't arrived yet are travelling more quickly than the portions of the wave in the medium. The result is that the angle of transmission is modified from the angle of incidence by the ratio of the speed of propagation in the two media:

$$\frac{1}{v_1} sin(\theta_1) = \frac{1}{v_2} sin(\theta_2) \tag{4}$$

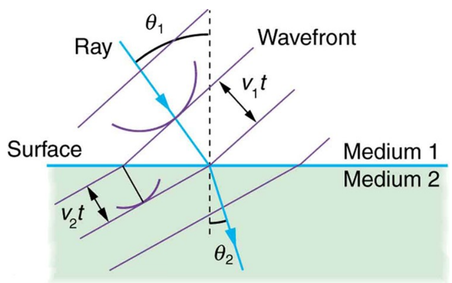

Figure 2: Huygens' principle applied to wave refraction

Wave Interference

When a wave front passes through two narrow openings in a barrier, each unobstructed point on the original wave front will act as a secondary source of new spherical waves. These "new" waves will propagate out expanding at the speed of the wave and will superimpose onto each other. The superposition of the two secondary waves will lead to a resulting wave at a distant point P (Figure 3) which can be represented by:

$$\psi(r_1, r_2, t) = A_o cos\,\omega(t - \frac{r_1}{v}) + A_o cos\,\omega(t - \frac{r_2}{v}) = 2A_o cos(\omega t) cos \frac{\pi(r_2 - r_1)}{\lambda} \tag{5}$$

Distances r_1 and r_2 correspond to Figure 3. The amplitude of the resulting wave depends on the path difference between the two waves arriving at point P on a distant screen, at the same time.

According to Figure 3, $\delta = r_2 - r_1 = dsin\theta$. When $L >> d$, $sin\theta \approx \theta \approx \frac{y}{L}$.

When the path difference is a maximum: $d\frac{y}{L} = m\lambda$, when δ is a minimum: $d\frac{y}{L} = m\frac{\lambda}{2}$ where m is an integer and λ is the wavelength.

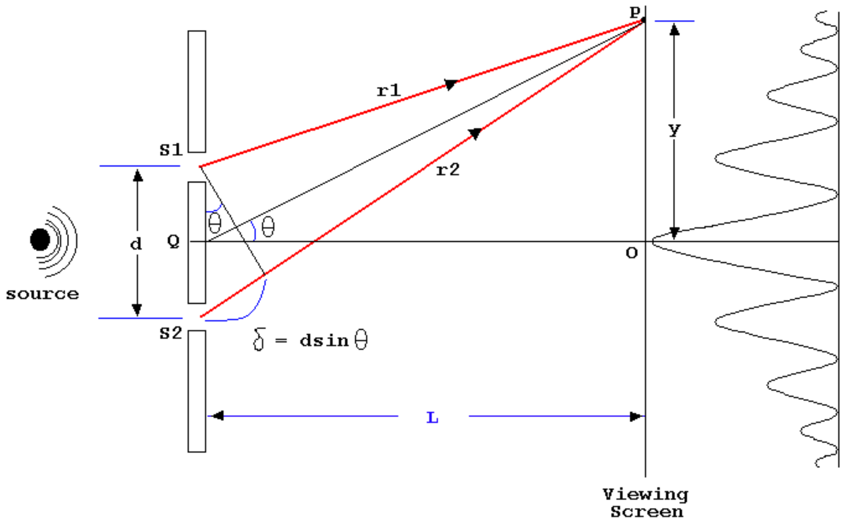

Figure 3: Double slit wave interference

In Figure 4, interference in a Young experiment can be seen. To the left of the slits there is a plane light wave with wavelength λ. To the right of the double slit, waves are spherical and look as if they were generated by the slits.

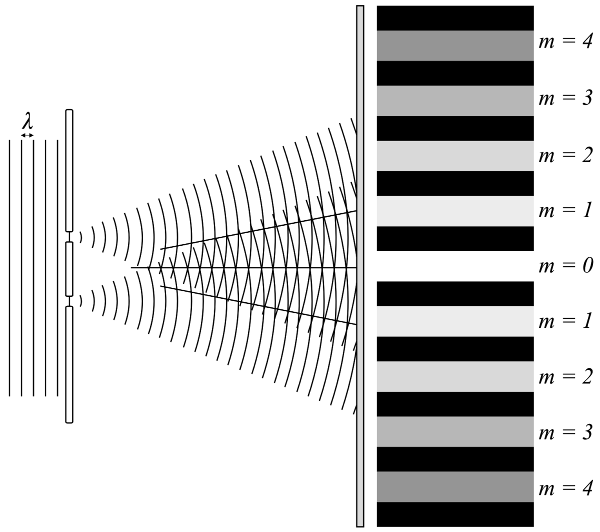

Figure 4: Double slit light interference

Wave Diffraction

Diffraction occurs when a wave "bends" as it passes around or through an object. Each point from the opening becomes a secondary source of waves. Only at large distances from the opening does the wave front become plane again. Huygens' Principle provides a powerful way of understanding the origin of diffraction. A condition can be found:

$$asin(\theta) = \lambda \tag{6}$$

Here a is the slit opening and θ is the angular spread of the first diffraction maximum.

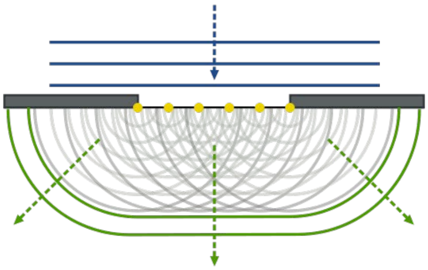

Figure 5: Wave diffraction past an opening

2. EXPERIMENTAL NOTES

The experimental setup consists of three main parts: the ripple tank and reflector, the light source, and the ripple generator (see Figure 6).

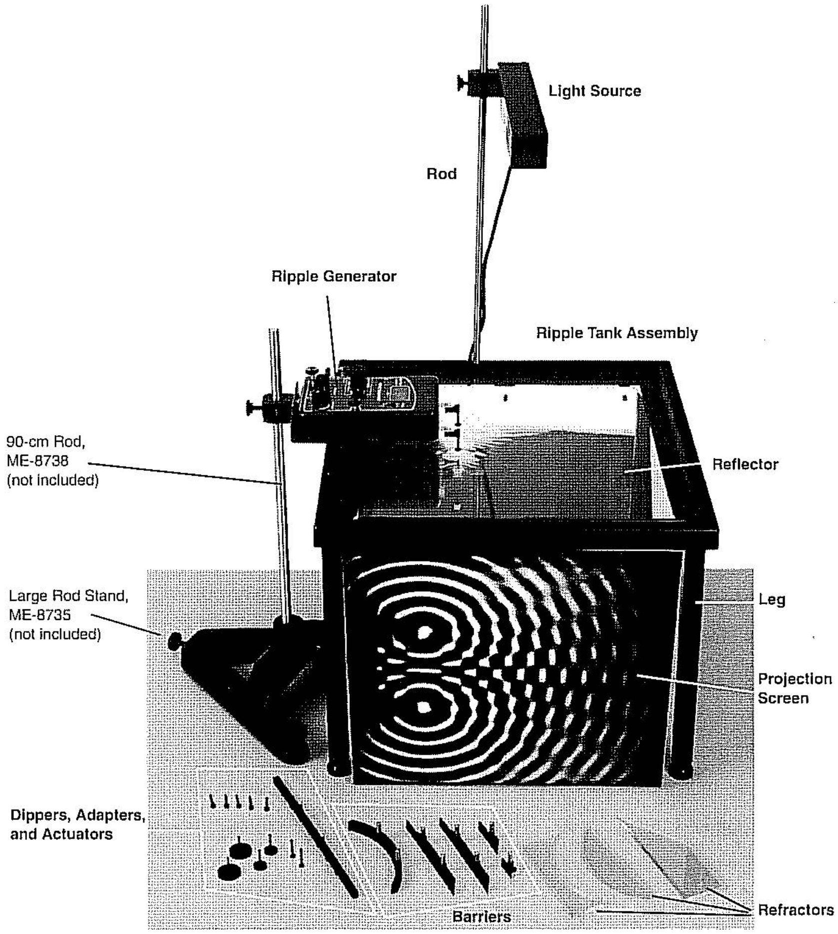

Figure 6: Apparatus

Ripple tank and light reflector

Water should be poured into the ripple tank until it is approximately 1cm deep and the height of each leg can be adjusted to ensure that the water depth is even. The tank drains into a bucket through an escape hose which is pinched off. When filling the tank with water first put a small amount of water in and look to see if it slides in any direction. This will indicate an uneven surface and should be corrected. Once the tank is even, pour ≈800mL into the tank. Make sure to press the foam around the edges into the water so it absorbs the water quickly. Failure to do so will result in a slow change in the height of the water as the foam slowly absorbs more water. Double check to make sure the hose is pinched shut and that water is not draining out. It is important to maintain a constant depth throughout the experiments.

The light source: The light source is turned on using the Pasco 9896 unit. The light setting should be set to strobe and the difference (delta) between the ripple generator and the strobe light should be set to zero.

The ripple generator: see figure below. Two fine knobs provide leveling control. There is an amplitude control which should be set to about 40% of maximum. There is also a frequency control which should be set to 15-20Hz.

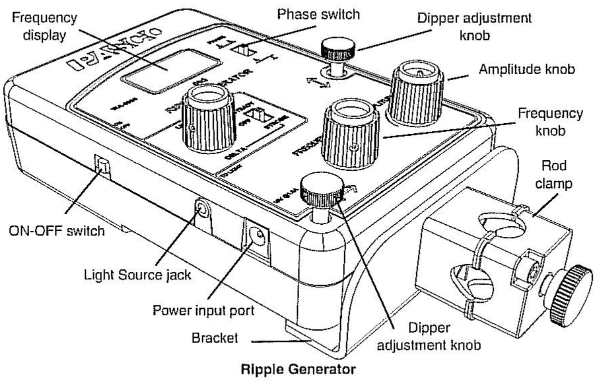

Figure 7: Ripple generator

The phase switch should be set so the two shakers move in parallel. It is very important to make sure the ripple generator is parallel (level). It is also very important that the dippers barely touch the water, for the plane dipper it is important that there be equal contact with the surface over its entire length (see Figure 8).

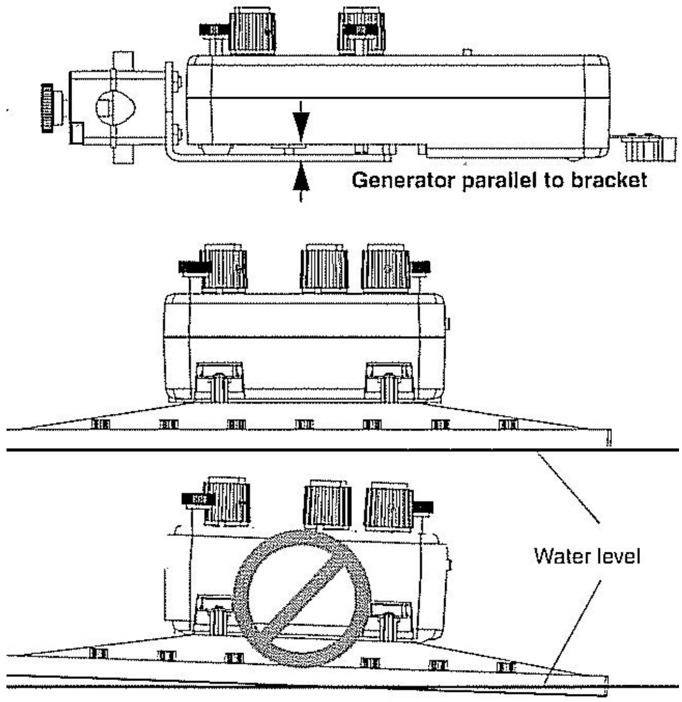

Figure 8: Ripple generator level

Image acquisition
To open the data acquisition program, click on the *Ripple Tank* shortcut located in the quick launch menu. The program will open and ask whether it is *OK* to launch a camera. If you click *Yes*, the camera will connect to the computer and will begin streaming the acquisition video. The image the camera records is a magnified version of the ripple tank which has already been calibrated. There are three different lines which can be drawn in order to measure distances and angles between wave fronts and other features. Perfectly horizontal or vertical lines can be drawn by holding down the Shift key. Angles are always measured as if the vectors on the screen were placed tail to tail, counter-clockwise from the higher numbered to the lower numbered line. It is possible to zoom in but not out. The print button grabs a screen shot and sends it to the printer so that your data can be included in your lab report.

3. **THE EXPERIMENT**
Exercise 1: Reflection
Straight Barrier
Place the long straight barrier at an angle in the middle of the tank. The water level should be high enough to rise halfway up the barrier's height. Position the ripple generator over the midpoint of one side of the tank; connect the plane wave dipper to the ripple arms. Adjust the height until the bottom of the plane dipper is barely in contact with the surface of the water. Set the frequency to approximately 20Hz and the amplitude to less than half full. Adjust the light source intensity to get the best possible viewing on the screen. Set the light source to STROBE. Adjust the amplitude of the ripple generator to get a clear pattern of plane waves.

Take measurements of incidence and reflection angles, verify equation: $\theta_i = \theta_r$.

Curved Barrier

Remove the long straight barrier and replace it with a curved barrier positioned so that it curves towards the plane wave generator. Record the reflected wave pattern. Estimate the focal distance and radius of the curved barrier. Turn the barrier around 180 degrees and repeat the measurements.

Exercise 2: Wave speed

Wave speed and frequency

Using the same setup as in Exercise 1, without any barrier, set the frequency to 5Hz and amplitude to less than half of maximum. Measure and record the wavelength 5 times. Calculate the average wavelength and the standard deviation. Repeat the measurements for 4-5 other frequencies and graph the average wavelength vs. frequency. Using equation: $v = \frac{\omega}{k} = f\lambda$, calculate the velocity of the wave in each case and discuss the sources of error.

Wave speed and water depth

Repeat the wavelength measurements at a fixed frequency 5-10Hz, but at different water depths between 2mm and 10mm. Calculate the wave speed for each water depth d. Note that this is the phase speed, as discussed before. Plot wave velocity vs. wavelength. Verify equation: $v = \sqrt{gd}$, and discuss the sources of error.

Exercise 3: Refraction

Place the trapezoidal refractor in the middle of the tank so the triangular end points towards the plane wave dipper. Adjust the amount of water in the tank so that water level covers the refractor by approximately 2mm. Set the light source to STROBE and adjust the frequency to 15Hz or less. Adjust the amplitude to get a clear pattern. Capture the refraction pattern. Explain how and why refraction appears in the ripple tank with the barrier in place. Measure the angles of incidence and refraction and using the result of your previous measurements (speed as a function of water depth) verify equation (4).

Exercise 4: Diffraction

Set up the ripple tank with the plane wave dipper and two straight barriers arranged to form a 3cm "slit". Set the generator to approximately 20Hz and amplitude to half maximum. Set the light source to STROBE. Vary the amplitude to get a clear pattern. Capture the diffraction pattern on the far side of the "slit". Determine the angular spread of the circular waves past the slit. Repeat for 4-5 different slit openings and verify the relation in equation (6).

Exercise 5: Interference

Turn off the ripple generator. Replace the plane wave dipper with two standard dippers. Adjust the generator so that the two standard dippers barely touch the surface of the water. Adjust the generator to 20Hz and set the light to strobe. Vary the amplitude until the interference pattern is visible.

Consider the two circular dippers to be point sources which generate expanding circular waves which overlap and interfere. The intersections of the wave fronts appear as dark and bright regions. In the far field consider a line parallel to the standard dippers along which to examine the interference pattern. On this line at the point equidistant between the two standard dippers there ought to be a point of constructive interference. This is the zeroth order point of constructive interference and it occurs because the two circular wave fronts have travelled the same distance to arrive at this point. On either side there will be a point of destructive interference. Measure the angles between the first and second points of constructive and destructive interference. Also measure the wavelength five times and find the average wavelength. Using the results of your measurements, find the separation between your dippers and confirm that it is 3cm. Repeat the measurement while varying the distance between the two point dippers. Discuss sources of error. Explain how the interference pattern changes as a function of the distance between the dippers. On the ripple generator set the dippers to be out of phase with each other. Explain the pattern and reformulate equation (5) and condition: $r_1 - r_2 = dsin(\theta_m) = m\lambda$.

This guide sheet has been revised by Ruxandra M. Serbanescu in 2020. Previous versions: rms-2005, j sinclair 2016.