The Way of the Physicist

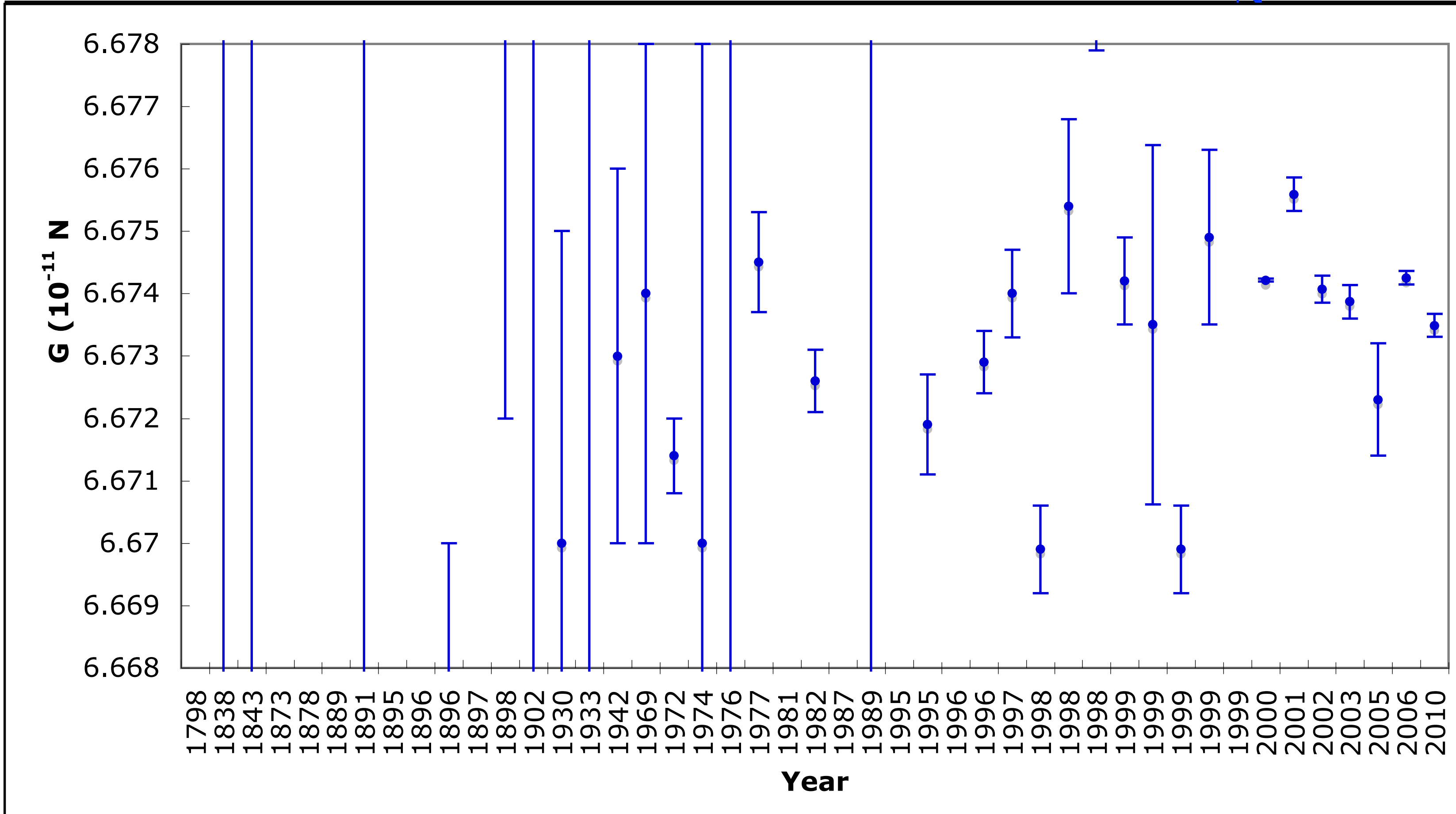
Physicists

- construct mathematical **models** of a physical system
- **solve** the model analytically or computationally
- make physical **measure**ments of the system
- **compare** the measurements with the expectations
- **communicate** results with others
- **improve** model, calculations, experiment; iterate

Experiments are tough

- What was the first fundamental constant measured in the lab?
- What is the worst measured fundamental constant?

Newton's Gravitational Constant: G_N



Ideal Analysis

Experimental measurements

↓

~~Determine whether physics model is correct and find true value of desired parameters.~~

Probability that hypothesis is correct and the probability distribution for the true value of desired parameters.

Unfortunately, this is also impossible

- Bayes Theorem: $P(H|D) = P(D|H)P(H)/P(D)$

Real Analysis

- Uncertainties are how we parameterize the probabilities.
- Uncertainties are defined by convention.

  e.g. Bayesian

  Frequentist

- As long as convention is a reasonable approximation to our ideal goal, and everyone uses the same convention, then we can compare results.

Normal Convention

"$x \pm \sigma$" means

- if other measurements of the same parameter are made, "$x_i \pm \sigma_i$", we expect

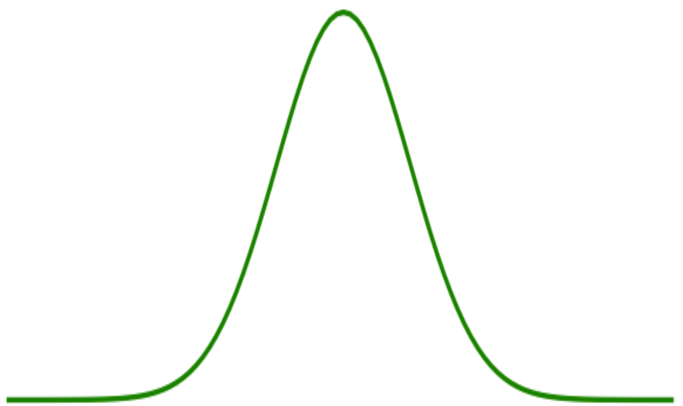
$$\left| x - x_i \right| < \sqrt{\sigma^2 + \sigma_i^2}$$

68.3% of the time.

Normal (Gaussian) Distributions

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Mean μ, RMS width σ



- Limit of Binomial distribution for large number of trials with mean not near zero.
- The **Central Limit Theorem** says (almost) everything averages out to a Gaussian.
- Many resolution functions are at least approximately Gaussian - a blob with a mean and a width.
- It is the only distribution many physicists really know.

Counting "Root N" Statistics

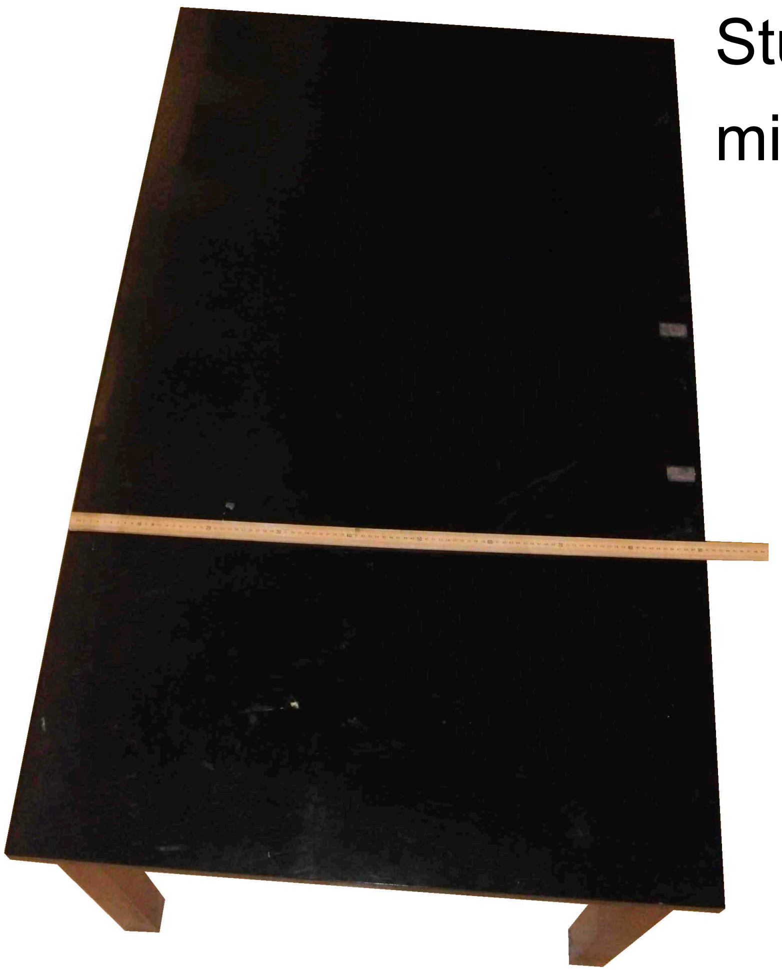
- When counting some random process, binomial statistics apply, but when the number of trials (N) is much larger than a not small mean value ($\mu=Np$), this reduces to the Gaussian distribution:

$$P(n) = \frac{1}{\sqrt{2\pi Np(1-p)}} e^{-\frac{(n-Np)^2}{2Np(1-p)}} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(n-\mu)^2}{2\sigma^2}}$$

In the Poisson limit ($p<<1\text{-}p$): $\mu=Np=\sigma^2$

- Gaussian statistics apply to most counting experiments, but Poisson statistics apply if the number of counts is small (e.g. < 10).

Width of a Table



Students with metre-sticks

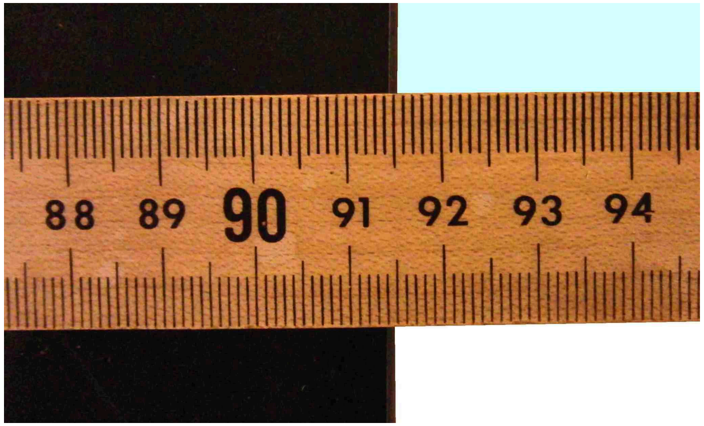
might all agree on the *cm*,

- estimate of *mm* would vary.
- if reading errors random, any reading is equally likely to be higher or lower than “true” value.
- Repeated measurements will increase precision and should improve accuracy.

Based on Peter Krieger 2009 talk, page 21
http://www.physics.utoronto.ca/~phy326/Introductory%20Talk%20by%20Peter%20Krieger.pdf

Width of Table (Cont'd)

If metre-stick is perfectly calibrated, *and* if the table is flat and has the same width at all points, *then* the average of the repeated measurements should provide a good estimate of the "true" value.



Statistical uncertainties decrease with repeated measurement: the fractional error on the mean of a set of N (independent) measurements (usually) decreases as $\sqrt{N}$.

Question: why did I specify a set of measurements made by individuals rather than a set made by a single person?

Based on Peter Krieger 2009 talk, page 22 http://www.physics.utoronto.ca/~phy326/Introductory%20Talk%20by%20Peter%20Krieger.pdf

Systematic Uncertainties

If the metre-stick is mis-calibrated, (e.g. it is actually only 0.996 m long) then the measurements will be systematically incorrect.

This type of uncertainty does NOT improve with repeated measurements, since each measurement is off by the same amount. Note, however, that calibration measurements can reduce systematic errors.

This applies to any measurement apparatus: voltmeters, ohmmeters, pressure gauges, neutrino time-of-flight detectors, ...

This is a correlated (rather than random) error; the error is the same on each measurement.

Based on Peter Krieger 2009 talk, page 23 http://www.physics.utoronto.ca/~phy326/Introductory%20Talk%20by%20Peter%20Krieger.pdf

Systematic Errors (cont'd)

Examples of other sources of systematic uncertainty:

- **Uncertain inputs**

    e.g. common lead samples often have a few percent of antimony, so the density of lead atoms in a Compton sample can't (easily) be determined better than ~%.

- **Model dependence**: the parameters you extract from your measurements depend on the model used.

    e.g. Determining the focal length of a lens using the thin lens equation $$\frac{1}{f} = \frac{1}{\text{object distance}} + \frac{1}{\text{image distance}}$$

    - Good model for most eye-glasses, not so good for a magnifying glass, terrible model for a crystal ball.

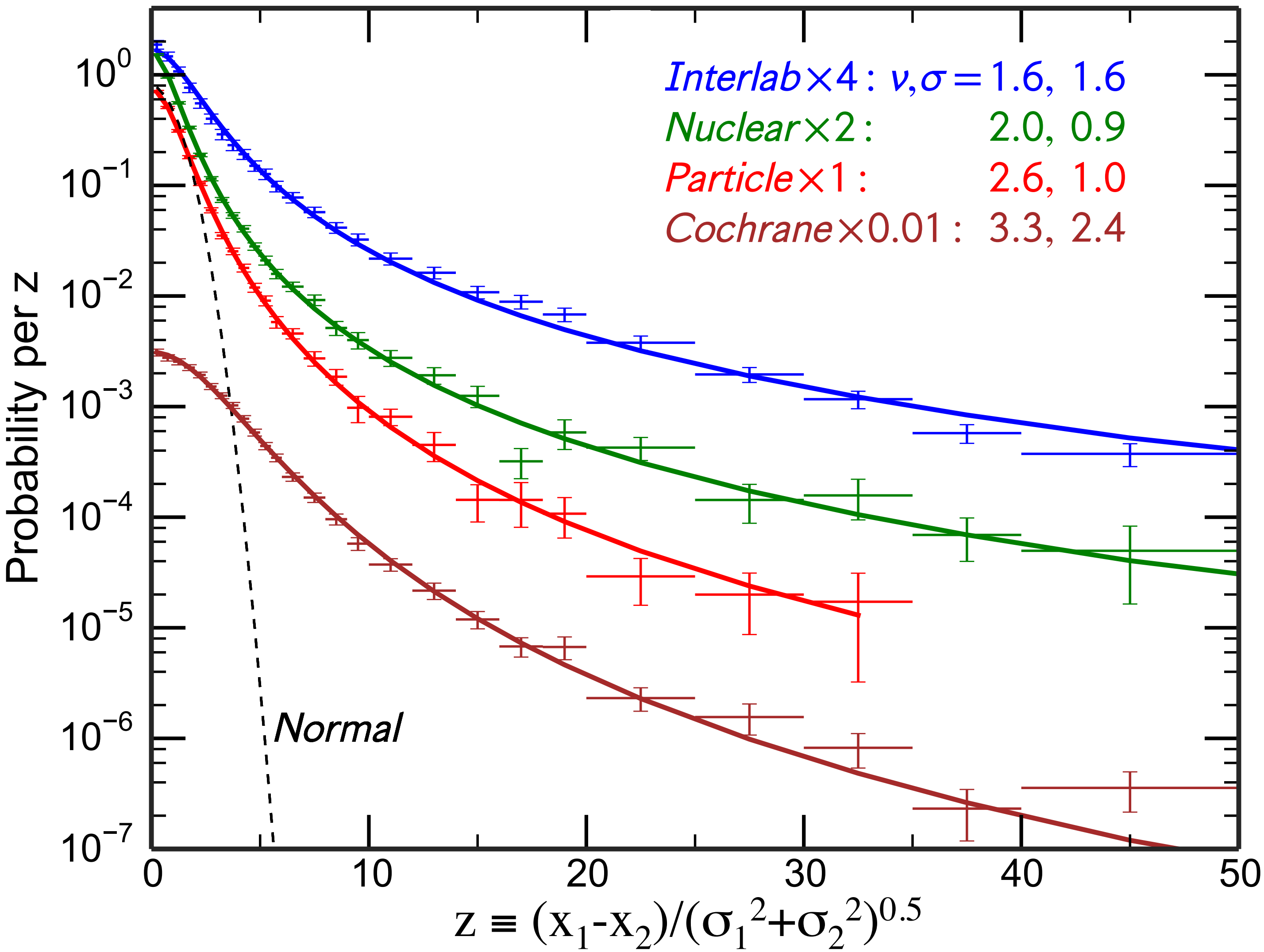
- **Detector** efficiencies, physics, backgrounds.

“Experimental errors” are not mistakes

- They are “experimental uncertainties”
- Mistakes are “illegitimate errors”, that can be eliminated by careful repeated observations and procedures.
- But mistakes do contribute to the uncertainties
  - **Real probability distributions always have larger tails than the ideal**
    - “s**t happens”

Not All (or even most) Probability Distributions are Gaussian

Uncertainties in different research fields:



Experimental Paranoia

Assume that the universe is conspiring to spoil your experiment.

e.g. Don't assume equipment is calibrated, that it is the same as the last time you used it, there are no typos, there is no noise, …

If you do make such assumptions, clearly state them in your notebook.

Fitting

- "Fitting" data means adjusting the variable parameters in the physics (mathematical) model so that it *best agrees* with the data.
- A *metric* must be used to measure the agreement between the model and the data. Fitting means minimizing the value of this metric.
- Most usual metric is χ^2 ("Chi-squared").

Chi-squared

- Consider a set of n independent random variables x_i, distributed as Gaussian densities with a theoretical means μ_i and standard deviations σ_i, respectively. The **chi-square** is the sum

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{x_i - \mu_i}{\sigma_i} \right)^2$$

- The mean value of the χ^2 is approximately the number of degrees of freedom, e.g. the number of bins less the number of fit parameters when comparing fit to data.

Software

- Matlab, Octave, Sage…
- Maple, Mathematica, Reduce, …
- Excel (for preliminary analysis)
- Faraday, DataStudio, Kaleidagraph, …
- **Python**, C, C++, …
- …

We don't care what you use, but we do care that you understand what you do.

But, if in doubt, use Python, since that is best supported for UofT UG Physics.

Python for the Advanced Lab

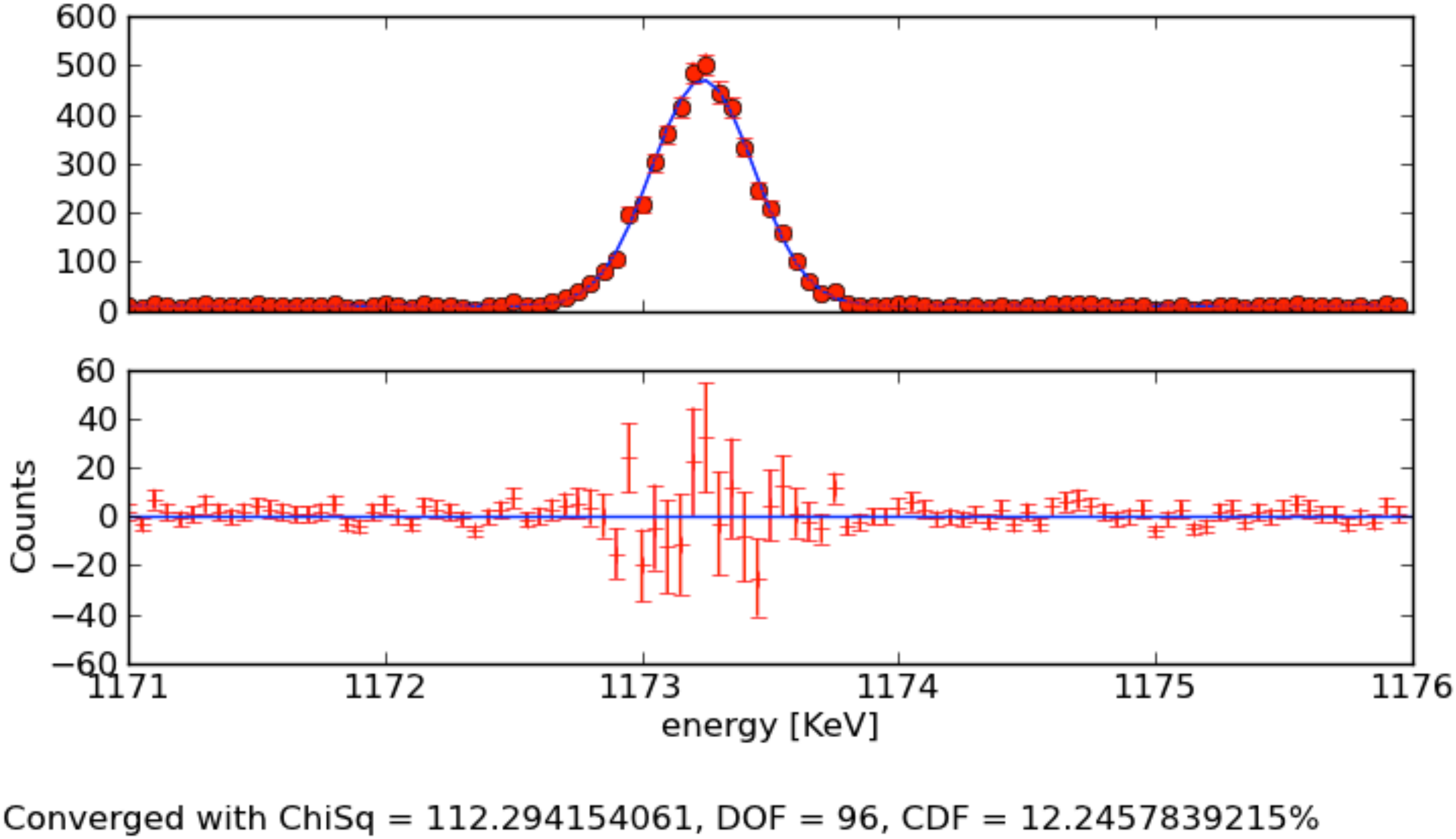
http://www.physics.utoronto.ca/~phy326/python/

Python Code Repository

- curve_fit_to_data.py or
  - simple_curve_fit_to_data.py
  - extended_curve_fit_to_data.py
- odr_fit_to_data.py
  - for errors in x and y

If you don't base your analysis on these examples, please be sure that you know what you are doing.

$\chi^2 \approx$ Sum of distance-squared between data and curve, measured in units of the uncertainty.



(Fit using APL Python example curve_fit_to_data:
http://www.physics.utoronto.ca/~phy326/python/curve_fit_to_data.py)

Least Squares

- If the uncertainties are all equal, then

$$\chi^2 = \frac{1}{\sigma^2}\sum_{i=1}^{n}\left(x_i - \mu_i\right)^2$$

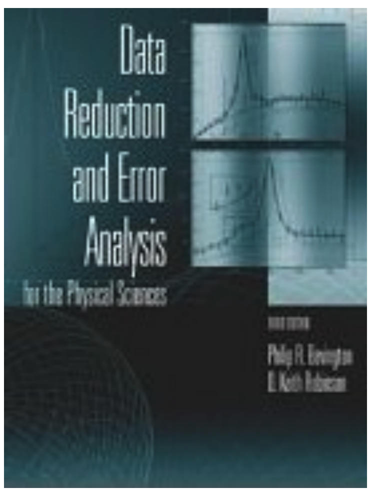
- So minimizing the χ^2 is same as minimizing

$$\sum_{i=1}^{n}\left(x_i - \mu_i\right)^2$$

i.e. an Ordinary Least Squares fit

- χ^2 minimization is an example of weighted least squares, where the weight is $^1/\sigma^2$.
- **Never, ever, use Ordinary Least Squares if the uncertainties are not equal!**
  - Fit should give best values for parameters with uncertainties, and χ^2 and

Data analysis in the Advanced Lab

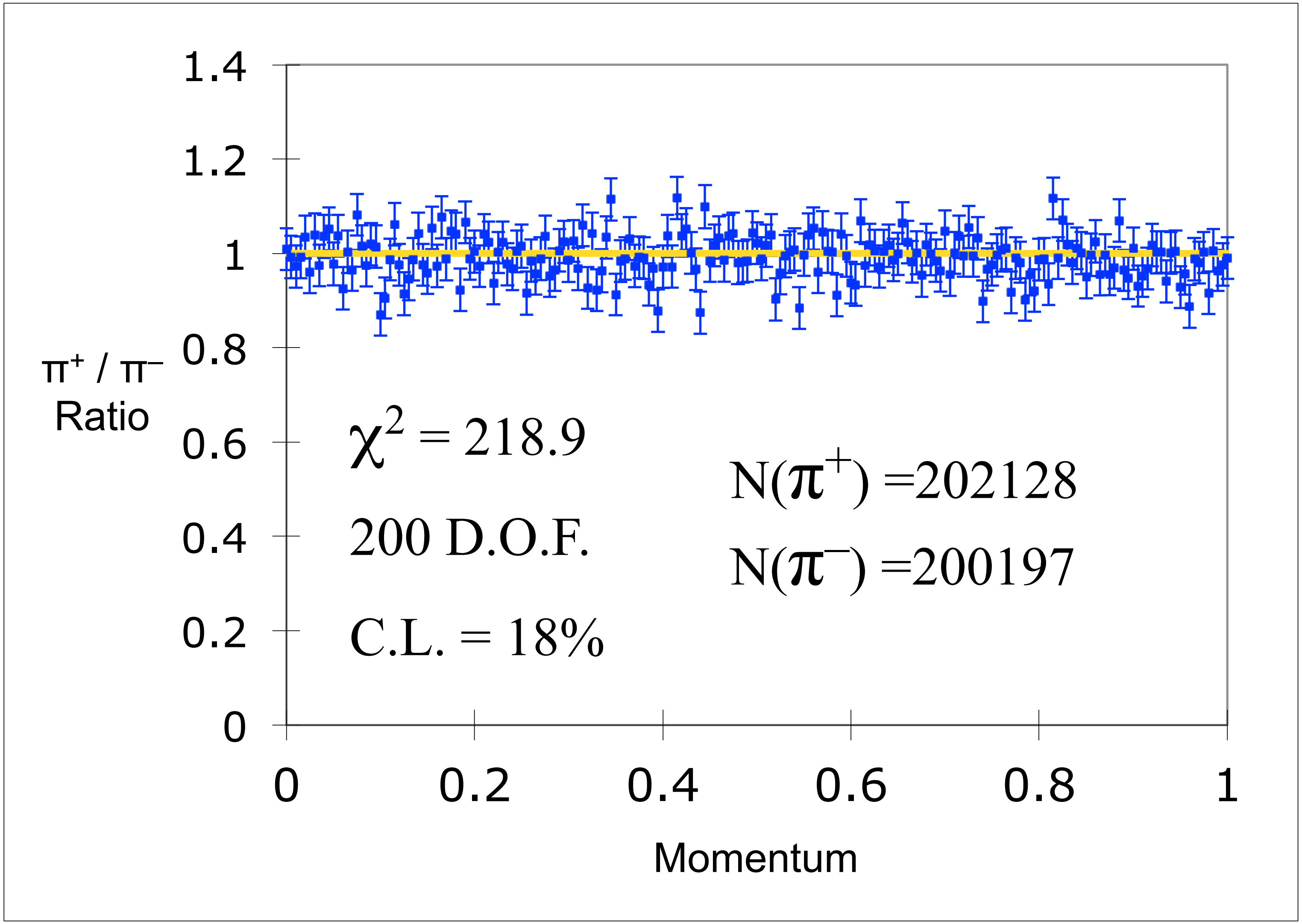


Reference: *P.R.**Bevington** and D.K.Robinson. Data Reduction and Error Analysis for the Physical Sciences (McGraw-Hill, New York, 2003) 3rd Edition***,** available at the U of T bookstore

See also lectures by Krasnopolskaia, Krieger, Thywissen, Harrison on course website/Materials

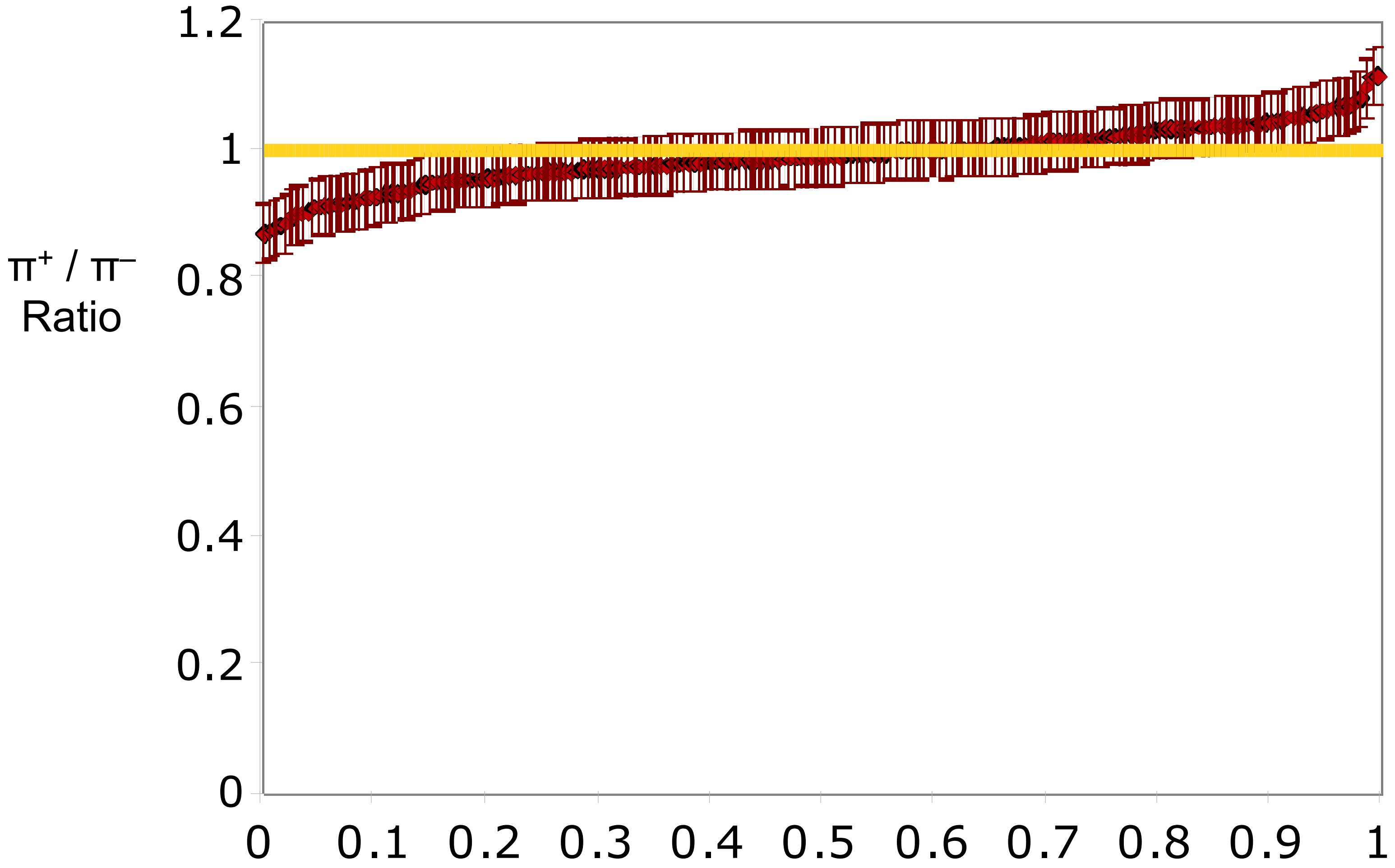
χ^2 does not tell you everything

- $p\bar{p} \rightarrow \pi^+\pi^0\pi^-$ Charge Conjugation Test



What about this data?

- Do (red) data and (yellow)model agree?



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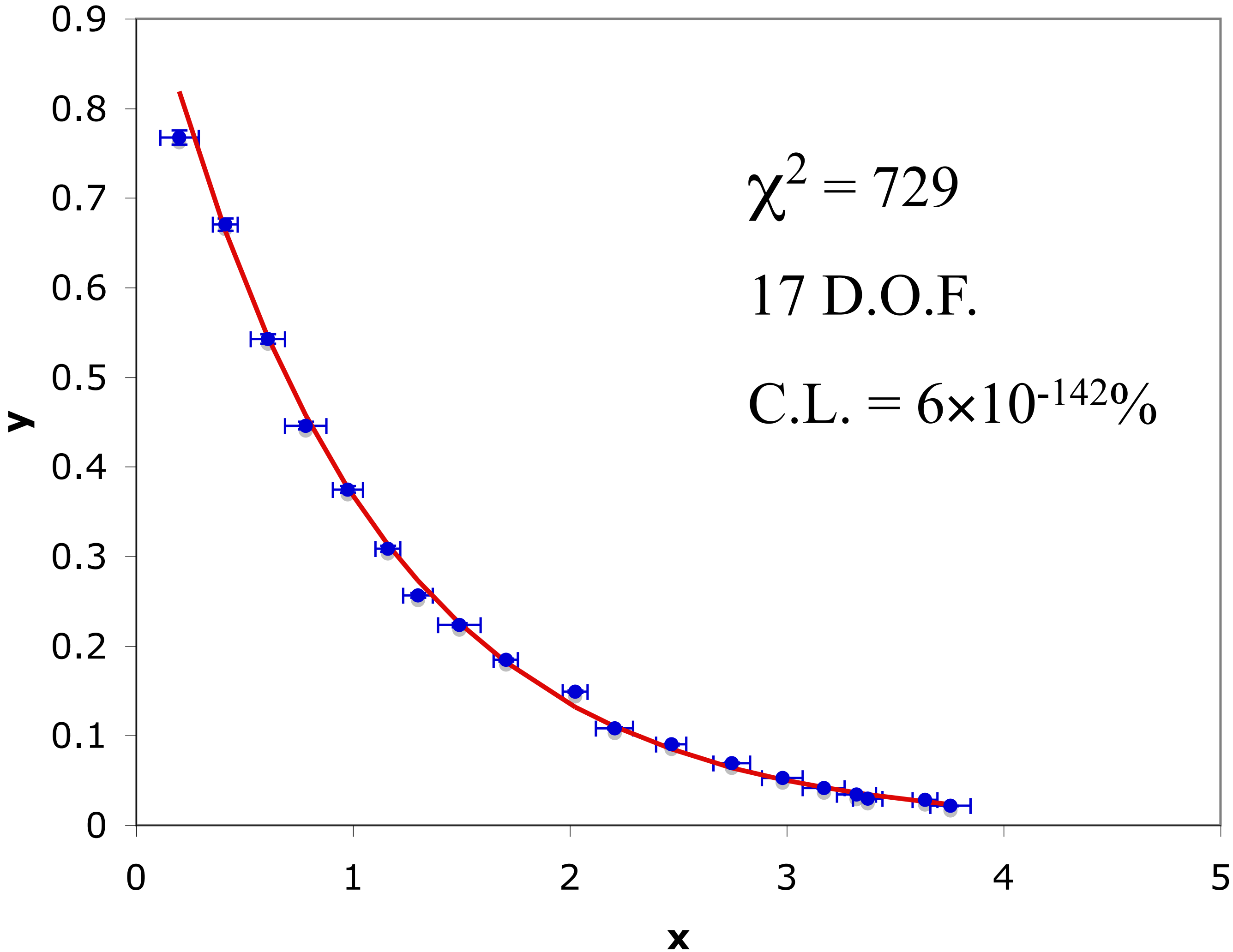

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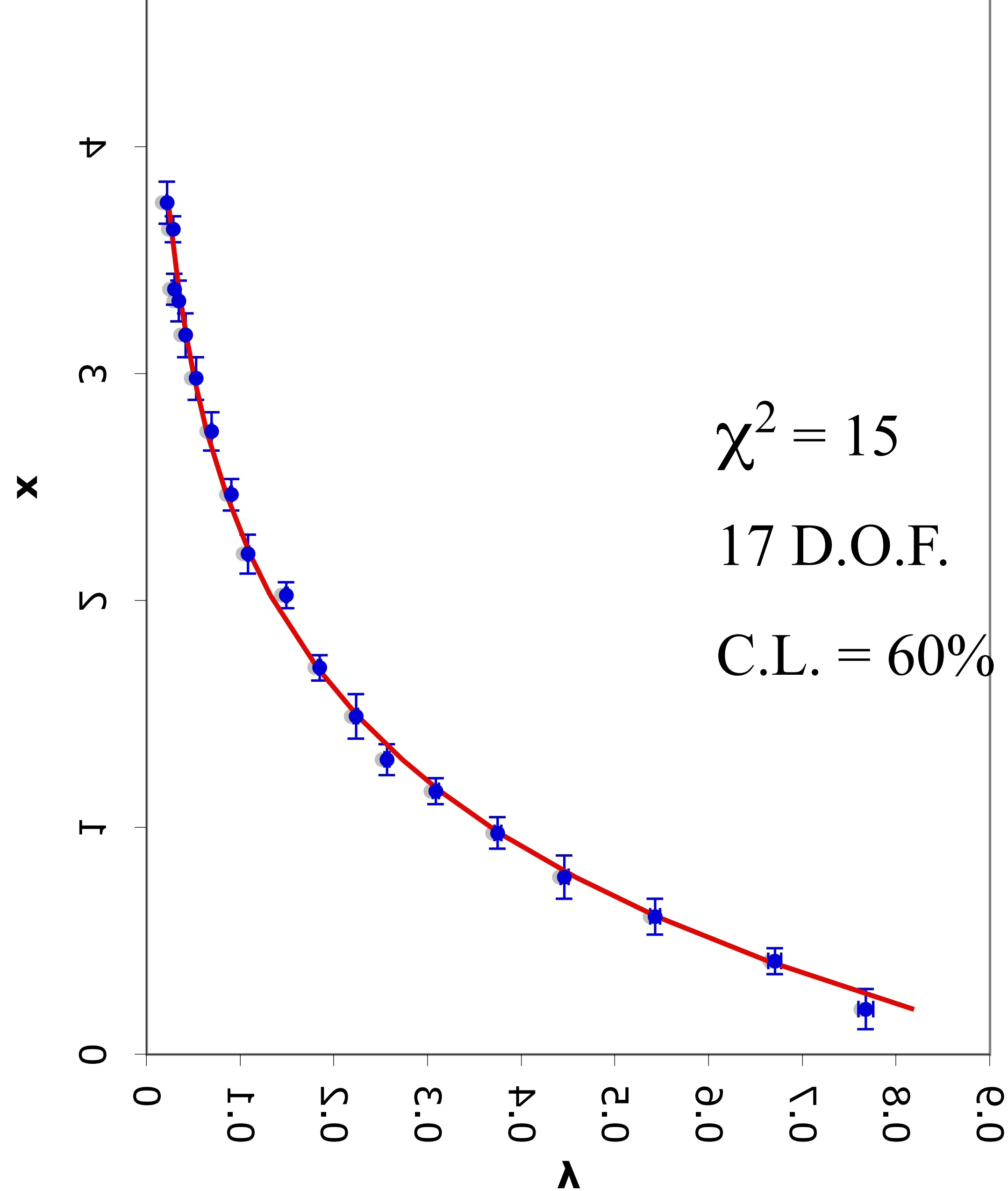

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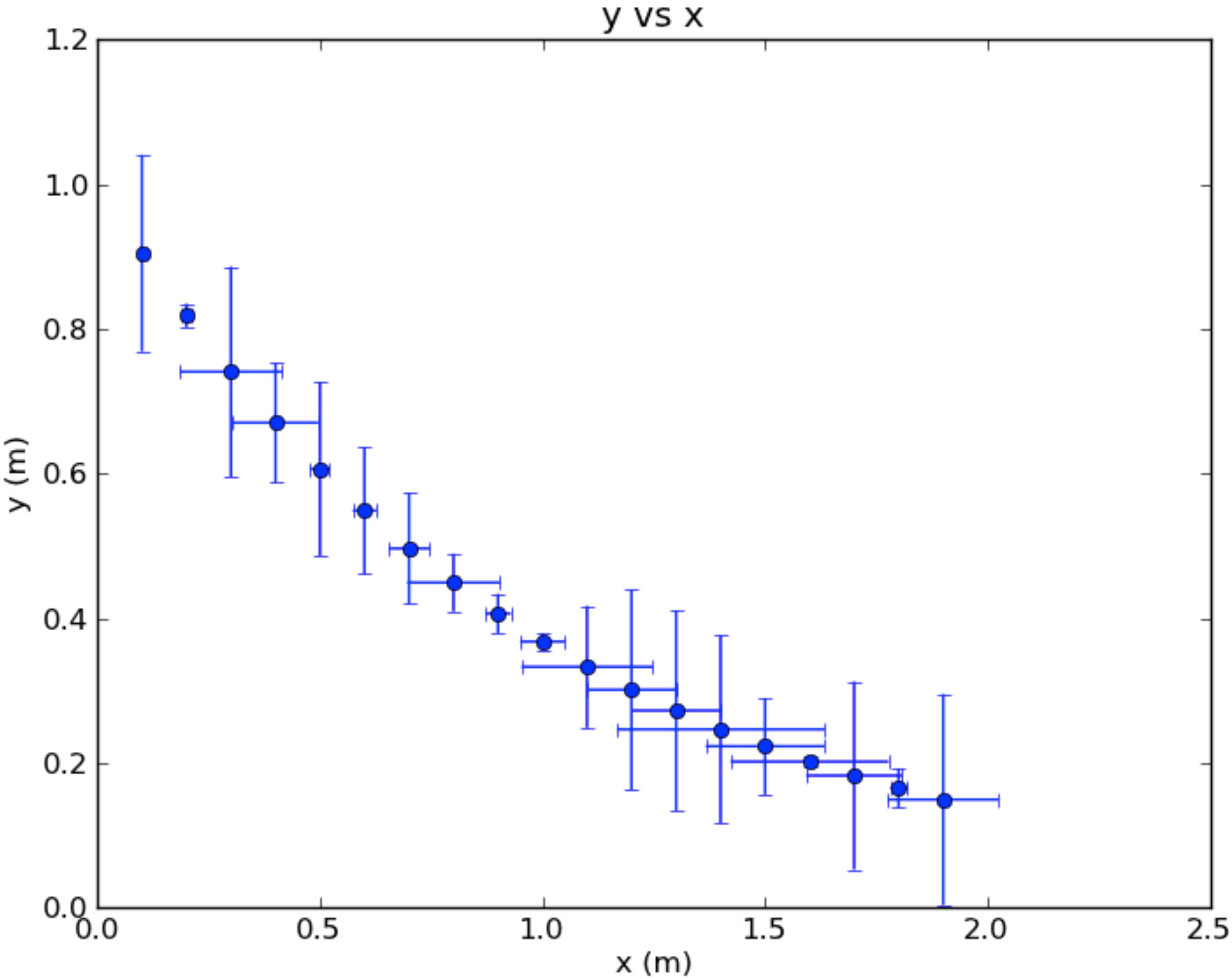
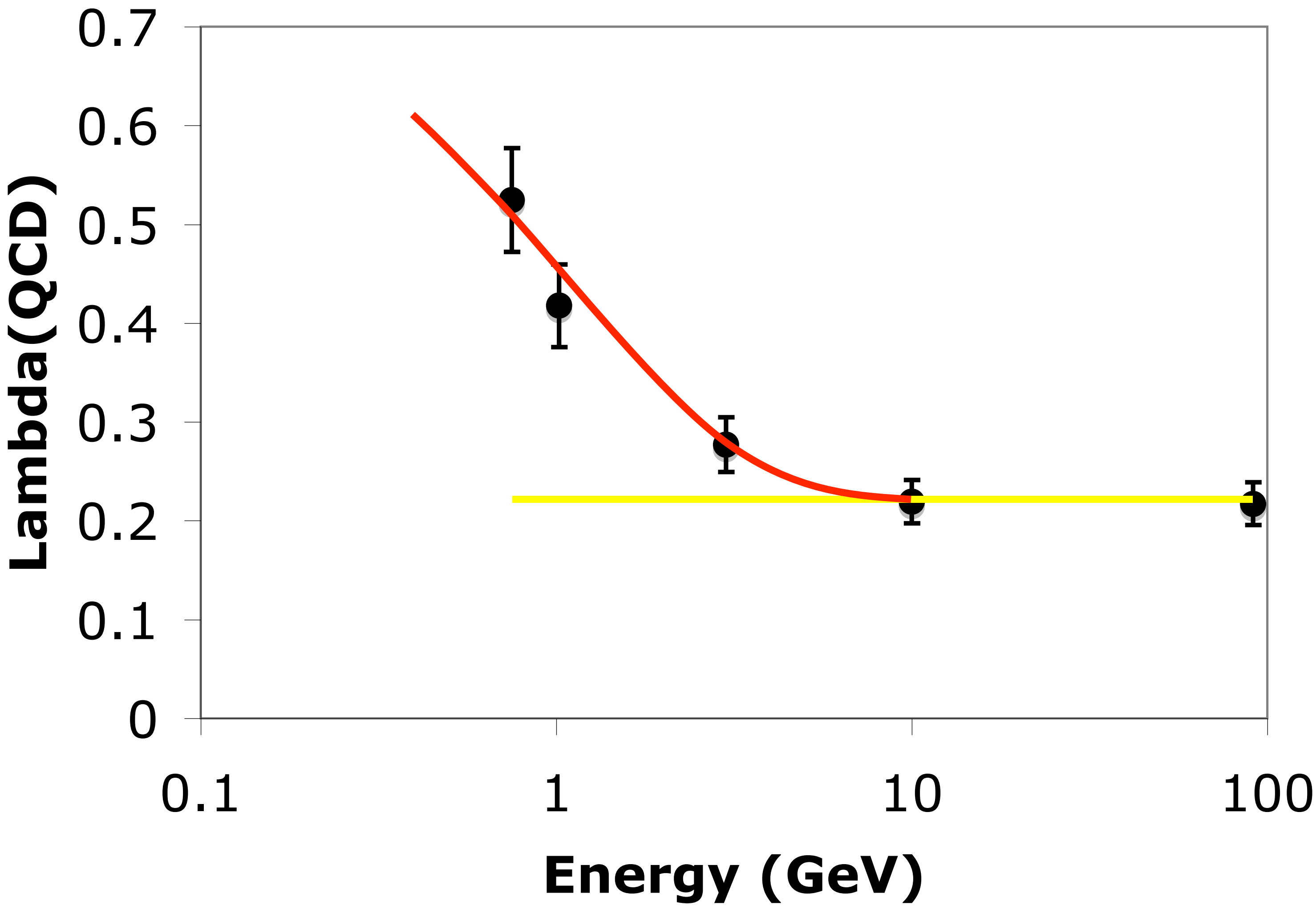


Where should you spend your time?



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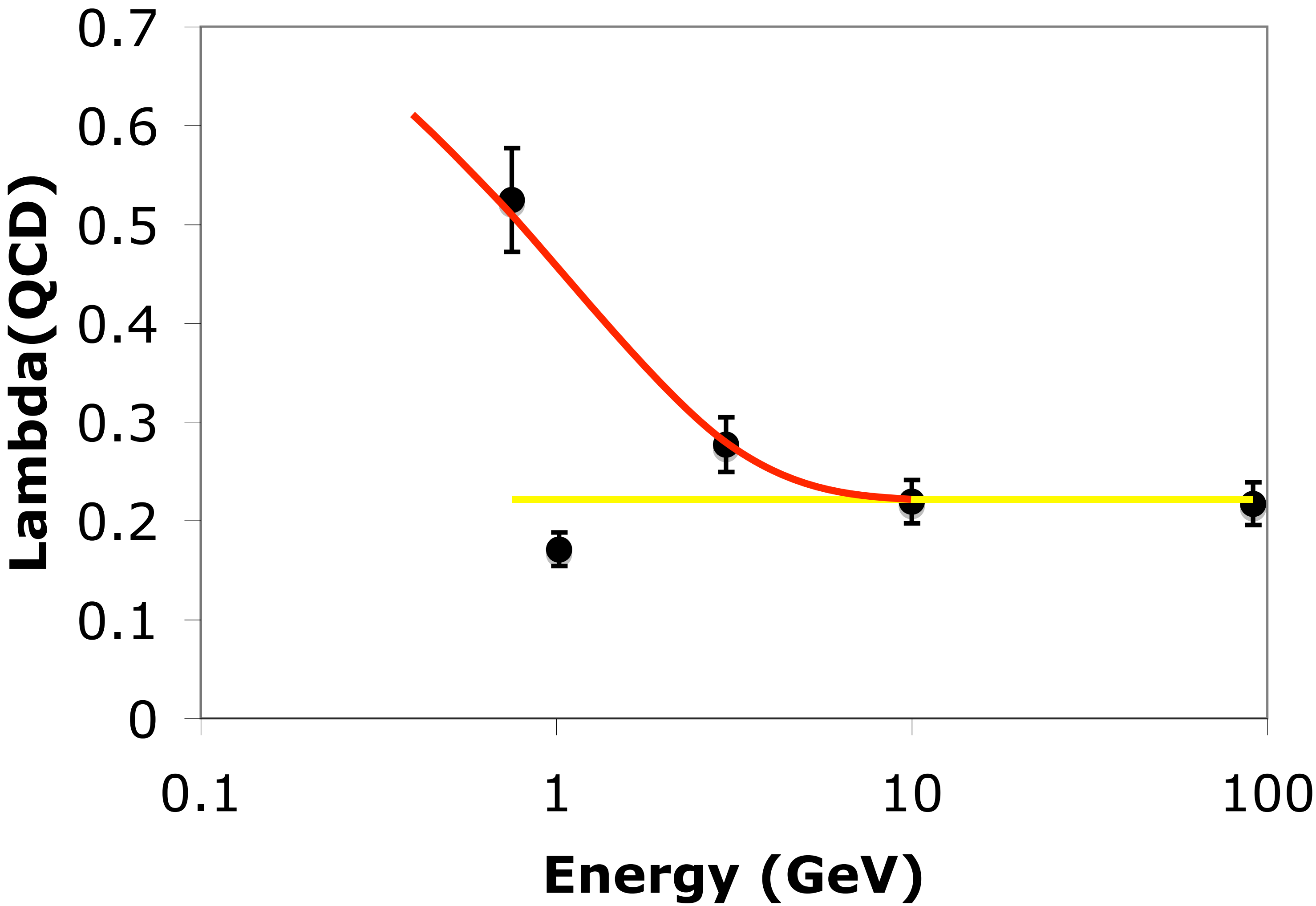

What is the Point?

What is my goal?

What am I trying to accomplish

- in this course?
- in this experiment?
- today?
- by making this measurement?
- by doing this instead of that?

Quality vs quantity

Don't just stand there, take some data.

It is better to have less data that is well understood rather than lots of data poorly analyzed.

You can't understand your data without taking some data, but it is usually a waste of time to take data without analyzing it as you go.

Flag Anomalies

That's weird, …

If there is some aspect of your analysis (or experiment) that doesn't make sense, say so in your lab notebook! Even if you don't have time or the tools to investigate further, you want to be the one that points out issues, not the professor.

The End